Statistical assessment of soil-water characteristic curve models for geotechnical engineering

W. Scott Sillers and Delwyn G. Fredlund

Abstract: A number of empirical equations have been proposed for the soil-water characteristic curve. A nonlinear, least squares method was used to determine best-fit parameters for several empirical equations that were best-fit to 230 water content versus soil suction data sets. In addition, two proposed correction methods to accommodate high soil suctions up to 1 000 000 kPa were applied to the various soil-water characteristic curve equations. The data sets of water content versus soil suction were arranged into one of the USDA soil classifications based on their relative amounts of sand, silt, and clay (only eight soil classifications had sufficient data for later analysis). The quality of fit for each model was compared using the Akaike Information Criterion. A series of conclusions were arrived at regarding (*i*) the relationship between two- and three-parameter equations, (*ii*) the relationship between exponential and sigmoidal type equations, and (*iii*) the value of correction factors for the high soil suction range.

Key words: soil-water characteristic curve, unsaturated soil, soil suction, regression analysis, SWCC models, Akaike Information Criterion.

Résumé : Un certain nombre d'équations empiriques ont été proposées pour la courbe caractéristique sol-eau. Une méthode non linéaire des moindres carrés a été utilisée pour déterminer les paramètres de la meilleure concordance pour plusieurs équations empiriques ayant la meilleure concordance pour 230 ensembles de teneur en eau en fonction de la succion du sol. De plus, deux méthodes de correction proposées pour traiter de fortes succions atteignant 1 000 000 kPa ont été appliquées à diverses équations de courbes caractéristiques sol-eau. Les ensembles de données de teneur en eau versus succion du sol ont été arrangés dans une des classifications de sols du USDA basée sur les quantités relatives de sable, silt et argile (seulement huit classifications de sols avaient suffisamment de données pour des analyses subséquentes). La qualité du lissage pour chaque modèle a été comparée en utilisant le critère d'information de Akaike. On en est arrivé à une série de conclusions concernant: (*i*) la relation entre les équations à 2 et 3 paramètres, (*ii*) la relation entre les équations de types exponentiel et sigmoïdal, et (*iii*) la valeur des facteurs de correction pour la plage des fortes succions du sol.

Mots clés : courbe caractéristique sol-eau, sol non saturé, succion du sol, analyse de régression, modèles SWCC, critère d'information de Akaike.

[Traduit par la Rédaction]

Introduction

Geotechnical engineers have become increasingly aware of problems unique to unsaturated soils and the possibility of using engineering analyses to provide solutions. Computer models are now routinely used to predict the movement of water and chemicals through unsaturated soils. Such models can most successfully be used when there is a mathematical equation to represent the soil-water characteristic curve and the unsaturated soil properties. The soil-water characteristic curve (i.e., the (gravimetric or volumetric) water content versus soil suction relationship) provides key information for the estimation of unsaturated soil property functions (Fredlund 1995).

The soil-water characteristic curve can be viewed as the continuous sigmoidal function describing the water storage capacity of a soil as it is subjected to various soil suctions. The soil-water characteristic curve contains important information regarding the amount of water contained in the pores at any soil suction and the pore size distribution related to the stress state in the soil-water.

The relationship of the soil-water characteristic curve to unsaturated soil behavior has been well established by numerous researchers (Fredlund 1996). The soil-water characteristic curve and the unsaturated coefficient of permeability functions are required when solving transient water and solute transport problems associated with the vadose zone. Many other soil properties such as shear strength, chemical diffusivity, chemical adsorption, water volume storage, volumetric water content, specific heat, thermal conductivity, and volume change can also be related to the soil-water characteristic curve.

Received October 31, 2000. Accepted April 16, 2001. Published on the NRC Research Press Web site at http://cgj.nrc.ca on January 10, 2002.

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 DOI: 10.1139/cgj-38-6-1297 © 2001 NRC Canada

Table 1. Soil-water characteristic models used in this study.

Model	Equation	
Brooks and Corey (1964)	$\Theta_d = 1$	$\psi < aev$
	$\Theta_d = (\psi / a_c)^{-n_c}$	$\psi > aev$
Brutsaert (1966)	$\Theta_d = 1 / (1 + (\psi / a_r)^{n_r})$	
McKee and Bumb 1984 (a Boltzman exponential form)	$\Theta_d = 1$	$\psi < aev$
	$\Theta_d = \exp((a_z - \psi) / n_z)$	$\psi > aev$
McKee and Bumb 1987 (Fermi)	$\Theta_d = 1 / (1 + \exp((\psi - a_e) / n_e))$	
Fredlund and Xing (1994)	$\Theta_d = (1 / \ln(e + (\psi / a_f)^{n_f}))^{m_f}$	
Gardner (1956)	$\Theta_d = 1 / (1 + a_g \psi^{n_g})$	
van Genuchten (1980)	$\Theta_d = (1 / (1 + (a_v \psi)^{n_v}))^{m_v}$	
van Genuchten (1980) – Burdine (1953)	$\Theta_d = (1 / (1 + (a_b \psi)^{n_b}))^{(1 - 2 / n_b)}$	
van Genuchten (1980) – Mualem (1976)	$\Theta_d = (1 / (1 + (a_m \psi)^{n_m}))^{(1 - 1 / n_m)}$	
Normalized water content form	$\theta_n = \theta_r + (1 - \theta_r)(\theta_f(\psi))$	
Fayer and Simmons (1995) correction	$\theta = \theta_a (1 - \ln(\psi) / \ln(1\,000\,000)) + (\theta_s - \theta_a (1 - \ln(\psi) / \ln(1\,000\,000)))(\theta_f(\psi))$	
Fredlund and Xing (1994) correction	$\theta = (1 - \ln(1 + \psi / \psi_r) / (1 + 1\,000\,000/\psi_r))(\theta_f(\psi))$	

Note: Definition of variables: θ is any volumetric water content; θ_s is the saturated volumetric water content; θ_r is the residual volumetric water content; Θ_d is the dimensionless water content (θ / θ_s); Θ_n is the normalized water content, $(\theta - \theta_r) / (\theta_s - \theta_r)$; ψ is the soil suction; aev is the air-entry value; a is the first soil fitting parameter; n is the second soil fitting parameter; m is the third soil fitting parameter; and $\theta_f(\psi)$ is the soil-water characteristic function.

Nonparametric and parametric methods for representing the soil-water characteristic curve

Nonparametric methods of representing the soil-water characteristic curve do not produce fitted parameters. The most basic nonparametric method of estimating the soil-water characteristic curve is to manually draw a series of straight lines between experimental points or a continuous curve through the data points. These procedures are inaccurate and cumbersome to use for numerical modeling purposes.

Another nonparametric method is the use of a spline function (Eubank 1988). There are a number of difficulties associated with the use of spline functions for modeling soil-water characteristic data.

(1) The first derivative of the spline function can often produce meaningless and erroneous results.

(2) Too much information is required for the spline function, and it is difficult to use the spline function data in the derivation of other soil properties.

(3) Experimental errors can cause the points to deviate from the true curve, and splines can become quite inaccurate.

A parametric model is a mathematical function that is fitted to experimental water content versus soil suction data points. The use of parametric models has several advantages.

(1) Parametric models inherently produce smooth curves from "noisy" soil-water characteristic curve data.

(2) Parametric models are relatively easy to use in the derivation of other soil properties.

(3) Parametric model parameters provide an efficient means of handling experimental data, which is particularly important to the computer modeling of large and complex problems.

(4) The model parameters give a quantitative description of the shape of the soil-water characteristic curve and can be correlated to soil classification properties.

Parametric methods appear to have a distinct advantage over nonparametric methods in representing soil-water characteristic curve information.

Objectives of the study

The objectives of this study are as follows:

(1) To determine the relative suitability of various mathematical equations previously proposed for the soil-water characteristic curve, particularly as the equations pertain to geotechnical engineering.

(2) To study the influence of two methods proposed for correcting the shape of the mathematical models to ensure that the soil-water characteristic curve approaches 1 000 000 kPa at zero water content. The mathematical correction procedures are applied to all of the proposed soil-water characteristic curve models.

Optimization of soil-water characteristic curve data

Optimization techniques are used to obtain the best-fit parameters for soil-water characteristic curve data sets. The curve fitting routine determines model parameters such that the mathematical function passes as close as possible to the experimental data points without necessarily going through any of the points.

Most automated optimization schemes are based on a least squares method (Sadler 1975). Details concerning the nonlinear procedure used for the best-fit analysis of soil-

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Fig. 1. The van Genuchten (1980) – Mualem (1976) equation fit to a sandy loam soil, with and without a correction factor applied in the high soil suction range.

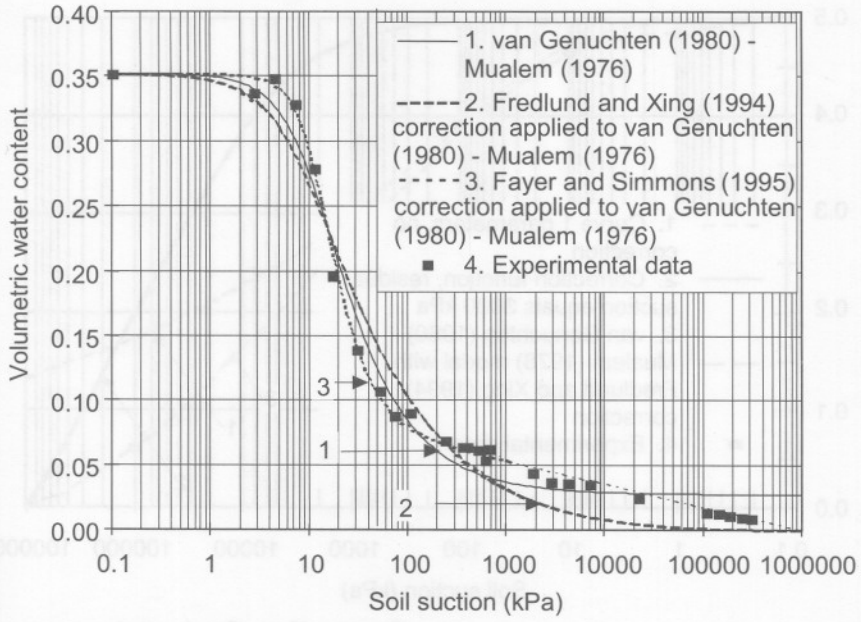
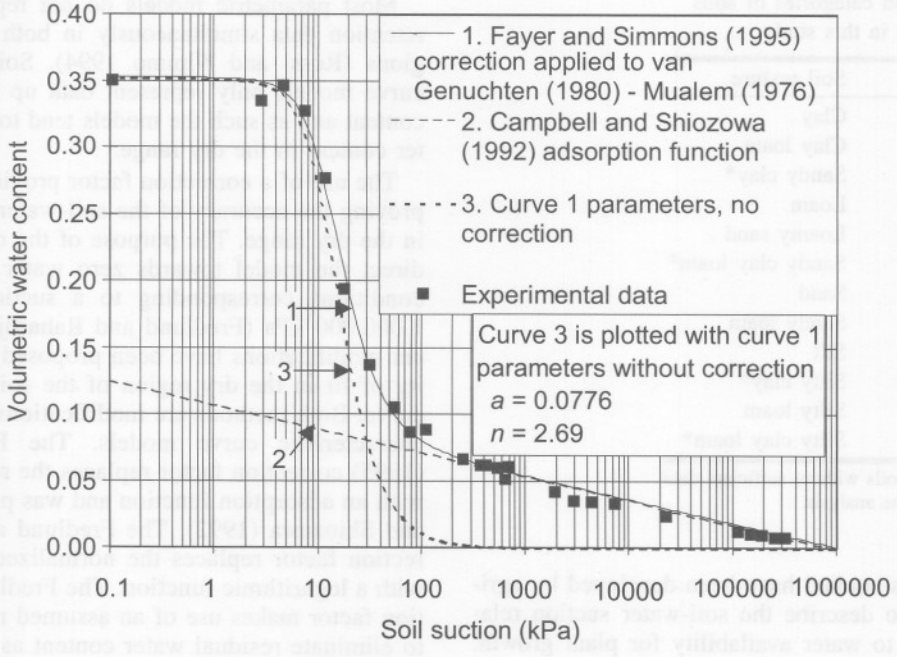


Fig. 2. Comparison of the fit for the sandy loam soil, with and without the Fayer and Simons (1995) correction applied to the van Genuchten (1980) – Mualem (1976) equation.



water characteristic curve data can be found in Fredlund and Xing (1994).

Micromechanical capillary models studied to represent the soil-water characteristic curve

The success of various soil-water characteristic curve models is related to the flexibility of the pore radius density distribution or to the pore size function upon which the model is based (Sillers 1996). The soil-water characteristic models examined in this study are parametric models based upon a pore size function and the capillary theory. The equations representing each model along with definitions of the variables used are given in Table 1. In general, the term *dimensionless water content,* Θ_d, will be used to represent the equations associated with the soil-water characteristic curve models.

$$[1] \qquad \Theta_d = (\theta / \theta_s)$$

where θ is the volumetric water content, and θ_s is the saturated volumetric water content.

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Fig. 3. Comparison of the fit for a clayey soil, with and without the Fredlund and Xing (1994) correction factor applied to the van Genuchten (1980) – Mualem (1976) equation.

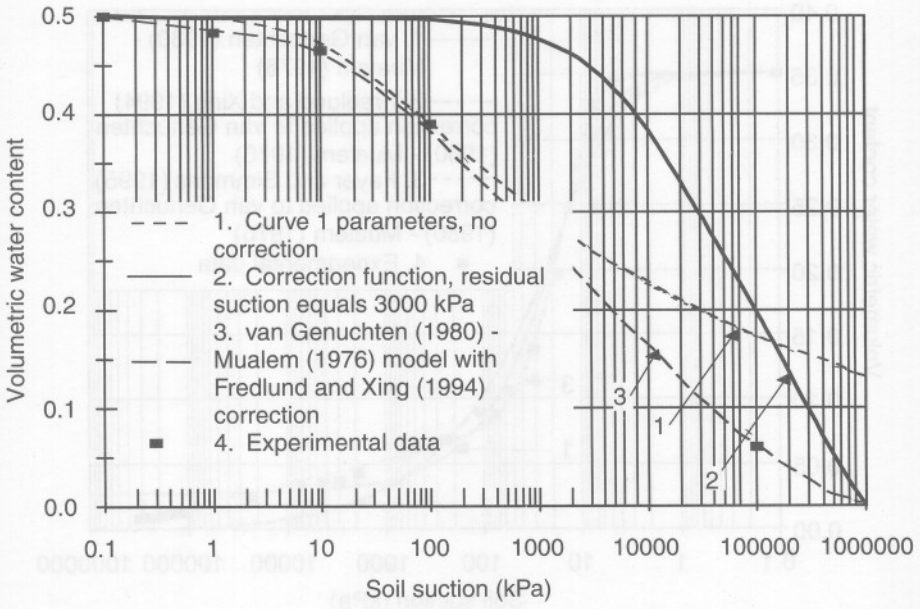

Table 2. Textures classification categories of soils used in this study.

No.	Soil texture
1	Clay
2	Clay loam
3	Sandy clay*
4	Loam
5	Loamy sand
6	Sandy clay loam*
7	Sand
8	Sandy loam
9	Silt
10	Silty clay*
11	Silty loam
12	Silty clay loam*

*Soils without sufficient data for the analysis.

Most of the models studied have been developed by agricultural researchers to describe the soil-water suction relationship as it relates to water availability for plant growth. The most common models in the agronomy literature appear to be those by Brooks and Corey (1964), Gardner (1956), van Genuchten (1980), and van Genuchten (1980) – Mualem (1976). There are existing databases of typical fitted parameters for these equations.

The Boltzman (McKee and Bumb 1984) and Fermi (McKee and Bumb 1987) models are less commonly used. The Brutsaert (1966) model was included since its form was similar to those of van Genuchten (1980) and Gardner (1956), and invites comparison. The Fredlund and Xing (1994) model was included due to its flexibility over a wide range of soil suctions and its physically meaningful soil parameters.

Correction factors to accommodate an extended soil-water characteristic curve

Most parametric models do not represent the soil-water retention data simultaneously in both the wet and dry regions (Rossi and Nimmo 1994). Soil-water characteristic curve models only represent data up to the residual water content and as such the models tend to overestimate the water content in the dry range.

The use of a correction factor provides one means of improving the accuracy of the soil-water characteristic model in the dry range. The purpose of the correction factor is to direct the model towards zero water content at oven dry conditions corresponding to a suction of approximately 1 000 000 kPa (Fredlund and Rahardjo 1993). Two different modifications have been proposed to improve the quality of fit in the dry region of the soil-water characteristic curve. Both methods are modifications of known soil-water characteristic curve models. The Fayer and Simmons (1995) correction factor replaces the residual water content with an adsorption function and was proposed by Campbell and Shiozawa (1992). The Fredlund and Xing (1994) correction factor replaces the normalized water content form with a logarithmic function. The Fredlund and Xing correction factor makes use of an assumed residual suction value to eliminate residual water content as a fitting parameter.

Figure 1 shows a data set for a sandy loam soil that is best fit with the van Genuchten (1980) – Mualem (1976) equation. The Fredlund and Xing (1994) correction and the Fayer and Simmons (1995) correction are then applied to improve the fit in the high soil suction range. The mathematical models with no correction factor show the suction extending to infinity at constant water content. The equations with a correction factor applied reach zero water content at a soil suction of 1 000 000 kPa. In order to test the accuracy of the proposed models, water content versus soil suction data is required within the zone of residual saturation, preferably extending to near oven-dry conditions.

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Fig. 4. Comparison of one set of soil-water characteristic curve data fitted using various models for a silty loam soil.

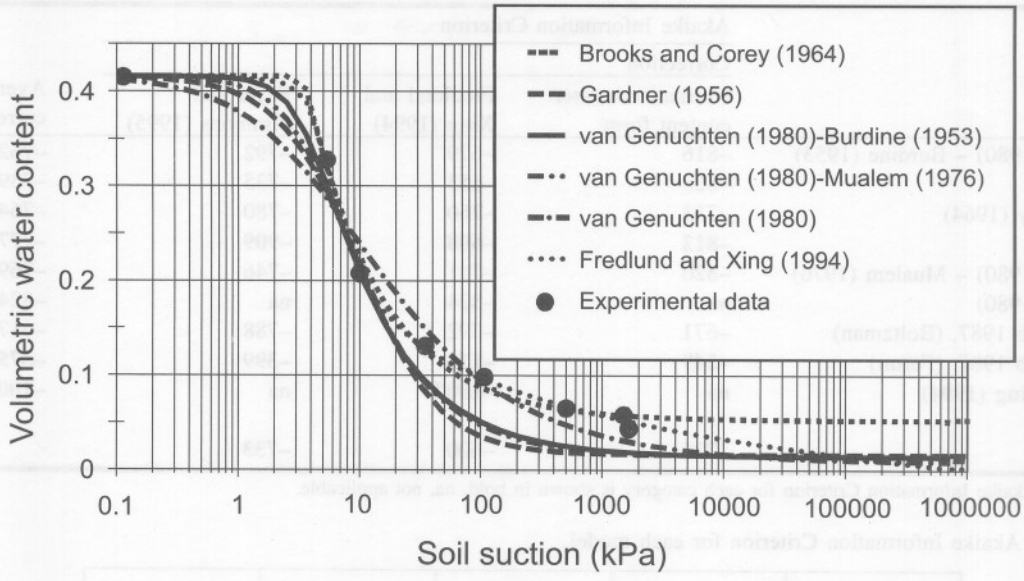

Fig. 5. Comparison of one set of soil-water characteristic curve data fitted using various models for a sandy loam soil.

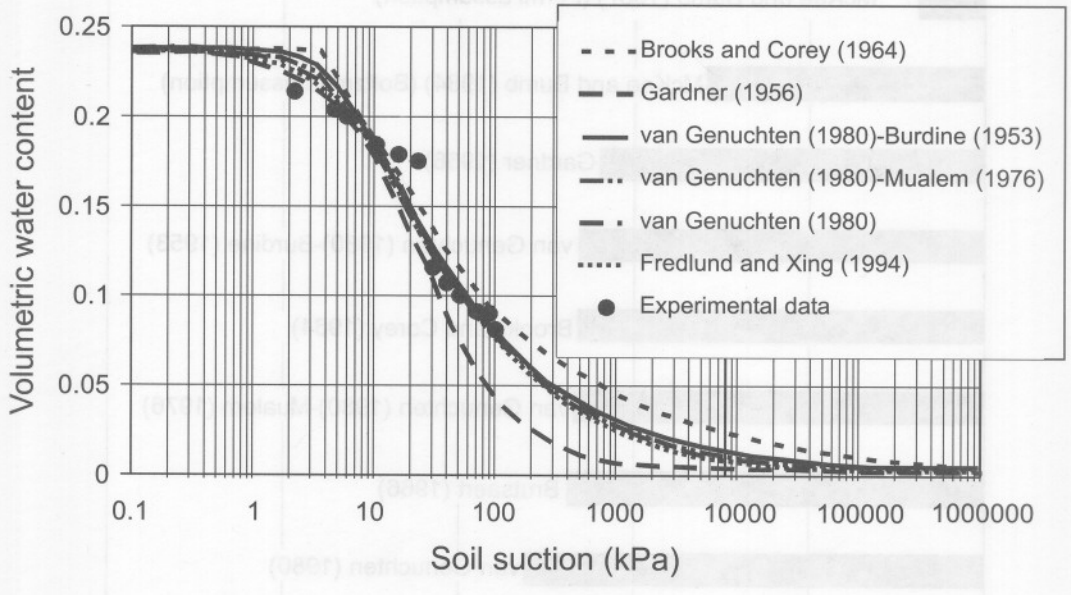

Table 3. Models and correction factors used and the number of fitting parameters.

Soil-water characteristic model	Number of fitting parameters* without correction factor	Number of fitting parameters* with correction factor		
		Normalized water content form	Fredlund and Xing (1994)	Fayer and Simmons (1995)
van Genuchten (1980) – Burdine (1953)	2	3	2	3
Gardner (1956)	2	3	2	3
Brooks and Corey (1964)	2	3	2	3
Brutsaert (1966)	2	3	2	3
van Genuchten (1980) – Mualem (1976)	2	3	2	3
van Genuchten (1980)	3	na	3	na
McKee and Bumb 1987, (Boltzman)	2	3	2	3
McKee and Bumb 1987, (Fermi)	2	3	2	3
Fredlund and Xing (1994)	3	na	3	na

Note: na, not applicable.
*The saturated water content, θ_s, is fixed as the water content at zero suction and not assumed to be a variable.

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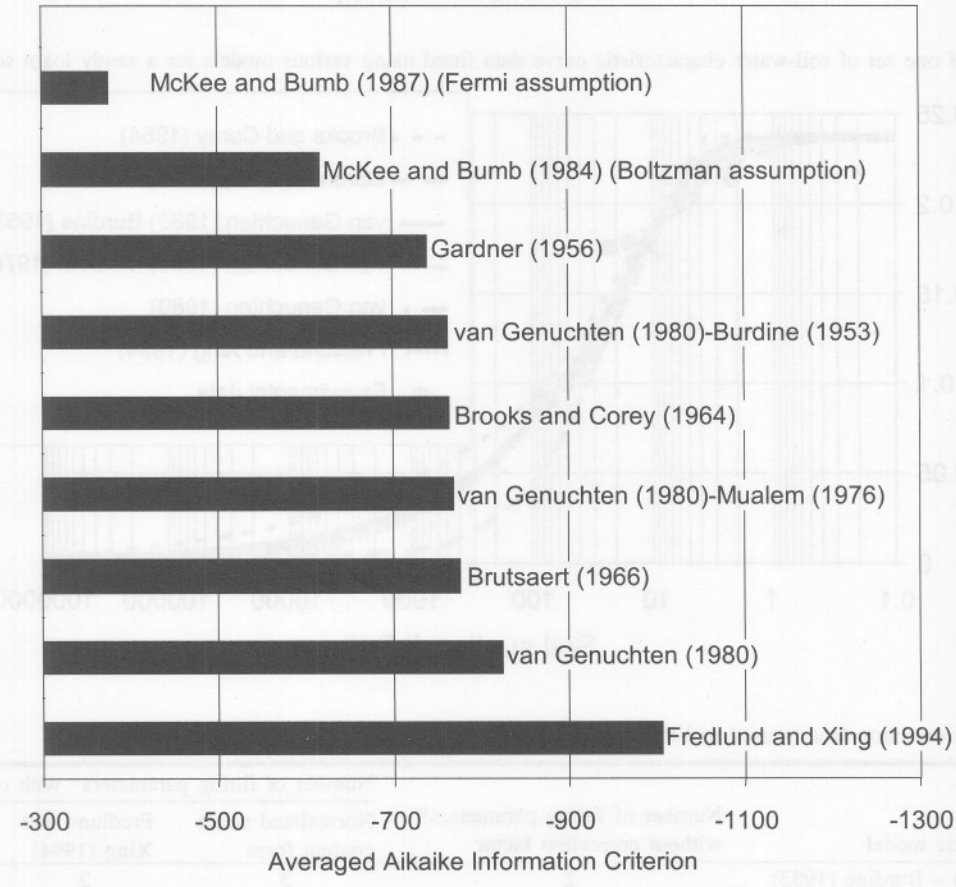
Table 4. Avarage Akaike Information Criterion for each model using all soil-water characteristic data sets.

	Akaike Information Criterion			
	Correction			
Model	Normalized water content form	Fredlund and Xing (1994)	Fayer and Simmons (1995)	Average of the corrected functions
van Genuchten (1980) – Burdine (1953)	–816	–679	–792	–762
Gardner (1956)	–828	–653	–735	–739
Brooks and Corey (1964)	–753	–760	–780	–764
Brutsaert (1966)	**–812**	**–608**	**–909**	**–777**
van Genuchten (1980) – Mualem (1976)	–820	–741	–746	–769
van Genuchten (1980)	na	–824	na	–824
McKee and Bumb 1987, (Boltzman)	–671	–392	–788	–617
McKee and Bumb 1987, (Fermi)	**–549**	**–178**	**–399**	**–375**
Fredlund and Xing (1994)	na	**–1007**	na	**–1007**
Average	–726	–600	**–733**	

Note: The best Akaike Information Criterion for each category is shown in bold. na, not applicable.

Fig. 6. Averaged Akaike Information Criterion for each model.



Fayer and Simmons (1995) correction

The Fayer and Simmons (1995) correction factor replaces the term for residual water content with an adsorption equation based on the adsorption of water on to soil particles (Campbell and Shiozawa 1992). The adsorption equation is a linear relationship between the water content and the logarithm of the soil suction in the dry zone of residual saturation.

$$[2] \quad \theta = \left[1 - \frac{\ln(\psi)}{\ln(\psi_0)}\right]\theta_a$$

where θ_a is the water content at 1 kPa suction (assumed to be equal to 6.3 times the air dry water content by Campbell et al. 1993), ψ is the soil suction, and ψ_0 is the soil suction at oven-dry conditions (can be assumed to be 1 000 000 kPa (Croney and Coleman 1961; Ross et al. 1991)).

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Table 5. Statistics on fitting parameters for three different forms of the van Genuchten (1980) – Burdine (1953) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	0.599	2.245	0.728	2.123	0.458	2.727
size = 12	Std dev	0.983	0.128	0.703	0.114	0.977	0.862
	Median	0.055	2.241	0.558	2.090	0.032	2.481
Clay loam	Mean	0.342	2.244	0.800	2.086	0.506	14.24
size = 24	Std dev	0.956	0.125	1.515	0.056	1.026	23.80
	Median	0.01	2.233	0.386	2.088	0.033	2.558
Loam	Mean	0.285	2.489	0.481	2.261	0.26	2.913
size = 18	Std dev	0.216	0.377	0.355	0.123	0.18	0.601
	Median	0.23	2.381	0.499	2.234	0.234	2.790
Loamy sand	Mean	0.656	2.943	0.878	2.574	0.443	4.398
size = 12	Std dev	0.999	0.914	1.516	0.288	0.371	3.576
	Median	0.363	2.627	0.391	2.468	0.356	3.106
Sand	Mean	0.541	5.084	0.579	4.174	0.534	6.120
size = 50	Std dev	0.855	2.089	0.854	1.581	0.872	4.021
	Median	5.196	0.715	0.384	3.607	0.325	5.283
Silt	Mean	0.353	3.637	0.465	2.533	0.390	3.018
size = 25	Std dev	1.375	4.623	1.519	0.768	1.540	1.031
	Median	0.037	2.414	0.079	2.300	0.040	2.579
Silty loam	Mean	0.403	2.464	0.609	2.202	0.484	5.005
size = 23	Std dev	0.572	0.374	0.642	0.114	0.813	10.40
	Median	0.199	2.312	0.443	2.225	0.177	2.550
Sandy loam	Mean	0.420	2.740	0.610	2.349	0.555	4.510
size = 36	Std dev	0.814	0.924	0.863	0.312	1.492	8.074
	Median	0.203	2.412	0.359	2.267	0.213	2.788

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

Figure 2 shows the van Genuchten (1980) – Mualem (1976) model for a sandy loam soil with and without the Fayer and Simmons (1995) correction factor. The Campbell and Shiozawa (1992) adsorption function is also shown in Fig. 2. The plot illustrates how the adsorption function controls the model in the dry region but does not affect the model in the wet region of the curve. The Fayer and Simmons correction factor can be applied to any soil-water characteristic curve equation, and takes the following general form.

$$[3] \qquad \theta = \left[1 - \frac{\ln\psi}{\ln\psi_0}\right]\theta_a + \left\{\theta_s - \left[1 - \frac{\ln\psi}{\ln\psi_0}\right]\theta_a\right\}(\theta_f(\psi))$$

where $\theta_f(\psi)$ is the soil-water characteristic curve function.

Equation [3] can be written in the normalized water content form such that the equation for the soil-water characteristic curve applies between the saturated water content and the residual water content.

$$[4] \qquad \theta = \theta_r + [\theta_s - \theta_r]\,(\theta_f(\psi))$$

where θ_r is the residual water content.

Curve 1 in Fig. 2 shows the volumetric water content, θ, represented by the following equation:

$$[5] \qquad \theta = \theta_s\left[\left[1 - \frac{\ln\psi}{\ln\psi_0}\right]\frac{\theta_a}{\theta_s} + \left[1 - \left[1 - \frac{\ln\psi}{\ln\psi_0}\right]\frac{\theta_a}{\theta_s}\right]\frac{1}{(1 + (a\psi^n))^{1-1/n}}\right]$$

where a is the soil parameter inversely related to the air-entry value, and n is the soil parameter related to the pore size distribution.

The second curve in Fig. 2 represents the Campbell and Shiozawa (1992) adsorption function, given by eq. [2], with $\theta_a = 0.107$. Curve 3 in Fig. 2 represents the van Genuchten (1980) – Mualem (1976) equation without the correction factor and can be written with $m = (1 - 1/n_m)$.

$$[6] \qquad \theta = \frac{\theta_s}{(1 + (a_m\psi^{n_m}))^m}$$

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Table 6. Statistics on fitting parameters for three different forms of the Gardner (1956) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	0.056	0.79	0.042	0.571	0.081	0.756
size = 12	Std dev	0.098	0.266	0.068	0.304	0.107	0.627
	Median	0.008	0.738	0.014	0.477	0.028	0.606
Clay loam	Mean	0.061	0.665	0.054	0.449	0.146	0.728
size = 24	Std dev	0.143	0.164	0.078	0.153	0.407	0.28
	Median	0.011	0.62	0.023	0.464	0.027	0.649
Loam	Mean	0.053	1.201	0.089	0.676	0.054	1.286
size = 18	Std dev	0.047	0.572	0.068	0.246	0.042	0.578
	Median	0.044	0.977	0.052	0.618	0.040	1.195
Loamy sand	Mean	0.102	1.885	0.119	1.199	0.116	1.736
size = 12	Std dev	0.151	1.032	0.139	0.582	0.178	0.781
	Median	0.06	1.703	0.069	1.078	0.063	1.591
Sand	Mean	0.721	3.892	0.744	2.921	1.056	3.025
size = 50	Std dev	2.987	1.836	2.972	1.626	4.006	1.419
	Median	0.013	3.732	0.056	2.908	0.048	2.642
Silt	Mean	0.022	1.347	0.027	1.063	0.027	1.079
size = 25	Std dev	0.05	0.772	0.054	0.682	0.051	0.548
	Median	0.001	1.152	0.009	0.932	0.007	0.973
Silty loam	Mean	0.057	1.085	0.078	0.663	0.072	1.011
size = 23	Std dev	0.053	0.552	0.057	0.360	0.073	0.539
	Median	0.056	0.906	0.078	0.585	0.040	0.878
Sandy loam	Mean	0.045	1.357	0.081	0.879	0.067	1.289
size = 36	Std dev	0.048	0.624	0.072	0.498	0.069	0.566
	Median	0.033	1.262	0.059	0.827	0.037	1.127

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

The Fayer and Simmons (1995) correction factor improves the fit of the data in the dry region, going towards zero water content.

Fredlund and Xing (1994) correction factor

The Fredlund and Xing correction factor eliminates the need for the residual water content as a fitting parameter and reduces the complexity of the model. The correction factor is somewhat dependent on the value selected for the residual suction ψ_r, but in general, it is possible to use a value of 3000 kPa as an estimate for most soils.

The Fredlund and Xing correction is based on mathematical considerations and is a logarithmic equation that is unity in the wet region and goes to zero water content at 1 000 000 kPa. The Fredlund and Xing correction factor equation can be written as follows:

$$[7] \quad C(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{\psi_0}{\psi_r}\right)}\right]$$

where ψ_0 is the soil suction at zero water content (e.g., 1 000 000 kPa).

The correction factor equation was presented as part of the Fredlund and Xing equation for the soil-water characteristic curve. However, it is also possible to use the Fredlund and Xing correction factor on other equations proposed for the soil-water characteristic curve.

$$[8] \quad \theta = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{\psi_0}{\psi_r}\right)}\right](\theta_f(\psi))$$

Figure 3 illustrates the effect of the Fredlund and Xing correction factor on a clay soil (Feddes et al. 1974). Curve 1 shows the van Genuchten (1980) – Mualem (1976) equation fit to the data set without any correction applied. Curve 3 has the Fredlund and Xing correction factor applied to the van Genuchten – Mualem) equation for the soil-water characteristic curve.

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Table 7. Statistics on fitting parameters for three different forms of the Brooks and Corey (1964) model for eight different soils.

Soil	Statistic	Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
		a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	325.1	0.207	66.62	0.127	114.9	0.260
size = 12	Std dev	744.3	0.108	139.6	0.095	198.8	0.148
	Median	43.30	0.197	6.342	0.104	36.19	0.258
Clay loam	Mean	507.7	0.219	455.3	0.111	376.7	0.382
size = 24	Std dev	2024	0.071	2033	0.04	1743	0.187
	Median	72.68	0.208	28.96	0.111	4.841	0.45
Loam	Mean	10.74	0.35	9.119	0.25	4.346	0.454
size = 18	Std dev	24.17	0.177	23.3	0.114	4.217	0.287
	Median	3.218	0.336	2.478	0.234	2.566	0.338
Loamy sand	Mean	3.886	0.657	3.547	0.470	4.230	0.789
size = 12	Std dev	6.522	0.516	6.105	0.220	6.493	0.610
	Median	1.912	0.516	1.911	0.401	2.065	0.748
Sand	Mean	5.523	1.601	5.313	1.345	5.574	1.747
size = 50	Std dev	12.16	0.917	11.920	0.847	12.14	0.957
	Median	2.109	1.358	1.962	1.112	2.230	1.584
Silt	Mean	33.62	0.749	29.95	0.432	16.92	0.843
size = 25	Std dev	52.71	1.406	46.25	0.480	13.38	1.437
	Median	18.43	0.306	17.47	0.268	16.44	0.394
Silty loam	Mean	23.16	0.402	14.77	0.242	21.37	0.555
size = 23	Std dev	66.75	0.326	37.78	0.14	63.9	0.535
	Median	4.174	0.296	3.634	0.244	4.165	0.325
Sandy loam	Mean	6.245	0.59	5.031	0.336	6.107	0.75
size = 36	Std dev	10.47	0.742	9.255	0.267	10.45	1.168
	Median	3.370	0.357	2.494	0.276	3.005	0.324

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

$$[9] \quad \theta = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{\psi_0}{\psi_r}\right)}\right] \frac{\theta_s}{(1 + (a_m \psi^{n_m}))^m}$$

Curve 2 in Fig. 3 shows the form of the Fredlund and Xing correction factor equation. The van Genuchten – Mualem model, without the Fredlund and Xing correction applied, tends to overestimate the water contents in the dry region but does not significantly affect the fit in the wet region. Combining the soil-water characteristic curve model with the correction factor equation significantly improves the fit over the entire soil suction range.

Combinations of soil-water characteristic curve models and correction factors used in the statistical study

The various soil-water characteristic curve models are treated as being independent from the proposed correction factors in this study. Therefore, it was possible to combine the soil-water characteristic curve models and the correction factor models in a variety of ways. For example, the van Genuchten (1980) three-parameter, soil-water characteristic curve model can readily be modified using the Fredlund and Xing (1994) correction or the Fayer and Simmons (1995) correction. It is also possible to apply the correction factors to the Boltzman model (McKee and Bumb 1984), the Brutsaert (1966) model, the Brooks and Corey (1964) model, the Fermi model from McKee and Bumb (1987), the Gardner (1956) model, the van Genuchten (1980) – Burdine (1953) model, and the van Genuchten (1980) – Mualem (1976) model. The soil-water characteristic curve models can also be written in the normalized water content form (Brooks and Corey 1964).

Normalized water content form limits the range of the model between the saturated water content and the residual water content. The normalized water content form, θ_n, applied to the Brooks and Corey (1964) model, as an example, can be written as follows:

$$[10] \quad \theta_n = \theta_r + (\theta_s - \theta_r)\left(\frac{\psi}{a_c}\right)^{-n_c}$$

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Table 8. Statistics on fitting parameters for three different forms of the Brutsaert (1966) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	2719	8.408	17 982	11.25	1579	1.719
size = 12	Std dev	3702	26.22	36 446	27.34	3126	2.740
	Median	849.1	0.939	992.1	0.794	216.6	0.666
Clay loam	Mean	2510	0.695	7 697	1.234	634.7	0.849
size = 24	Std dev	2441	0.174	20 622	2.630	2036	0.573
	Median	2434	0.688	2 499	0.625	82.62	0.733
Loam	Mean	155.5	1.198	382.7	0.707	24.89	1.705
size = 18	Std dev	364.9	0.541	789.1	0.274	36.46	0.994
	Median	15.26	1.011	50.19	0.618	7.580	1.562
Loamy sand	Mean	11.86	1.946	382.7	0.707	10.29	2.210
size = 12	Std dev	13.80	1.167	789.1	0.274	12.58	1.579
	Median	5.891	1.739	50.19	0.618	5.373	1.741
Sand	Mean	15.11	4.346	22.64	3.131	13.93	4.689
size = 50	Std dev	54.08	2.294	73.18	1.842	53.62	2.276
	Median	3.710	4.064	4.339	2.907	3.634	4.655
Silt	Mean	544	2.698	5 485	3.182	83.48	2.545
size = 25	Std dev	901.8	4.978	12 645	7.29	147.3	5.197
	Median	75.20	1.176	208.7	0.985	40.51	1.257
Silty loam	Mean	272.7	1.122	637.0	0.700	38.67	1.398
size = 23	Std dev	642.8	0.604	1 010	0.413	46.63	0.973
	Median	52.16	0.906	87.36	0.596	19.21	1.019
Sandy loam	Mean	115.7	1.476	155.5	0.888	55.76	1.859
size = 36	Std dev	491.4	1.002	442.1	0.494	198.7	1.529
	Median	18.63	1.319	40.45	0.827	14.62	1.495

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

Sources of information for the soil-water characteristic curve

A sizeable database of soil-water characteristic curve data is required in order for meaningful correlations to be drawn between the fitted model parameters and the soil properties. Journals were the main source for soil-water characteristic curve data, because of their general availability and reliability. All of the data in the journals was assumed to be correct, regardless of the method used to measure the soil-water characteristic curve. The publications that contained the most soil-water characteristic curve data were the Soil Science Society of America Journal and the Water Resources Research Journal. Other journals that provided information were: the Journal of Hydrology, the Soil Science Journal, the Canadian Geotechnical Journal, and the Transactions of the American Society of Agricultural Engineering Journal. The Proceedings of the International Workshop on Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils (1992) also provided information on the soil-water characteristic curve, as well as the Proceedings of the First International Conference on Unsaturated Soils (1995).

Criteria for inclusion in the database

Three criteria had to be met in order for soil-water characteristic data to be included in the database. The first criterion was that a substantial portion of the soil-water characteristic curve must be present. The curve must be experimentally defined beyond the air-entry value and over much of the unsaturated region. The second criterion was that it must be possible to reduce the experimental laboratory data to a tabular form. Any curve that was represented by fitted parameters or a curve without experimental points was excluded. The third criterion was that adequate information describing the soil must be provided within the text. Unfortunately, the laboratory method used to measure the soil-water characteristic curve was rarely mentioned. All soil-water characteristics curve information was recorded within the database as volumetric water content and soil suction in kPa.

Over 200 sets of water content versus soil suction data were collected. Each fitted model was plotted over the soil suction range from 0.1 to 1 000 000 kPa. Twelve different textures of soils, as indicated by the USDA soil classification system, were used in this study (Table 2).

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Table 9. Statistics on fitting parameters for three different forms of the van Genuchten (1980) – Mualem (1976) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	59.22	1.233	0.778	1.176	2.67	1.927
size = 12	Std dev	203.1	0.139	1.455	0.144	7.577	2.201
	Median	0.093	1.251	0.079	1.141	0.139	1.343
Clay loam	Mean	0.218	1.274	0.401	1.128	0.319	1.469
size = 24	Std dev	0.604	0.147	1.247	0.060	0.663	0.243
	Median	0.011	1.265	0.024	1.130	0.130	1.472
Loam	Mean	0.200	1.604	0.317	1.312	0.301	1.496
size = 18	Std dev	0.149	0.471	0.290	0.152	0.253	0.484
	Median	0.162	1.449	0.213	1.250	0.207	1.396
Loamy sand	Mean	0.412	2.196	0.600	1.721	0.488	2.100
size = 12	Std dev	0.482	1.071	0.935	0.411	0.507	0.806
	Median	0.295	1.728	0.289	1.584	0.302	1.812
Sand	Mean	0.474	4.6	0.494	3.545	0.482	3.924
size = 50	Std dev	0.702	2.293	0.704	1.744	0.705	1.943
	Median	0.32	4.333	0.33	3.158	0.33	3.493
Silt	Mean	0.312	1.872	0.337	1.722	2.059	1.834
size = 25	Std dev	1.304	0.961	1.375	0.918	9.943	0.695
	Median	0.032	1.463	0.034	1.358	0.043	1.695
Silty loam	Mean	0.289	1.55	0.401	1.329	0.356	1.455
size = 23	Std dev	0.404	0.492	0.528	0.299	0.466	0.596
	Median	0.141	1.354	0.183	1.274	0.146	1.382
Sandy loam	Mean	0.305	1.853	0.412	1.471	0.340	1.933
size = 36	Std dev	0.486	1.015	0.57	0.403	0.509	1.481
	Median	0.156	1.483	0.23	1.377	0.208	1.453

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

Designations and combinations of models used in this study

Three designations of the soil-water characteristic curve models were used in this study. First, a normalized water content form was used. In this case, the predicted water content becomes a constant at residual conditions, and soil suction is allowed to increase to infinity. Second, the Fayer and Simmons (1995) correction was applied to the soil-water characteristic curve data sets. Third, the Fredlund and Xing (1994) correction was applied to the various soil-water characteristic curve data sets. Figure 4 shows a typical comparison of soil-water characteristic curves fitted using various models for a silty loam soil. Figure 5 shows a typical comparison of soil-water characteristic curves fitted using various models for a sandy loam soil.

Method of evaluating each model performance

The Akaike Information Criterion (Akaike 1971, 1972, 1974) was selected for the evaluation of the performance of each model. The Akaike Information Criterion takes into account the number of fitting parameters of the model, the residual sum of squares, and the number of experimental data points. A low or negative Akaike Information Criterion indicates a better fit of the model to the data. The selection of a model should not be based solely on the goodness of fit. The error estimation technique must compare the error associated with different models having a different number of fitting parameters, on an unbiased scale.

The curve fitting routine designed by Fredlund and Xing (1994) was modified and updated by Fredlund (1996) and used to fit parameters for 23 possible combinations (or forms) of models.

Presentation and analysis of fitted model parameters results

Each model analyzed is limited to either two or three fitting parameters. The correction factors used in conjunction with the soil-water characteristic model add either one or no additional fitting parameters. Table 3 shows the combinations of models and correction factors and the number of fit-

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Table 10. Statistics on fitting parameters for the van Genuchten (1980) model for eight different soils.

		Fredlund and Xing (1994) correction*		
Soil	Statistic	a (kPa)	n	m
Clay	Mean	0.968	9.883	0.902
size = 12	Std dev	1.973	27.43	2.511
	Median	0.070	1.450	0.043
Clay loam	Mean	0.700	3.554	0.092
size = 24	Std dev	1.821	7.282	0.070
	Median	0.030	1.400	0.081
Loam	Mean	0.427	5.99	0.102
size = 18	Std dev	0.367	5.59	0.106
	Median	0.341	3.252	0.052
Loamy sand	Mean	1.188	8.695	0.186
size = 12	Std dev	2.581	10.698	0.174
	Median	0.448	3.242	0.147
Sand	Mean	0.418	9.621	2.967
size = 50	Std dev	0.250	7.207	7.843
	Median	0.399	7.441	0.197
Silt	Mean	0.095	7.41	2.371
size = 25	Std dev	0.162	8.827	10.30
	Median	0.051	3.35	0.075
Silty loam	Mean	0.420	3.323	0.142
size = 23	Std dev	0.468	2.815	0.132
	Median	0.266	2.136	0.080
Sandy loam	Mean	0.410	3.899	2.280
size = 36	Std dev	0.609	4.223	11.88
	Median	0.222	2.320	0.156

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

ting parameters associated with each of these combinations. Over 200 sets of water content versus soil suction data were fitted to each of the three forms of soil-water characteristic curve models (i.e., van Genuchten (1980) – Burdine (1953), Gardner (1956), Brooks and Corey (1964), Brutsaert (1966), Mualem (1976), van Genuchten (1980), Boltzman from McKee and Bumb (1984), Fermi from McKee and Bumb (1987), and Fredlund and Xing (1994)).

The water content versus soil suction data sets were sorted into eight USDA soil classifications (Table 2) and were used for the Akaike Information Criterion analysis.

An attempt was made to assess the relative benefits of the two correction factors, when applied to each of the models. The relationship between the fitted parameters and the effect that each parameter has on the shape and position of the soil-water characteristic model was also studied.

The mathematical form of the soil-water characteristic models studied can be divided into three basic categories as follows:

(1) exponentially based models such as Boltzman (McKee and Bumb 1984) and Fermi (McKee and Bumb 1987),

(2) models that have two fitting parameters and are not exponentially based; such as the Gardner (1956), Brooks and Corey (1964), Brutsaert (1966), van Genuchten (1980), van Genuchten (1980) – Burdine (1953), and van Genuchten (1980) – Mualem (1976) models, and

(3) models having three fitting parameters; such as the van Genuchten (1980) and Fredlund and Xing (1994) models.

Each of the three categories of models gives a different quality of fit as shown by the Akaike Information Criterion presented in Table 4. A comparison of the Akaike Information Criterion can be made because each model combination was fitted to a similar data set. Any experimental error in the data is consistent for each of the models. The average Akaike Information Criterion for each model was calculated by adding the Akaike Information Criterion for each model combination that was fitted, and dividing by the number of combinations.

Summary of the results from the Akaike Information Criterion analysis

The Fredlund and Xing (1994) model, along with the Fredlund and Xing (1994) correction factor, had the lowest Akaike Information Criterion, i.e., –1007. The lowest Akaike Information Criterion indicates the best fit of the given data sets. The van Genuchten (1980) model modified using the Fredlund and Xing (1994) correction factor had an Akaike Information Criterion of –824. The Brutsaert (1966) model was found to have an average Akaike Information Criterion of –777. The Brutsaert model was found to have the lowest Akaike Information Criterion of any two-parameter model tested. However, the difference in the calculated Akaike Information Criterion for the Brutsaert (1966), van Genuchten (1980) – Mualem (1976), Brooks and Corey (1964), van Genuchten (1980) – Burdine (1953), and Gardner (1956) models was small.

Figure 6 is a plot of the average Akaike Information Criterion versus the model type. In general, the three-parameter models performed best for fitting the soil-water characteristic data. Each of the two-parameter models that were not exponential based, performed approximately the same. The exponential based models generally provided the poorest fit to the soil-water characteristic data.

Table 4 also shows the average Akaike Information Criterion with the correction factor added. The Fayer and Simmons (1995) correction factor had the lowest Akaike Information Criterion of –733. The Fredlund and Xing (1994) correction had the largest Akaike Information Criterion of all of the correction factors. In other words, the Fredlund and Xing (1994) correction does not add a fitting parameter, however, this does not appear to compensate for the increased residual sum of squares. The difference in the Akaike Information Criterion between the Fayer and Simmons (1995) correction factor and the normalized water content form of the models can be considered to be inconclusive. The Fayer and Simmons (1995) correction had the lowest calculated Akaike Information Criterion. However, the normalized water content form was only slightly less ef-

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Table 11. Statistics on fitting parameters for three different forms of the McKee and Bumb 1984 (Boltzman) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	115.3	2324	1057.3	3202	–12.63	1107
size = 12	Std dev	734.4	3773	2494	3636	31.97	2818
	Median	–2.617	335.6	–11.15	1632	–0.180	334.9
Clay loam	Mean	230.5	2610	241.900	3226	417.0	667.9
size = 24	Std dev	2096	2431	877.5	2451	2041	2027
	Median	–93.63	2693	–1.060	3164	0.110	81.53
Loam	Mean	–21.44	272.5	–22.19	400.5	–0.482	34.70
size = 18	Std dev	72.73	618.6	49.38	894.6	8.300	58.98
	Median	–0.177	21.28	–6.729	72.26	0.065	11.07
Loamy sand	Mean	2.563	13.03	–1.161	37.19	2.458	11.00
size = 12	Std dev	6.473	13.22	3.969	45.98	6.425	10.14
	Median	0.793	6.885	0.323	20.67	0.111	6.991
Sand	Mean	3.788	16.38	1.961	32.34	3.559	14.19
size = 50	Std dev	7.497	77.41	10.38	124	7.532	71.05
	Median	1.741	2.553	1.313	3.781	1.642	2.488
Silt	Mean	–32.62	697.4	–5.924	871.9	4.914	231.4
size = 25	Std dev	102.0	1110	42.84	1306	14.88	563.5
	Median	5.758	111.8	2.645	203.5	0.531	48.97
Silty loam	Mean	–24.64	330.2	–11.53	881.1	–1.937	100.1
size = 23	Std dev	85.09	677.5	387.3	1498	22.26	268.5
	Median	–0.858	79.73	–7.850	178.4	0.219	25.34
Sandy loam	Mean	–1.266	59.61	–35.69	228.6	–2.527	38.38
size = 36	Std dev	9.274	113.6	119.1	559.1	15.65	83.65
	Median	0.243	26.77	–2.428	52.03	0.491	16.72

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

ficient at modeling the given data sets than when using the Fayer and Simmons (1995) correction factor.

The normalized water content form that performed best was the Gardner (1956) equation. The two-parameter model that performed best with the Fredlund and Xing (1994) correction was the Brooks and Corey (1964) model. The Brutsaert (1966) model, modified using the Fayer and Simmons (1995) correction factor, was the overall best two-parameter model.

The models that gave the lowest Akaike Information Criterion tended to require the least effort to find best-fit parameters. The Fredlund and Xing (1994) model that had the lowest Akaike Information Criterion also tended to require less iterations to compute the best-fit parameters compared to the other models. Also, the Fredlund and Xing (1994) model required less accuracy in the initial parameter guesses than other models in order to ensure convergence to the best-fit parameters. The equations with the lowest Akaike Information Criterion number tended to provide solutions with the fewest number of iterations and had fewer singularity problems.

The models that were the easiest to fit tended to have a form similar to the Gardner (1956) model (i.e., Brutsaert (1966); van Genuchten (1980); van Genuchten (1980) – Burdine (1953); van Genuchten (1980) – Mualem (1976), and Fredlund and Xing (1994)). The Gardner (1956) model has parameters that have an independent effect on the soil-water characteristic curve. For example, the n parameter, or pore size distribution parameter, is an exponent that determines the general shape or slope of the curve. The a parameter bears a relationship to the air-entry value of the soil and determines the lateral position of the curve.

The models that were the most difficult to fit were the exponential based models. These models usually required many trials using different initial guesses for the parameters and required increased iterations before convergence to the best-fit parameters. The parameters of the exponential based models affect both the shape and the position of the curve. The interdependence of the parameters may cause uniqueness problems that increase the number of iterations required for the parameters to converge and the number of trials required in order to find a global minimum.

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Table 12. Statistics on fitting parameters for three different forms of the McKee and Bumb 1987 (Fermi) model for eight different soils.

		Normalized water content form		Fredlund and Xing (1994) correction*		Fayer and Simmons (1995) correction	
Soil	Statistic	a (kPa)	n	a (kPa)	n	a (kPa)	n
Clay	Mean	2088	1152	904	330.2	194.5	387.3
size = 12	Std dev	3772	1955	2865	1051	555.4	1295
	Median	168.1	59.96	52.11	16.19	23.73	3.767
Clay loam	Mean	2013	1174	545.5	1748	434.7	145.6
size = 24	Std dev	2247	1587	1330	5267	1938	666.2
	Median	1509	703.4	48.64	1.254	18.73	3.626
Loam	Mean	118.7	56.5	207.3	106.6	17.06	4.562
size = 18	Std dev	327.3	149.3	474.8	188.6	25.48	5.736
	Median	15.52	7.687	54.48	39.52	11.56	2.475
Loamy sand	Mean	11.32	4.115	22.54	12.43	11.41	4.147
size = 12	Std dev	13.87	4.917	25.53	12.32	23.07	9.184
	Median	5.57	1.879	8.467	5.884	4.844	1.023
Sand	Mean	15.38	4.967	21.42	8.057	15.93	6.567
size = 50	Std dev	56.16	23.13	82.49	36.21	79.16	34.44
	Median	3.661	0.945	4.643	1.213	3.662	0.600
Silt	Mean	1387.8	83.11	544.3	204.2	106.6	62.32
size = 25	Std dev	2776	216	1161	460.7	404.7	189.6
	Median	69.08	15.76	78.06	33.14	16.35	3.582
Silty loam	Mean	351.9	87.13	612.7	317.8	14.63	8.971
size = 23	Std dev	1055	169.9	874.0	479.6	10.12	23.55
	Median	47.78	15.77	216.7	114.6	11.92	2.250
Sandy loam	Mean	38.38	19.27	106.0	71.49	8.780	2.815
size = 36	Std dev	69.82	40.61	313.7	280.0	5.597	2.809
	Median	18.31	7.345	38.04	19.98	7.636	1.932

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

Statistics of fitted parameters for different soils and forms of model

The texture of a soil indicated by the USDA soil classification bears a relationship to the shape of the soil-water characteristic curve. The tables of best-fit model parameters versus soil texture provide an estimate for the initial parameter guesses, as well as a range of reasonable results for the fitting routine. The statistics of the fitted curve for each fitted model parameter are shown in Tables 5–13. Each of the tables provides the statistical properties of mean, median, and standard deviation for each of the soil fitting parameters corresponding to each of the soil-water characteristic curve models. The best-fit analyses were also performed with the Fredlund and Xing (1994) correction applied and with the Fayer and Simmons (1995) correction applied. The best-fit analyses were performed for eight of the soil classifications.

Coefficients of variation can be computed for each of the soil models. These were found to be quite large in most cases, which means that it is not possible to fix soil parameters for soils falling into a particular category. Rather, it is necessary to use other analytical means of obtaining suitable soil parameters when analyzing geotechnical engineering problems.

Observations based on the statistical study of the data

The following observations were made using the statistical studies on the soil-water characteristic data and the various proposed models.

(1) The fitting of the water content versus soil suction data to the various models indicated that the Fredlund and Xing (1994) model had the lowest calculated Akaike Information Criterion and would appear to most accurately model the soil-water characteristic curve data.

(2) The van Genuchten (1980) model, modified using the Fredlund and Xing (1994) correction factor, showed the second lowest average Akaike Information Criterion.

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Table 13. Statistics on fitting parameters for the Fredlund and Xing (1994) model for eight different soils.

		Fredlund and Xing (1994) correction*		
Soil	Statistic	a (kPa)	n	m
Clay	Mean	506.3	9.27	0.626
size = 12	Std dev	790.9	27.57	0.648
	Median	30.95	1.075	0.327
Clay loam	Mean	172.6	2.418	0.492
size = 24	Std dev	210.3	6.308	0.280
	Median	92.30	0.864	0.535
Loam	Mean	20.3	2.365	0.562
size = 18	Std dev	43.12	1.394	0.283
	Median	4.971	1.941	0.413
Loamy sand	Mean	6.684	3.398	0.874
size = 12	Std dev	8.284	3.436	0.343
	Median	3.667	2.026	0.836
Sand	Mean	17.27	6.537	4.428
size = 50	Std dev	84.49	4.286	10.72
	Median	2.814	5.776	0.885
Silt	Mean	188.4	4.286	0.949
size = 25	Std dev	370.7	9.650	1.381
	Median	33.58	1.676	0.714
Silty loam	Mean	63.14	2.188	0.665
size = 23	Std dev	153.6	1.987	0.323
	Median	9.656	1.294	0.626
Sandy loam	Mean	16.37	5.53	2.608
size = 36	Std dev	21.24	16.44	10.76
	Median	8.953	1.729	0.58

*A residual suction of 3000 kPa was used when obtaining the best-fit soil parameters.

(3) The Brutsaert (1966) model had the lowest Akaike Information Criterion of the two-parameter models when considering all three forms of the equation. The difference in the calculated Akaike Information Criterion for the Brutsaert (1966), Mualem (1976), Brooks and Corey (1964), van Genuchten (1980) – Burdine (1953), and Gardner (1956) models is small. The accuracy, simple form, and meaningful parameters associated with the Brutsaert (1966) model make it a desirable two-parameter model. Also, the Brutsaert (1966) model converged to the best-fit parameters with the least number of iterations, had fewer uniqueness problems, and was not sensitive to the initial parameter guesses.

(4) The van Genuchten (1980) and van Genuchten (1980) – Mualem (1976) models appear to be among the most commonly used models, particularly in agricultural related disciplines.

(5) The Boltzman (McKee and Bumb 1984) and Fermi (McKee and Bumb 1987) equations were based upon an exponential function, were the most difficult to fit, and had the largest Akaike Information Criterion values. One reason for the difficulty in performing the fit is the overlap in the domain of each parameter. Both fitting parameters affect the shape and position of the curve, and this results in uniqueness difficulties for the fitting routine.

(6) The Brutsaert (1966), van Genuchten (1980), van Genuchten (1980) – Mualem (1976), Brooks and Corey (1964), van Genuchten (1980) – Burdine (1953), and Gardner (1956) models produced an Akaike Information Criterion that was approximately equal for each model.

(7) The van Genuchten (1980) model with the m parameter equal to 1.0 is essentially the same as the Brutsaert (1966) model. The Brutsaert (1966) model had a lower Akaike Information Criterion than the van Genuchten (1980) – Burdine (1953) and van Genuchten (1980) – Mualem (1976) models for the given data. The relationship between n and m for the van Genuchten (1980) model, which treats the m parameter as a fitting parameter, varied with soil texture. Therefore, setting the m parameter equal to 1.0 appears to be a better assumption than fixing the relationship between the n and m parameters as in the van Genuchten (1980) – Burdine (1953) and van Genuchten (1980) – Mualem (1976) models. Also, allowing m to be a fitting parameter, as in the van Genuchten (1980) model, appears to be a superior solution to defining a relationship between n and m.

(8) Correction factors for the soil-water characteristic curve models provide a useful improvement when modeling the water content versus soil suction relationship in the high suction region. The extension of the soil-water characteristic models to zero water content at 1 000 000 kPa appears to agree with the available data and theoretical considerations.

(9) The normalized water content form for the equations limits the range of water content that can be fit by the model. Therefore, the soil-water characteristic model may appear to show an improved accuracy over a limited range of the water content versus soil suction data.

(10) The lowest measured water contents and soil suctions appear to influence the magnitude of the residual water content. The fitting routine tends to overestimate the residual water content if the experimental data does not extend well into the zone of desaturation. The fitted residual water content parameter tends to be overestimated when the last measured water content is larger than the water content at which the zone of residual saturation begins.

(11) The operation of the fitting routine indicated that models with the lowest Akaike Information Criterion tended to converge with the least average number of iterations and have the least sensitivity to the initial parameter guesses.

(12) Best-fit parameters from one model do not, in general, appear to have a close correlation with the fitted parameters of another model. Likewise, the fitted parameters from a model do not appear to have a close correlation to the best-fit parameters of the same model when a different correction factor is used.

(13) Converting fitted parameters of one model into parameters of another model does not appear to be reliable at this point. Likewise, converting the fitted parameters of one form of a model into parameters of another form of the same model does not appear to be reliable. The soil-water characteristic curve data should be independently fitted to each model.

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Discussion on the findings regarding normalized water content form and the use of correction factors

Normalized water content form

The normalized water content form is a method that limits the range of water content over which the soil-water characteristic model is fit (i.e., between the saturated water content and the residual water content). Most models were originally proposed in the normalized water content form and provided some advantages:

(1) The normalized water content form increases the accuracy of the fit in the low suction region, (i.e., the capillary saturation zone and the desaturation zone).

(2) The normalized water content form appears to be a commonly used form for soil-water characteristic curve models. Databases of fitted model parameters generally use the normalized water content form of the equation for various soils.

Some of the disadvantages of the normalized water content form are as follows:

(1) There is uncertainty regarding the definition of residual water content.

(2) The normalized water content form does not adequately model the entire soil-water characteristic curve. In the high suction region it tends to overestimate the water content.

The Fredlund and Xing (1994) correction factor

The Fredlund and Xing correction factor was proposed in order to improve the quality of fit in the high suction region of the soil-water characteristic curve. The advantages of using the Fredlund and Xing correction factor are as follows:

(1) The correction factor does not add an additional fitting parameter for the correction of the soil-water characteristic curve in the high soil suction range.

(2) The correction factor improves accuracy in the high soil suction range.

Some of the disadvantages of this correction factor are as follows:

(1) The Fredlund and Xing correction factor may slightly reduce the accuracy in the overall fit of the soil-water characteristic curve when compared to the normalized water content form.

(2) The correction is in the form of a defined mathematical function that may apply a restriction on the model and make it more difficult to fit to the experimental data.

The Fayer and Simmons (1995) correction factor

The Fayer and Simmons correction factor provides an alternative means of modeling the soil-water characteristic curve and has the following advantages:

(1) The correction factor allows the modeling of the entire water content versus soil suction range and provides a physical explanation for water retention in the zone of residual saturation.

(2) The correction factor bypasses the controversy over the residual water content parameter used in the normalized water content form.

Some of the disadvantages of the Fayer and Simmons correction factor are as follows:

(1) One additional fitting parameter is required, which is related to a new empirical variable called the air-dried water content.

(2) Fitted model parameters cannot be converted to other forms of the same model.

Acknowledgement

The authors would like to acknowledge the assistance of Mrs. Noshin Zakerzadeh in the preparation of this paper.

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