User Notes on SACS Ring Analysis

1. Introduction

Ring program is an interactive program where users define the ring geometry and loading conditions, perform analysis and view results in a single interface.

2. Understanding the limitations of ring program,

a. Only English units supported currently, the loads can be input in **kips** for concentrated force, **inch-kips** for moments and **kips/inch** for uniform force; the angles are input in degrees and are positive clockwise.

b. The present version of the Ring program allows the analysis of Roark ring cases 1, 2, 3, 6, 8, 12, 16, 17 and 18. See Roark and Young, “Formulas for Stress and Strain”, 5th edition, McGraw-Hill Book Company, 1975.

c. Three types of rings are currently supported: a rectangular section (no stiffener), external and internal tee stiffeners.

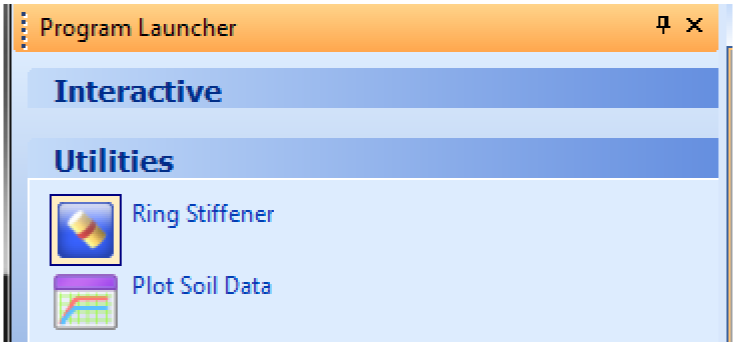
3. Understanding the assumptions for the ring formulas

The following are assumptions applicable to the cases ring program supported from section 8.3, Roark’s Formulas for Stress and Strain, 5th edition.

1) The ring is of uniform section;
2) It is such large radius in comparison with its radial thickness that the deflection theory for straight beams is applicable;
3) It is nowhere stressed beyond elastic limit;
4) It is not so severely deformed as to lose its essentially circular shape;
5) Its deflection is due primarily to bending, but it is desired, the deflections due to deformations caused by axial tension or compression the ring and/or by transverse shear stresses in the ring may be included.

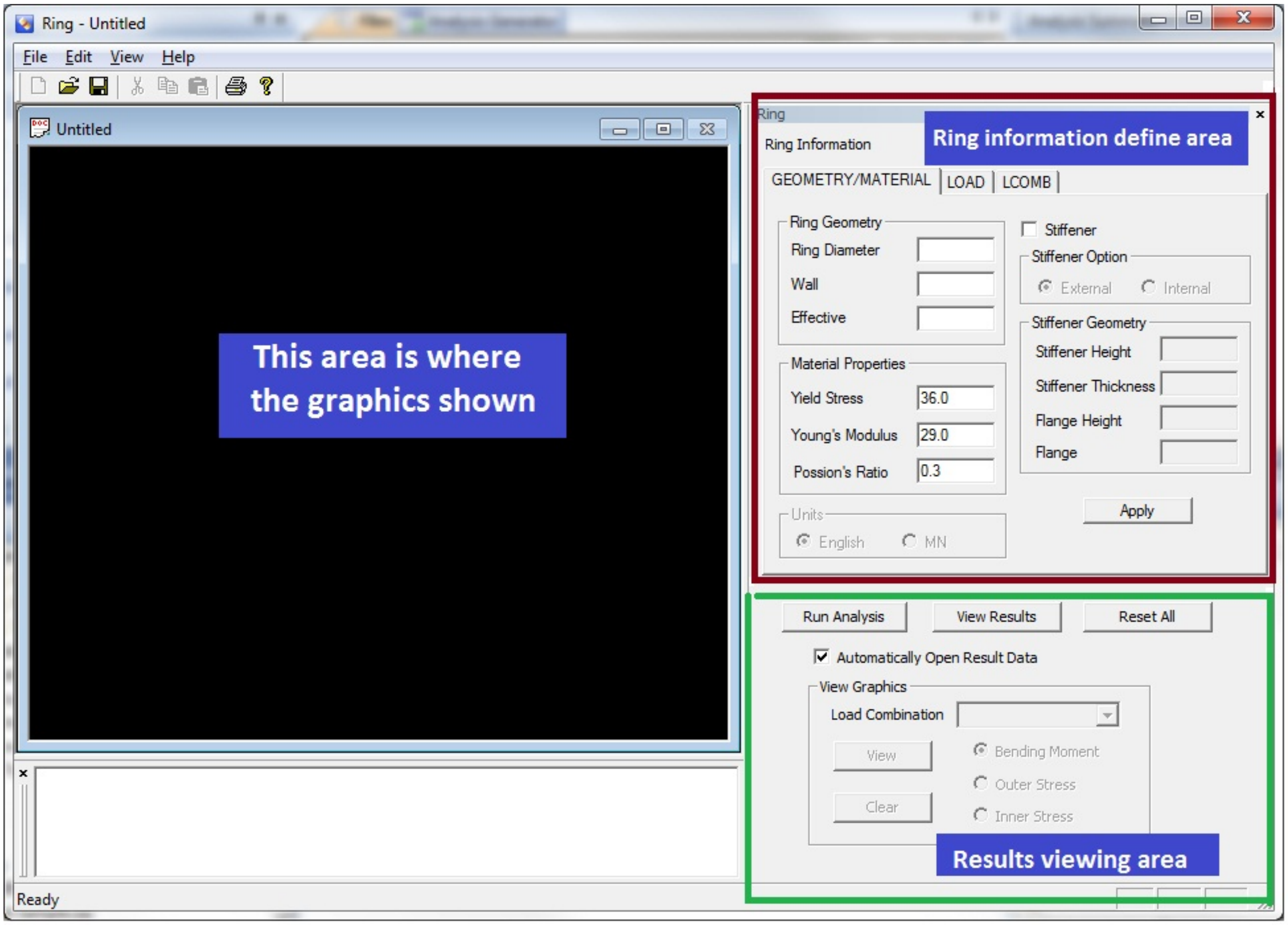
4. Launch the ring program

Under SACS 5.3 Executive, click “**Ring Stiffener**” in “**Utilities**”, this will launch the ring program user interface.



Or optionally user can browse into SACS system directory C:\Program Files\SACS53, find the program SACWRNG.exe, and double click to launch the ring program interface.

5. Understanding the front end interface of ring programs



☐ Stiffener

Yield Stress 36.0

Young's Modulus 29.0

Possion's Ratio 0.3

Units: ● English ○ MN

☑ Automatically Open Result Data

6. Understanding input of Geometry and Material Properties of rings

a. **Ring Geometry**:

   i. Ring Diameter **D** (units in inch) is the outside diameter of the ring;
   ii. Wall thickness **T** (units in inch) of the ring;
   iii. Effective width **BE** (units in inch):

   For rings with stiffeners, this should be the effective (or associated) flange width of the ring section. After user input ring diameter and wall thickness, program will automatically suggest an effective flange width based on the following formula from API RP 2A.

$$BE = 1.1\sqrt{DT}$$

   For rings without stiffeners, this should be the unit calculation width corresponding to the input of loads. The program suggested value should **NOT** be used in this case.

b. **Stiffener**:

   i. When Stiffener option checked, user has options to define the stiffener as External stiffener or Internal stiffener;
   ii. Stiffener Web Height **DS** (units in inch);
   iii. Stiffener Web Thickness **TS** (units in inch);
   iv. Stiffener Flange Width **BF** (units in inch);
   v. Stiffener Flange Thickness **TF** (units in inch).

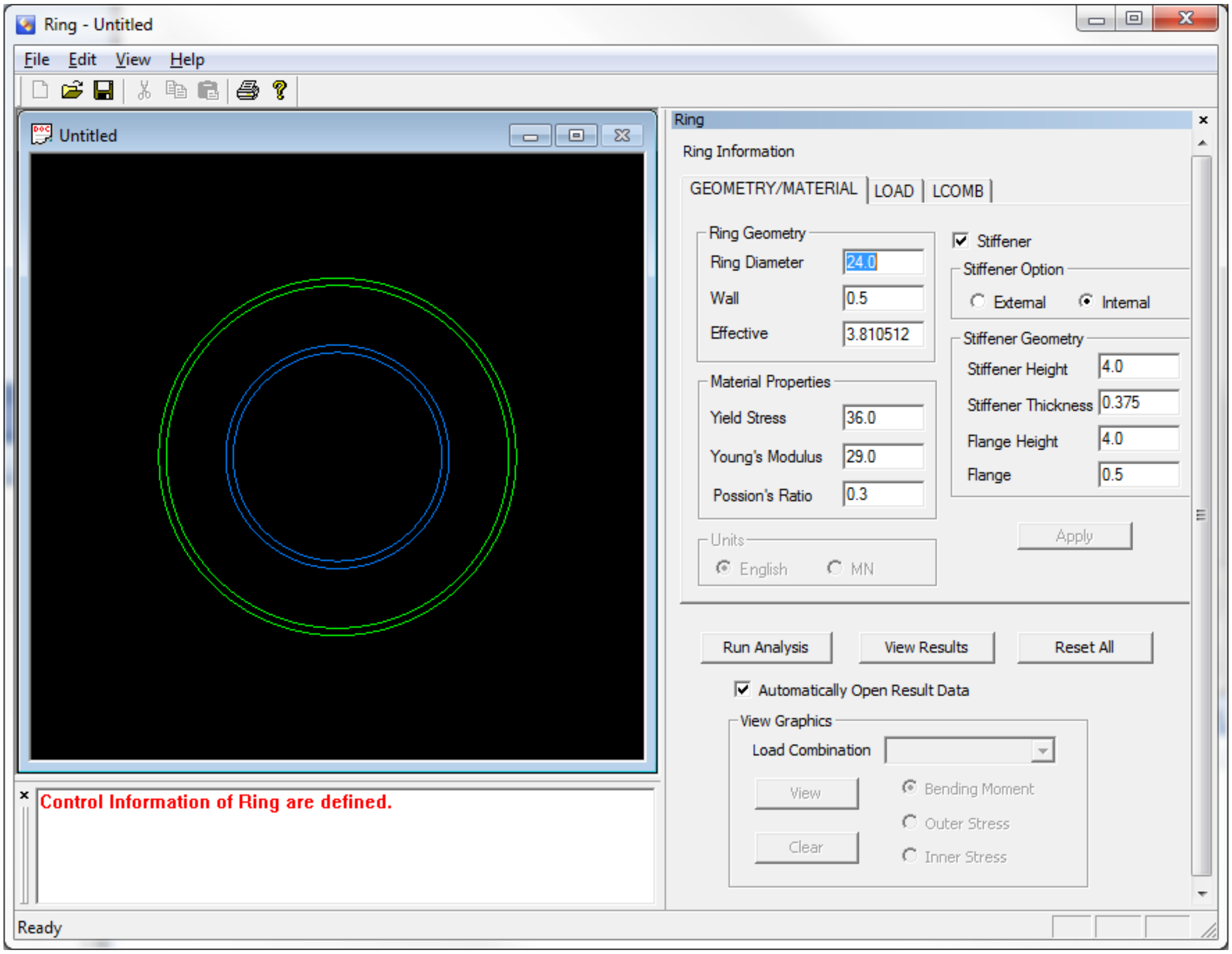
c. **Material Properties**:

   i. Program assumes the material properties are the same for both ring and its stiffeners (if any);
   ii. Yield Stress Fy (units in ksi), default to 36ksi;
   iii. Young’s Modulus (units in 1000ksi), default to 29000ksi;
   iv. Possion’s Ratio, default to 0.3;

d. Display the geometry

   After the ring geometry input, click “**Apply**”, the ring and its stiffener (if any) will be displayed graphically at the main window.

e. The geometry and material property input window looks like following,



Control Information of Ring are defined.

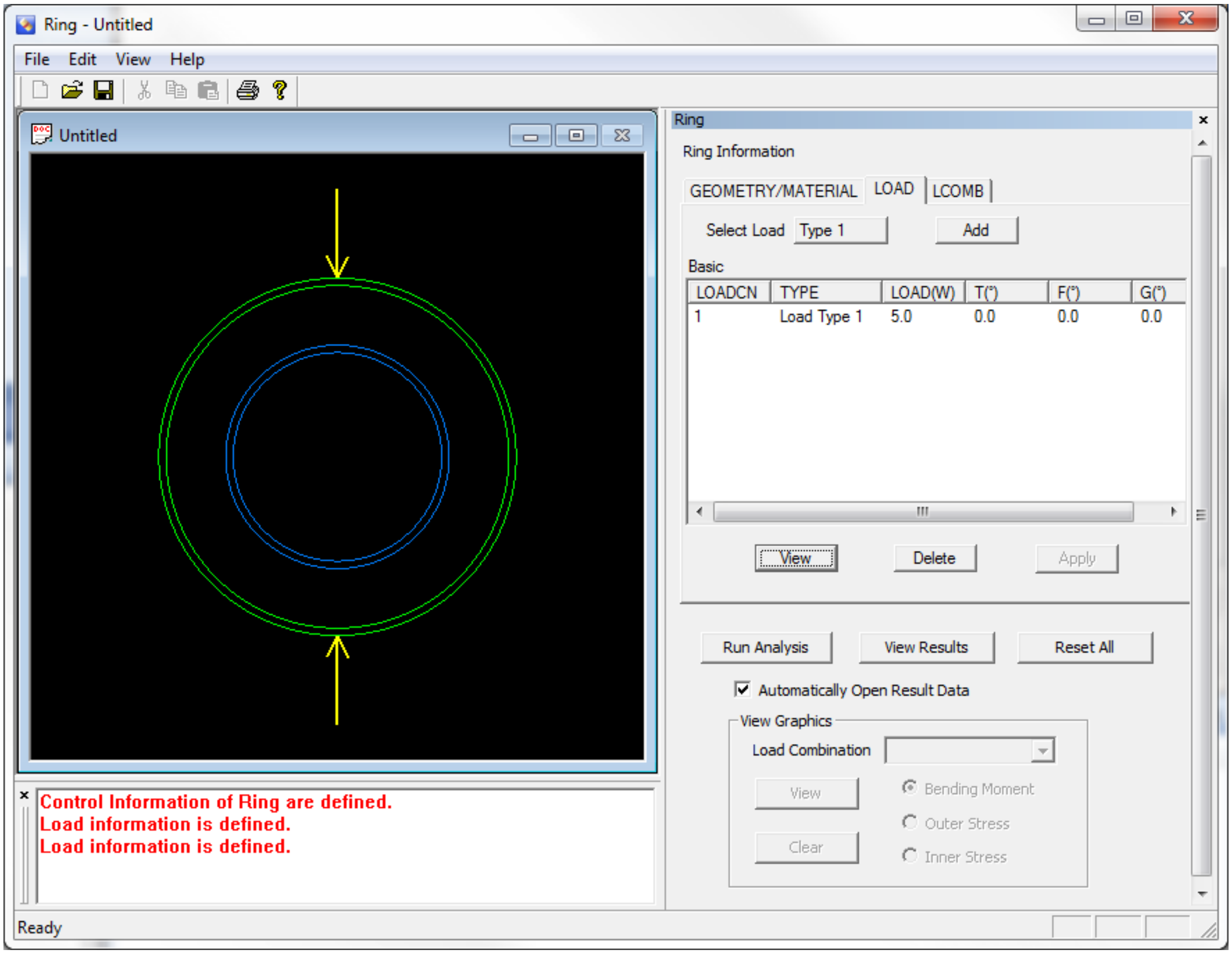
Ring Diameter 24.0
Wall 0.5
Effective 3.810512
Yield Stress 36.0
Young's Modulus 29.0
Possion's Ratio 0.3
☑ Stiffener
○ External ● Internal
Stiffener Height 4.0
Stiffener Thickness 0.375
Flange Height 4.0
Flange 0.5
● English ○ MN
☑ Automatically Open Result Data
● Bending Moment

7. Understanding input of basic load conditions

A basic load condition consists of a load condition number (program automatically assigned), a ring case number (corresponding load type 1, 2, 3, 6, 8, 12, 16, 17 and 18), a load value (could be applied force, applied moment and applied distributed force), load position angles “Theta” T and “PHI” F and the rotation angle of the axis of symmetry of the loading pattern “GAMMA” G.

To input a basic load case, select load type first then click “**Add**”; input the load value and its position according to the corresponding load type selected. The loads are input in **kips** for concentrated forces, in **in-kips** for concentrated moments and in **kips/in** for distributed forces. The load position angles are input in degrees and are positive clockwise. Click “**Apply**” to add the load condition.

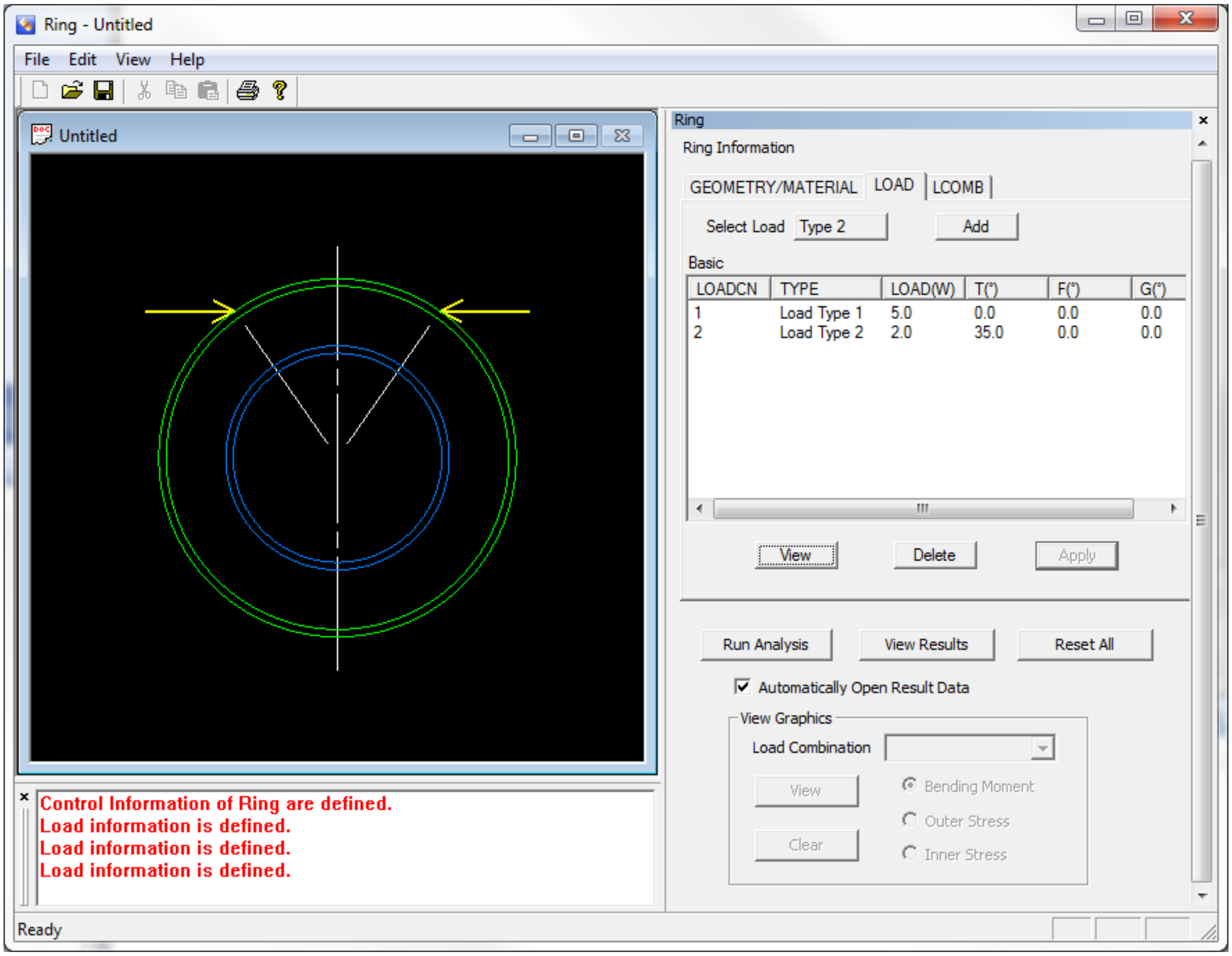
After a basic load case is added, click “**View**” to view the added load case.



LOADCN	TYPE	LOAD(W)	T(°)	F(°)	G(°)
1	Load Type 1	5.0	0.0	0.0	0.0

Control Information of Ring are defined.
Load information is defined.
Load information is defined.

Repeat above procedures to add more basic load conditions. After a basic load case is added, click “**View**” to view the added load case. An example basic load condition 2 with load type 2 and a load value of 2.0kips at a “Theta” angle T = 35.0 degrees looks like following,



Select Load: Type 2

LOADCN	TYPE	LOAD(W)	T(°)	F(°)	G(°)
1	Load Type 1	5.0	0.0	0.0	0.0
2	Load Type 2	2.0	35.0	0.0	0.0

☑ Automatically Open Result Data

```
Control Information of Ring are defined.
Load information is defined.
Load information is defined.
Load information is defined.
```

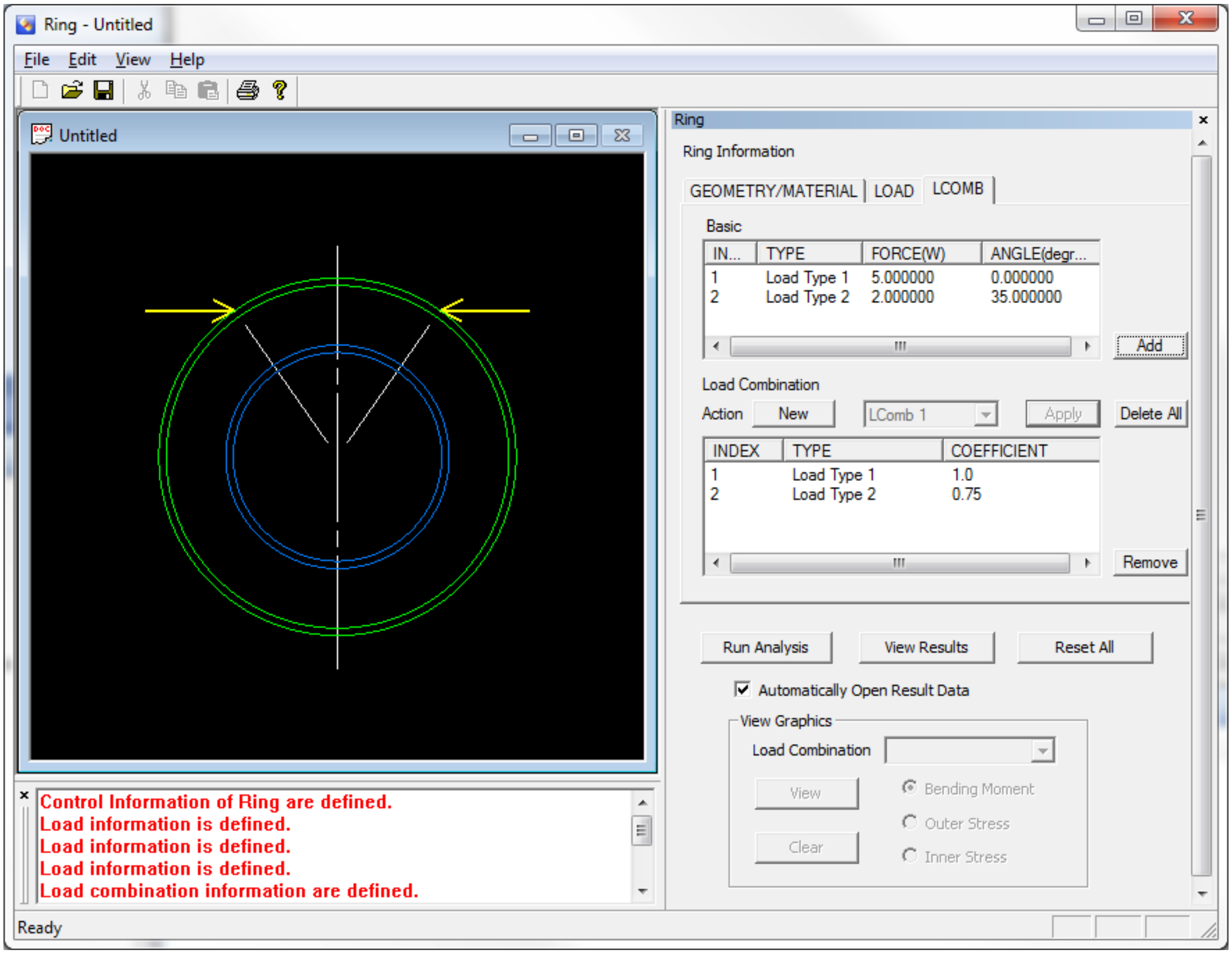
8. Understanding input of load combinations

A load combination case consists of the basic load condition numbers and the factors of the basic load to be applied. Up to 10 basic load conditions can be used in one load combination case. Total 25 load combination cases are allowed.

Select the basic load condition and click “**Add**”, the selected load condition will be automatically put into the load combination with a default load factor to 1.0. After all the basic load conditions selected, click “**Apply**” to add the load combination, the names of the load combinations will be assigned automatically.

User has the options to change load factors and to add or remove basic load conditions from the defined load combinations. After a change is made, click “**Apply**” to accept the change.

A example load combination combines two basic load conditions are shown in following, the load combination consists 100% basic load condition 1 and 75% basic load condition 2.



Ring Information

Basic

IN...	TYPE	FORCE(W)	ANGLE(degr...
1	Load Type 1	5.000000	0.000000
2	Load Type 2	2.000000	35.000000

Load Combination

Action: New LComb 1

INDEX	TYPE	COEFFICIENT
1	Load Type 1	1.0
2	Load Type 2	0.75

☑ Automatically Open Result Data

View Graphics

Load Combination

● Bending Moment

○ Outer Stress

○ Inner Stress

Control Information of Ring are defined.
Load information is defined.
Load information is defined.
Load information is defined.
Load combination information are defined.

9. Running the ring analysis

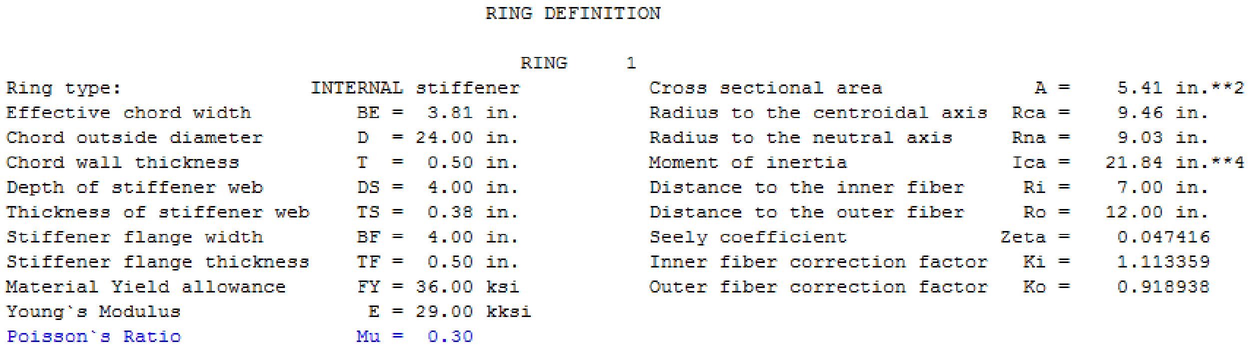
After all the ring information, load information and load combination information defined, click “**Run Analysis**” button, program will collect all the input and run the ring analysis.

10. Understanding the analysis listing results

The output listing results are divided into following four parts,

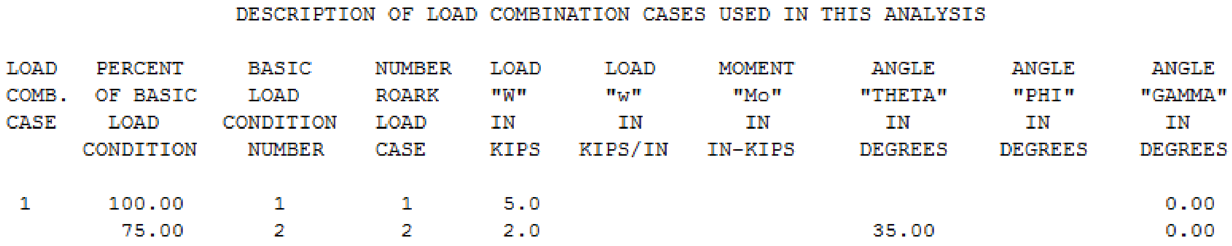
Ring Definition part is a description of all the geometry and properties user input and the calculated ring properties used in the analysis such as cross section area etc. An example output of this part looks like following,

RING DEFINITION



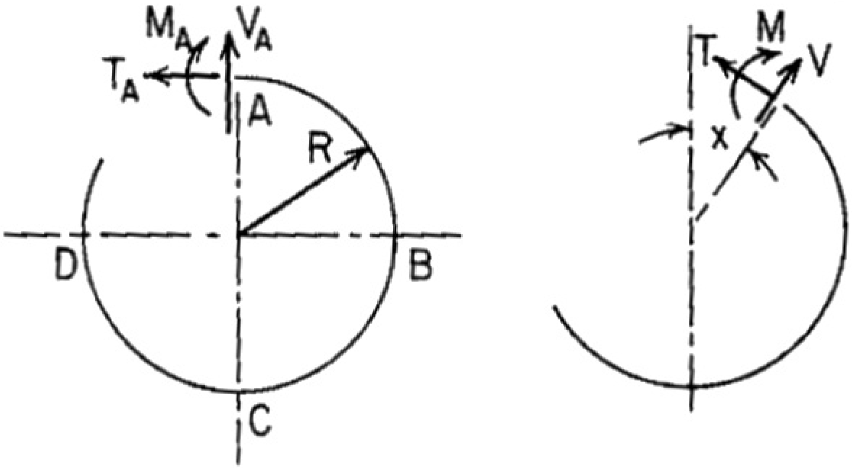
```
                                 RING DEFINITION

                                    RING     1
Ring type:                 INTERNAL stiffener          Cross sectional area          A =    5.41 in.**2
Effective chord width         BE =  3.81 in.           Radius to the centroidal axis  Rca =    9.46 in.
Chord outside diameter        D  = 24.00 in.           Radius to the neutral axis     Rna =    9.03 in.
Chord wall thickness          T  =  0.50 in.           Moment of inertia              Ica =   21.84 in.**4
Depth of stiffener web        DS =  4.00 in.           Distance to the inner fiber     Ri =    7.00 in.
Thickness of stiffener web    TS =  0.38 in.           Distance to the outer fiber     Ro =   12.00 in.
Stiffener flange width        BF =  4.00 in.           Seely coefficient             Zeta =    0.047416
Stiffener flange thickness    TF =  0.50 in.           Inner fiber correction factor   Ki =    1.113359
Material Yield allowance      FY = 36.00 ksi           Outer fiber correction factor   Ko =    0.918938
Young`s Modulus                E = 29.00 kksi
Poisson`s Ratio               Mu =  0.30
```

Description of load combination cases used in this analysis is a summary of the basic information of basic load conditions and load combinations. An example of output of this part looks like following,

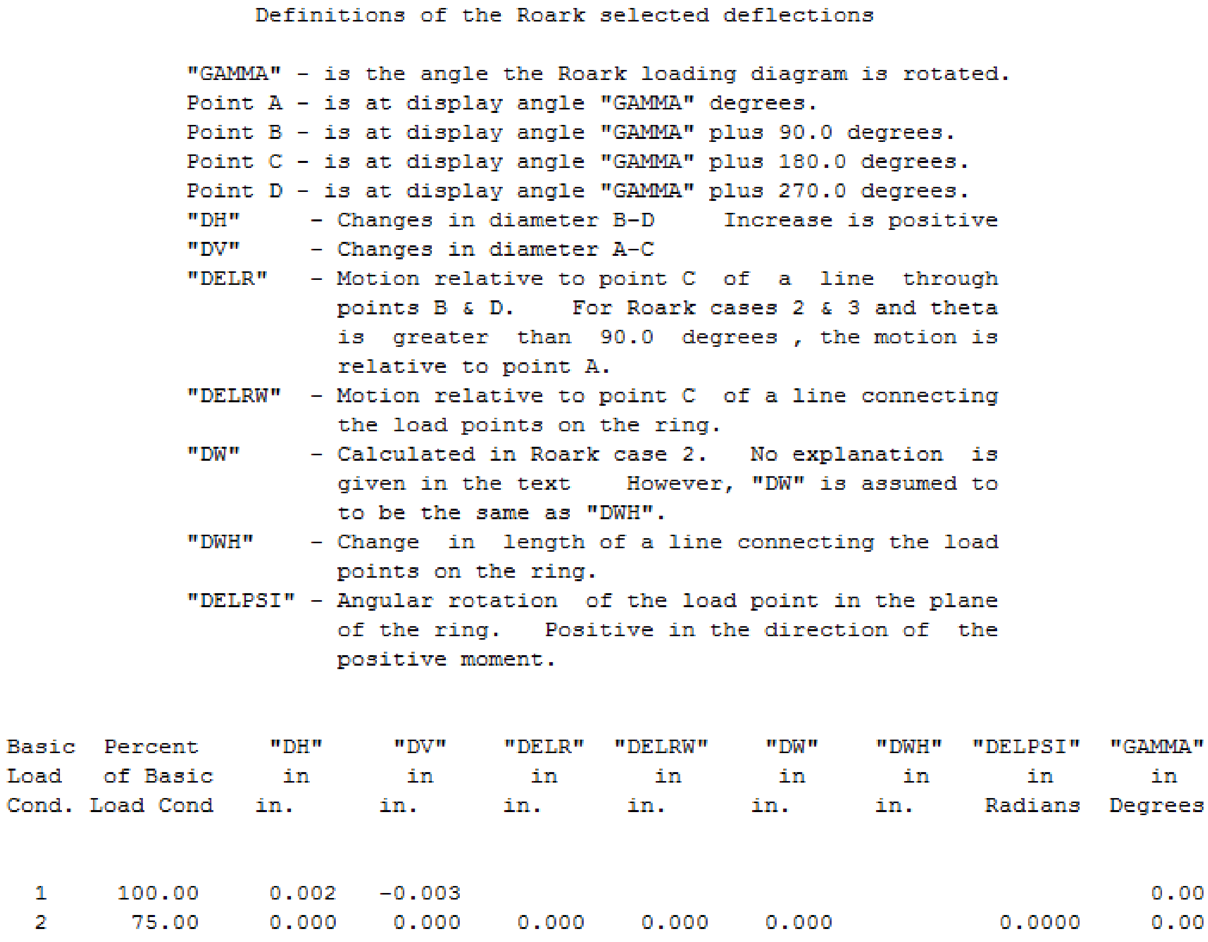


```
                 DESCRIPTION OF LOAD COMBINATION CASES USED IN THIS ANALYSIS

 LOAD    PERCENT     BASIC     NUMBER   LOAD     LOAD    MOMENT     ANGLE     ANGLE     ANGLE
 COMB.  OF BASIC     LOAD      ROARK    "W"      "w"      "Mo"     "THETA"    "PHI"    "GAMMA"
 CASE     LOAD     CONDITION   LOAD     IN       IN       IN        IN        IN        IN
        CONDITION   NUMBER     CASE     KIPS   KIPS/IN  IN-KIPS   DEGREES   DEGREES   DEGREES

  1       100.00      1          1       5.0                                            0.00
           75.00      2          2       2.0                       35.00                0.00
```

The third part is a **deflection report** part. The definitions in this part are based on the diagram from Roark' formulas,

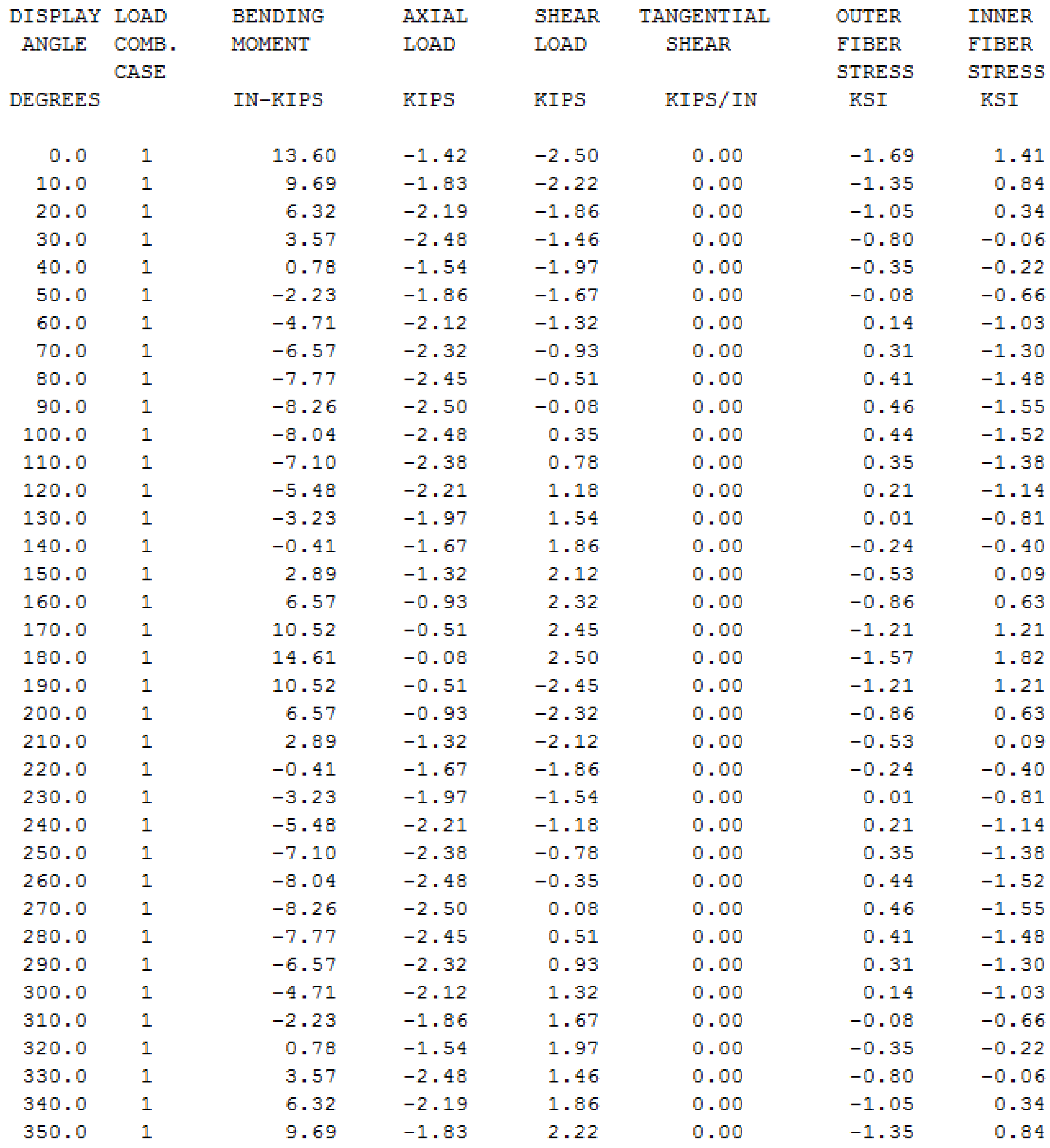


An example output of the third part looks like following,



```
                  Definitions of the Roark selected deflections

             "GAMMA" - is the angle the Roark loading diagram is rotated.
             Point A - is at display angle "GAMMA" degrees.
             Point B - is at display angle "GAMMA" plus 90.0 degrees.
             Point C - is at display angle "GAMMA" plus 180.0 degrees.
             Point D - is at display angle "GAMMA" plus 270.0 degrees.
             "DH"     - Changes in diameter B-D     Increase is positive
             "DV"     - Changes in diameter A-C
             "DELR"   - Motion relative to point C  of  a  line  through
                        points B & D.    For Roark cases 2 & 3 and theta
                        is  greater  than  90.0  degrees , the motion is
                        relative to point A.
             "DELRW"  - Motion relative to point C  of a line connecting
                        the load points on the ring.
             "DW"     - Calculated in Roark case 2.   No explanation  is
                        given in the text    However, "DW" is assumed to
                        to be the same as "DWH".
             "DWH"    - Change  in  length of a line connecting the load
                        points on the ring.
             "DELPSI" - Angular rotation  of the load point in the plane
                        of the ring.   Positive in the direction of  the
                        positive moment.

Basic  Percent     "DH"     "DV"     "DELR"  "DELRW"    "DW"     "DWH"  "DELPSI"  "GAMMA"
Load   of Basic     in       in        in       in        in       in       in        in
Cond. Load Cond   in.      in.      in.      in.      in.      in.     Radians   Degrees


  1     100.00     0.002   -0.003                                                     0.00
  2      75.00     0.000    0.000    0.000    0.000    0.000            0.0000        0.00
```

The fourth part of the output results is **the internal loads and stresses report**. For each ring analysis, the report will be based on 36 positions with each position 10 degrees apart to cover the full 360 degrees. An example output of this part looks like following,



```
DISPLAY LOAD      BENDING      AXIAL     SHEAR   TANGENTIAL    OUTER     INNER
 ANGLE  COMB.     MOMENT       LOAD      LOAD      SHEAR       FIBER     FIBER
        CASE                                                   STRESS    STRESS
DEGREES           IN-KIPS      KIPS      KIPS      KIPS/IN      KSI       KSI

   0.0    1         13.60      -1.42     -2.50      0.00       -1.69      1.41
  10.0    1          9.69      -1.83     -2.22      0.00       -1.35      0.84
  20.0    1          6.32      -2.19     -1.86      0.00       -1.05      0.34
  30.0    1          3.57      -2.48     -1.46      0.00       -0.80     -0.06
  40.0    1          0.78      -1.54     -1.97      0.00       -0.35     -0.22
  50.0    1         -2.23      -1.86     -1.67      0.00       -0.08     -0.66
  60.0    1         -4.71      -2.12     -1.32      0.00        0.14     -1.03
  70.0    1         -6.57      -2.32     -0.93      0.00        0.31     -1.30
  80.0    1         -7.77      -2.45     -0.51      0.00        0.41     -1.48
  90.0    1         -8.26      -2.50     -0.08      0.00        0.46     -1.55
 100.0    1         -8.04      -2.48      0.35      0.00        0.44     -1.52
 110.0    1         -7.10      -2.38      0.78      0.00        0.35     -1.38
 120.0    1         -5.48      -2.21      1.18      0.00        0.21     -1.14
 130.0    1         -3.23      -1.97      1.54      0.00        0.01     -0.81
 140.0    1         -0.41      -1.67      1.86      0.00       -0.24     -0.40
 150.0    1          2.89      -1.32      2.12      0.00       -0.53      0.09
 160.0    1          6.57      -0.93      2.32      0.00       -0.86      0.63
 170.0    1         10.52      -0.51      2.45      0.00       -1.21      1.21
 180.0    1         14.61      -0.08      2.50      0.00       -1.57      1.82
 190.0    1         10.52      -0.51     -2.45      0.00       -1.21      1.21
 200.0    1          6.57      -0.93     -2.32      0.00       -0.86      0.63
 210.0    1          2.89      -1.32     -2.12      0.00       -0.53      0.09
 220.0    1         -0.41      -1.67     -1.86      0.00       -0.24     -0.40
 230.0    1         -3.23      -1.97     -1.54      0.00        0.01     -0.81
 240.0    1         -5.48      -2.21     -1.18      0.00        0.21     -1.14
 250.0    1         -7.10      -2.38     -0.78      0.00        0.35     -1.38
 260.0    1         -8.04      -2.48     -0.35      0.00        0.44     -1.52
 270.0    1         -8.26      -2.50      0.08      0.00        0.46     -1.55
 280.0    1         -7.77      -2.45      0.51      0.00        0.41     -1.48
 290.0    1         -6.57      -2.32      0.93      0.00        0.31     -1.30
 300.0    1         -4.71      -2.12      1.32      0.00        0.14     -1.03
 310.0    1         -2.23      -1.86      1.67      0.00       -0.08     -0.66
 320.0    1          0.78      -1.54      1.97      0.00       -0.35     -0.22
 330.0    1          3.57      -2.48      1.46      0.00       -0.80     -0.06
 340.0    1          6.32      -2.19      1.86      0.00       -1.05      0.34
 350.0    1          9.69      -1.83      2.22      0.00       -1.35      0.84
```

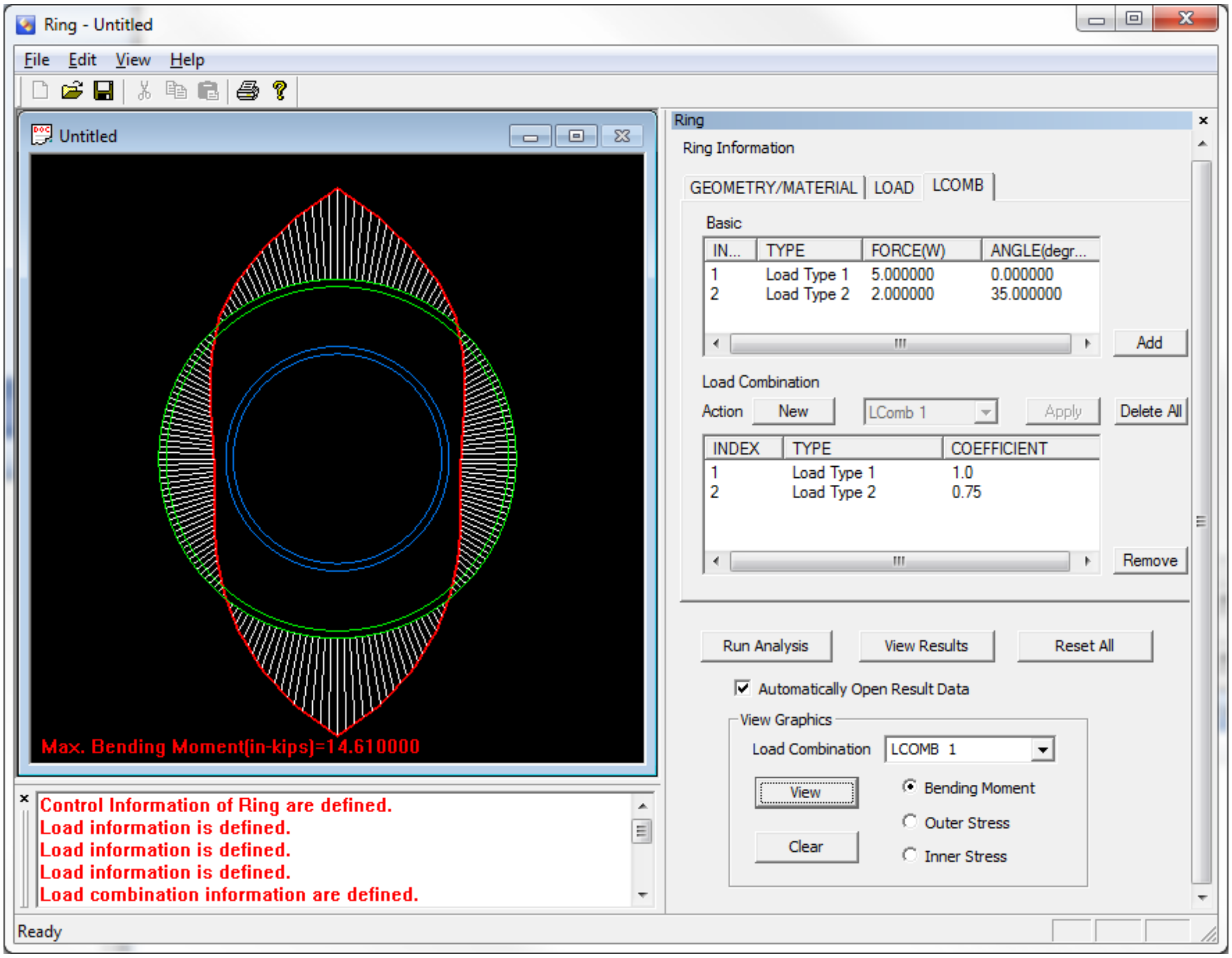
The analysis results will not be automatically saved. User must use the browser to save the output listing file for future use or reference.

11. Viewing the analysis results graphically

The ring program has a built-in graphic results viewer. The user has the option to view the bending moments, outer stresses and inner stresses for each load combination analyzed.

Under “**View Graphics**” part, select the load combination and one of the three variables (either bending moments or outer stresses or inner stresses), then click “**View**” to view the results graphically with maximum value shown at the bottom.

An example **Bending Moment** diagram for a ring analysis shown here,



```
Max. Bending Moment(in-kips)=14.610000
```

IN...	TYPE	FORCE(W)	ANGLE(degr...
1	Load Type 1	5.000000	0.000000
2	Load Type 2	2.000000	35.000000

Action: LComb 1

INDEX	TYPE	COEFFICIENT
1	Load Type 1	1.0
2	Load Type 2	0.75

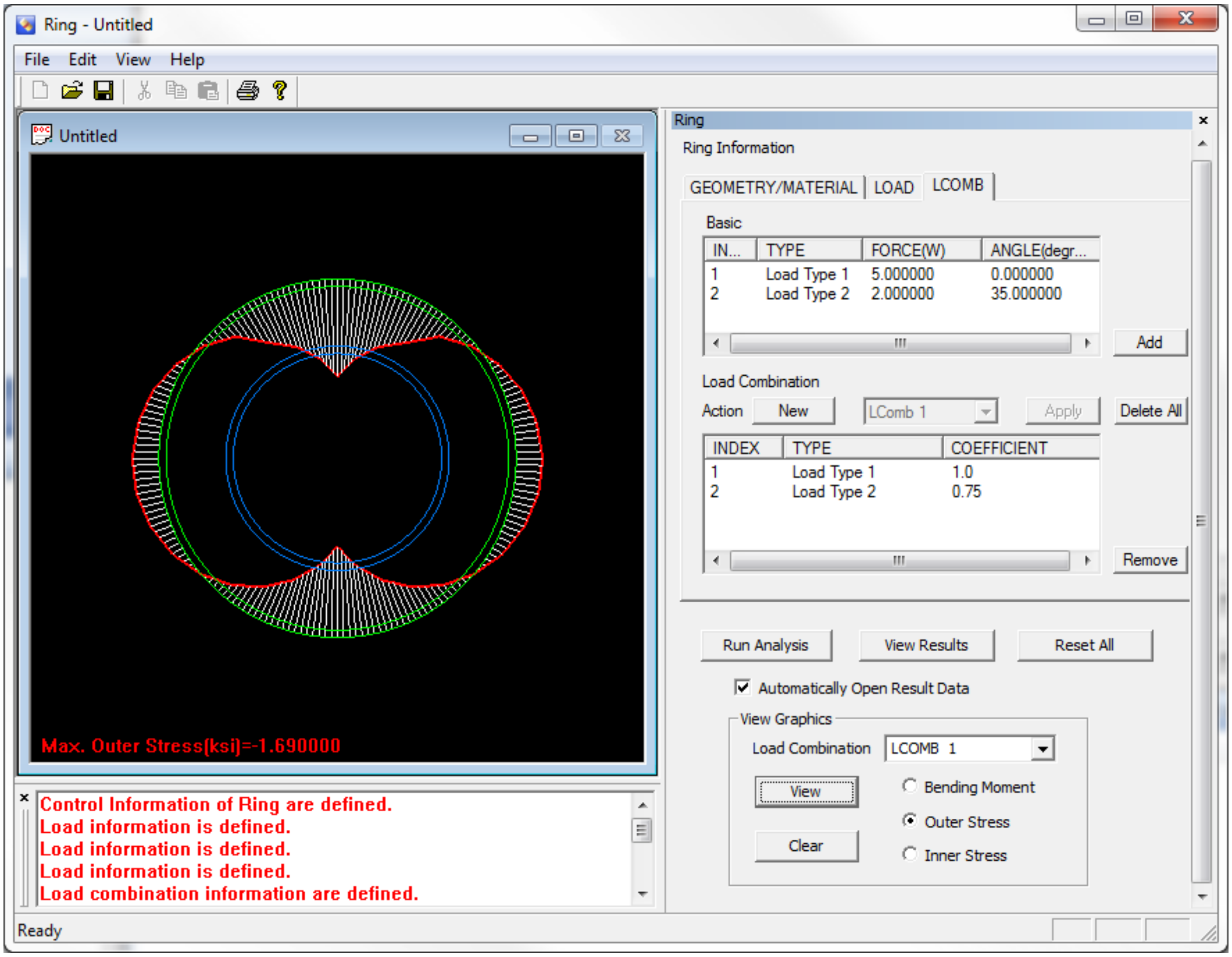
☑ Automatically Open Result Data

Load Combination: LCOMB 1

● Bending Moment
○ Outer Stress
○ Inner Stress

```
Control Information of Ring are defined.
Load information is defined.
Load information is defined.
Load information is defined.
Load combination information are defined.
```

An example **Outer Stresses** diagram for a ring analysis shown here,



Max. Outer Stress(ksi)=-1.690000

Basic

IN...	TYPE	FORCE(W)	ANGLE(degr...
1	Load Type 1	5.000000	0.000000
2	Load Type 2	2.000000	35.000000

Load Combination

Action: New LComb 1

INDEX	TYPE	COEFFICIENT
1	Load Type 1	1.0
2	Load Type 2	0.75

☑ Automatically Open Result Data

View Graphics

Load Combination: LCOMB 1

○ Bending Moment
● Outer Stress
○ Inner Stress

Control Information of Ring are defined.
Load information is defined.
Load information is defined.
Load information is defined.
Load combination information are defined.

Ready

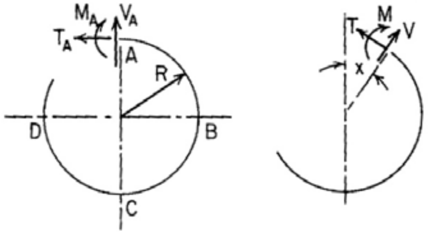
12. Supported Ring Cases from Roark’s Formulas For Stress and Strain

Totally nine (9) ring cases supported from Table 17 – Formulas for Circular Rings, from Roark’s Formulas For Stress and Strain, 5th edition. These supported cases 1, 2, 3, 6, 8, 12, 16, 17 and 18 are listed in the following pages for reference. If there is any doubt or problem in the attached cases, the original print of Formulas For Stress and Strain should prevail.

TABLE 17 *Formulas for circular rings*

NOTATION: $W =$ load (pounds); w and $v =$ unit loads (pounds per linear inch); $\rho =$ weight of contained liquid (pounds per cubic inch); $M_o =$ applied couple (inch-pounds); M_A and M are internal moments at A and x, respectively, positive as shown. T_A, T, V_A, and V are internal forces, positive as shown. $E =$ modulus of elasticity (pounds per square inch); $I =$ area moment of inertia of ring cross section (inches to the fourth). [Note that for a pipe or cylinder a representative 1-in segment may be used by replacing EI by $Et^3/12(1-\nu^2)$.] θ, x, and ϕ are angles (radians); $s = \sin\theta$, $c = \cos\theta$, $z = \sin x$, $u = \cos x$, $n = \sin\phi$, and $e = \cos\phi$. D_V and D_H are changes in the vertical and horizontal diameters, respectively, and an increase is positive. ΔR is the change in the lower half of the vertical diameter or the vertical motion relative to point C of a line connecting points B and D on the ring. Similarly ΔR_W is the vertical motion relative to point C of a horizontal line connecting the load points on the ring. D_{WH} is the change in length of a horizontal line connecting the load points on the ring. $\Delta\psi$ is the angular rotation (radians) of the load point in the plane of the ring and is positive in the direction of a positive moment at that point

The hoop stress deformation factor is $\alpha = I/AR^2$, where A is the cross-sectional area and R is the radius to the centroid of the cross section. The transverse (radial) shear deformation factor is $\beta = FEI/GAR^2$, where G is the shear modulus of elasticity and F is a shape factor for the cross section (see page 185). The following constants are hereby defined in order to simplify the expressions which follow. Note that all of these constants are unity if no correction for hoop stress or shear stress is necessary or desired. $k_1 = 1 + \alpha + \beta$, $k_2 = 1 - \alpha + \beta$, $k_3 = 1 + \alpha - \beta$, $k_4 = k_2/k_1$, $k_5 = k_2^2/k_1$



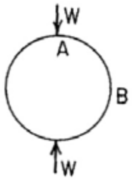
General formulas for moment, hoop load, and radial shear

$$M = M_A - T_A R(1-u) + V_A R z + LT_M$$
$$T = T_A u + V_A z + LT_T$$
$$V = -T_A z + V_A u + LT_V$$

where LT_M, LT_T, and LT_V are load terms given below for several types of load

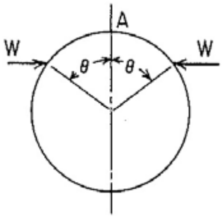
Note: Due to symmetry in most of the cases presented, the loads beyond 180° are not included in the load terms. Only for cases 16, 17, and 19 should the equations for M, T, and V be used beyond 180°.

Note: The use of the bracket $\langle x - \theta \rangle^0$ is explained on page 94 and has a value of zero unless $x > \theta$

<table>
<tr><th>Reference no., loading, and load terms</th><th colspan="2">Formulas for moments, loads, and deformations and some selected numerical values</th></tr>
<tr><td>1.


$LT_M = \frac{-WRz}{2}$ $LT_T = \frac{-Wz}{2}$
$LT_V = \frac{-Wu}{2}$</td><td>$M_A = \frac{WR}{\pi}$
$T_A = 0$
$V_A = 0$
$D_H = \frac{WR^3}{EI}\left(\frac{2}{\pi} - \frac{k_3}{2}\right)$
$D_V = \frac{-WR^3}{EI}\left(\frac{\pi k_1}{4} - \frac{2}{\pi}\right)$</td><td>Max $+M = M_A = 0.3183WR$ Max $-M = M_B = -0.1817WR$
If $\alpha = \beta = 0$, $D_H = 0.137\frac{WR^3}{EI}$ and $D_V = -0.149\frac{WR^3}{EI}$
For greater accuracy when the ring is relatively thick, multiply D_H by k_H and D_V by k_V, where k_H and k_V depend upon the ratio of outer radius R_o to inner radius R_i and have the following values:
<table><tr><th>R_o/R_i</th><th>1.3</th><th>1.4</th><th>1.5</th><th>1.6</th><th>1.7</th><th>1.8</th><th>1.9</th></tr><tr><td>k_H</td><td>1.05</td><td>1.115</td><td>1.175</td><td>1.225</td><td>1.275</td><td>1.325</td><td>1.360</td></tr><tr><td>k_V</td><td>1.03</td><td>1.055</td><td>1.090</td><td>1.114</td><td>1.155</td><td>1.180</td><td>1.225</td></tr></table>(Ref. 19)</td></tr>
</table>

2.



$$LT_M = -WR(c-u)\langle x-\theta\rangle^0$$
$$LT_T = Wu\langle x-\theta\rangle^0$$
$$LT_V = -Wz\langle x-\theta\rangle^0$$

$$M_A = -WR\left[\left(1-\frac{\theta}{\pi}\right)(1-c) - \frac{s}{\pi}(1-ck_4)\right]$$

$$T_A = -W\left(1-\frac{\theta}{\pi}+\frac{sck_4}{\pi}\right)$$

$$V_A = 0$$

$$D_H = \frac{-WR^3}{EI}\left[\frac{\theta}{2}k_1 - \frac{sc}{2}k_2 - \frac{2}{\pi}(s-\theta c)\right] \qquad \text{if } \theta \le \frac{\pi}{2}$$

$$D_V = \frac{WR^3}{EI}\left[\frac{2}{\pi}(s-\theta c) + c - 1 + \frac{s^2}{2}k_2\right]$$

$$\Delta R = \frac{WR^3}{EI}\left[\frac{k_2}{2\pi}(\theta - sck_4) - 0.1817(s-\theta c)\right] \qquad \text{if } \theta \le \frac{\pi}{2}$$

$$\Delta R_W = \frac{WR^3}{EI}\left\{\frac{\theta}{\pi}\left[sc(\pi-\theta) - 1 - c + \frac{s^2}{2}(4+k_2)\right] + \frac{s}{\pi}(1+c) - s^2 - \frac{s^3c}{2\pi}k_5\right\}$$

$$D_W = \frac{-WR^3}{EI}\left\{\frac{\theta}{\pi}[(\pi-\theta)(2c^2+k_1) + 2sc(2+k_2)] - sc(2+k_2) - \frac{s^2}{\pi}(2+c^2k_5)\right\}$$

$$\Delta\psi = \frac{WR^2}{EI}\left\{s - \frac{\theta}{\pi}[2s + (\pi-\theta)c] + \frac{s^2c}{\pi}k_4\right\}$$

$$\text{Max } +M = \frac{WRs}{\pi}(1-c^2k_4) \qquad \text{at } x = \theta$$

$$\text{Max } -M = M_A \qquad \text{if } \theta \le \frac{\pi}{2}$$

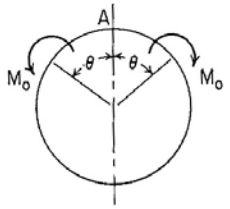
If $\alpha = \beta = 0$, $M = K_M WR$, $T = K_T W$, $D = K_D WR^3/EI$, $\Delta\psi = K_{\Delta\psi} WR^2/EI$, etc.

θ	30°	45°	60°
K_{M_A}	−0.0903	−0.1538	−0.1955
K_{M_θ}	0.0398	0.1125	0.2068
K_{T_A}	−0.9712	−0.9092	−0.8045
K_{D_H}	−0.0157	−0.0461	−0.0891
K_{D_V}	0.0207	0.0537	0.0930
$K_{\Delta R}$	0.0060	0.0179	0.0355
$K_{\Delta R_W}$	0.0119	0.0247	0.0391
K_{D_W}	−0.0060	−0.0302	−0.0770
$K_{\Delta\psi}$	0.0244	0.0496	0.0590

TABLE 17 *Formulas for circular rings* (*Cont.*)

Reference no., loading, and load terms | **Formulas for moments, loads, and deformations and some selected numerical values**

3.



$LT_M = M_o\langle x - \theta\rangle^0$

$LT_T = 0$

$LT_V = 0$

$$M_A = -M_o\left(1 - \frac{\theta}{\pi} - \frac{2s}{\pi k_1}\right)$$

$$T_A = \frac{M_o}{R}\frac{2s}{\pi k_1}$$

$$V_A = 0$$

$$D_H = \frac{M_oR^2}{EI}\left(\frac{2\theta}{\pi} - s\right) \qquad \text{if } \theta \le \frac{\pi}{2}$$

$$D_V = \frac{M_oR^2}{EI}\left(\frac{2\theta}{\pi} - 1 + c\right)$$

$$\Delta R = \frac{M_oR^2}{EI}\left[\frac{1}{\pi}(\theta + sk_4) - \frac{\theta}{2}\right] \qquad \text{if } \theta \le \frac{\pi}{2}$$

$$\Delta R_W = \frac{M_oR^2}{EI}\left\{\frac{\theta}{\pi}[1 + c - (\pi - \theta)s] + \frac{s^3}{\pi}k_4\right\}$$

$$D_{WH} = \frac{-M_oR^2}{EI}\left\{2s - \frac{2\theta}{\pi}[(\pi - \theta)c + 2s] + \frac{2s^2c}{\pi}k_4\right\}$$

$$\Delta\psi = \frac{M_oR}{EI}\left[\theta\left(1 - \frac{\theta}{\pi}\right) - \frac{2s^2}{\pi k_1}\right]$$

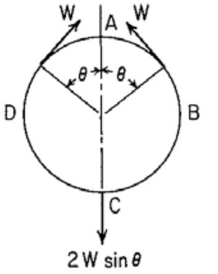
Max $+M = M_o\left(\frac{\theta}{\pi} + \frac{2sc}{\pi k_1}\right)$ at x just greater than θ

Max $-M = -M_o\left(1 - \frac{\theta}{\pi} - \frac{2sc}{\pi k_1}\right)$ at x just less than θ

If $\alpha = \beta = 0$, $M = K_MM_o$, $T = K_TM_o/R$, $D = K_DM_oR^2/EI$, $\Delta\psi = K_{\Delta\psi}M_oR/EI$, etc.

θ	30°	45°	60°	90°
K_{M_A}	−0.5150	−0.2998	−0.1153	0.1366
K_{T_A}	0.3183	0.4502	0.5513	0.6366
K_{M_θ}	−0.5577	−0.4317	−0.3910	−0.5000
K_{D_H}	−0.1667	−0.2071	−0.1994	0.0000
K_{D_V}	0.1994	0.2071	0.1667	0.0000
$K_{\Delta R}$	0.0640	0.0824	0.0854	0.0329
$K_{\Delta R_W}$	0.1326	0.1228	0.1022	0.0329
$K_{D_{WH}}$	−0.0488	−0.0992	−0.1180	0.0000
$K_{\Delta\psi}$	0.2772	0.2707	0.2207	0.1488

6.



$$LT_M = -WR[1 - \cos(x-\theta)]\langle x-\theta\rangle^0$$

$$LT_T = W\cos(x-\theta)\langle x-\theta\rangle^0$$

$$LT_V = -W\sin(x-\theta)\langle x-\theta\rangle^0$$

$$M_A = -WR\left[\frac{s}{\pi}(1+k_4) - \left(1-\frac{\theta}{\pi}\right)(1-c)\right]$$

$$T_A = -W\left[\frac{s}{\pi}k_4 + \left(1-\frac{\theta}{\pi}\right)c\right]$$

$$V_A = 0$$

$$D_H = \frac{WR^3}{EI}\left[\frac{s}{2}(2+k_3) - \frac{2}{\pi}(\theta+s) - \frac{\theta c}{2}k_1\right] \quad \text{if } \theta \le \frac{\pi}{2}$$

$$D_H = \frac{WR^3}{EI}\left[2 - \frac{2}{\pi}(\theta+s) - \frac{\pi-\theta}{2}ck_1 - \frac{s}{2}k_2\right] \quad \text{if } \theta \ge \frac{\pi}{2}$$

$$D_V = \frac{WR^3}{EI}\left[1 - c - \frac{2}{\pi}(\theta+s) + \frac{\pi-\theta}{2}sk_1\right]$$

$$\Delta R = \frac{WR^3}{EI}\left[\frac{1}{2}(\theta-s) - \frac{1}{\pi}\left(\theta + s - \frac{\theta c}{2}k_2 + \frac{s}{2}k_5\right) + \frac{\pi s}{4}k_1\right] \quad \text{if } \theta \le \frac{\pi}{2}$$

$$\Delta R = \frac{WR^3}{EI}\left[\frac{1}{2}(\pi - \theta - ck_2 - s - 2c) - \frac{\theta}{\pi}\left(1 - \frac{c}{2}k_2\right) - \frac{s}{2\pi}(2+k_5) + \frac{\pi-\theta}{2}sk_1\right] \quad \text{if } \theta \ge \frac{\pi}{2}$$

$$\Delta R_W = \frac{WR^3}{EI}\left\{\frac{\theta}{\pi}\left[(\pi-\theta)s - 1 - c - s^2\left(1-\frac{c}{2}k_2\right)\right] - \frac{s}{\pi}\left(1 + c + \frac{s^2}{2}k_5\right) - s\left[\frac{sc}{2}(2-k_3) - \frac{\pi-\theta}{2}k_1\right]\right\}$$

$$D_{WH} = \frac{-WR^3}{EI}\left\{\frac{1}{\pi}[\theta(\pi-\theta)(2+k_1)c - 2\theta sc + \theta sk_2 + 2s^2 - s^2ck_5] - s(2+c^2k_2)\left(1-\frac{\theta}{\pi}\right)\right\}$$

$$\Delta\psi = \frac{-WR^2}{EI}\left[\frac{\theta}{\pi}(\pi - \theta - s + sc) - sc - \frac{s^2}{\pi}k_4\right]$$

$$\text{Max } -M = M_C = -WR\left[\frac{s}{\pi}(1-k_4) + \frac{\theta}{\pi}(1+c)\right]$$

Max $+M$ occurs at an angular position $x_1 = \arctan\dfrac{-\pi s}{sk_4 - \theta c}$

(x_1 is always greater than θ and also greater than 90°)

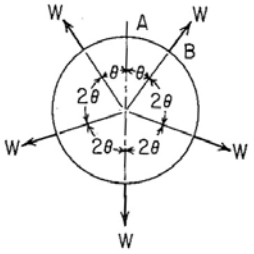
If $\alpha = \beta = 0$, $M = K_M WR$, $T = K_T W$, $D = K_D WR^3/EI$, $\Delta\psi = K_{\Delta\psi}WR^2/EI$, etc.

θ	30°	60°	90°	120°	150°
K_{M_A}	−0.2067	−0.2180	−0.1366	−0.0513	−0.0073
K_{T_A}	−0.8808	−0.6090	−0.3183	−0.1090	−0.0148
K_{M_C}	−0.3110	−0.5000	−0.5000	−0.3333	−0.1117
K_{D_H}	−0.1284	−0.1808	−0.1366	−0.0559	−0.0083
K_{D_V}	0.1368	0.1889	0.1488	0.0688	0.0120
$K_{\Delta R}$	0.0713	0.1073	0.0933	0.0472	0.0088
$K_{\Delta R_W}$	0.1129	0.1196	0.0933	0.0460	0.0059
$K_{D_{WH}}$	−0.0170	−0.1063	−0.1366	−0.0548	−0.0036
$K_{\Delta\psi}$	0.0874	0.1180	0.0329	−0.0264	−0.0123

TABLE 17 *Formulas for circular rings* *(Cont.)*

Reference no., loading, and load terms	Formulas for moments, loads, and deformations and some selected numerical values

7. Ring under any number of equal radial forces equally spaced



For $0 < x < \theta$ $\quad M = \frac{WR}{2}\left(\frac{u}{s} - \frac{1}{\theta}\right) \quad T = \frac{Wu}{2s} \quad V = \frac{-Wz}{2s}$

Max $+M = M_A = \frac{WR}{2}\left(\frac{1}{s} - \frac{1}{\theta}\right)$ $\quad$ Max $-M = \frac{-WR}{2}\left(\frac{1}{\theta} - \frac{c}{s}\right)$ at each load position

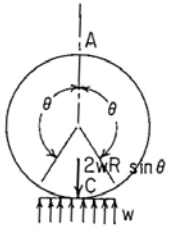
Radial displacement at each load point $= \frac{WR^3}{EI}\left[\frac{1}{4s^2}(\theta k_1 + sck_3) - \frac{1}{2\theta}\right] = R_B$

Radial displacement at $x = 0,\ 2\theta$, and so on $= \frac{-WR^3}{EI}\left[\frac{1}{2\theta} - \frac{1}{4s^2}(sk_3 + \theta ck_1)\right] = R_A$

If $\alpha = \beta = 0$, $M = K_M WR$, $R = K_R WR^3/EI$,

θ	15°	30°	45°	60°	90°
K_{M_A}	0.02199	0.04507	0.07049	0.09989	0.18169
K_{M_B}	−0.04383	−0.08890	−0.13662	−0.18879	−0.31831
K_{R_B}	0.00020	0.00168	0.00608	0.01594	0.07439
K_{R_A}	−0.00018	−0.00148	−0.00539	−0.01426	−0.06831

8.



$\left(\text{Note: } \theta \geq \frac{\pi}{2}\right)$

$LT_M = \frac{-wR^2}{2}(z - s)^2\langle x - \theta\rangle^0$

$LT_T = -wRz(z - s)\langle x - \theta\rangle^0$

$LT_V = -wRu(z - s)\langle x - \theta\rangle^0$

$$M_A = wR^2\left[\frac{1}{4} + \frac{s^2}{2} - \frac{1}{\pi}\left(s + \frac{3}{4}sc + \frac{s^3}{3}k_4 + \frac{\theta}{4} + \frac{\theta s^2}{2}\right)\right]$$

$$T_A = -wR\left(\frac{s^3}{3\pi}k_4\right)$$

$$V_A = 0$$

$$D_H = \frac{-wR^4}{EI}\left[\frac{1}{\pi}\left(2s + \frac{3sc}{2} + \theta s^2 + \frac{\theta}{2}\right) - \frac{1}{2} - s^2 + \frac{s^3}{6}k_2\right]$$

$$D_V = \frac{wR^4}{EI}\left[\frac{k_1}{2}(\pi s - \theta s - 1 - c) + \frac{k_2}{6}(1 + c^3) + \frac{1}{2}(1 + s^2) - \frac{1}{\pi}\left(2s + \frac{3sc}{2} + \theta s^2 + \frac{\theta}{2}\right)\right]$$

$$\Delta R = \frac{-wR^4}{EI}\left[\frac{1}{\pi}\left(s + \frac{3sc}{4} + \frac{\theta s^2}{2} + \frac{k_5 s^3}{6} + \frac{\theta}{4}\right) - \frac{1}{4} + \frac{s}{2} + \frac{3sc}{8} - \frac{\pi - \theta}{8}(1 + 2s^2) - \frac{k_2}{6}(1 + c^3) - \frac{k_1}{2}(\pi s - \theta s - 1 - c)\right]$$

Max $-M = M_C = -wR^2\left[-\frac{1}{4} + \frac{1}{\pi}\left(s + \frac{3sc}{4} - \frac{s^3}{3}k_4 + \frac{\theta}{4} + \frac{\theta s^2}{2}\right)\right]$

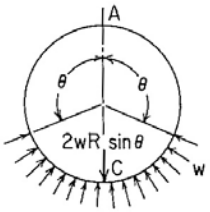
If $\alpha = \beta = 0$, $M = K_M wR^2$, $T = K_T wR$, $D = K_D wR^4/EI$, etc.

θ	90°	120°	135°	150°
K_{M_A}	−0.0494	−0.0329	−0.0182	−0.0065
K_{T_A}	−0.1061	−0.0689	−0.0375	−0.0133
K_{M_C}	−0.3372	−0.2700	−0.1932	−0.1050
K_{D_H}	−0.0533	−0.0362	−0.0204	−0.0074
K_{D_V}	0.0655	0.0464	0.0276	0.0108
$K_{\Delta R}$	0.0448	0.0325	0.0198	0.0080

TABLE 17 ***Formulas for circular rings (Cont.)***

Reference no., loading, and load terms	Formulas for moments, loads, and deformations and some selected numerical values

12.



$$LT_M = -wR^2[1-\cos(x-\theta)]\langle x-\theta\rangle^0$$

$$LT_T = -wR[1-\cos(x-\theta)]\langle x-\theta\rangle^0$$

$$LT_V = -wR\sin(x-\theta)\langle x-\theta\rangle^0$$

$$M_A = -wR^2\left[\frac{1}{\pi}(\theta+2s-\theta c)-1+c\right]$$

$$T_A = -wR\left[\frac{1}{\pi}(s-\theta c)+c\right]$$

$$V_A = 0$$

$$D_H = \frac{-wR^4}{EI}\left[\frac{2}{\pi}(\theta+s)-2+\frac{k_1}{2}(2-s+\theta c)+k_3(1-s)\right] \quad \text{if } \theta \leq \frac{\pi}{2}$$

$$D_H = \frac{-wR^4}{EI}\left[\frac{k_1}{2}(\pi c-\theta c+s)-\frac{2}{\pi}(\pi-\theta-s)\right] \quad \text{if } \theta \geq \frac{\pi}{2}$$

$$D_V = \frac{wR^4}{EI}\left[1-c-\frac{2}{\pi}(\theta+s)+\frac{s}{2}(\pi-\theta)k_1-\alpha(1+c)\right]$$

$$\Delta R = \frac{wR^4}{EI}\left[\frac{1}{2}(\theta-s)-\frac{1}{\pi}(\theta+s)-\frac{k_1}{4}(2-\pi s) + \frac{k_2}{2\pi}(\pi-s+\theta c)\right] \quad \text{if } \theta \leq \frac{\pi}{2}$$

$$\Delta R = \frac{wR^4}{EI}\left[0.8183(\pi-\theta-s)-1-c+\frac{k_2}{2\pi}(\pi-s+\theta c) - \frac{k_1}{2}(1+c-\pi s+\theta s)\right] \quad \text{if } \theta \geq \frac{\pi}{2}$$

$$\text{Max } -M = M_C = -wR^2\left[\frac{\theta}{\pi}(1+c)\right]$$

Max $+M$ occurs at an angular position $x_1 = \arctan\frac{-s\pi}{s-\theta c}$

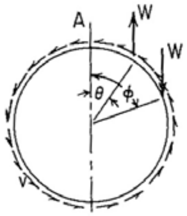
If $\alpha = \beta = 0$, $M = K_M wR^2$, $T = K_T wR$, $D = K_D wR^4/EI$, etc.

θ	30°	60°	90°	120°	150°
K_{M_A}	−0.2067	−0.2180	−0.1366	−0.0513	−0.0073
K_{T_A}	−0.8808	−0.6090	−0.3183	−0.1090	−0.0148
K_{M_C}	−0.3110	−0.5000	−0.5000	−0.3333	−0.1117
K_{D_H}	−0.1284	−0.1808	−0.1366	−0.0559	−0.0083
K_{D_V}	0.1368	0.1889	0.1488	0.0688	0.0120
$K_{\Delta R}$	0.0713	0.1073	0.0933	0.0472	0.0088

TABLE 17 *Formulas for circular rings* (*Cont.*)

Reference no., loading, and load terms | Formulas for moments, loads, and deformations and some selected numerical values

16.



$$v = \frac{W}{2\pi R}(\sin\phi - \sin\theta)\ \text{lb/in}$$

$$LT_M = \frac{-WR}{2\pi}(n - s)(x - z) + WR(z - s)\langle x - \theta\rangle^0 - WR(z - n)\langle x - \phi\rangle^0$$

$$LT_T = \frac{W}{2\pi}(n - s)z + Wz\langle x - \theta\rangle^0 - Wz\langle x - \phi\rangle^0$$

$$LT_V = \frac{-W}{2\pi}(n - s)(1 - u) + Wu\langle x - \theta\rangle^0 - Wu\langle x - \phi\rangle^0$$

$$M_A = \frac{-WR}{2\pi}[c - e - (\pi - \theta)s + (\pi - \phi)n + k_4(s^2 - n^2)]$$

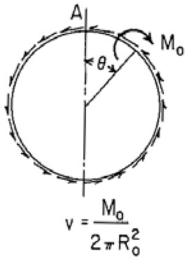
$$T_A = \frac{-W}{2\pi}[k_4(s^2 - n^2)]$$

$$V_A = \frac{W}{2\pi}[\theta - \phi + s - n + k_4(sc - ne)]$$

If $\alpha = \beta = 0$, $M = K_M WR$, $T = K_T W$, $V = K_V W$,

θ	ϕ	0°	30°	60°	90°	120°	150°	180°
0°	K_{M_A}	0.0000	−0.1899	−0.2489	−0.2500	−0.2637	−0.2989	−0.3183
	K_{T_A}	0.0000	0.0398	0.1194	0.1592	0.1194	0.0398	0.0000
	K_{V_A}	0.0000	−0.2318	−0.3734	−0.4092	−0.4023	−0.4273	−0.5000
30°	K_{M_A}	0.1899	0.0000	−0.0590	−0.0601	−0.0738	−0.1090	−0.1284
	K_{T_A}	−0.0398	0.0000	0.0796	0.1194	0.0796	0.0000	−0.0398
	K_{V_A}	0.2318	0.0000	−0.1416	−0.1773	−0.1704	−0.1955	−0.2682
45°	K_{M_A}	0.2322	0.0423	−0.0167	−0.0178	−0.0315	−0.0667	−0.0861
	K_{T_A}	−0.0796	−0.0398	0.0398	0.0796	0.0398	−0.0398	−0.0796
	K_{V_A}	0.3171	0.0853	−0.0563	−0.0920	−0.0851	−0.1102	−0.1829
60°	K_{M_A}	0.2489	0.0590	0.0000	−0.0011	−0.0148	−0.0500	−0.0694
	K_{T_A}	−0.1194	−0.0796	0.0000	0.0398	0.0000	−0.0796	−0.1194
	K_{V_A}	0.3734	0.1416	0.0000	−0.0357	−0.0288	−0.0539	−0.1266
90°	K_{M_A}	0.2500	0.0601	0.0011	0.0000	−0.0137	−0.0489	−0.0683
	K_{T_A}	−0.1592	−0.1194	−0.0398	0.0000	−0.0398	−0.1194	−0.1592
	K_{V_A}	0.4092	0.1773	0.0357	0.0000	0.0069	−0.0182	−0.0909

17.



$$LT_M = \frac{-M_o}{2\pi}(x - z) + M_o\langle x - \theta\rangle^0$$

$$LT_T = \frac{M_o z}{2\pi R}$$

$$LT_V = \frac{-M_o}{2\pi R}(1 - u)$$

$$M_A = \frac{-M_o}{2\pi}(\pi - \theta - 2k_4 s)$$

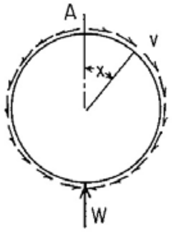
$$T_A = \frac{M_o}{\pi R}k_4 s$$

$$V_A = \frac{-M_o}{2\pi R}(1 + 2k_4 c)$$

If $\alpha = \beta = 0$, $M = K_M M_o$, $T = K_T M_o/R$, $V = K_V M_o/R$,

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
K_{M_A}	−0.5000	−0.2575	−0.1499	−0.0577	0.0683	0.1090	0.1001	0.0758	0.0000
K_{T_A}	0.0000	0.1592	0.2251	0.2757	0.3183	0.2757	0.2250	0.1592	0.0000
K_{V_A}	−0.4775	−0.4348	−0.3842	−0.3183	−0.1592	0.0000	0.0659	0.1165	0.1592

TABLE 17 *Formulas for circular rings* (*Cont.*)

Reference no., loading, and load terms	Formulas for moments, loads, and deformations and some selected numerical values	
18. Bulkhead or supporting ring in pipe, supported at bottom and carrying total load W transferred by tangential shear of v lb/linear in distributed as shown  $v = \frac{W \sin x}{\pi R}$ $LT_M = \frac{WR}{\pi}\left(1 - u - \frac{xz}{2}\right)$ $LT_T = \frac{-W}{2\pi} xz$ $LT_V = \frac{W}{2\pi}(z - xu)$	$M_A = \frac{WR}{2\pi}\left(k_4 - \frac{1}{2}\right)$ $T_A = \frac{W}{2\pi}\left(k_4 + \frac{1}{2}\right)$ $V_A = 0$ $D_H = \frac{WR^3}{EI}\left(\frac{1}{\pi} - \frac{k_3}{4}\right)$ $D_V = \frac{-WR^3}{EI}\left(\frac{\pi}{8}k_1 - \frac{1}{\pi}\right)$ $\Delta R = \frac{-WR^3}{EI}\left[\frac{1}{4\pi}(1 - k_5 + \beta) + \frac{3\pi k_1}{32} - \frac{1}{4}\right]$	If $\alpha = \beta = 0$, $M_A = 0.0796WR$ $T_A = 0.2387W$ $V_A = 0$ $D_H = 0.0683\frac{WR^3}{EI}$ $D_V = -0.0744\frac{WR^3}{EI}$ $\Delta R = -0.0445\frac{WR^3}{EI}$ Max $+M = 0.2387WR$ at $x = \pi$ Max $-M = -0.1028WR$ at $x = 1.84$ rad (105.2°)