The 1999 R.M. Hardy Lecture: The implementation of unsaturated soil mechanics into geotechnical engineering

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Abstract: The implementation of unsaturated soil mechanics into geotechnical engineering practice requires that there be a paradigm shift from classical soil mechanics methodology. The primary drawback to implementation has been the excessive costs required to experimentally measure unsaturated soil properties. The use of the soil-water characteristic curve has been shown to be the key to the implementation of unsaturated soil mechanics. Numerous techniques have been proposed and studied for the assessment of the soil-water characteristic curves. These techniques range from direct laboratory measurement to indirect estimation from grain-size curves and knowledge-based database systems. The soil-water characteristic curve can then be used for the estimation of unsaturated soil property functions. Theoretically based techniques have been proposed for the estimation of soil property functions such as (*i*) coefficient of permeability, (*ii*) water storage modulus, and (*iii*) shear strength. Gradually these estimations are producing acceptable procedures for geotechnical engineering practices for unsaturated soils. The moisture flux ground surface boundary condition is likewise becoming a part of the solution of most problems involving unsaturated soils. The implementation process for unsaturated soils will still require years of collaboration between researchers and practicing geotechnical engineers.

Key words: unsaturated soil mechanics, soil suction, unsaturated soil property functions, negative pore-water pressure, matric suction, soil-water characteristic curve.

Résumé : L'implantation de la mécanique des sols non saturés dans la pratique de la géotechnique requiert qu'il y ait un changement de paradigme par rapport à la méthodologie de la mécanique des sols classique. Le principal inconvénient à cette implantation a été les coûts excessifs requis pour mesurer expérimentalement les propriétés des sols non saturés. Il a été démontré que l'utilisation de la courbe caractéristique sol-eau est la clef de l'implantation de la mécanique des sols saturés. De nombreuses techniques ont été proposées et étudiées pour déterminer les courbes caractéristiques sol-eau. Ces techniques s'étendent de la mesure directe en laboratoire à l'estimation indirecte en partant des courbes granulométriques et basée sur la connaissances des systèmes de banques de données. La courbe caractéristique sol-eau peut alors être utilisée pour l'estimation des fonctions de propriétés des sols non saturés. Des techniques fondées sur la théorie ont été proposées pour l'estimation des fonctions de propriétés des sol telles que: (*i*) coefficient de perméabilité, (*ii*) module d'entreposage de l'eau, et (*iii*) résistance au cisaillement. Ces estimations produisent graduellement des procédures acceptables pour les pratiques de la géotechnique dans les sols non saturés. La condition de frontière de l'écoulement de l'humidité à la surface du terrain devient également une partie de la solution de la plupart des problèmes impliquant les sols non saturés. Le processus d'implantation des sols non saturés va nécessiter encore des années de collaboration entre les ingénieurs géotechniciens en recherche et en pratique.

Mots clés : mécanique des sols non saturés, succion du sol, fonctions de propriétés du sol non saturé, pression interstitielle négative, succion matricielle, courbe caractéristique sol-eau.

[Traduit par la Rédaction]

Introduction

It is a long road from the discovery of the basic science related to an engineering phenomenon to its implementation into standard engineering practice. There can be a sound theoretical basis for material behavior, as well as mathematical solutions, and still this may not bring about a change in en-

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gineering practice. Implementation is a unique and important step that brings theories and analytical solutions into engineering practice. There are several stages in the development of a science that must be brought together in an efficient and appropriate manner in order for implementation to become a reality. This paper focuses on the stages related to the implementation of unsaturated soil mechanics.

The engineering behavior of an unsaturated soil is commonly viewed as being more complex and more difficult to understand than that of a saturated soil. Unsaturated soils have negative pore-water pressures, but it is the wide range of associated degrees of saturation that produces a broad spectrum of soil behavior. Figure 1 shows that an unsaturated soil can be near to 100% saturation in the capillary

Fig. 1. Classification of the regions within a saturated–unsaturated soil profile. S, degree of saturation.

Fig. 2. Primary stages leading towards the successful implementation of unsaturated soil mechanics.
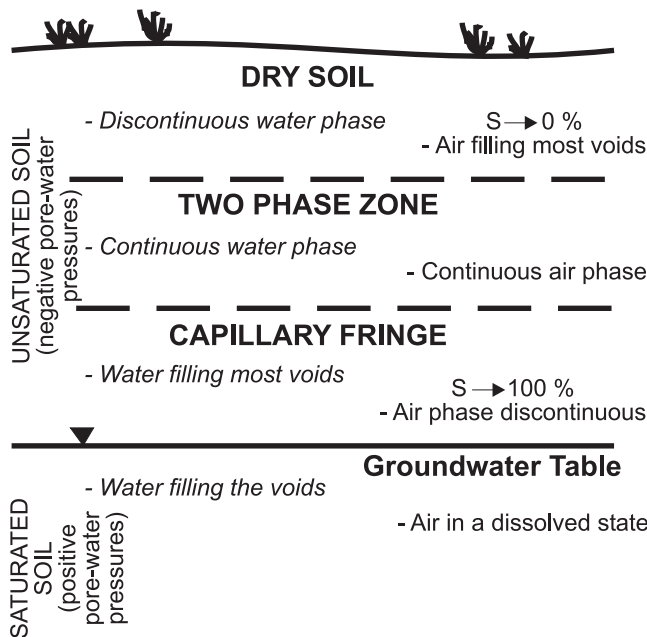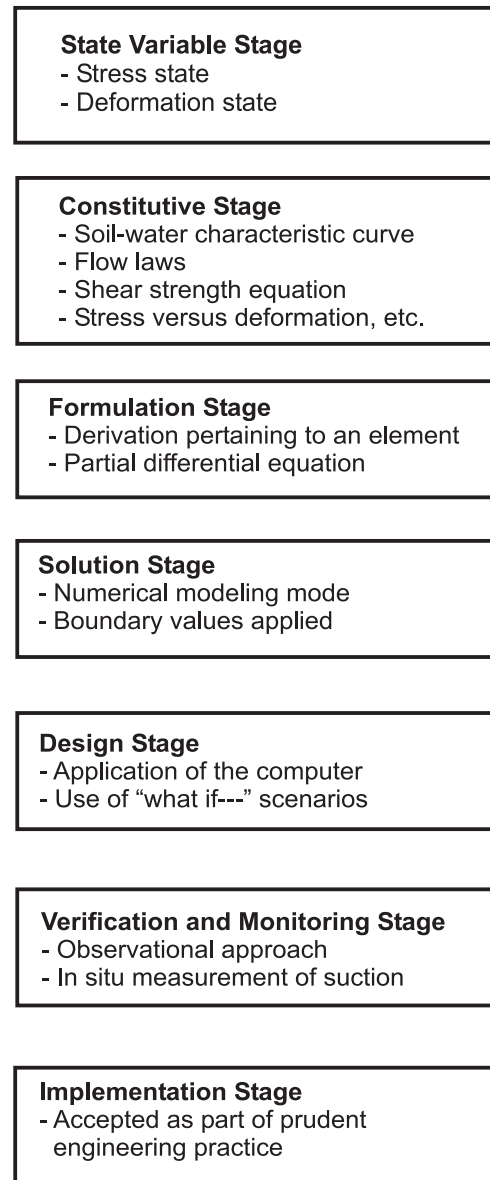

zone and completely dry near the ground surface. The behavioral science for an unsaturated soil has been primarily developed for the case where the air and water phases are continuous (i.e., two-phase zone). The degree of saturation for the two-phase zone generally ranges from about 20 to 80%. However, it has been found that the proposed theories can be extended throughout the entire unsaturated soil spectrum (Fredlund and Rahardjo 1993).

A cursory review of research into the behavior of unsaturated soils shows that the 1970s (and earlier years) were a period when the fundamental theories and concepts for unsaturated soil mechanics were formulated. To have a scientific basis for unsaturated soil mechanics, it was necessary that the state variables (in particular, the stress state variables) be defined for an unsaturated soil. The use of two sets of independent stress state variables was firmly established with a supporting analysis based on multiphase continuum mechanics during the 1970s (Fredlund and Morgenstern 1977).

The development of theories for unsaturated soil behavior can be viewed from the standpoint of the classic constitutive relationships established in saturated soil mechanics, namely seepage, shear strength, and volume change. Although thermal and chemical constitutive relations are of importance, this paper will use the above-mentioned classic areas to illustrate the steps required in moving towards the implementation of unsaturated soil mechanics into geotechnical engineering practice.

The 1980s were a period when boundary conditions were assumed for a variety of geotechnical engineering problems and the emphasis was on solving problems. It became apparent that the soil properties took the form of nonlinear mathematical functions that added to the difficulty in obtaining solutions. Iterative, numerical models became a common, useful tool for solving most geotechnical engineering problems. The numerical solutions were of particular value for unsaturated soil problems because of the difficulty in obtain-

ing closed-form solutions. One example is the solution for saturated–unsaturated seepage problems where flow-net solutions became no longer relevant. Fortunately, our computational abilities were increasing at a pace similar to that of our understanding of unsaturated soil behavior (Fredlund 1996). The ability to produce more realistic solutions for various classes of soil mechanics problems might lead to the inference that unsaturated soil mechanics was ready for application in standard engineering practice. Unfortunately, that was not the case and much more research was required.

The 1990s were a period in which the emphasis was more focused on the implementation of unsaturated soil mechanics. Implementation has proven to be difficult and it is apparent that a paradigm shift in methodology is required to facilitate the implementation of unsaturated soil mechanics. The primary drawback to implementation has been the excessive time (and therefore costs) required to experimentally

Fig. 3. Separation of saturated and unsaturated soil mechanics based on the stress state description.
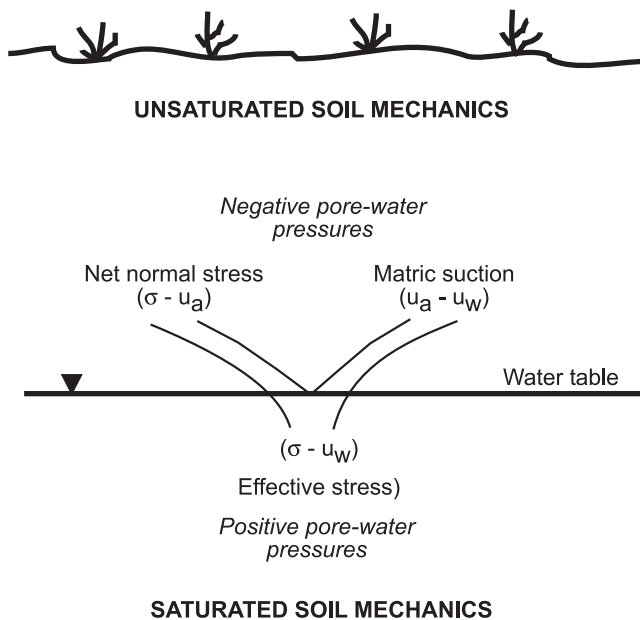
measure unsaturated soil properties. The implementation process for unsaturated soil mechanics will extend well into the next millennium.

What is the implementation of unsaturated soil mechanics?

Implementation can be described as bringing a particular science or technology into standard engineering practice. There are a series of stages involved in the process leading to implementation. Failure to accomplish or achieve a satisfactory solution at any stage in the development of an applied science or technology may result in the failure to achieve full implementation into engineering practice.

The primary stages involved in bringing unsaturated soil mechanics from a basic science to the implementation phase are summarized in Fig. 2. The stages leading towards implementation progress from the most fundamental theoretical stage to the most practical stage. Research is required at all stages leading towards implementation. Research studies directed at all stages must lead towards an appropriate technology for implementation. An appropriate technology must meet requirements of being sufficiently accurate while at the same time being practical and cost effective.

Stages leading towards implementation

The desired end result is to have an appropriate technology that produces sufficiently accurate solutions such that it can readily become part of standard, prudent engineering practice. A survey of the research literature shows that there has been a slow but continual development of a science-based technology emerging for unsaturated soil mechanics (Clifton et al. 1999). The following sections briefly illustrate how each of the stages leads towards a more general acceptance of unsaturated soil mechanics in engineering practice.

State variable stage

The state variable stage is the most basic and fundamental level at which a science for unsaturated soil behavior can be initiated. It is most basic and fundamental because the variables so defined are generally embedded within the conservative laws of mass and energy. As such, these variables are independent of the physical properties of the material.

The most important state variables for an unsaturated soil are the stress state variables. The net normal stress, $(\sigma - u_a)$, where σ is the total stress and u_a is the pore-air pressure, and the matric suction, $(u_a - u_w)$, where u_w is the pore-water pressure, have become widely accepted stress state designations for an unsaturated soil. Figure 3 illustrates the need to separate the effects of total stress and pore-water pressure when the pore-water pressures are negative. There is also a smooth transition between the saturated and unsaturated states. The complete stress state acting in three dimensions, at a point, is illustrated in Fig. 4. The matrix form of the stress tensors is shown in the following:

$$[1] \quad \begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix} \\ \begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix}$$

where τ_{xy} is the shear stress on the x plane in the y direction.

There are two components of soil suction, ψ, namely matric suction, $(u_a - u_w)$, and osmotic suction, π. Both components are important to unsaturated soil mechanics. Matric suction is of greater relevance for low suctions (i.e., less than residual water content conditions) and total suction is of greater relevance at high suctions (i.e., greater than residual water content conditions).

The stress state variables are to *unsaturated* soils what effective stress variables are to *saturated* soils. The description of the stress state provides an important tool for sharing and comparing engineering experiences from around the world. A description of the stress state changes soil mechanics from an empirical discipline to an engineering science.

The state variables used to "map" the relative movement of a point associated with any phase, due to a change in stress state, are called the deformation state variables. Changes in void ratio, water content, or degree of saturation are generally used in soil mechanics as deformation state variables. Normal strains associated with the soil structure are also part of the deformation state variables (Fredlund and Morgenstern 1976).

Constitutive stage

The constitutive stage becomes the point at which empirical, semiempirical, and possibly theoretical relationships between state variables are proposed and verified. The verification of proposed constitutive relations must be conducted for a wide range of soils to ensure uniqueness, and subsequently confidence on the part of the practicing engineer.

Several constitutive relationships for unsaturated soils were studied during the 1970s (Fredlund and Morgenstern

Fig. 4. Elements showing the stress state at a point in a saturated soil and an unsaturated soil.
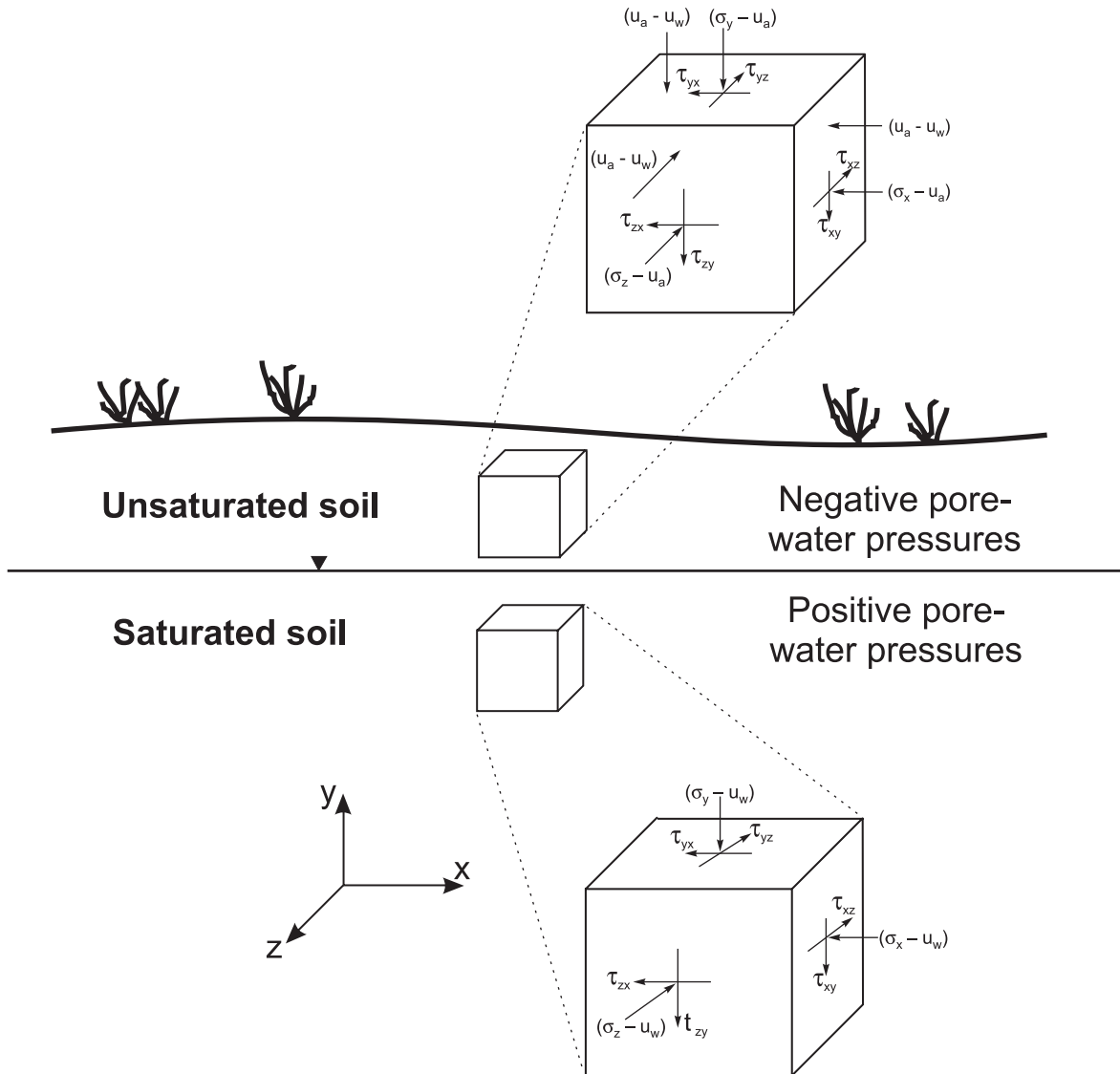
1976; Fredlund et al. 1978; Fredlund 1979). As well, earlier and later developments have also contributed to our understanding of unsaturated soil behavior (Blight 1965; Matyas and Radhakrishna 1968).

Constitutive relations are generally proposed on the basis of an understanding of the phenomenological behavior of the soil. Experimental programs are then undertaken in an attempt to verify the constitutive relations. The verification process may be extremely demanding and require the testing of several types of soil. Verification studies are generally undertaken with little regard for how demanding and costly it may be to obtain the soil parameters.

Demanding laboratory test procedures have proven to be a significant "hurdle" in bringing unsaturated soil mechanics through the implementation stage. The constitutive stage is important and must stand the tests of rigor and careful experimental analysis. However, it may be necessary to use alternate procedures to more economically quantify the soil properties in engineering practice. The later section on the implementation stage discusses a number of alternate proce-

dures that are feasible for the indirect quantification of unsaturated soil properties.

Volume change constitutive relationship

The overall volume change of an unsaturated soil can be defined as a change in void ratio in response to a change in the stress state:

$$[2] \qquad \mathrm{d}e = \frac{\partial e}{\partial(\sigma - u_a)}\, \mathrm{d}(\sigma - u_a) + \frac{\partial e}{\partial(u_a - u_w)}\, \mathrm{d}(u_a - u_w)$$
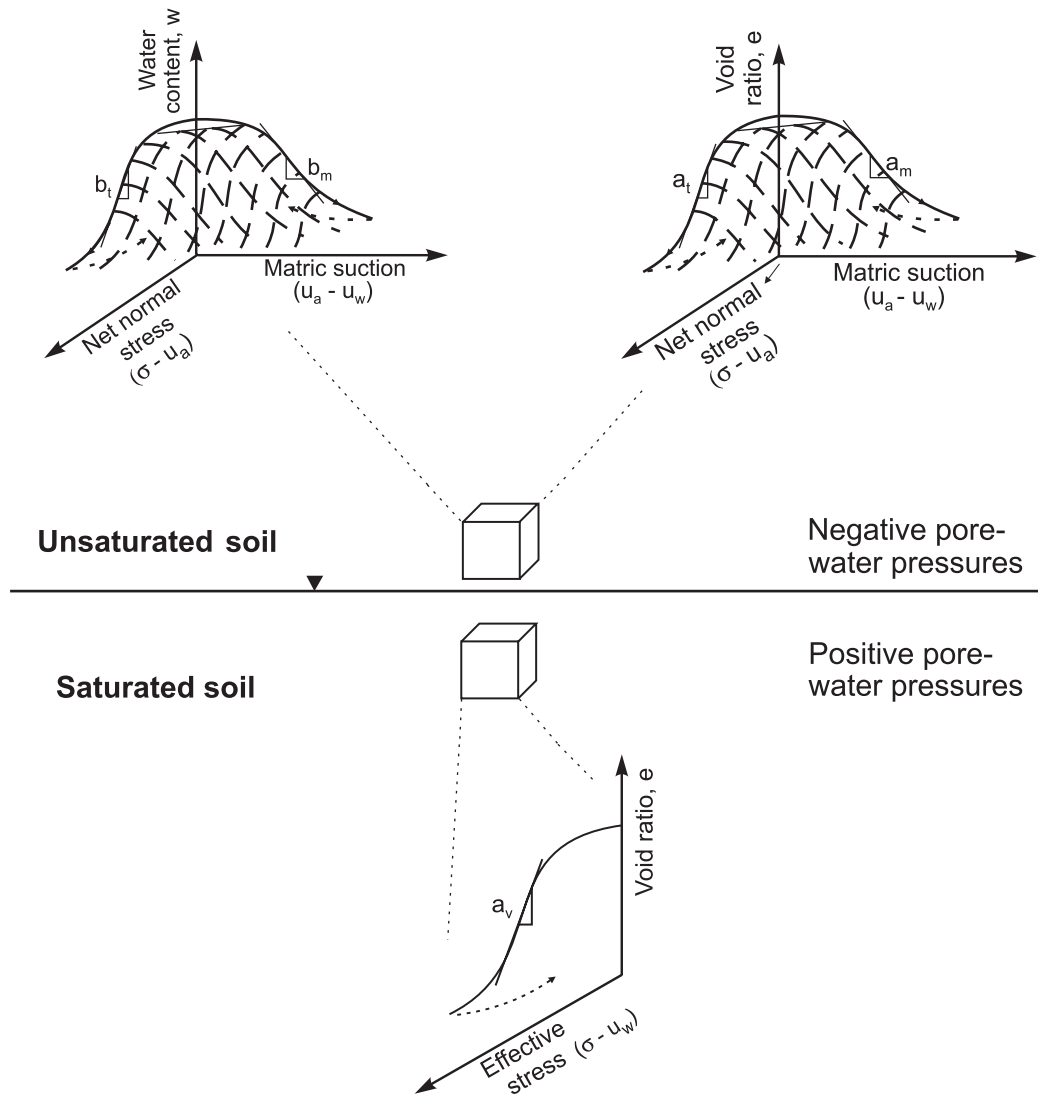
where
e is the void ratio; and
σ is the total normal confining stress (e.g., isotropic confining pressure).

Equation [2] can be viewed as having two parts, namely a part that is the designation of the stress state (i.e., $(\sigma - u_a)$ and $(u_a - u_w)$), and a part that is a designation of the soil properties (i.e., $(\partial e/\partial(\sigma - u_a))$ and $(\partial e/\partial(u_a - u_w))$). The soil properties can be viewed as the slope of the void ratio constitutive surfaces as shown in Fig. 5. The soil properties are

Fig. 5. Void ratio and water content constitutive surfaces for an unsaturated soil and a saturated soil. a_t, coefficient of compressibility with respect to a change in the net normal stress, $d(\sigma_{mean} - u_a)$; a_m, coefficient of compressibility with respect to a change in matric suction, $d(u_a - u_w)$; b_t, coefficient of water content change with respect to a change in the net normal stress, $d(\sigma_{mean} - u_a)$; b_m, coefficient of water content change with respect to a change in matric suction, $d(u_a - u_w)$; and a_v, coefficient of compressibility for a saturated soil.

moduli that vary as a function of the stress state. The soil moduli associated with the net normal stress, $(\sigma - u_a)$, can be written in a general functional form:

$$[3] \qquad \frac{\partial e}{\partial(\sigma - u_a)} = \text{func}\,[(\sigma - u_a),\,(u_a - u_w)\,]$$

where func means that the soil property is a function of the stress state. At a particular stress state, the compressibility modulus, m_1^s, for the void ratio constitutive surface with respect to $(\sigma - u_a)$, can be designated as a constant:

$$[4] \qquad \frac{\partial e}{\partial(\sigma - u_a)} = m_1^s$$

Similarly, the soil moduli associated with soil suction, $(u_a - u_w)$, can be written in a general functional form:

$$[5] \qquad \frac{\partial e}{\partial(u_a - u_w)} = \text{func}\,[(\sigma - u_a),\,(u_a - u_w)\,]$$

At a particular stress state, the compressibility modulus, m_2^s, for the void ratio constitutive surface with respect to $(u_a - u_w)$ can be designated as a constant:

$$[6] \qquad \frac{\partial e}{\partial(u_a - u_w)} = m_2^s$$

Each of the soil moduli is a function of both stress state variables. To define the magnitude of the soil moduli corresponding to any stress state, there needs to be a constitutive equation describing the entire void ratio constitutive surface. The equation then needs to be differentiated with respect to each of the stress state variables to obtain the compressibility moduli. At present, no equations have been published

Fig. 6. Definition of variables associated with the soil-water characteristic curve.
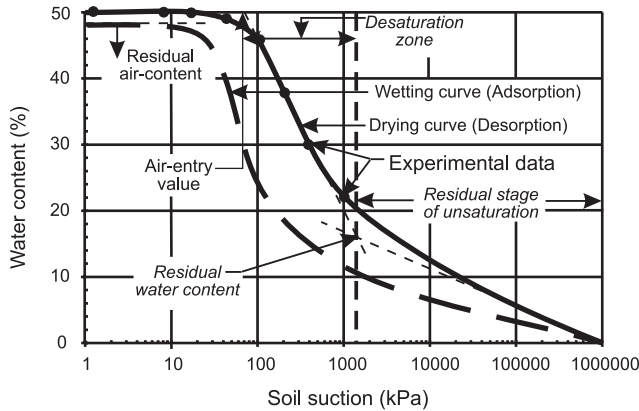
to represent the entire void ratio constitutive surface in terms of the stress state variables.

The volume-change behavior of an unsaturated soil has been the focus of numerous research studies. To date, however, volume-change behavior remains the most difficult unsaturated soil behavior to characterize.

Unsaturated soil property functions

The soil moduli on the void ratio constitutive surface can be referred to as an *unsaturated soil property function*. Essentially all soil properties related to the behavior of an unsaturated soil become a function of the stress state and are therefore nonlinear in nature (Fredlund 1995, 1998). *Saturated* soil properties are also a function of the stress state; however, in general it has been possible to either linearize the soil property or use a constant parameter to characterize soil behavior.

Once the compressibility moduli values corresponding to any stress state can be predicted, it is then necessary to convert these values into soil parameters acceptable to numerical computer models. Generally this means converting the compressibility moduli values into incremental elastic parameters (i.e., an elastic parameter functional).

The practice of *saturated* soil mechanics requires the characterization of a number of soil parameters, but the practice of *unsaturated* soil mechanics requires the characterization of a similar number of *unsaturated soil property functions*. The increased difficulty in experimentally measuring unsaturated soil property functions becomes a primary challenge in the implementation of unsaturated soil mechanics.

Water content constitutive relationship

Two constitutive relationships are required to define the volume–mass variables in terms of the stress state variables. The need for two independent constitutive relations for an unsaturated soil can be demonstrated through the differentiation of the basic volume–mass relationship (i.e., $Se = wD_r$):

$$[7] \quad \int_{e_o}^{e_f} S\,de + \int_{S_o}^{S_f} e\,dS = D_r \int_{w_o}^{w_f} dw$$

where

w is the water content;
S is the degree of saturation;
D_r is the relative density of the soil solids; and
subscripts o and f represent the initial and final states, respectively.

The water content constitutive surface can be used as a second relationship for defining the volume–mass behavior of an unsaturated soil (Fig. 5). The water content constitutive relationship can be written in the following general form:

$$[8] \quad dw = \frac{\partial w}{\partial(\sigma - u_a)}\,d(\sigma - u_a) + \frac{\partial w}{\partial(u_a - u_w)}\,d(u_a - u_w)$$

Once again, eq. [8] has a part that designates the stress state and a part that designates an unsaturated soil property that is a function of the stress state. The soil moduli associated with the net normal stress variable, $(\sigma - u_a)$, can be written as a general function:

$$[9] \quad \frac{\partial w}{\partial(\sigma - u_a)} = \text{func}\,[(\sigma - u_a),\,(u_a - u_w)\,]$$

At a particular stress state, the compressibility modulus, m_1^w, for the water content constitutive surface, with respect to $(\sigma - u_a)$, can be designated as a constant:

$$[10] \quad \frac{\partial w}{\partial(u_a - u_w)} = m_1^w$$

Similarly, the soil moduli associated with the soil suction, $(u_a - u_w)$, can be written as a general function of the stress state:

$$[11] \quad \frac{\partial w}{\partial(u_a - u_w)} = \text{func}\,[(\sigma - u_a),\,(u_a - u_w)\,]$$

At a particular stress state, the compressibility modulus, m_2^w, for the water content constitutive surface, with respect to $(u_a - u_w)$, can be designated as a constant:

$$[12] \quad \frac{\partial w}{\partial(u_a - u_w)} = m_2^w$$

At present, there is no published equation to represent the entire water content constitutive surface. Once an appropriate equation is formulated, the derivatives will provide the soil moduli values corresponding to any stress state.

A portion of the entire water content constitutive surface has emerged to have of central importance in the development of unsaturated soil mechanics. This portion is the soil-water characteristic curve that relates water content to the applied soil suction under conditions where the net normal stress, $(\sigma - u_a)$, is zero or a small value (Fig. 6). The soil-water characteristic curve becomes a special case of the entire water content constitutive surface. Numerous mathematical equations have been proposed to represent the soil-water characteristic curve. The primary application for the soil-water characteristic curve has been in the estimation of unsaturated soil property functions. The most commonly used unsaturated soil property functions in engineering practice are those related to seepage and shear strength.
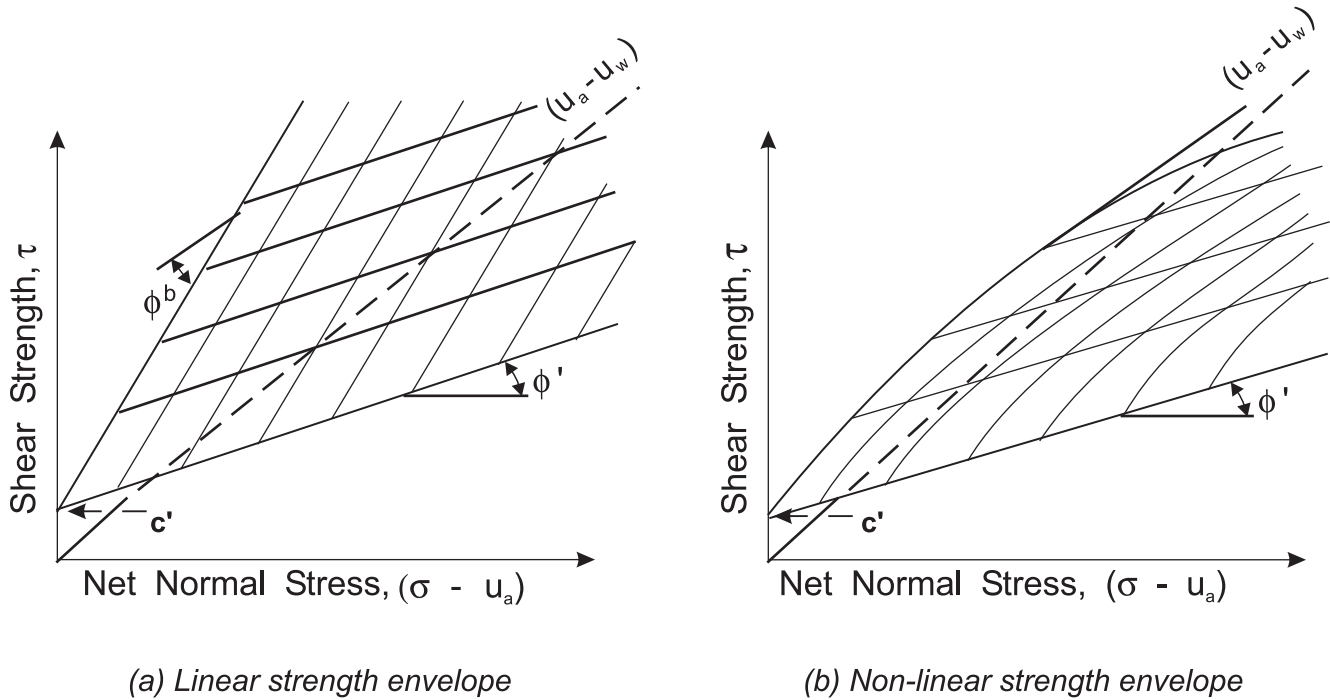
Fig. 7. A soil-water characteristic curve showing different stages of desaturation (modified after White et al. 1970).
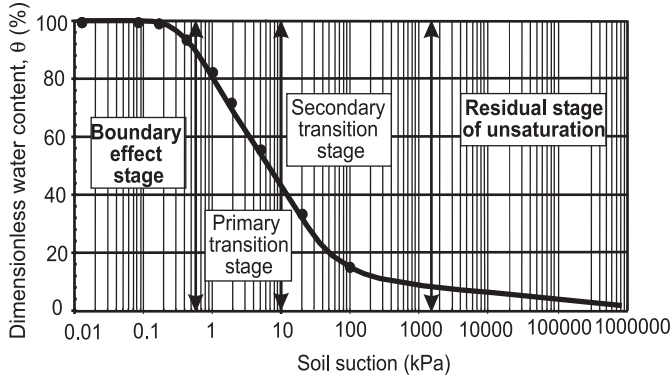
Fig. 8. Typical soil-water characteristic curves (i.e., desorption branches) for four soils from Saskatchewan, Canada.
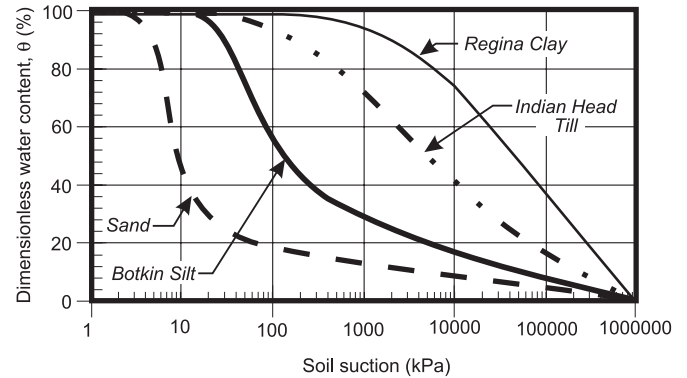
Fig. 9. Extended Mohr–Coulomb shear strength envelope showing the strength parameters for a saturated–unsaturated soil.

(a) Linear strength envelope

(b) Non-linear strength envelope

**Nature, characterization, and theory of the soil-water characteristic curve**

The soil-water characteristic curve has played a dominant role in the study of unsaturated soils in disciplines such as soil science, soil physics, agronomy, and agriculture (Barbour 1998). The soil-water characteristic curve is a relationship between the amount of water in the soil and soil suction. The amount of water in the soil is generally quantified in terms of gravimetric water content w, degree of saturation S, or volumetric water content θ. The results are plotted as matric suction in the lower suction range and total suction in the higher suction range, and usually the term *soil suction* is used as the abscissa of the plot. Typical features of the drying and wetting portions of the soil-water characteristic curves are defined in Fig. 6. The hysteresis loop associated with the wetting or drying of a soil is the first indication that the soil-water characteristic curve is not unique.

An initially saturated soil specimen begins to desaturate when it is subjected to soil suction beyond the air-entry value. Figure 7 shows the desaturation stages along the desorption branch of a soil-water characteristic curve (White et al. 1970). Similar stages apply to the adsorptive branch. There are three identifiable stages of desaturation, namely the boundary-effect stage, the transition stage (i.e., with primary and secondary transition stages), and the residual stage of desaturation. Typical desorption branches of the soil-water characteristic curves for several soils are shown in Fig. 8.

Shear strength constitutive relationship

The shear strength equation is a constitutive relationship defining the shear strength of a soil in terms of the stress state variables and soil properties. Fredlund et al. (1978) proposed a linear form for the shear strength τ of an unsaturated soil (Fig. 9a):

Fig. 10. Character of the permeability function defined in terms of soil suction.
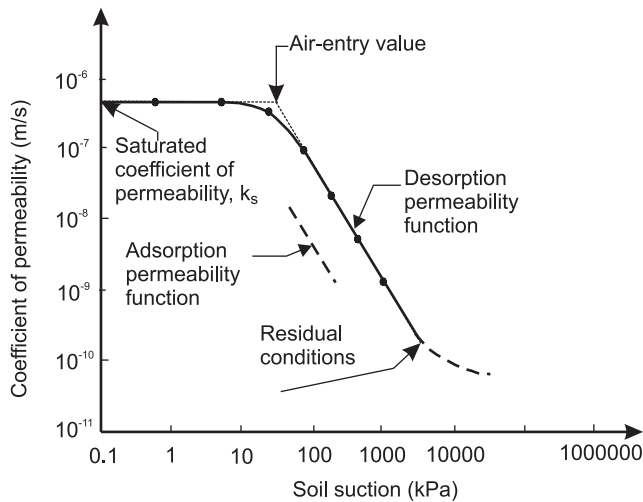
[13]　　$\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$

where

σ_n is the total normal stress on the failure plane at failure;
c' is the effective cohesion intercept;
ϕ' is the effective angle of internal friction; and
ϕ^b is the angle defining the rate of increase in shear strength with respect to soil suction.

The angle of friction associated with the soil suction variable, $(u_a - u_w)$, was originally assumed to be a constant soil parameter (Fredlund et al. 1978). However, further laboratory studies over a wide range of soil suctions have revealed that the friction angle should be written as an unsaturated soil property function (Fredlund et al. 1987, Gan et al. 1988):

[14]　　$\tan \phi^b = \text{func}[\tan \phi', (\sigma_n - u_a), (u_a - u_w)]$

The dimensionless water content of an unsaturated soil, (i.e., $w(u_a - u_w)/w_s$), is a function of the complete stress state but is generally simplified as a function of soil suction. Therefore, the increase in strength with respect to soil suction can be written in terms of the normalized water content used to describe the soil-water characteristic curve. It is also necessary to include an additional fitting parameter, p, to account for deficiencies in the one-to-one fit between water content and shear strength:

[15]　　$\tan \phi^b = \text{func}[\tan \phi', (w(u_a - u_w)/w_s), p]$

where

$w(u_a - u_w)$ is the water content at a particular soil suction;
w_s is the water content under saturated conditions; and
p is a soil-fitting parameter.

The shear strength surface becomes curvilinear in shape because the increase in strength changes nonlinearly with respect to suction. There are, however, mathematical equations that have been proposed and verified for the soil-water characteristic curve and as a result it is possible to write a closed-form equation for the shear strength constitutive surface for an unsaturated soil, as shown later in this paper.

Seepage constitutive relationship

The driving potential for the conductive flow of water through a saturated or unsaturated soil is the hydraulic head gradient, dh/dy (Childs and Collis-George 1950):

[16]　　$h = \dfrac{u_w}{\rho_w g} + Y$

where

h is the hydraulic head;
ρ_w is the density of water;
g is the acceleration due to gravity; and
Y is the elevation head.

The constitutive relationship to describe flow through a saturated or unsaturated soil is Darcy's law:

[17]　　$v = -k_w \dfrac{dh}{dy}$

where

v is the flow velocity over the discharge area;
k_w is the coefficient of permeability; and
y is the depth in the y direction.

The proportionality variable between velocity and hydraulic gradient is assumed to be a constant for saturated soils, k_s, but becomes a permeability function for an unsaturated soil. The coefficient of permeability of an unsaturated soil is a function of the amount of water in the soil which, in turn, can be written in terms of the stress state of the soil (Huang et al. 1998):

[18]　　$k_w = \text{func}[k_s, (\sigma - u_a), (u_a - u_w)]$

It is generally considered sufficient, however, to quantify the amount of water in the soil as a function of matric suction, $(u_a - u_w)$. The unsaturated coefficient of permeability can then be written as a function of the saturated coefficient of permeability and the normalized water content (Fig. 10). It is again necessary to include an additional fitting parameter, q, to complete the functional relationship:

[19]　　$k_w(u_a - u_w) = \text{func}[(w(u_a - u_w)/w_s), q, k_s]$

Numerous analyses have been proposed for the estimation of the permeability function for unsaturated soils (Fredlund et al. 1994; Leong and Rahardjo 1997b). Common to all methods is the existence of a mathematical relationship between the coefficient of permeability and the soil-water characteristic curve.

Formulation stage

The formulation stage involves combining the constitutive behavior of a material with the conservation laws of physics applied to an elemental volume. The result is generally a partial differential equation that describes a designated process for an element of the continuum. A two-dimensional, transient seepage partial differential equation for a saturated–unsaturated soil is shown to illustrate the formulation stage:
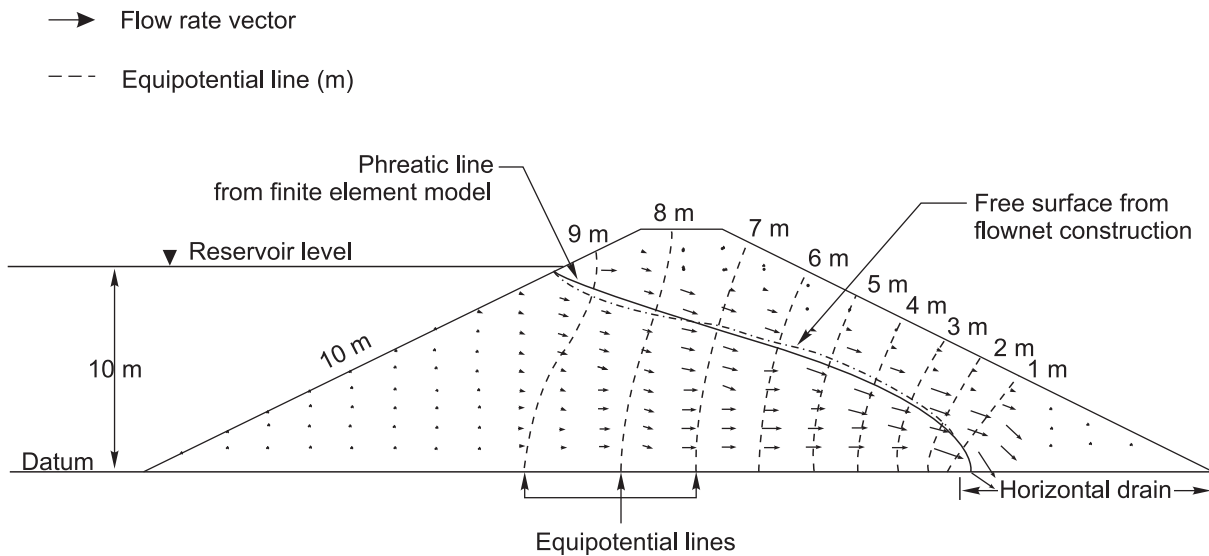
[20]　　$k_x \dfrac{\partial^2 h}{\partial x^2} + k_y \dfrac{\partial^2 h}{\partial y^2} + \dfrac{\partial k_x}{\partial x} \dfrac{\partial h}{\partial x} + \dfrac{\partial k_y}{\partial y} \dfrac{\partial h}{\partial y} = \dfrac{m_2^w}{\rho_w g} \dfrac{\partial h}{\partial t}$

where
m_2^w is the water storage coefficient;

Fig. 11. Two-dimensional, steady state solution of a saturated–unsaturated formulation of seepage through an earthfill dam.

k_x and k_y are permeability functions in the x and y directions, respectively; and

t is the time.

The partial differential equation is nonlinear because the coefficient of permeability in the flow law is a function of the pore-water pressure, which in turn is part of the hydraulic head. Therefore, an unsaturated soil is similar to a saturated soil with a constantly changing coefficient of permeability.

Appropriate boundary conditions must be placed on the region under consideration with the result that a "boundary-value problem" is defined. The physics at an elemental level is solved for all elements within the boundaries while at the same time satisfying the designated boundary conditions.

The ground surface moisture flux boundary condition has proven to be an important but challenging boundary to define for engineering purposes. The ground surface becomes the plane of interaction between the soil and the atmosphere. The local climatic conditions need to be converted into a net moisture flux at the ground surface. This problem has proven to be a particular challenge, but recent developments in this area have resulted in methodologies for new applications in geotechnical engineering (Wilson 1990). The design of soil covers is an example that illustrates the importance and use of ground surface moisture flux conditions.

Solution stage

The solution stage involves solving specific examples representative of a class of problems. At the solution stage, the partial differential equations are converted to a numerical solution that becomes known as a software package. An example of the solution stage is the analysis of seepage through an earth-fill dam using the finite element method (Fig. 11). Essentially the same type of solution can be used for a wide variety of seepage problems. The classic reference solutions have given rise to the general area of saturated–unsaturated seepage modeling.

There are generally accepted procedures for modeling a wide variety of partial differential equations. The most common procedure involves writing the partial differential equation in the weighted residual form and using the Galerkin

method to solve a series of linear equations. The weighted residual – Galerkin solution to the saturated–unsaturated, partial differential seepage equation can be written as an integral over the area and boundary surface of an element:

$$[21] \quad \int_A [B]^T [k_w] dA\{h_{wn}\} + \int_A [L]^T \lambda [L] dA \frac{\partial\{h_{wn}\}}{\partial t}$$
$$- \int_s [L]^T \bar{v}_w \, ds = 0$$

where

$[B]$ is the matrix of the derivatives of the area coordinates of a finite element;

A is the area of the element;

$[k_w]$ is the tensor of the water coefficients of permeability;

$\{h_{wn}\}$ is the matrix of hydraulic heads at the nodal points;

$[L]$ is the matrix of the element area coordinates;

$\lambda = \rho_w g m_2{}^w$;

$\bar{v}_w$ is the external water flow rate in a direction perpendicular to the boundary;

s is the perimeter of the element; and

T is the transpose of matrix.

The finite element procedure has become routine and there are computer programs dedicated to solving specific partial differential equations and general partial differential equation solvers (e.g., PDEase (MacSyma Inc. 1996) and FlexPDE (PDE Solutions Inc. 1999)). Linear or nonlinear partial differential equations can also be solved in a coupled or uncoupled manner using the general partial differential equation solver. These capabilities are particularly attractive for solving problems involving saturated–unsaturated soil systems that require mathematical functions to describe the soil properties.

The formulation and solution stages were the focus of considerable research during the 1980s. Numerical methods such as the finite element technique have become a necessary and routine tool for solving saturated–unsaturated problems in engineering practice.

Fig. 12. Illustration of "What if … ?" scenario with respect to the stability of a slope subjected to various magnitudes of steady state seepage.
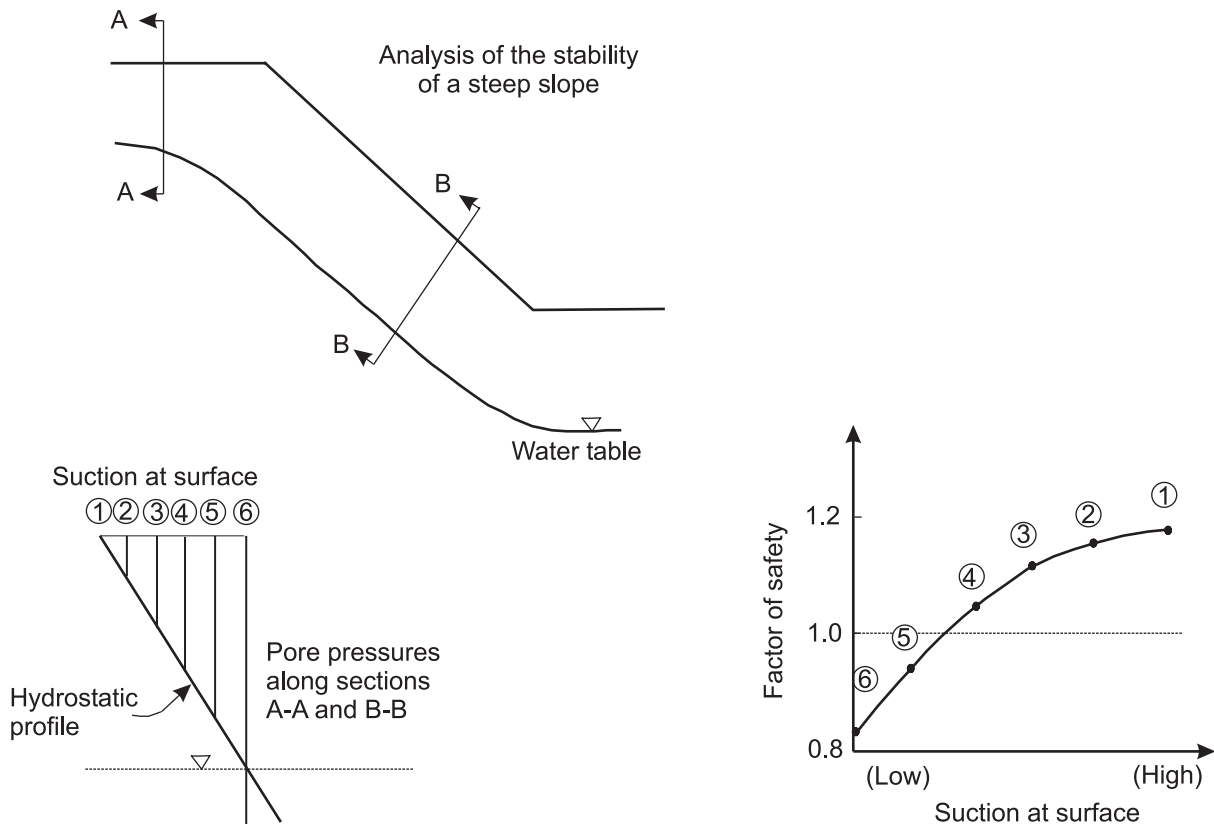
Design stage

There is a gradual increase in engineering confidence as research progresses from the formulation stage to the solution stage and on to the design stage. The design stage focuses on the primary unknowns that must be quantified from a practical engineering standpoint. The design stage generally involves a quantification of geometric and soil property variables that become part of an engineering design.

The computer has an important role in the design of earth systems and has changed the way in which geotechnical designs are conducted. The design stage generally takes the form of a parametric-type study. There are many variables, as well as ranges of variables, related to soil behavior which might need to be estimated or approximated. Each set of variables means another series of analyses. Eventually there is a matrix of solutions from which the engineer must select a design solution. The engineer repeatedly asks the question, "How would the design be affected if certain soil parameters were changed in the following manner?" Each "What if … ?" scenario can be studied in a matter of seconds through another run on the computer. In the final analysis, engineering judgement and experience must be used to decide upon the most suitable and preferable engineering design.

The design stage has taken the solution stage, along with other information, and determined the geometric (and other) variables required for the engineered structure. Figure 12 illustrates how a variety of possible pore-water pressure profiles can be analyzed to show the relationship between the stability of a slope and the amount of suction in the soil in the upper portion of the profile. Repeated studies for various slope geometries become part of the design stage.

There may be significant benefits associated with new engineering design procedures, but the momentum related to the use of past procedures is often hard to overcome. This is true even when the new design procedures are more reliable and accurate.

Verification and monitoring stage

There is need to "observe" the behavior of any infrastructure during and subsequent to construction to provide feedback to the designer. Only through field monitoring and feedback can confidence be firmly established in the design procedures. The "observational method" as defined by Peck (1969) goes even beyond the verification of design and is considered to be a part of the design process.

Case histories play an important part in the practice of geotechnical engineering. Many conferences have been held where geotechnical engineers report on investigative studies, soil testing programs, and the design procedures that have been used, along with an evaluation of the performance of the structure. These case histories are particularly necessary for situations involving unsaturated soils, just as they have proven necessary for saturated-soils cases. The limited number of case histories involving unsaturated soil conditions is one of the factors contributing to the slow implementation of unsaturated soil design procedures.

The engineer needs to have techniques that can be used to monitor, evaluate, and ensure the adequacy of the engineering design. The measurement of positive pore-water pressures is often used for this purpose when monitoring *saturated soils*. There is a similar need to measure in situ negative pore-water pressures when monitoring *unsaturated soils*. Negative

Fig. 13. Suction measurements using thermal conductivity sensors installed in the test track, Regina, Saskatchewan.
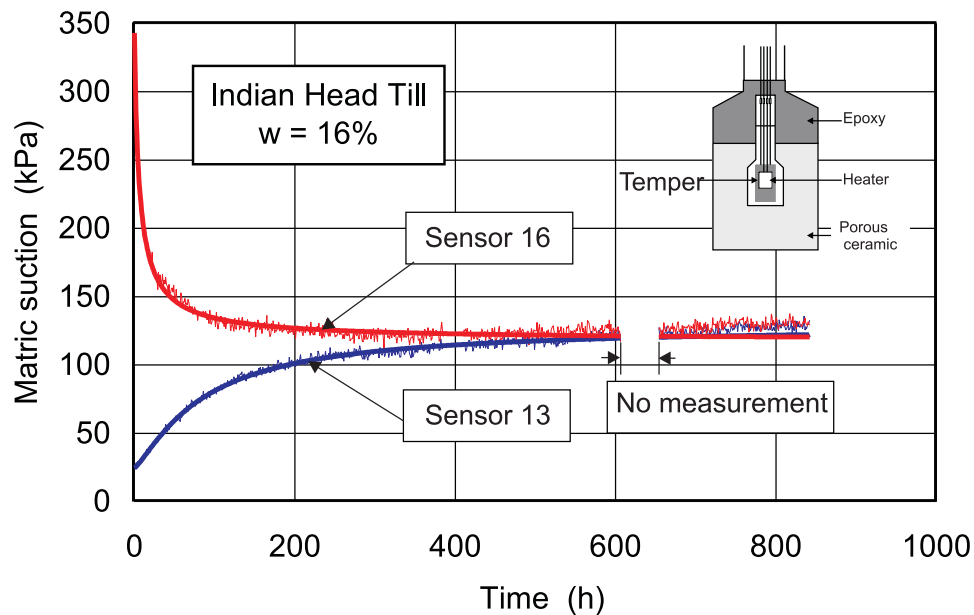
pore-water pressure measurements have proven to be a challenge to geotechnical engineers. There have been several new technological advances related to devices that can measure highly negative pore-water pressures. The thermal conductivity soil suction sensor has been improved in recent years and shows promise for use in engineering practice (Fredlund 1992).

Figure 13 shows a cross section illustrating the components of a thermal conductivity soil suction sensor. The thermal conductivity measurement on a standard ceramic that contains air and water is calibrated against applied matric suction. Recent thermal conductivity soil suction sensors have been shown to have durable ceramics and reliable electronics for the measurement of soil suctions as high as 1000 kPa over an extended period. Sensors can be initially wet or dry, and then installed into a sample of soil, or in situ. In Fig. 13, sensor 16 was initially dry and then inserted into the soil, while sensor 13 was initially water saturated. The final equilibrium suctions differed by about 45 kPa because one sensor (i.e., sensor 16) needed to absorb water from the soil when coming to equilibrium and the other sensor (i.e., sensor 13) underwent desorption in coming to equilibrium. It is possible, however, to measure the hysteresis associated with the drying and wetting of the ceramic sensor and to take the hysteretic effects into consideration in the calibration of the sensor. Figure 13 shows that the suction values become essentially equal once the effect of hysteresis is taken into account. Thermal conductivity sensors are receiving increasing usage in monitoring the performance of engineered structures involving unsaturated soils.

Implementation stage

The unsaturated soil mechanics area requires research on the subject of implementation. Failure to give serious study to the implementation stage will mean the loss of an opportunity to expand the scope of geotechnical engineering.

The implementation stage may not be realized in engineering practice even when the theoretical formulations and related design procedures have been fully studied and verified. Implementation is the final stage in bringing an engineering science into standard engineering practice. Other factors that need to be addressed at the implementation level are (*i*) the cost of undertaking any special site investigations, soil testing, and engineering analyses; (*ii*) the human resistance to change; and (*iii*) the political, regulatory, and litigation factors that may be involved.

The slowness in the implementation of unsaturated soil mechanics appears to be related to the cost of soil testing for the quantification of soil properties. The old soil mechanics paradigm involving the direct measurement of soil properties becomes extremely costly when measuring unsaturated soil property functions. However, there are a number of other procedures that provide a new paradigm for evaluating unsaturated soil property functions. These procedures differ in some respects from classical saturated soil mechanics procedures but provide the necessary accuracy required for analyzing most unsaturated soil mechanics problems (Fredlund 1996). Figure 14 provides a qualitative illustration of the benefits derived from using estimated unsaturated soil property functions. Estimates of the unsaturated soil property functions are shown to provide a significant increase in the accuracy of the engineered design, for a nominal increase at the soil investigation and testing stage. The accuracy of the output from an analysis depends strongly upon the independent variable being computed. The proposed procedures for the estimation of unsaturated soil property functions should not be considered as sufficient and satisfactory for all modeling situations.

Moving towards the implementation of unsaturated soil mechanics

The quantification of unsaturated soil property functions, more than any other single factor, becomes the key to the implementation of unsaturated soil mechanics in geotechnical engineering practice. As shown in the previous

Fig. 14. Qualitative representation of the benefits derived from using estimated unsaturated soil property functions.

Fig. 15. Approaches that can be used in the laboratory to determine the unsaturated soil property functions.
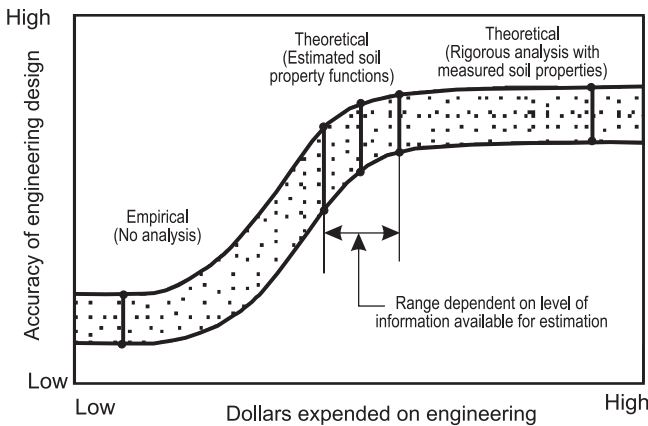
section, all of the stages related to bringing an engineering science into practice have been successfully addressed for unsaturated soil mechanics. The main remaining challenge is to determine economically viable procedures for the assessment of unsaturated soil property functions.

The estimation of unsaturated soil property functions provides a new philosophical framework that could greatly assist in the implementation of unsaturated soil mechanics. It is important not to remain in a fixed soil mechanics paradigm that is going to deter the implementation of unsaturated soil mechanics. The challenge is to find new procedures that can result in a more sound engineering approach with respect to the unsaturated soil portion of the profile.
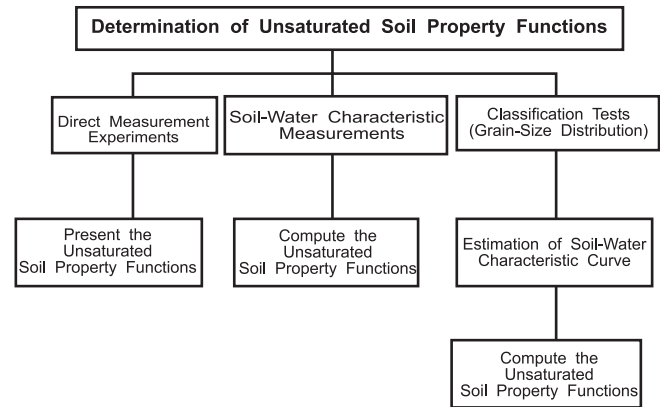
The upper portion of the soils profile plays a dominant role in water storage and the transmission of water to underlying soil strata. The shear strength and volume-change behavior of the unsaturated soil portion of the profile also change significantly in response to the ground surface moisture flux. To model the behavior of the upper portion of the soil strata it is necessary to be able to compute or estimate relevant unsaturated soil property functions.

Assessment of unsaturated soil property functions

One of several approaches can be taken to determine unsaturated soil property functions as shown in Fig. 15. Laboratory tests can be used as a direct measure of the required unsaturated soil property. Let us consider the determination of the shear strength properties of an unsaturated soil. For example, a modified direct shear test can be used to measure the relationship between matric suction and shear strength. These tests are costly to perform and the necessary equipment may not be available. However, it may be sufficient to perform an indirect laboratory test (i.e., pressure plate test) to obtain the unsaturated soil property function or it may be sufficient to estimate the function from the results of another test or from classification soil properties.

Measurement of the soil-water characteristic curve for a soil can be used as an indirect laboratory test to compute an unsaturated soil property function. The soil-water characteristic curve can then be used in conjunction with the saturated shear strength properties of the soil to estimate the relation-

ship between shear strength and soil suction to an acceptable level for most engineering projects.

Figure 15 also suggests the use of a classification test for the prediction of the desired unsaturated soil property function. A grain-size analysis can be used to estimate the soil-water characteristic curve, which is then used to determine the unsaturated soil property function (Fredlund et al. 1997). There may be a reduction in the accuracy of the estimated unsaturated soil property function when using this procedure. The engineer must assess whether or not the approximated unsaturated soil property function is satisfactory for the analyses to be performed.

Mathematical form for the soil-water characteristic curve

Several mathematical equations have been proposed to describe the soil-water characteristic curve. The equation of Gardner (1958) was originally proposed for defining the unsaturated coefficient of permeability function, and its application to the soil-water characteristic curve is inferred. The mathematical equations proposed by Burdine (1953) and Maulem (1976) are two-parameter equations that become special cases of the more general three-parameter equation proposed by van Genuchten (1980). These equations are asymptotic to horizontal lines in the low soil suction range and a suction beyond residual conditions. As such, these equations are not forced through zero water content at 1 000 000 kPa of suction. A correction factor, C_r, has been applied to the mathematical equation proposed by Fredlund and Xing (1994). The correction factor forces the soil-water characteristic curve function through a suction of 1 000 000 kPa at a water content of zero. Some of the common equations proposed for the soil-water characteristic curve are summarized in Table 1. A more complete summary of proposed equations can be found in Sillers (1997).

All of the proposed equations provide a reasonable fit of soil-water characteristic data in the low- and intermediate-suction ranges (Leong and Rahardjo 1997a). In all cases, the *a* parameter bears a relationship to the air-entry value of the soil and usually refers to the inflection point along the curve. The *n* parameter corresponds to the slope of the straight-line portion of the main desorption (or adsorption) portion of the soil-water characteristic curve.

Table 1. Summary some of the mathematical equations proposed for the soil-water characteristic curve.

Author(s)	Equation	Soil parameter
Gardner 1958	$w = \dfrac{w_s}{1 + \left(\dfrac{\psi}{a_g}\right)^{n_g}}$	$a_g,\ n_g$
Van Genuchten 1980	$w = \dfrac{w_s}{\left[1 + \left(\dfrac{\psi}{a_{vg}}\right)^{n_{vg}}\right]^{m_{vg}}}$	$a_{vg},\ n_{vg},\ m_{vg}$
Maulem 1976	$w = \dfrac{w_s}{\left[1 + \left(\dfrac{\psi}{a_m}\right)^{n_m}\right]^{m_m}}$	$a_m,\ n_m,\ m_m = 1/(1 - n_m)$
Burdine 1953	$w = \dfrac{w_s}{\left[1 + \left(\dfrac{\psi}{a_b}\right)^{n_b}\right]^{m_b}}$	$a_b,\ n_b,\ m_b = 2/(1 - n_b)$
Fredlund and Xing 1994	$w = C(\psi)\dfrac{w_s}{\left[\ln\left(e + \left(\dfrac{\psi}{a_f}\right)^{n_f}\right)\right]^{m_f}}$	$a_f,\ n_f,\ m_f,\ C(\psi)$

The Fredlund and Xing (1994) mathematical function applies over the entire range of soil suctions from 0 to 1 000 000 kPa. The relationship is essentially empirical and, similar to earlier models, is based on the assumption that the soil consists of a set of interconnected pores that are randomly distributed. Subsequent discussions on the soil-water characteristic curve are restricted to the Fredlund and Xing equation.

The Fredlund and Xing (1994) equation, written in terms of gravimetric water content, w, is as follows:

$$[22] \quad w = C(\psi)\frac{w_s}{\left[\ln\left(e + \left(\dfrac{\psi}{a}\right)^{n}\right)\right]^{m}}$$

where

w_s is the saturated gravimetric water content;

a is a suction value corresponding to the inflection point on the curve and is somewhat greater than the air-entry value;

n is a soil parameter related to the slope of the soil-water characteristic curve at the inflection point;

ψ is the soil suction (i.e., matric suction at low suctions and total suction at high suctions);

m is a fitting parameter related to the results near to residual water content;

e is the natural number, 2.71828… ; and

$C(\psi)$ is the correction function that causes the soil-water characteristic curve to pass through a suction of 1 000 000 kPa at zero water content.

The correction factor is defined as

$$[23] \quad C(\psi) = \left[1 - \frac{\ln\left(1 + \dfrac{\psi}{\psi_r}\right)}{\ln\left(1 + \dfrac{1\,000\,000}{\psi_r}\right)}\right]$$

where ψ_r is the suction value corresponding to residual water content, w_r. Residual suction can be estimated as 1500 kPa for most soils, unless the actual value is known.

Equation [23] can be written in a dimensionless form by dividing both sides of the equation by the saturated gravimetric water content (i.e., $\Theta = w/w_s$, where Θ is the dimensionless water content):

$$[24] \quad \Theta = C(\psi)\frac{1}{\left[\ln\left(e + \left(\dfrac{\psi}{a}\right)^{n}\right)\right]^{m}}$$

Equation [24] can be used to best-fit the desorption or adsorption branches of soil-water characteristic curve data over the entire range of suctions. The fitting parameters (i.e., a, n, and m values) can be determined using a nonlinear regression procedure such as the one proposed by Fredlund and Xing (1994).

The character of the Fredlund and Xing (1994) equation (eq. [24]) can be observed by varying each of the curve-fitting parameters (i.e., a, n, and m). Figure 16 illustrates the lateral translation of the soil-water characteristic curves as a result of varying the a parameter with the n parameter fixed at 1.5. Figure 17 illustrates the change in slope of the soil-water characteristic curve as a result of changing the n parameter with the a parameter fixed at 25 kPa. Figure 18 illustrates the rise in the soil-water characteristic curve as the

Fig. 16. Plot of the Fredlund and Xing (1994) equation with *m* and *n* constant and *a* varying.

Fig. 17. Plot of the Fredlund and Xing (1994) equation with *m* and *a* constant and *n* varying.
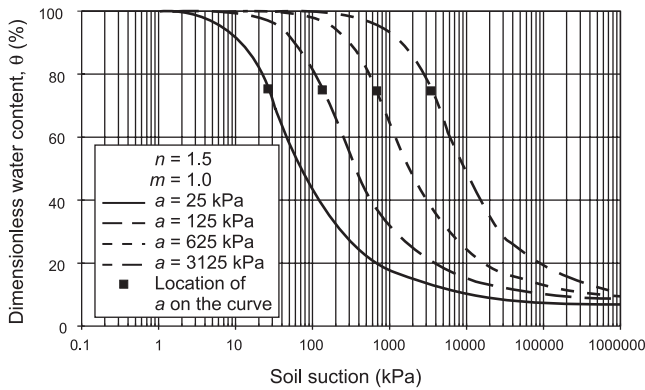
m parameter is varied. The correction factor, $C(\psi)$, was set at 1.0 for each of the three illustrative examples. If the correction factor, $C(\psi)$, is computed using eq. [23], each of the soil-water characteristic curves would pass through a soil suction of 1 000 000 kPa at zero water content.

Fig. 18. Plot of the Fredlund and Xing (1994) equation with *a* and *n* constant and *m* varying.

Laboratory measurement of the soil-water characteristic curve

The soil-water characteristic curve can be measured in the laboratory with relative ease. The test equipment is commonly found in soil science laboratories and has also found its way into some soil mechanics laboratories. The cost of performing the tests is somewhat less than the costs associated with performing a one-dimensional consolidation test.

The experimental measurement of the soil-water characteristic curve can be divided into two parts, namely the regions where the suctions are less than approximately 1500 kPa, and the regions where the suctions are greater than 1500 kPa. Suctions greater than 1500 kPa are generally established using an osmotic desiccator. The water content corresponding to high suction values (i.e., suctions greater than 1500 kPa) is determined by allowing small soil specimens to come to equilibrium in an osmotic desiccator containing a salt solution.

Water contents corresponding to low suction values are usually determined using an acrylic pressure plate device (often referred to as a Tempe Cell, Soilmoisture Equipment Inc., Santa Barbara, Calif.), with a 1 bar (100 kPa) high-air-entry disk. A second commercially available pressure plate device is the Volumetric Pressure Plate (Soilmoisture Equipment Inc. Santa Barbara, Calif.), which has a 2 bar (200 kPa) high-air-entry disk. A third pressure plate device is available (Soilmoisture Equipment Inc.) with a 15 bar (1500 kPa) ceramic or a pressure membrane. Each of the above pieces of equipment was designed for use in areas other than geotechnical engineering. As such, each apparatus has certain limitations, and several attempts have been made to develop an apparatus more suitable for geotechnical engineering. One such device is the pressure plate cell developed at the University of Saskatchewan, Saskatoon (Fig. 19).
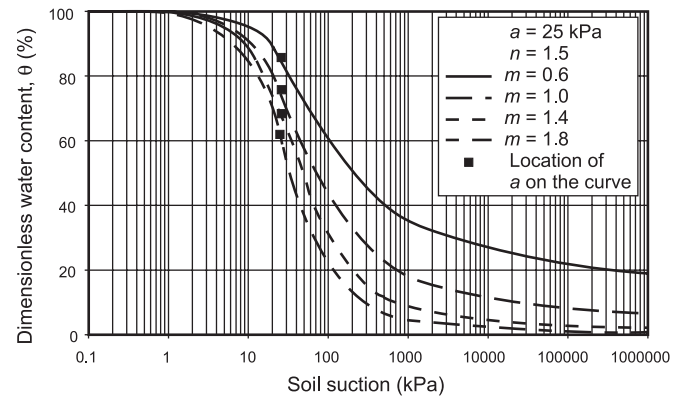
Test procedure for measuring the soil-water characteristic curve

The test procedure for measuring the soil-water characteristic curve was originally developed in soil science and
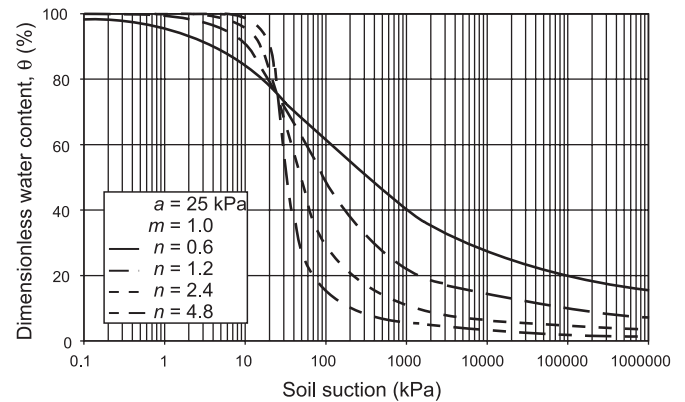
agronomy, but the test procedures adopted within geotechnical engineering have remained similar. The attention given to the initial preparation of soil specimens is quite different between geotechnical engineering and soil science. The agriculture-related disciplines have generally not paid much attention to the initial state (or structure) of the soil specimens. On the other hand, many of the soil mechanics theories used in geotechnical engineering are predicated on the assumption that it is possible to obtain undisturbed soil samples that can be tested to measure in situ physical soil properties. Soil-water characteristic curve data collected from a variety of sources will generally have used several different specimen preparation procedures.

Initial preparation states for a soil specimen can be categorized as follows: (*i*) undisturbed samples that retain the in situ soil structure, (*ii*) completely remolded specimens where the soil is mixed with water to form a semiliquid paste, and (*iii*) remolded and compacted samples where the initial water content is near to the plastic limit. Regardless of which of the above procedures is used, the soil specimens are placed into the pressure plate apparatus, covered with water, and allowed to saturate. Therefore, the soil suction is reduced to zero prior to commencing the formal test.

The difference between the various specimen preparation procedures may not be of great concern for a sandy soil but the opposite is true for a clayey soil. The soil structure and secondary macrostructure become of increasing concern for

Fig. 19. A single-specimen, pressure plate cell developed at the University of Saskatchewan, Saskatoon, Canada.
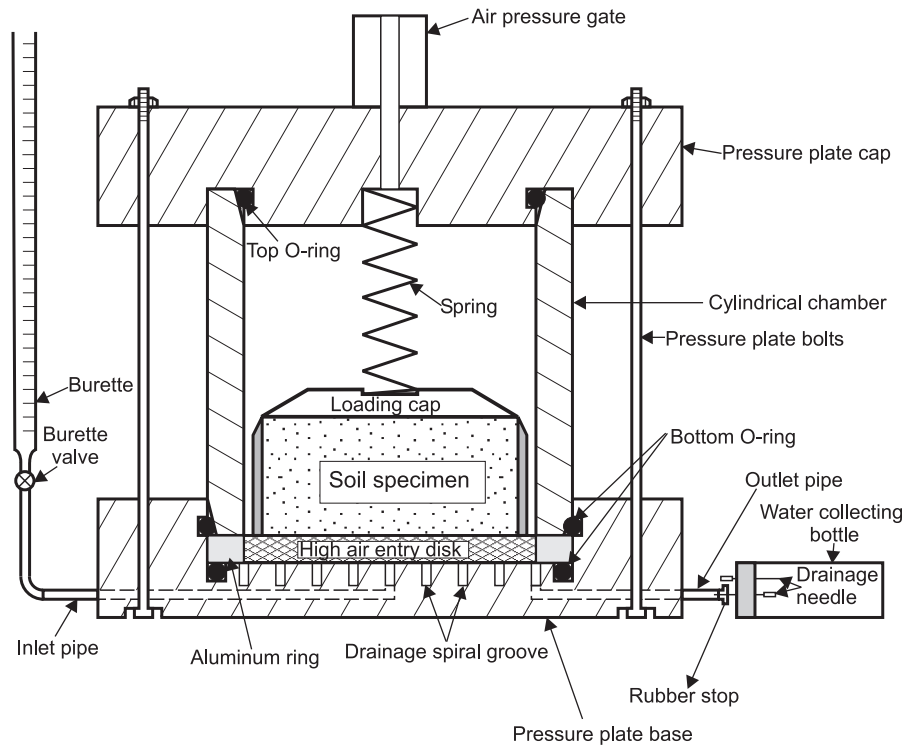
Fig. 20. Illustration of the influence of initial state on the soil-water characteristic curve.

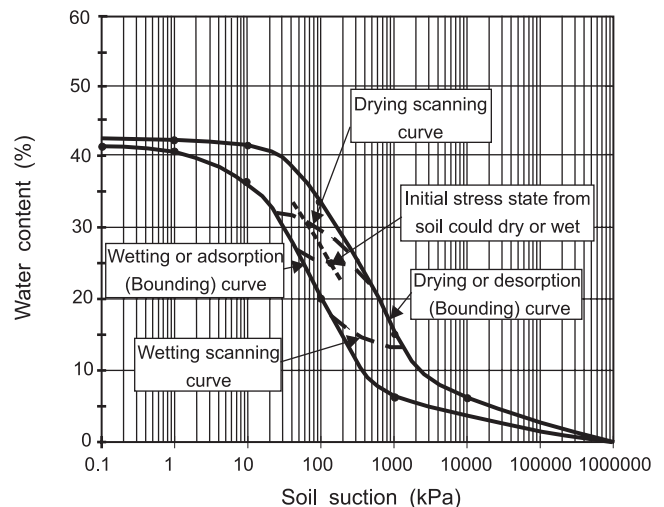**Fig. 21.** Description of wetting and drying scanning curves and initial stress state.
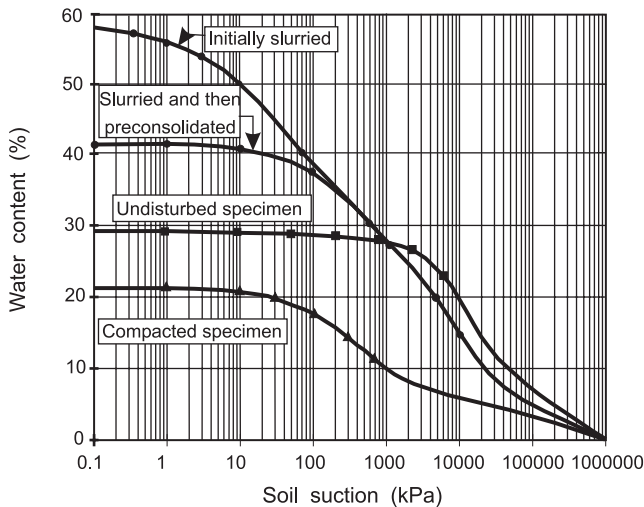
clay-rich soils. Figure 20 shows the effect that the above initial states can have on the desorption curves for a clayey soil. It is well recognized that there is no single or unique soil-water characteristic curve for a particular soil.

Interpretation of soil-water characteristic curve data

The hysteresis associated with the wetting and drying of a soil also illustrates that there is no unique soil-water characteristic curve. The wetting and drying branches form the extreme bounds for the soil-water characteristic curve. In

addition, there are an infinite number of intermediate (drying or wetting) scanning curves. The drying and wetting scanning curves become asymptotic to the bounding curves as shown in Fig. 21. An undisturbed soil sample from the field will have a soil suction that lies somewhere between the bounding curves. Let us assume that the effect of total confining stress on the wetting and drying curves is negligible. For interpretation purposes, the stress state can be on any one of the scanning curves or on either of the bounding curves.

Once the soil specimen is placed in the pressure plate apparatus, it is immersed in water and time is allowed for the

Fig. 22. Stress path followed at the start of a pressure plate test of a clayey soil.

Fig. 23. Stress path followed during the initial stages of pressure plate testing of a sandy soil.
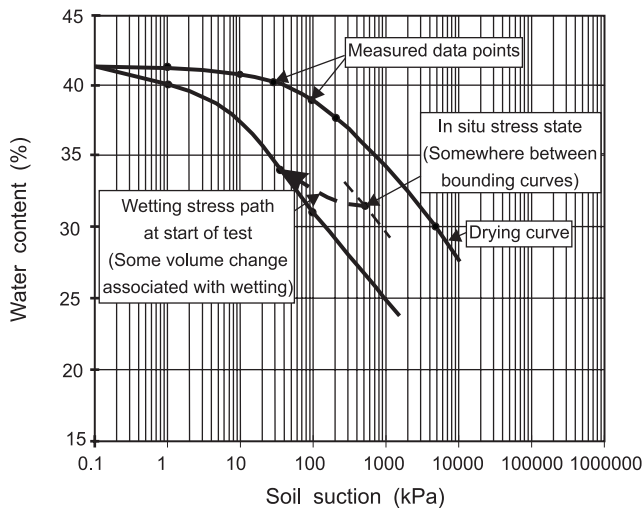

soil suction to go to zero. The stress state of the soil has now been altered and the stress path followed is shown in Fig. 22. If the soil is clayey, it is likely that the specimen will undergo a change in volume. Once the specimen is saturated, the test data for the soil-water characteristic curve is obtained by applying a series of suction values while allowing the specimen to come to equilibrium. In so doing, the desorption (bounding) branch of the water content versus soil suction relationship is measured. Figure 23 illustrates the change in state experienced by a sandy soil as it is wetted prior to testing. The effects of initial wetting have the appearance of being quite dramatic.

The changes in state experienced by the soil being tested have been outlined to assist in the interpretation of the test data. For example, it is obvious that the laboratory data do not represent a soil being dried from the in situ stress state. Rather, the laboratory-measured soil-water characteristic curve data correspond to a particular test procedure that does not accurately represent in situ conditions and may not represent conditions applied later in an engineering analysis. The fact that soil-water characteristic curve data have proven to be of significant value in engineering applications would indicate that a high level of accuracy is not required for the characterization of unsaturated soil property functions. The above description of the pressure plate test procedure also helps explain why the soil-water characteristic curve cannot be used to provide an indication of in situ suction.

It would appear to be difficult to propose a new test procedure for obtaining the soil-water characteristic curve data. Possibly there would be some merit in more accurately representing the in situ total stresses when performing the test. There appears to have been considerable success in using data obtained from the above-mentioned test procedure. It would appear reasonable to make as much use as possible of existing data sets and the present test procedure (Fredlund et al. 1996). It must be recognized that most of the existing data represent the desorption curve, subsequent to saturating a soil specimen. It would be of value to always record the details regarding specimen preparation.
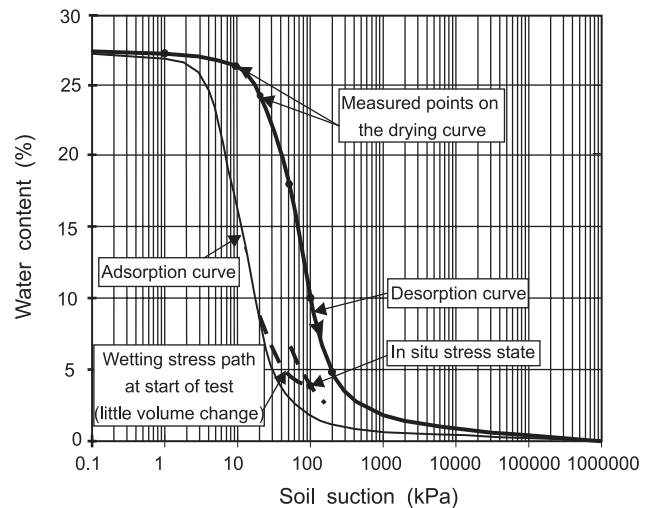
Estimation of the soil-water characteristic curve

One of several approaches can be used to estimate a soil-water characteristic curve. Some attempts have been made to correlate the fitting parameters for a soil-water characteristic curve with the plasticity and (or) grain-size distribution classification properties of a soil (Ahuja et al. 1985). These correlations are based on limited data, but further studies may prove these relationships to be of value in engineering practice.

The use of operations research techniques holds promise in the search for a suitable soil-water characteristic curve. A large volume of soil-water characteristic curve data has been collected in several disciplines (e.g., soil science, agronomy, agriculture, and engineering) and in many countries. A compiled database can be used to select an approximate soil-water characteristic curve. The grain-size distribution curves for a soil can be matched to other grain-size curves to select an approximate soil-water characteristic curve.

Figure 24 illustrates several approaches that can be used to obtain a soil-water characteristic curve that can subsequently be used for the determination of unsaturated soil property functions. The classification and (or) soil-water characteristic curves can be used in conjunction with a knowledge-based database to assist the user in arriving at a reasonable soil-water characteristic curve (Fredlund et al. 1996; Fredlund 1997).

The first suggested procedure compares a measured soil-water characteristic curves with soil-water characteristic curves already in the database. The measured soil-water characteristic curve can be used either to compute unsaturated soil property functions or to select unsaturated soil property functions already in the database.

The second suggested procedure involves matching measured classification properties (i.e., grain-size curves) with classification properties already in the database. Once one or more similar soils have been found, corresponding soil-water characteristic curves can be retrieved from the database. The soil-water characteristic curve data can be used to compute suitable unsaturated soil property functions, or existing

Fig. 24. Approaches that can be used to estimate soil-water characteristic curves for the determination of unsaturated soil property functions when using classification tests and a data base.
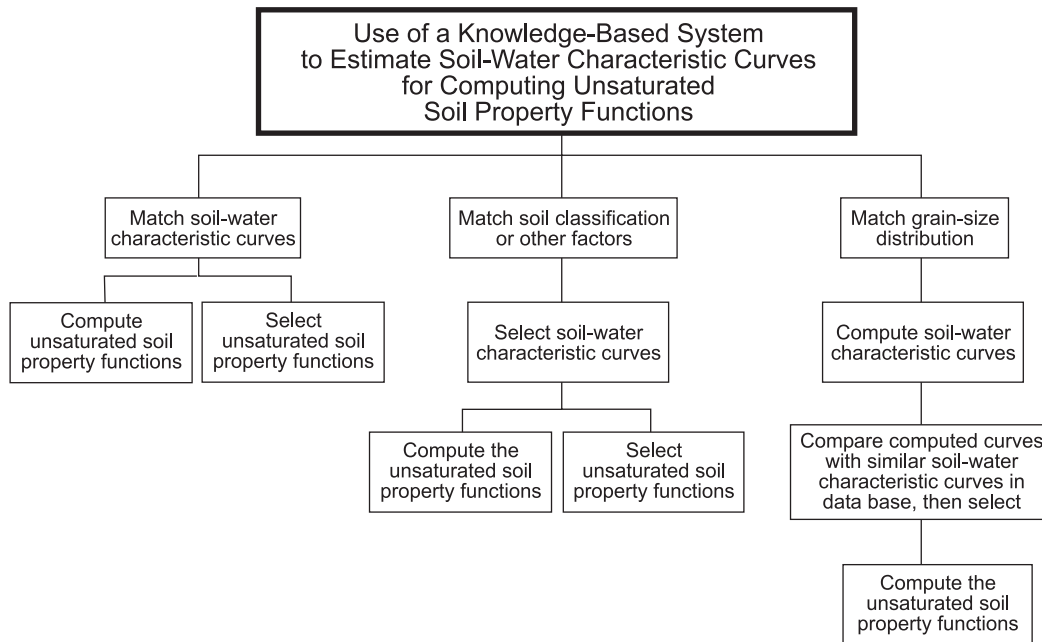
unsaturated soil property functions can be retrieved from the database.

The third suggested procedure makes direct use of the measured grain-size distribution curve. The grain-size distribution curve for a given soil is compared to grain-size curves already in the database. Soil-water characteristic curves can then be computed from the grain-size curves and compared to soil-water characteristic curves in the database. An engineering decision must be made regarding a reasonable soil-water characteristic curve and then the unsaturated soil property functions can be computed. In general, each of the above procedures becomes increasingly less precise in the estimation of the unsaturated soil property function.

The third procedure involves computing the soil-water characteristic curve directly from the grain-size distribution curve (Arya and Paria 1981). The mathematical equation that is used for describing the soil-water characteristic curve can also be used for fitting the grain-size curve. The form of the Fredlund and Xing (1994) equation for the soil-water characteristic curve can be modified to fit the grain-size distribution curve, since it has the ability to independently characterize the two extremes of the function (Fredlund et al. 1997). Three parameters and a correction factor are used to define the mathematical function:

$$[25] \quad P = C(d_r) \frac{100}{\left[\ln \left(e + \left(\frac{d}{a_s} \right)^{n_s} \right) \right]^{m_s}}$$

where

P is the percent passing;

100 represents 100% passing;

e is the natural number 2.71828… ;

d is the diameter of the particles in millimetres;

a_s is a parameter corresponding to the inflection point on the grain-size curve, related to the largest particles in the distribution (on a semilogarithmic scale);

n_s is a parameter related to the uniformity of the particle-size distribution;

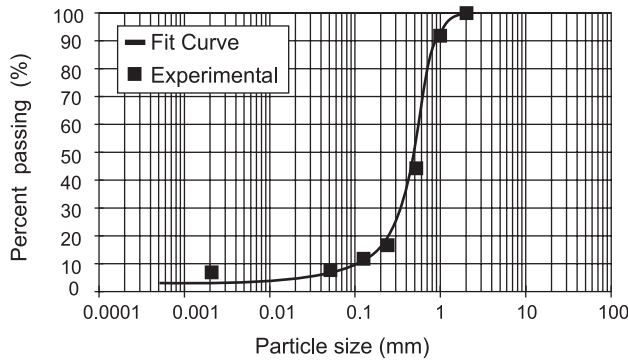m_s is a parameter related to residual particle sizes; and

$C(d_r)$ is a correction factor to ensure that the function goes through a lower limit particle diameter (e.g., 0.00001 mm).

The grain-size distribution curve parameters (i.e., a_s, n_s, and m_s) can be obtained by performing a best-fit regression analysis on particle-size data. The grain-size distribution data may have been measured in the laboratory or obtained from an existing database. The grain-size curve parameters allow the soil to be represented as a continuous mathematical function for further analysis. The grain-size distribution curve provides information on the pore-size distribution of the soil. This is true only to a degree, and the volume–mass properties must also be taken into consideration in the form of a "packing factor" (Fredlund et al. 1997).
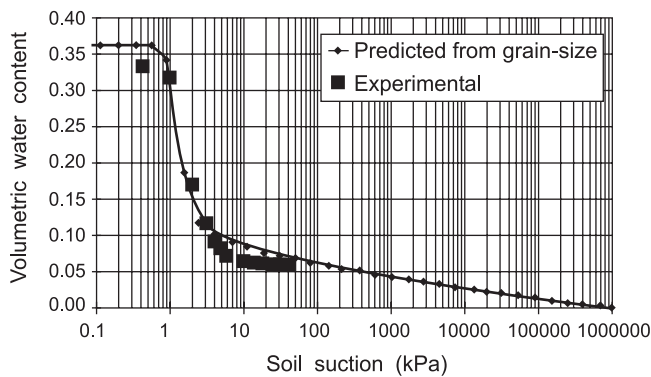
Mathematically characterizing the grain-size curve provides a function that forms the basis for a theory to estimate the pore-size distribution. Several attempts have been made to characterize the pore-size distribution from the grain-size curve (Gupta and Larson 1979; Ghosh 1980; Arya and Paria 1981; Ahuja et al. 1985; Haverkamp and Parlange 1986).

Fredlund et al. (1997) showed that it is possible to estimate the soil-water characteristic curve from a grain-size distribution curve provided the procedure is "trained" with the assistance of a knowledge base. Figure 25a shows a best-fit of the grain-size curve for a sand. This curve is then used to predict the soil-water characteristic curve that can be compared with experimental data (Fig. 25b). The results are encouraging for sands and silts, but more research is required when using this procedure for structured and clayey soils.

Fig. 25. Comparison between experimental and predicted soil-water characteristic curves for sand.

(a) Grain-size distribution for sand

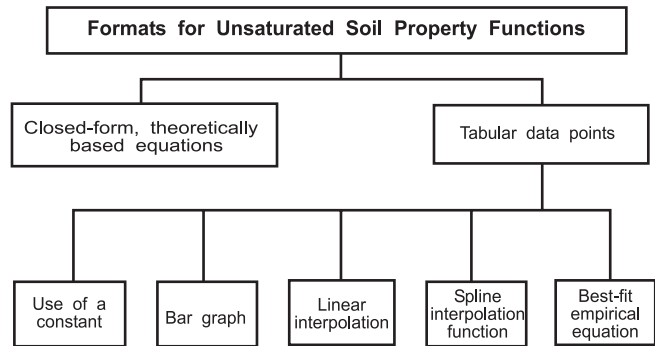(b) The direct soil-water characteristic curve for sand
(after Fredlund 1987)

Fig. 26. Summary of formats for presenting unsaturated soil property data.

The remaining sections illustrate the general forms for shear strength and seepage unsaturated soil property functions. A number of theoretically based unsaturated soil property functions have been presented for shear strength and seepage properties, but it is beyond the scope of this paper to summarize all the equations.

Shear strength of unsaturated soils

It is possible to mathematically represent the nonlinear form of the unsaturated shear strength envelope through the use of the soil-water characteristic curve and the saturated shear strength parameters of the soil. The angle ϕ^b begins to deviate from the effective angle of internal friction, ϕ', as the soil desaturates at suctions greater than the air-entry value (Fig. 28). As the soil suction reaches a value corresponding to the residual water content, ϕ^b appears to approach a value close to zero degrees (or it may even be negative).

The shear strength function can be obtained through the use of the soil-water characteristic curve. Fredlund and Xing (1994) proposed the following shear strength expression as a function of matric suction and the effective shear strength parameters:

$$[26] \quad \tau = (u_a - u_w)\Theta^p \tan \phi'$$

where
Θ is the normalized water content = $w(u_a - u_w)/w_s$;
$w(u_a - u_w)$ is the gravimetric water content at any suction, which can be represented by the equation for the soil-water of characteristic curve; and
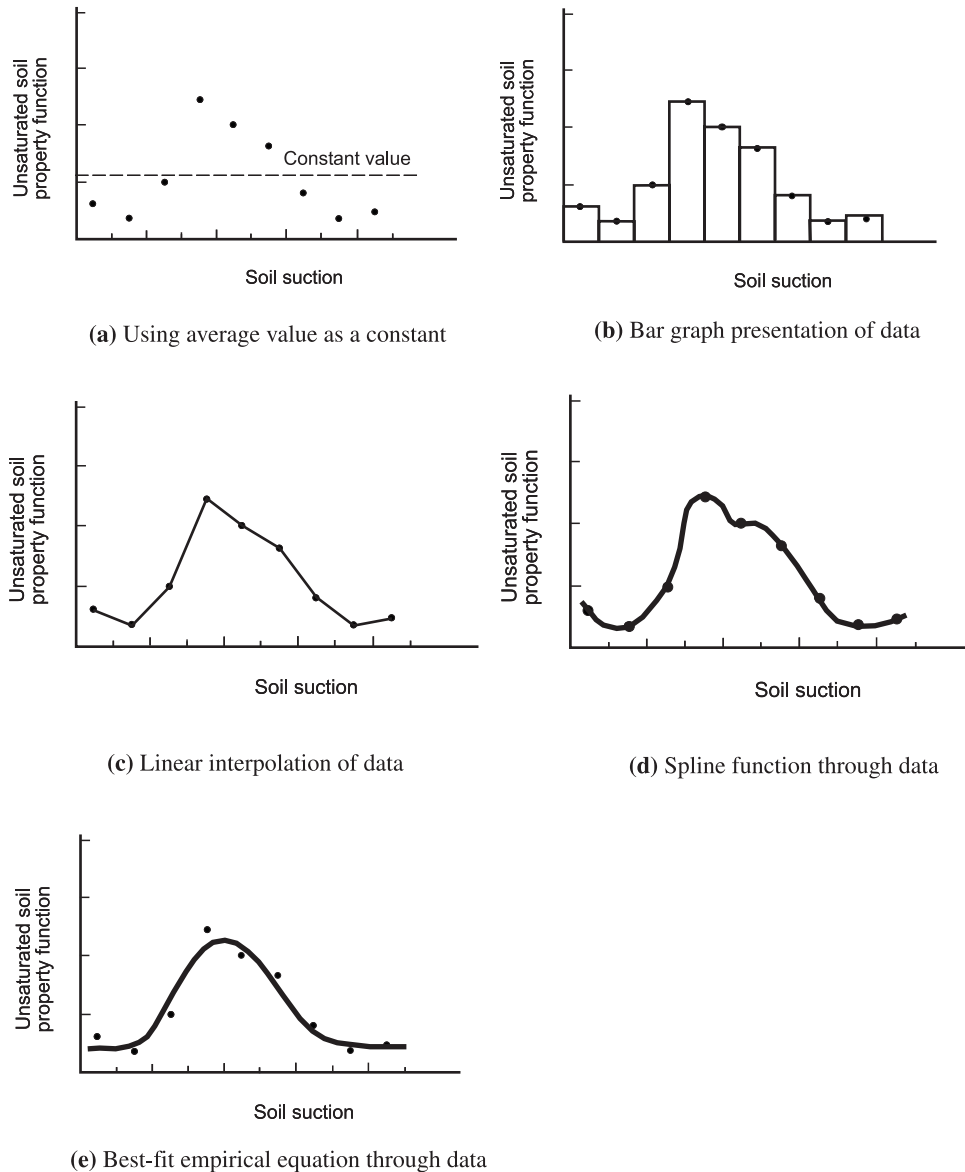p is a fitting parameter.

The unsaturated shear strength prediction depends on the fitting parameter p. A value of 1 can be assumed for p for most inactive soils such as sands, silts, and some fine-grained soils in the soil suction range between 0 and 500 kPa. Comparisons between predictions and measured shear strength data will provide a better understanding of the parameter p.

The entire shear strength equation for an unsaturated soil can be written as follows:

$$[27] \quad \tau = c' + (\sigma_n - u_a)\tan \phi' + (u_a - u_w)\Theta^p \tan \phi'$$

The unsaturated soil now appears to have one friction angle, ϕ', but the area over which soil suction acts (i.e., water phase) is reduced as the suction increases. The soil-water characteristic curve quantifies the amount of water on any section through the soil.

Application of unsaturated soil property functions

The unsaturated soil property functions take on a variety of mathematical forms for shear strength, seepage, and volume change. The general graphical form is consistent for a particular unsaturated soil property. However, there are a variety of mathematical functions that can be used to represent a particular unsaturated soil property. A particular class of geotechnical problems may require several mathematical functions.

It is beyond the scope of this paper to present all proposed unsaturated soil property functions. Also, it is not always possible to present the soil property functions as simple closed-form solutions. Figure 26 illustrates some common formats that can be used for unsaturated soil property functions. The functional data is used as input when solving an engineering problem. A closed-form, theoretically based equation is the preferable format for unsaturated soil property functions. It is not always possible to present the data in this format and therefore other formats must be used.

Figure 27 illustrates the different ways in which tabular data can be input to an analysis. In some cases, a bar-graph format may be satisfactory; however, in general it is better to fit the data points with a meaningful empirical equation. The spline function may pass through the data points, but the best-fit empirical equation may produce more meaningful results with less convergence difficulties in a subsequent numerical analysis.

Fig. 27. Various formats for inputting tabular data when analyzing a problem.

(a) Using average value as a constant

(b) Bar graph presentation of data

(c) Linear interpolation of data

(d) Spline function through data

(e) Best-fit empirical equation through data

Seepage through unsaturated soils

The nature of the coefficient of permeability function can be visualized by comparing its shape to that of the soil-water characteristic curve. Figure 29 shows typical plots of the soil-water characteristic curve and the coefficient of permeability function for a sand and a clayey silt. The coefficient of permeability for both soils remains relatively constant until the air-entry value of the soil is reached. Beyond this point, the coefficient of permeability decreases rapidly for both soils.

The function appears to remain essentially linear until the residual suction of the soil is reached. Beyond this point, the coefficient of permeability appears to remain essentially constant but there are a lack of data to confirm this portion of the function. The relationship between soil suction and the coefficient of permeability of an unsaturated soil (i.e., permeability function) can be predicted with sufficient accuracy for many engineering problems through a knowledge of the saturated coefficient of permeability and the soil-water characteristic curve.

Childs and Collis-George (1950) proposed a statistical model to predict the coefficient of permeability based on a random variation of pore sizes. This model was modified by Marshall (1958) and further modified by Kunze et al. (1968). These calculations are performed by dividing the volumetric water content versus suction relationship into several water-content increments. This is equivalent to integration with respect to volumetric water content.

The accuracy of the prediction of the coefficient of permeability function depends not only on the closeness of the best-fit curve to the experimental soil-water characteristic curve data, but also on the prediction model adopted. Mualem (1986) concluded that there was no single model that fits every soil type. The proposed models have been found to be most satisfactory for sandy soils, whereas agreement with experimental data may prove to be unsatisfactory

Fig. 28. Relationship between soil-water characteristic curve and shear strength for a sand and a clayey silt.

Fig. 29. Typical permeability functions for a sand and a clayey silt with the suction plotted on a logarithmic scale.
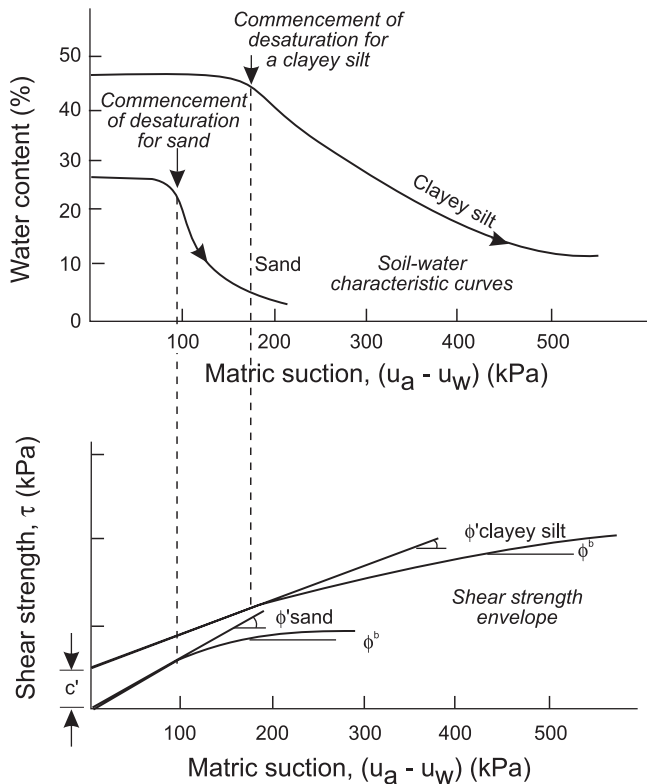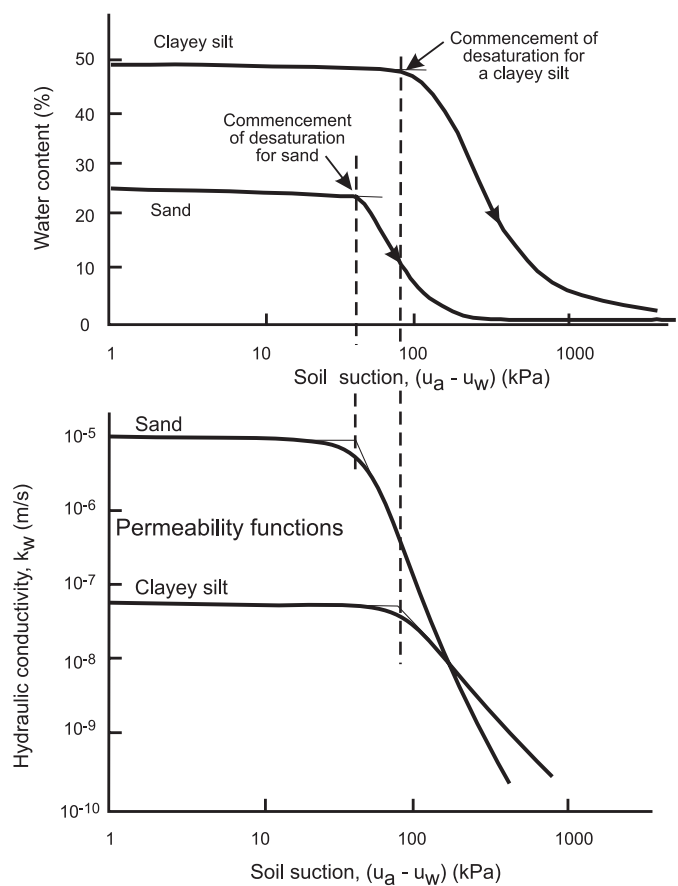

for finer grained soils. The accuracy of the permeability function prediction can be improved by incorporating a correction factor for tortuosity, Θ^q (Mualem 1986). The integration form of the permeability function is as follows:

$$[28] \quad k(\psi) = \frac{\Theta^q(\psi) \displaystyle\int_{\ln(\psi)}^{\psi_r} \frac{w(e^y) - w(\psi)}{e^y} w'(e^y) \, dy}{\displaystyle\int_{\ln(\psi_{aev})}^{\psi_r} \frac{w(e^y) - w_s}{e^y} w'(e^y)}$$

where

e is the natural number 2.71828… ;
$k(\psi)$ is the coefficient of permeability at any soil suction;
y is a variable of integration representing the logarithm of section;
w' is the derivative of the soil-water characteristic curve;
ψ is the variable of integration representing soil suction;
ψ_{aev} is the soil suction corresponding to the air-entry value;
ψ_r is the soil suction corresponding to residual water content; and
q is a correction factor to account for tortuosity and produce close fits between experimental data and the theory.

The value of the power, q, can be assumed to be 1 unless there is reason to assume otherwise (Kunze et. al. 1968). The numerical integration of eq. [28] can be performed using the procedure of numerical integration presented by Fredlund et al. (1994). A series of data points are computed that can be placed into the form of a table of soil suction versus coefficient of permeability. This information can be used in one of the forms suggested in Fig. 27.

Leong and Rahardjo (1997b) showed that results similar to the above integration procedure (i.e., eq. [28]), can be obtained using the Fredlund and Xing (1994) equation for the soil-water characteristic curve along with one additional fitting parameter. The suggested extension to the soil-water characteristic curve equation is as follows:

$$[29] \quad k(\psi) = C(\psi) \frac{k_s}{\left[\ln\left(e + \left(\dfrac{\psi}{a} \right)^n \right) \right]^{mq}}$$

The soil parameters in the permeability function are now the same as those in the soil-water characteristic curve. The fitting parameter, q, can be assumed equal to 1 for silts and sands. Comparisons between predictions and measured data sets will provide a little understanding of the parameter q.

The procedure suggested by Leong and Rahardjo (1997b) provides a form similar to that used for shear strength and allows the user to more clearly visualize the role of the soil-water characteristic curve. In each case, the soil parameters for the soil-water characteristic curve are used in the unsaturated soil property function.

The water storage function is required in conjunction with the coefficient of permeability function when performing a transient flow analysis in an unsaturated soil. The water storage function can be defined as the slope of the soil-water characteristic curve. The differentiation of the equation for

Fig. 30. Typical coefficient of water storage, m_2^w, function computed from the slope of the soil-water characteristic curve for a coarse sand.

a)   Soil-water characteristic curve

b)   Coefficient of water storage function
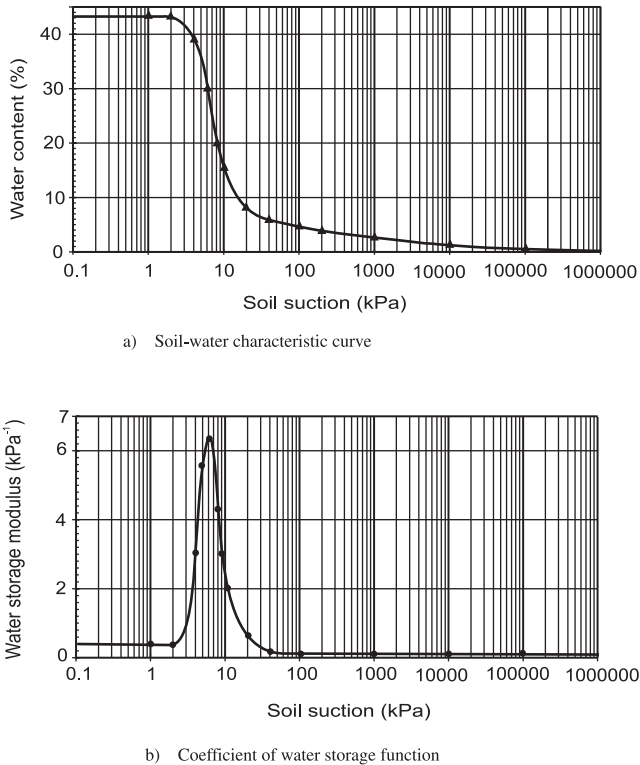
the soil-water characteristic curve yields the water storage function for an unsaturated soil. Figure 30 shows a soil-water characteristic curve along with a plot of the water storage function. The water storage function becomes highly nonlinear as the soil desaturates.

Ground surface moisture flux boundary conditions

Discussions regarding the implementation of unsaturated soil mechanics would be incomplete without some mention of the paramount role to be played by the ground surface interaction with the atmosphere. The solutions require real-time moisture and temperature flux boundary conditions.

The dynamic nature of climatic conditions makes the ground surface flux conditions difficult to quantify. However, the climate must be quantified, since design conditions require answers to questions that are strongly controlled by weather conditions. The performance of a soil-cover system is predominantly controlled by climatic conditions (Wilson 1998). A soil-cover system can be specifically designed to accommodate virtually any type of climatic conditions. The analytical tools required for the design of a soil-cover system involve the coupling of two nonlinear partial differential equations. The moisture-flow portion of the analysis is subjected to a changing net moisture flux condition. One of the partial differential equations is for computing moisture flux and it is coupled with a partial differential equation for thermal analysis. Design technologies for soil-cover systems that have emerged over the past one or two decades provide an

Fig. 31. Cover design and associated soil-water characteristic curves for a tailings impoundment in an arid climatic environment (Wilson 1998).
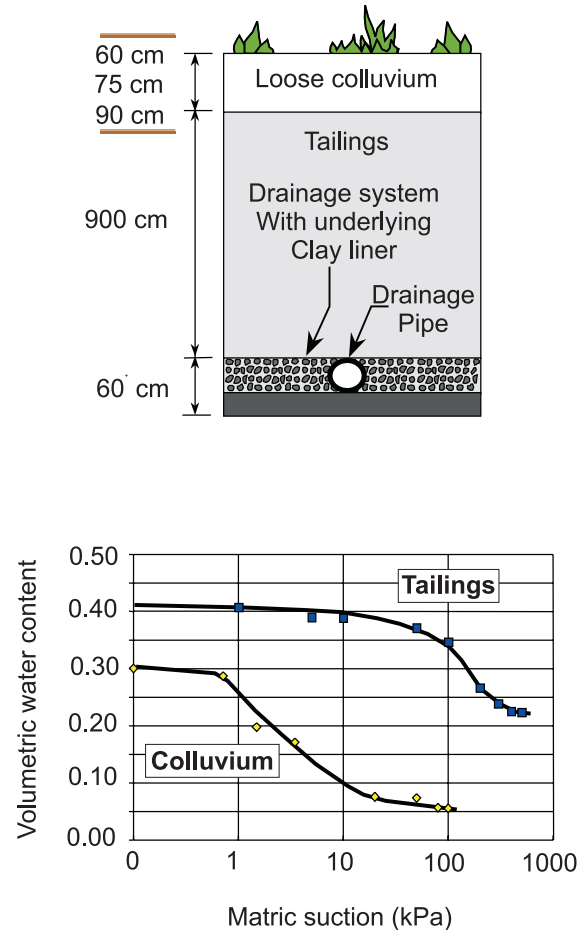
impressive example of how the climate conditions can be quantified and, along with unsaturated soil properties, yield the primary information required in soil-cover design. Soil-cover design procedures have been established for a number of climatic conditions, namely (*i*) arid climates, (*ii*) temperate regions, (*iii*) high-rainfall terrains, and (*iv*) tropical monsoon conditions with wet and dry seasons (Wilson 1998). In all cases a coupled soil–atmospheric numerical model (i.e., SoilCover; see Wilson 1997) implements the theoretical aspects of heat and mass flow (Wilson et al. 1994, 1997).

The theoretical analysis for the design of soil-cover systems requires two key pieces of information: (*i*) an assessment of net moisture flux conditions, and (*ii*) an assessment of the soil-water characteristic curve information for each soil involved in the problem. In the case of a cover for a reclamation of a tailings site, a soil-water characteristic curve is required for the tailing material and for potential cover materials. The key characteristic of the soil-water characteristic curve that must be examined are (*i*) the air-entry value, and (*ii*) the residual conditions. These states are defined in terms of a water content and a soil suction stress state. Figure 31 shows the soil-water characteristic curves for the tailings and the proposed cover material to be used at an arid climatic site (Wilson 1998). The upward rate of evaporation was to be controlled by the colluvium, which had an air-entry value

Fig. 32. Illustration of the effect of climate conditions on the pore-water pressure profile near the ground surface.
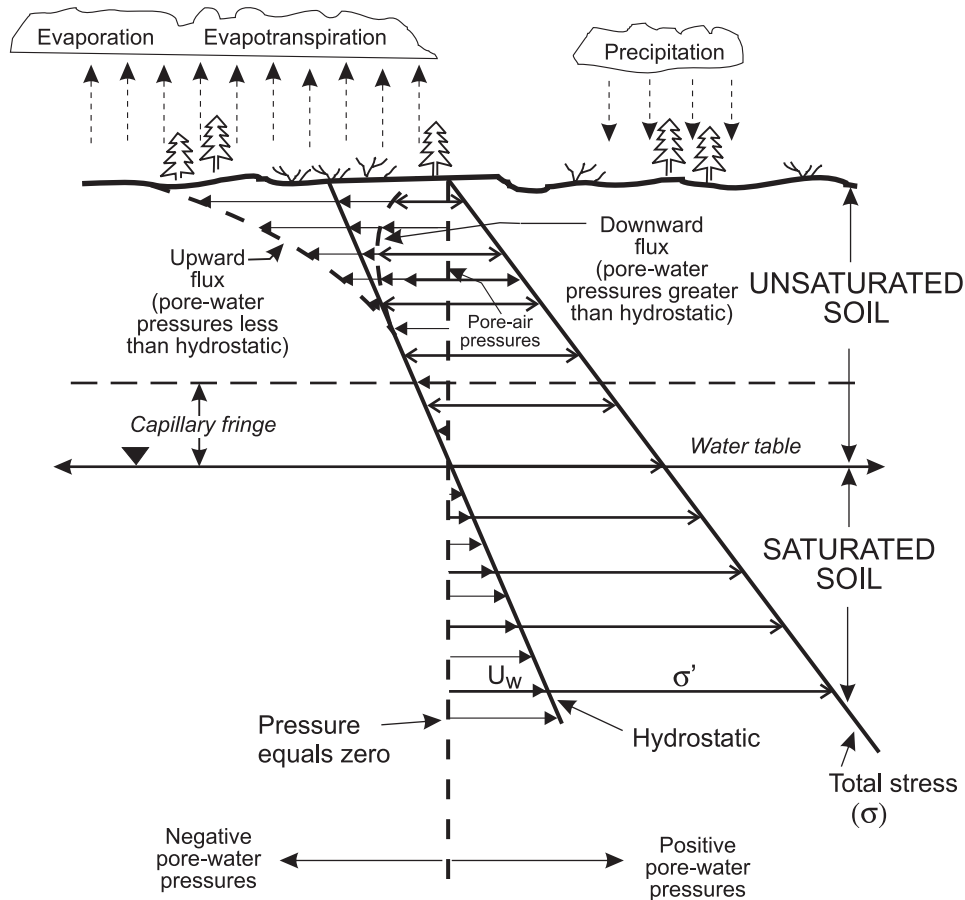
of about 1 kPa and a residual soil suction of 10 kPa. These proved to be excellent soil properties to minimize the upward migration of water from the tailings in an arid environment.

The quantification of the moisture flux boundary conditions arises from a study of the water balance at the ground surface. Precipitation enters the soil surface through the process of infiltration. Precipitation must be quantified through the statistical analysis of weather station data. Alternatively, soil water leaves the soil surface as water vapor through the process of evaporation and evapotranspiration. The actual evaporative flux can be computed based on a soil–atmospheric model (e.g., SoilCover; see Wilson 1997) that requires thermal data input in addition to information on the soil-water characteristic curve for the soil. This is the primary input information required; however, other data are also required.

Surface hydrologists have mainly focused on predicting *potential* evaporation from a water surface (Hillel 1980), but it is the *actual* evaporation from the soil surface that is required when assessing the ground surface moisture flux. It is also the *actual* evaporative flux that controls the negative pore-water pressure profile below the ground surface, as shown in Fig. 32.

Wilson (1990) provided experimental evidence to confirm that the rate of evaporation from a soil surface was uniquely related to the suction in the soil. This relationship proved to be essentially unique for sand, silt, and clay soils (Fig. 33).

The finding provided important confirmation for the development of the theoretical soil–atmosphere numerical model.
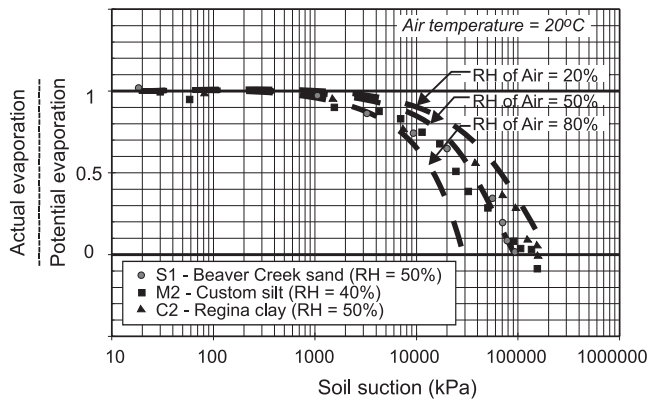
Figure 34 shows a plot of the components of ground surface flux for a monsoon dry–wet climate setting (Wilson 1998). The figure shows the difference between the potential and actual evaporative flux components. The analysis is for the case of bare tailings. There is considerably more information that can be obtained from the design procedure, but the focus remains on the key elements of input for design.

In summary, it is the soil-water characteristic curves that become the primary information required to assess the actual evaporative moisture flux and the storage and hydraulic conductivity characteristics of the cover materials. All materials involved in the design must be analyzed as unsaturated soils and appropriate unsaturated soil property functions need to be assessed. It is important to be able to characterize the unsaturated soil property functions in an economical and reasonably accurate manner for engineering design purposes.

Steps towards implementation of unsaturated soil mechanics

The primary stages leading towards the implementation of unsaturated soil mechanics into geotechnical engineering were outlined at the start of the paper. Substantial research has been undertaken at all stages and the emphasis is for engineers to take advantage of the potential for applying unsaturated soil mechanics to all classes of geotechnical

Fig. 33. Relationship between soil suction and the ratio of the actual evaporation to the potential evaporation from the ground surface (Wilson et al. 1994).

Fig. 34. Water balance analysis using SoilCover for bare tailings in a tropical wet–dry climatic setting (Wilson 1998).

problems. The soil-water characteristic curve, even with its limitations and assumptions, appears to hold the key to an economical implementation of unsaturated soil mechanics.

Further research is required on many aspects of unsaturated soil mechanics. Listed below are suggested areas of research and other needs that would encourage the implementation of unsaturated soil mechanics.

(1) There are limitations in unsaturated soil theories at various stages leading towards implementation. Research should be directed towards better understanding and clarifying the issues involved.

(2) The research community needs to encourage and facilitate the application of research findings in unsaturated soil mechanics wherever possible.

(3) The "observational method" needs to be promoted more often when working with unsaturated soils. Monitoring of the performance of structures will advance our understanding and assume a greater confidence level in unsaturated soil mechanics.

(4) There needs to be more technical training and a general upgrade in the understanding of unsaturated soil behavior. Along with the training should come a clearer understanding of the array of potential applications in geotechnical engineering.
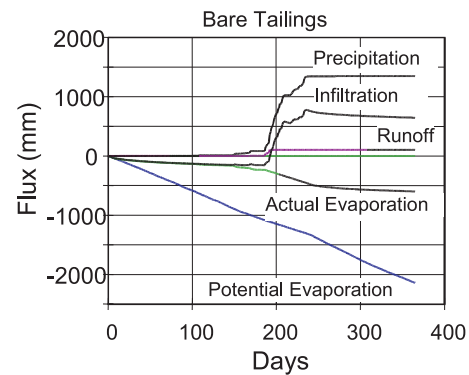
(5) There needs to be a focus on research related to implementation procedures. For example, the procedures need to be defined for the assessment of climatic conditions as it pertains to establishing moisture flux boundary conditions. The same is true for procedures related to the assessment of unsaturated soil property functions and design criteria.

(6) There needs to be a focus on various classes of geotechnical problems and the role of unsaturated soil mechanics. Some classes of problems are slope stability, coupled stress and seepage analyses, and the prediction of heave and collapse. There also need to be studies devoted to specific near-ground-surface infrastructure elements, such as pipelines, roads, and sidewalks. Unsaturated soil mechanics has an important role to play in the engineering of all structures forming a part of the infrastructure.

Acknowledgements

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