Hydrodynamic pressure in dynamic analysis: Westergaard's added mass

MODEL VALIDATION CASES

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Note: This feature was released as Technology Preview in PLAXIS CONNECT EDITION V20.1

This document describes the newly implemented Westergaard's added-mass feature to deal with the hydrodynamic effects induced by the impounded water on the response of a dam subjected to earthquake excitation.

Theory and Formulation

During an earthquake, the interaction between the stiff structure and the outer water (reservoir, river, lake, sea) creates additional pressures on the upstream face of the dam. These hydrodynamic pressures may be approximated by the Westergaard (1933) formula, which uses a parabolic approximation for the additional pressures due to earthquake motion. *Figure 1* illustrates the forces due to the total water pressures during an earthquake. Note that the hydrodynamic forces act in both directions. The water pressure is regarded as an added mass, acting on the upstream surface of a dam structure and the rest of the water is assumed to be inactive.

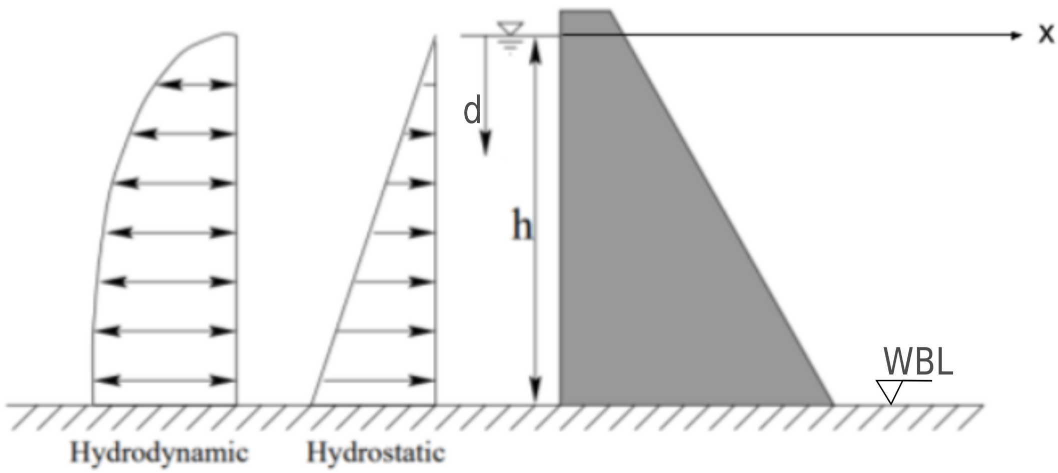

Figure 1: Hydrostatic and hydrodynamic forces during earthquake excitation.

Westergaard's approximate solution

The dynamic pressure is approximated as:

$$p = \frac{7}{8}\rho a_x \sqrt{hd}$$

where

ρ	=	Density of water
a_x	=	Horizontal acceleration

This equation indicates that the hydrodynamic pressure exerted normally on the upstream face of the dam, at depth *d*, due to ground acceleration axis is equivalent to the inertia force of a prismatic body of water of unit cross-section and length $\frac{7}{8}\sqrt{hd}$ attached firmly to the face of the dam, and moving with the dam back and forth in the direction normal to the face of the dam (that is, horizontally) without friction. This body of water attached to the dam face and moving with the dam, is the added-mass applied by the reservoir to the dam, a concept first introduced by Westergaard.

In the PLAXIS implementation, the Westergaard equation has also been generalized to take into account the hydrodynamic pressure acting on inclined faces considering acceleration normal to the face (Generalized Westergaard formulation).

Finite Element Analysis in PLAXIS

A simple example is presented to illustrate the Westergaard's added mass implementation in PLAXIS to model the effect of hydrodynamic pressure on a dam (with fixed displacements in the vertical direction). *Figure 2* shows the geometry of the model in PLAXIS 2D and PLAXIS 3D. The dam is modelled taking into account the following parameters (*Table 1* and *Table 2*):

Table 1: Material parameters

Parameter	Symbol	Value	Unit
Material model	Model	Linear Elastic	[-]
Soil Unit weight above phreatic level	γ_{unsat}	20.0	kN/m^3
Soil Unit weight below phreatic level	γ_{sat}	20.0	kN/m^3
Young's Modulus	E'	$81.82 \cdot 10^3$	[-]
Poissons ratio	v'	0.3	[-]

Table 2: Dynamic conditions

Condition	Value	Unit
Signal direction	Horizontal	[-]
Signal type (based on velocities)	Harmonic	[-]
Amplitude	1	[-]
Frequency	1	Hz
Dynamic boundary conditions	None	[-]

For the simulation two phases are considered. In the Initial Phase, the dam is subjected to hydrostatic loading and in the following phase, the dam is subjected to hydrodynamic loading using a dynamic velocity signal applied at the base of the dam. This hydrodynamic load is applied as an added mass at the interface of the dam and water. In the **Structures** mode the added mass option is activated using the *Create added mass* option as shown in *Figure 3 (a)*. Once the added mass is created the corresponding drop-down menu is displayed inside the **Model explorer** (see *Figure 3 [b])*.

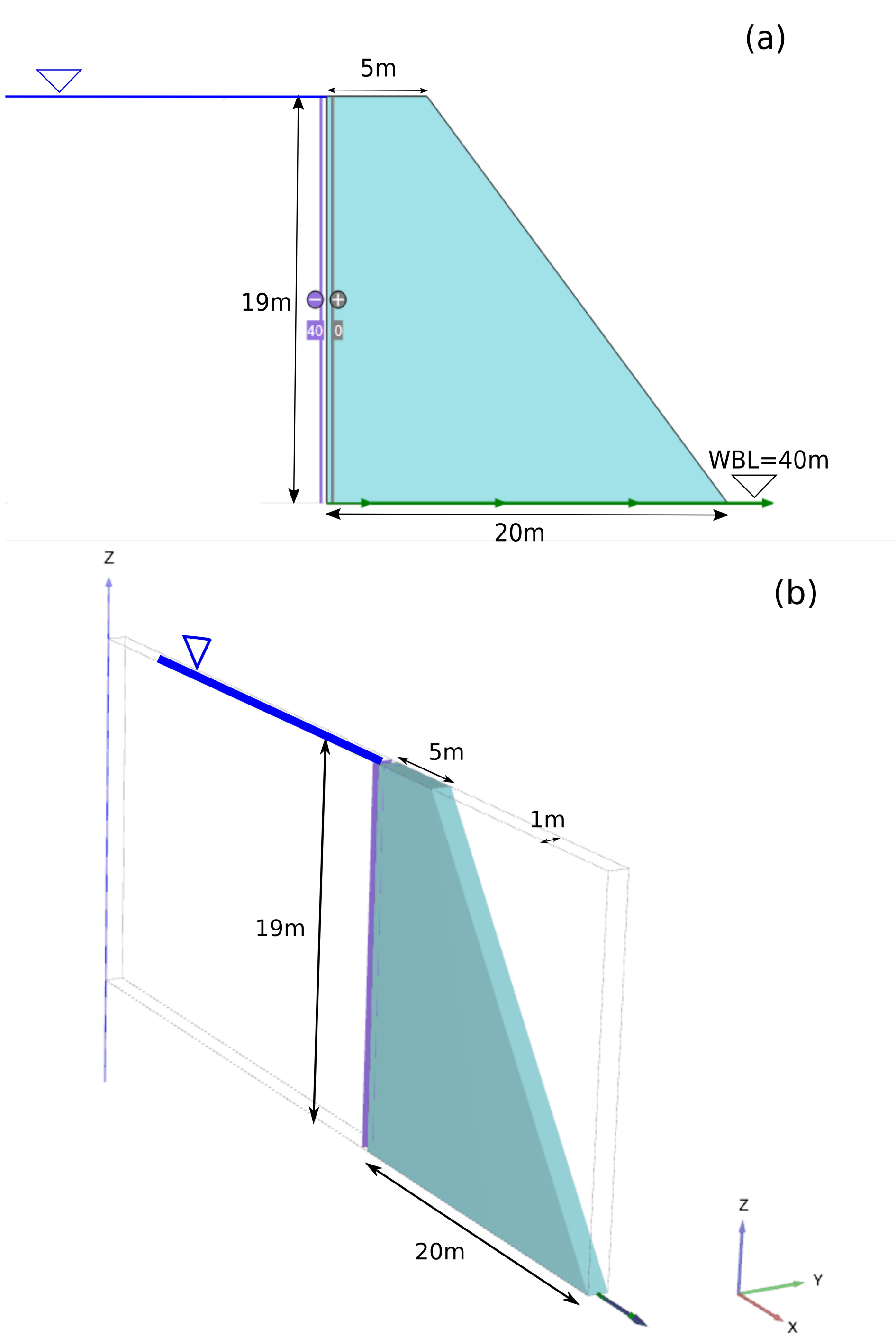

Figure 2: Model Geometry PLAXIS 2D (a) and PLAXIS 3D (b). In the example the base of the dam is located at d=40m (see Figure 3 [b]).

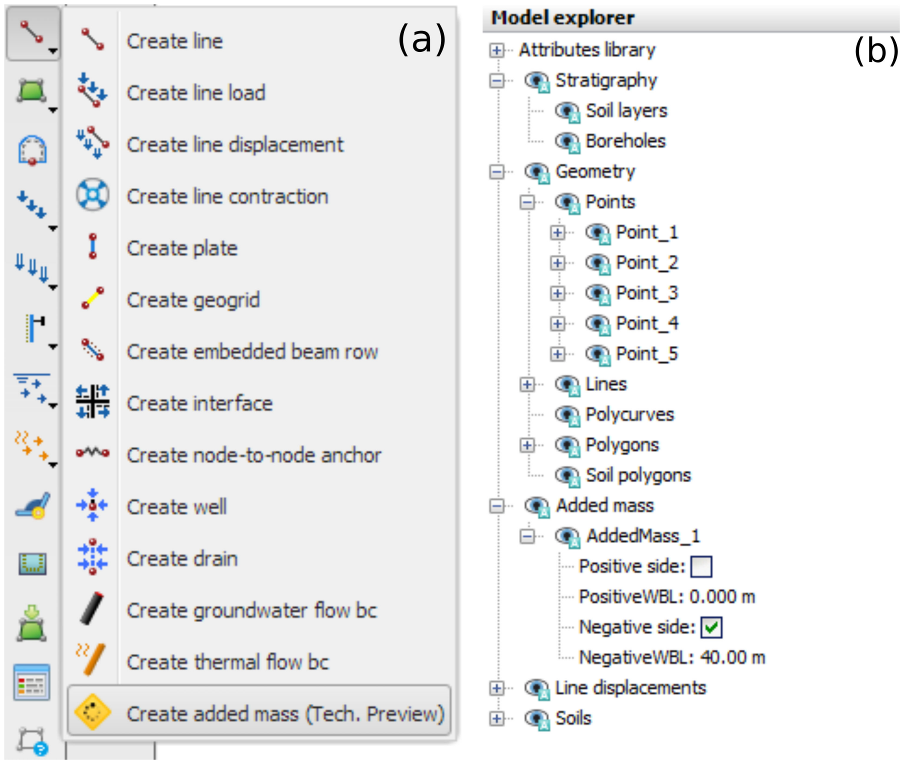

Figure 3: Create Added mass option in the Model Explorer of the Structures mode

Note: It is necessary to take into account that the drawing direction of the added mass will display a positive or negative sign in one or the other side of the dam. For this reason, it is necessary that in the drop-down menu for the **Added mass** in the **Model explorer** the side where the water is located is activated.

Note: The *Water Base Level*(*WBL*) corresponds to the bottom elevation of the free water body acting on the structure under investigation, which is in this model at the base of the structure.

In the presented case, as shown *Figure 3 (b)*, a *Water base level* (**NegativeWBL**) of 40m was selected based on the coordinate system used for the base of the dam.

For this example, the horizontal displacement at the top of the dam at the upstream face was monitored. The results, in *Figure 4*, show an agreement between the simulations of PLAXIS 2D and PLAXIS 3D.

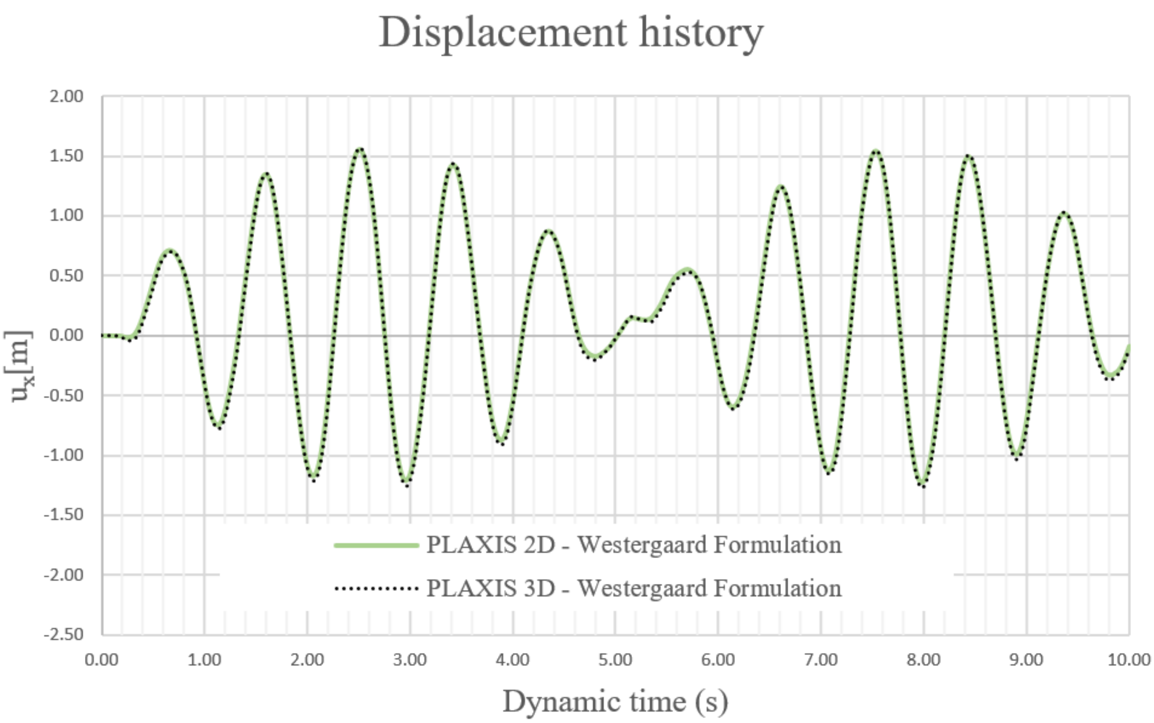

Figure 4: Comparison in displacement history PLAXIS 2D and PLAXIS 3D with Westergaard added mass implementation

References

1. Shaw - Han Kuo, J. (1982). Fluid - Structure Interactions: Added mass computations for incompressible fluid. Report to the National Science Foundation, University of California Berkley, Earthquake Engineering Research Center