Crack Width Calculation for Columns Subject to Biaxial Bending

R. Gong*, S. Cao**

**AECOM, 540 Wickham Street, Fortitude Valley, QLD 4006, Australia (Email: Richard.Gong@aecom.com)*
***YWL Engineering, 230 Orchard Road, #08-232 Faber House, Singapore 238854 (Email: shengfa.cao@ywlgroup.com)*

ABSTRACT
Crack evaluation is required for reinforced concrete columns subject to large bending moments. Section analysis for columns under uniaxial bending is similar to that for beams and slabs, with the axial force taken into account. For columns subject to biaxial bending, the neutral axis is no longer parallel to the column sides, and the evaluation becomes difficult. This paper presents two approaches for crack width calculation for columns subject to biaxial bending, which were developed in the alternative design for Route 8 - Ngong Shuen Chau Viaduct in Hong Kong. One approach simplifies the biaxial bending into uniaxial bending, with an increased moment obtained from the formula recommended in BS8110 for ultimate limit state design. The other approach is based on rigorous section analysis. Orientation of the neutral axis is determined from the ratio of the moments in the two directions. Formulae for the internal forces and bending moments are derived from integration of stresses over the section. Equilibrium equations are established and solved, and crack width is calculated based on the correlation with the outermost tensile strain. Results of the two approaches are compared, and the simplified approach is found to be overly conservative.

KEYWORDS
Biaxial bending; Crack width; Equilibrium equations, Neutral axis; Section analysis

INTRODUCTION
Crack evaluation is required for reinforced concrete flexural members such as beams and slabs. Columns are compression members, for which cracking is usually not deemed a problem. However, in some cases where columns are subject to large bending moments, cracking would need to be evaluated. Crack width calculation for columns subject to uniaxial bending can follow the same approach as for beams and slabs, plus taking the axial force into account in establishing force and moment equilibrium equations. For columns subject to biaxial bending, section analysis becomes complicated as the neutral axis is no longer parallel to the column sides. Solutions to this problem are not readily available.

This paper presents two approaches for crack width calculation for biaxially bent columns. One approach simplifies the biaxial bending into uniaxial bending, and the other based on rigorous section analysis. The two approaches were developed in the alternative design for Route 8 - Ngong Shuen Chau Viaduct in Hong Kong. Results from the two approaches are compared.

THE PROBLEM

The Route 8 - Ngong Shuen Chau Viaduct in Hong Kong was approximately 2.6 km long. The superstructure comprised precast segmental box girders constructed by the balanced cantilever method. The substructure was made of reinforced concrete piers, most with crossheads. In the conforming design the superstructure was in the form of a continuous box girder supported on bearings. The alternative design modified the structure form through a monolithic connection between piers and girder, thus dispensing with the need for bearings, except at movement joints.

The Route 8 piers typically have solid or hollow rectangular sections, with the major axis along the bridge alignment. In the conforming design, bending of a pier was mainly in the transverse direction (about the major axis). Moment in the longitudinal direction (about the minor axis) was generally negligible. The alternative design, replacing bearings with monolithic connections, resulted in moment transfer between girder and pier. The longitudinal bending moment in the pier increased significantly as a result, and was even larger than the transverse moment in some situations.

Ultimate strength design for the bridge piers was carried out in accordance with British Standard BS5400. However, this did not give a method of crack width calculation for biaxially bent columns. An appropriate solution was required for the alternative design.

THE SIMPLIED APPROACH

BS8110-1997 recommends an approach for ultimate limit state design for biaxially bent columns, in the absence of more rigorous calculations. The approach simplifies biaxial bending to uniaxial bending with an increased moment about one axis given by the following equations:

for $M_x / h' \geq M_y / b'$, $M_x^{'} = M_x + \beta \dfrac{h'}{b'} M_y$

for $M_x / h' < M_y / b'$, $M_y^{'} = M_y + \beta \dfrac{b'}{h'} M_x$

where b’ and h’ are the effective width and depth, and β is the coefficient as defined in the standard.

This approach was initially also adopted for crack width calculation. The serviceability moments M_x and M_y were combined into one moment, either $M_x^{'}$ or $M_y^{'}$, based on the equations above, and the crack width was calculated for the combined moment. During the design process it was found that in many cases the crack width design was more critical than the ultimate strength, and thus controlled the pier design. The simplified approach was surmised to be conservative. A more rigorous approach for crack width calculation was desired to produce a more efficient design.

THE RIGOROUS APPROACH

Neutral axis and coordinate system

A typical rectangular column section of width b and depth h is shown in Figure 1. When the column is subject to uniaxial bending, the neutral axis is parallel to the x or the y axis. Under biaxial bending, the neutral axis is in a direction such that $\tan(\alpha) = M_x / M_y$, where α is the angle between the overall bending direction and the x axis.

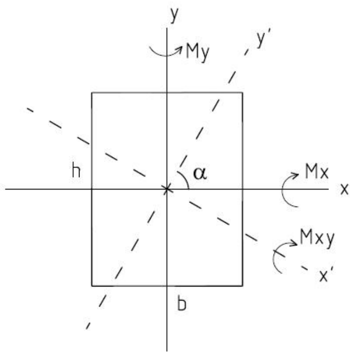


Figure 1. Typical column section and coordinate system

A new coordinate system (x', y') is introduced to facilitate section analysis, with the x' axis parallel to the neutral axis, as shown in Figure 1. The relations between the two coordinate systems are:

$$x' = x\sin\alpha - y\cos\alpha$$

$$y' = y\sin\alpha + x\cos\alpha$$

Section analysis for rectangular solid section

The following assumptions are made in the section analysis, similar to those for uniaxial bending:

i) Plane sections remain plane
ii) Elastic behaviour exists for concrete and steel
iii) The tensile strength of concrete is ignored

The typical strain distribution diagram and the forces/moments on the section are shown in Figure 2. For equilibrium

$$F_{st} - F_{sc} - F_{cc} + N = 0$$

$$M_{st} + M_{sc} + M_{cc} + M_N - M = 0$$

where M_{st}, M_{sc}, M_{cc} and M_N are moments of F_{st}, F_{sc}, F_{cc} and N about the neutral axis, respectively.

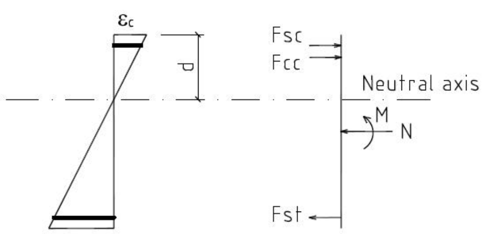
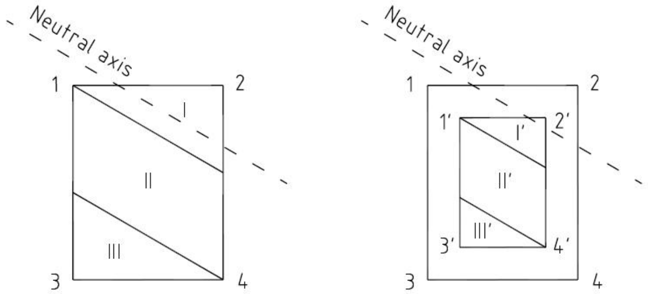


Figure 2. Strain distribution diagram and internal forces/moments

Internal force and bending moment of concrete are derived by integration of stresses over the section. To facilitate the analysis, the section is divided into three zones (I, II, and III), as illustrated in Figure 3(a). The interfaces between the adjacent zones are parallel to the neutral axis. The length of the interface is given by $a = b/\sin\alpha$.



(a) (b)

Figure 3. Zoning of the column section

Four corner points of the section are noted as 1, 2, 3 and 4. Considering Point 2 as the outermost compressive point, depth of Point i (i=1, 3, 4) to Point 2 in the direction perpendicular to the neutral axis equals $y_2^{'}$ - $y_i^{'}$. Accordingly the depths of Points 1, 3 and 4 are:

$$d_1 = y_2^{'} - y_1^{'} = b\cos\alpha$$
$$d_3 = y_2^{'} - y_3^{'} = h\sin\alpha + b\cos\alpha$$
$$d_4 = y_2^{'} - y_4^{'} = h\sin\alpha$$

By assuming the strain at Point 2 (ε_2) and the neutral axis depth (d), the strains at the other points (ε_1, ε_3, ε_4) can be related with ε_2, d, and their respective depths (d_1, d_3, d_4) based on the plane section assumption. The stresses at the points (f_1, f_2, f_3, f_4) are calculated by multiplying the strains by ϕE_c, where E_c is the elastic modulus of concrete, and ϕ the creep coefficient.

Depending on the orientation of the neutral axis (tanα) and the configuration of the column section (b/h), d_1 can be smaller or greater than d_4. Section analysis is performed in this paper for the case $d_1 < d_4$. However, the formulae derived can also be applied to the other case by simply switching the parameters for Point 1 and Point 4.

Typical concrete stress distributions on the three zones are illustrated in Figure 4. For each zone two stress distributions are considered, i.e. the neutral axis located within or below the zone. Concrete compressive force on each zone ($F_{cc\text{-}I}$, $F_{cc\text{-}II}$, or $F_{cc\text{-}III}$) is equivalent to the volume of the corresponding stress block, and can be calculated by integration of stresses over the zone. The force is acting at the centroid of the stress block, whose distance to the neutral axis can also be calculated by integration. Moment caused by the concrete force ($M_{cc\text{-}I}$, $M_{cc\text{-}II}$, or $M_{cc\text{-}III}$) is then obtained by multiplying the force by the distance to the neutral axis. The formulae derived for each stress distribution in the three zones are summarised in Table 1.

The total forces and moments of concrete and steel on the section are

$$F_{cc} = F_{cc-I} + F_{cc-II} + F_{cc\_III}$$
$$M_{cc} = M_{cc-I} + M_{cc-II} + M_{cc-III}$$
$$F_{st} - F_{sc} = \sum F_i$$
$$M_{st} + M_{sc} = \sum M_i$$

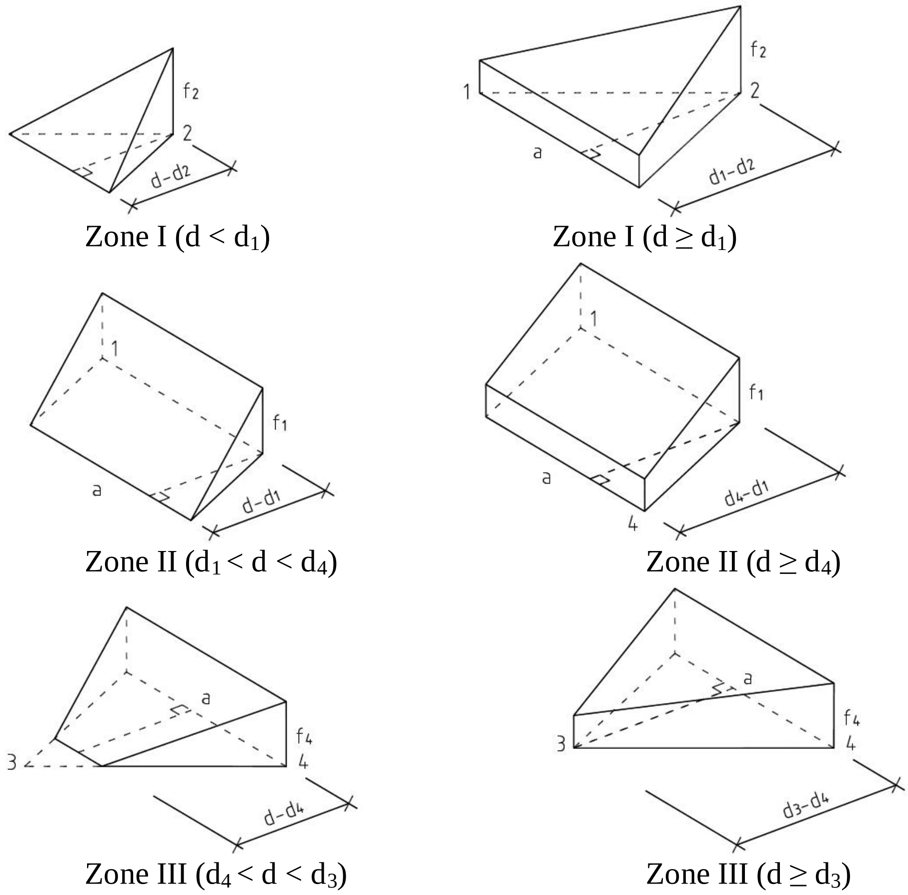


Figure 4. Stress distributions on the section zones

Table 1. Concrete forces and distances to neutral axis

Zone	Location of neutral axis	Concrete force	Distances to neutral axis
I	Within the zone ($d<d_1$)	$\dfrac{af_2d^2}{6d_1}$	$\dfrac{1}{2}d$
	Below the zone ($d\geq d_1$)	$\dfrac{1}{6}d_1a\left(f_2+2f_1\right)$	$\dfrac{d_1\left(f_2+f_1\right)}{2\left(f_2+2f_1\right)}+d-d_1$
II	Within the zone ($d_1<d<d_4$)	$\dfrac{1}{2}af_1\left(d-d_1\right)$	$\dfrac{2}{3}\left(d-d_1\right)$
	Below the zone ($d\geq d_4$)	$\dfrac{1}{2}a\left(f_1+f_4\right)\left(d_4-d_1\right)$	$d-d_1-\dfrac{f_1+2f_4}{3\left(f_1+f_4\right)}\left(d_4-d_1\right)$
III	Within the zone ($d_4<d<d_3$)	$\dfrac{1}{6}af_4\left(d-d_4\right)(3-\dfrac{d-d_4}{d_3-d_4})$	$\dfrac{4\left(d-d_4\right)\left(d_3-d_4\right)-\left(d-d_4\right)^2}{6\left(d_3-d_4\right)-2\left(d-d_4\right)}$
	Below the zone ($d\geq d_3$)	$\dfrac{1}{6}af_4\left(d_3-d_4\right)\left(3-\dfrac{d_3-d_4}{d-d_4}\right)$	$\dfrac{4\left(d-d_4\right)\left(d_3-d_4\right)-\left(d_3-d_4\right)^2}{6\left(d-d_4\right)-2\left(d_3-d_4\right)}$

The moment of the axial force N about the neutral axis is

$$M_N = N\left(y_2^{'} - d\right)$$

Substituting the forces and moments into the two equilibrium equations, the two variables, i.e. the strain at point 2 (ε_2) and the neutral axis depth (d), can be obtained by solving the equations. The strain distribution on the section can thus be determined.

Section analysis for rectangular hollow section

The rectangular hollow section is equivalent to the external solid section minus the internal solid section. The internal rectangular section is also divided into three zones (I', II' and III'), and the four corner points are noted as 1', 2', 3' and 4', as shown in Figure 3(b). Coordinates and depths of 1', 2', 3' and 4' can be established, the same as for Points 1, 2, 3 and 4. The equations summarised in Table 1 are also applicable to the internal rectangular section, with the following modifications:

i) f_i is replaced by $f_{i'}$ which can also be related with the strain at Point 2 (ε_2) and the neutral axis depth (d) based on the plane section assumption.
ii) d_i is replaced by $d'_{i'}$ which is the depth from Point 2'.
iii) a is replaced by a' which is the length of the internal zone interface.
iv) d is replaced by $d\text{-}d_{2'}$, where $d_{2'}$ is the depth from Point 2.

The forces ($F_{cc\text{-}I'}$, $F_{cc\text{-}II'}$, $F_{cc\text{-}III'}$) on the internal rectangular section zones I', II' and III' and their moments ($M_{cc\text{-}I'}$, $M_{cc\text{-}II'}$, $M_{cc\text{-}III'}$) about neutral axis can thus be obtained. The total concrete force and moment on the rectangular hollow section are:

$$F_{cc} = F_{cc-I} + F_{cc-II} + F_{cc\_III} - F_{cc-I'} - F_{cc-II'} - F_{cc-III'}$$
$$M_{cc} = M_{cc-I} + M_{cc-II} + M_{cc-III} - M_{cc-I'} - M_{cc-II'} - M_{cc-III'}$$

Calculation of steel forces and moments is the same as for solid section. By solving the force and moment equilibrium equations, the two variables (ε_2 and d) can be obtained, and the strain distribution on the section can be determined.

Crack width calculation

Clark (1983) discussed the following two equations for crack width calculation. Equation (1) was for heavily reinforced concrete members such as bridge beams and columns, and equation (2) for relatively lightly reinforced members such as slabs.

$$w = 2.3 a_{cr} \varepsilon \qquad (1)$$

$$w = \frac{3 a_{cr} \varepsilon_m}{1 + 2\frac{(a_{cr} - c)}{(h - d)}} \qquad (2)$$

where

ε = strain at where crack is considered ignoring concrete tension stiffening
ε_m = strain at where crack is considered allowing for concrete tension stiffening
a_{cr} = the distance from where crack is considered to the nearest main bar surface
c = concrete cover

Crack widths calculated from equations (1) and (2) are similar. In BS5400-4:1990 equation (2) is used for beams and columns as well as for slabs. Considering its simplicity the equation (1) was adopted in the Route 8 alternative design for biaxially bent columns. With the strain at the outmost tensile point, Point 3, obtained from the section analysis, the crack width is calculated by $w = 2.3 a_{cr} \varepsilon_3$.

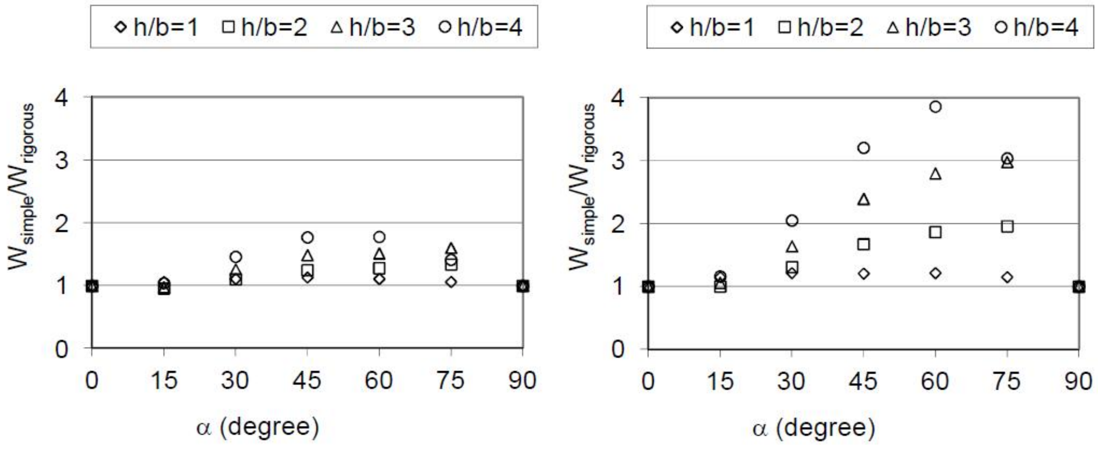
COMPARISON OF THE TWO APPROACHES

Spreadsheets of crack width calculation were developed for the uniaxial and biaxial bending columns, respectively. Analyses were performed to compare results of crack width calculated from the two approaches. Table 2 shows four solid rectangular sections considered in the analysis, together with reinforcement and assumed axial forces and bending moments.

Table 2. Column sections and load cases considered in the analysis

Section (b x h)	Reinforcement		Load Case 1		Load Case 2	
	along b	along h	N (kN)	M (kN m)	N (kN)	M (kN m)
1.2m x 1.2m	8 N32	8 N32	0	1500	5760	3000
1.2m x 2.4m	8 N32	16 N32	0	3000	5760	5000
1.2m x 3.6m	8 N32	24 N32	0	5000	5760	7000
1.2m x 4.8m	8 N32	32 N32	0	7000	5760	9000

Concrete grade was taken to be 40MPa, and concrete cover 40mm. The angle of the overall moment M to the x axis (α) was assumed to be 0, 15, 30, 45, 60, 75 and 90 degrees respectively. Ratios of the crack width calculated from the simplified and the rigorous approaches ($W_{simple}/W_{rigorous}$) are plotted in Figure 5.



(a) Load Case 1: N=0 (b) Load Case 2: N=5760 kN

Figure 5 Ratios of crack widths calculated from the simple and the rigorous approaches

As shown in Figure 5, the crack widths calculated from the simplified approach are larger than those from the rigorous approach. The difference increases with h/b, and becomes more apparent when α is between 30 and 75 degrees and axial force exists. The simplified approach was initially adopted in the Route 8 project for crack width calculation, but later abandoned due to its over conservatism. The rigorous approach proved to be cost effective for the design.

CONCLUSIONS

References of crack evaluation for columns subject to biaxial bending are not readily available. Two approaches for crack width calculation for solid or hollow rectangular columns are presented. One simplifies biaxial bending into uniaxial bending, and the other based on rigorous section analysis. The simplified approach was found to be overly conservative by comparison. The rigorous approach produced a more efficient design and was adopted in the alternative design for Route 8 - Ngong Shuen Chau Viaduct in Hong Kong.

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