Benchmarking Multi-Dimensional Large Strain Consolidation Analyses

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ABSTRACT

Analyzing the consolidation of tailings slurries and dredged fills requires a more extensive formulation than is used for common (small strain) consolidation problems. Large strain consolidation theories have traditionally been limited to 1-D formulations. SoilVision Systems has developed the capacity to analyze large strain consolidation problems in 2 and 3-D. The benchmarking of such formulations is not a trivial task. This paper presents several examples of modeling large strain consolidation in the beta versions of the new software. These examples were taken from the literature and were used to benchmark the large strain formulation used by the new software. The benchmarks reported here are: a comparison to the consolidation software application CONDES0, Townsend's Scenario B and a multi-dimensional analysis of long-term column tests performed on oil sands tailings. All three of these benchmarks were attained using the SVOffice suite.

INTRODUCTION

The consolidation of soil/liquid slurries plays a large role in the design of tailings impoundments and dredged fills. It is necessary to be able to predict the settlement of the material over time, as well as changes in the strength and hydraulic characteristics of the material.

This paper presents several models used to benchmark the consolidation features available in a beta version of the SVOffice suite of SoilVision Systems. It provides the results from a series of initial benchmarks consisting of 1-D models to test the validity of the 1-D formulation before it is applied to 2-D and 3-D models. The benchmarks included: a comparison of results obtained using the program CONDES0 to those from SVOffice [1]; Townsend's Scenario B representing a mature tailings deposit that is filled intermittently [2]; and a multi-dimensional analysis of the results obtained from long term, column testing of oil sands tailings [3]. A variety of formulations used to model the effective stress-void ratio curve were examined, as well as different methods of filling, including:

- Instantaneous and staged filling
- Use of various effective stress curves including: power function, extended power function and the Weibull function
- Extremely high initial void ratios at the time of deposition

Large Strain Consolidation

The sedimentation and consolidation of soft soils is a complex process that has challenged researchers over much of the last century. Like much of soil mechanics, engineers must use their judgement and make approximations in order to estimate consolidation of soils. Karl Terzaghi established the first general formulation when he developed his Theory of Consolidation [4]. Terzaghi assumes that hydraulic conductivity remains constant and that settlement is small.

In 1967, Gibson, England and Hussey refined the formulation to allow for large strain consolidation where the layers are able to move and where hydraulic conductivity isn't assumed to be constant [5]. Their formulation (equation 1) is written in terms of void ratio, similar to Terzaghi's original formulation.

$$+\left(\frac{\rho_s}{\rho_f}-1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right]\frac{\delta e}{\delta z} + \frac{\delta}{\delta z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{d\sigma'}{de}\frac{\delta e}{\delta z}\right]+\frac{\delta e}{\delta t}=0 \qquad (1)$$

Where ρ_s is the density of the solid phase, ρ_f is the density of water, e is the void ratio, k(e) is the hydraulic conductivity written in terms of void ratio, z is the reduced Lagrangian height coordinate, σ' is the effective stress and t is time

The advantage of using the void ratio as a basis of calculation is that the effective stress and hydraulic conductivity can be written in terms of the void ratio based upon the results obtained from oedometer and other laboratory consolidation tests. Formulations for the effective stress relationship are generally written as void ratio in terms of stress. Common formulations include: the Power Function (equation 2), the Extended Power Function (equation 3) and the Weibull Function (equation 4). The Hydraulic Conductivity is generally provided as a Power Function (equation 5) with hydraulic conductivity written in terms of void ratio.

$$e = A(\sigma')^B \qquad (2)$$

$$e = A(\sigma' + Z)^B \qquad (3)$$

$$e = A - B\exp(E\sigma'^{F}) \quad (4)$$

$$k = Ce^{D} \quad (5)$$

Lagrangian Coordinate System

The Gibson formulation is based upon a Lagrangian coordinate system [5]. Most soil mechanics analyses uses the Eulerian method whereby the plane of reference is fixed and the rate of flux and soil movement are measured [6]. With regards to consolidation, this becomes difficult as the volume of the slurry will change over time as water leaves the system decreasing the void ratio and thus the overall volume.

This is solved using the Lagrangian system whereby a fixed volume of dry soils is used as a basis of measurement. The soil within the volume remains constant, while the volume of water and the height of that column vary over time. The z term is used to indicate the unit volume of the dry soil fraction of the slurry. Figure 1 shows how z remains constant while the slurry height, a, varies with void ratio.

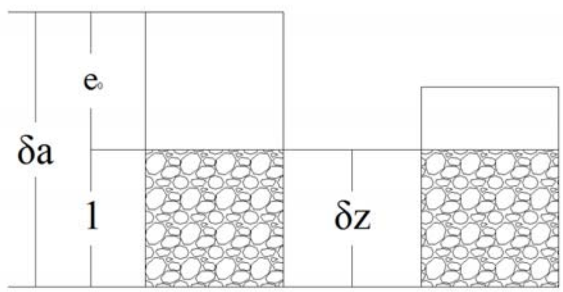


Figure 1 - Comparison of Eulerian and Lagrangian Coordinate Systems [*after* 6]

SVOffice Consolidation Software

SVOffice uses a coupled deformation flux analysis to solve large strain problems. By coupling their deformation package (SVSolid) with the flux package (SVFlux), they were able to solve for large strain applications. The program requires much the same input data as is required for the Gibson, England and Hussey formulation but performs the calculations using a stress-strain formulation rather than a void ratio term representing the volume [7].

This has proven to be extremely useful when solving for consolidation problems that occur in multidimensional space as it accounts for the multidimensional aspects of both groundwater flux and deformation. This was not possible with the previous formulations.

In using a stress-strain formulation it is required to specify a value of Poisson's ratio as additional input to the analyses. This parameter is not considered in other consolidation formulations, with the exception of Cryer, 1963 [9]. This formulation allows for the calculation ofthe excess porewater pressure due to the so-called Mandel Cryer effect at various Poisson's ratios. The details of this, including the selection of appropriate values of this parameter, are part of the ongoing research at this time.

2-D and 3-D Consolidation Modeling

Until recently there have been relatively few cases where multi-dimensional consolidation has been investigated. While one-dimensional analyses are considered suitable approximations they fail to take into account several key factors in the process. These include [7]:

- Lateral flux
- Sidewall boundary conditions
- Mandel-Cryer effect
- Spatially variable material properties

Fredlund et al. (2009) presents several preliminary analyses and benchmarks of multidimensional modeling including [7]:

- Comparison between coupled and uncoupled formulations
- Townsend et al.'s (1990) Scenario A [2]
- Mandel Cryer effect in 2-D
- An idealised 2-D model of tailings deposited into a pit

BENCHMARKING

CONDES0

CONDES0 is a software application compiled in 1997 to solve the 1-D consolidation and desiccation of tailings materials [1]. It has been applied in a variety of situations and is considered to reliably model 1-D large strain problems for one material under various boundary and filling scenarios.

GEOMETRY AND BOUNDARY CONDITIONS

For this benchmark a one-dimensional column of material with a height of 10.0 m is used. The material in the column is assumed to be deposited in five 2 meter increments every 200 days for a period of 800 days. This is followed by a further period of consolidation of 200 days. It is assumed that the slurry is homogeneous at the time of deposition. Figure 2 shows the model geometry.

The top boundary is allowed to deform, while the bottom boundary is fixed. Flow is only allowed out of the system through the top surface. As this is a 1-D model no lateral flux or deformation is allowed. The water level is held constant at a height of 10.0 m.

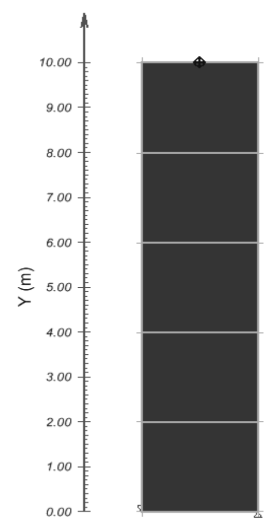


Figure 2 - Example geometry

MATERIAL CHARACTERISTICS

The material characteristics are assumed based on typical slurry characteristics in the literature. CONDES0 requires that the effective stress-void ratio relationship be defined in terms of the extended power function (3). For this function it is assumed that the initial void ratio occurs at the boundary between sedimentation and consolidation and at an effective stress which is theoretically the lowest that should occur for the system. The Z term is defined as this effective stress at the initial void ratio [8]. Equation 6 provides the power function used for the analysis.

The hydraulic conductivity and void ratio relationship is defined using an ordinary power function, as shown in equation (equation 7). These effective stress-void ratio and hydraulic-conductivity-void ratio functions are shown in Figure 3a and b.

$$e = 4(\sigma' + 0.615)^{-0.08} \, (kPa) \tag{6}$$

$$k = 6 \times 10^{-5} e^{4.0} \, (m/day) \tag{7}$$

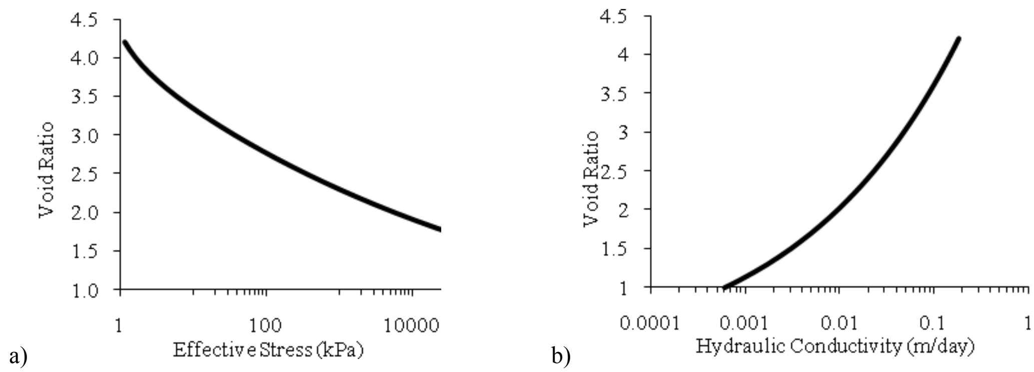


Figure 3 – a) Effective Stress – Void Ratio Curve b) Hydraulic Conductivity - Void Ratio Relationship

As indicated above SVOffice requires the user to define the Poisson's ratio for the materials. For this 1-D benchmark it was found that a theoretical maximum Poisson's ratio of 0.495 provided the most comparable results between CONDES0 and SVOffice.

RESULTS AND DISCUSSIONS

The results of the 1-D benchmark are shown in Figures 4 and 5. Figure 4 shows the change in material column height over time. The CONDES0 and SVOffice results are almost identical. A total consolidation of 1.8 meters is predicted after a period of 1000 days. At this time the final void ratio is 3.1 as shown on Figure 5.

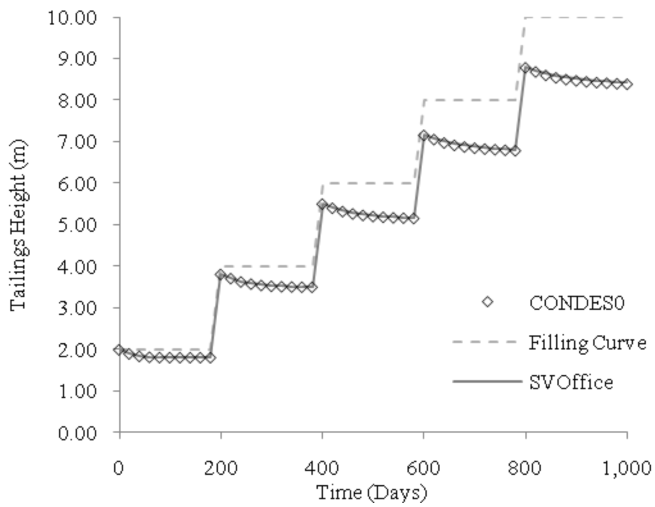


Figure 4 - Column Height Comparison between SVOffice and CONDES0

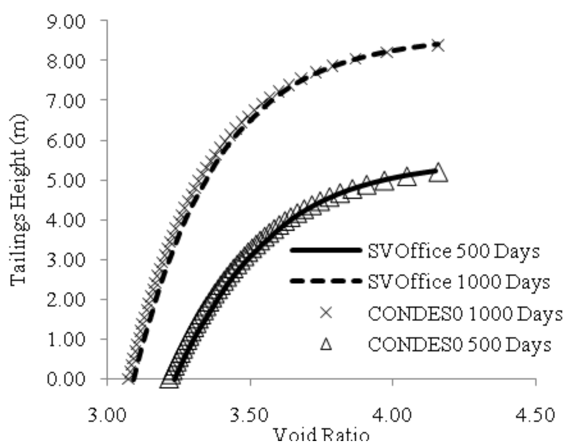


Figure 5 - Void Ratio Profile Comparison at 500 and 1000 Days

Townsend Scenario B

In 1989 Townsend and McVay present a study on predicting large strain consolidation of materials under a number of scenarios [2]. Researchers were given four consolidation scenarios and were asked to come up with an estimate of several outcomes, including tailings height as a function of time and time-dependent pore pressure distribution in the profile. Scenario A is the basic case of instantaneous deposition of a material, which is then allowed to consolidate under its own self weight. This scenario was previously modeled in SVOffice by Fredlund et al (2009) [7].

Scenario B is more complicated as it is set up to evaluate the consolidation in a 7.2 m deep pond filled in two 6 month stages (with a 6 month latency period between) and at different initial void ratios. This scenario simulates mature tailings ponds that have been filled intermittently with phosphate slurries.

GEOMETRY AND BOUNDARY CONDITIONS

The model consists of two 3.6 m lifts of clay tailings from a phosphate mine placed over a period of 6 months, with a 6 month latency period between lifts. The initial lift is placed at a uniform void ratio

(e_o) of 14.8 and allowed to consolidate for a period of six months. The second lift is placed at an initial void ratio of 22.8. The column is allowed to drain only through the top surface requiring a no flow boundary on the bottom and sides of the column. Deformation is completely fixed (x, y and z directions) on the bottom plane and fixed horizontally (x and y directions) along the vertical boundaries. Initial conditions are defined by the water level, which is maintained at 8.2 m for the entire trial. Figure 6a) shows a diagram of the model, while Figure 6b) presents the filling curve.

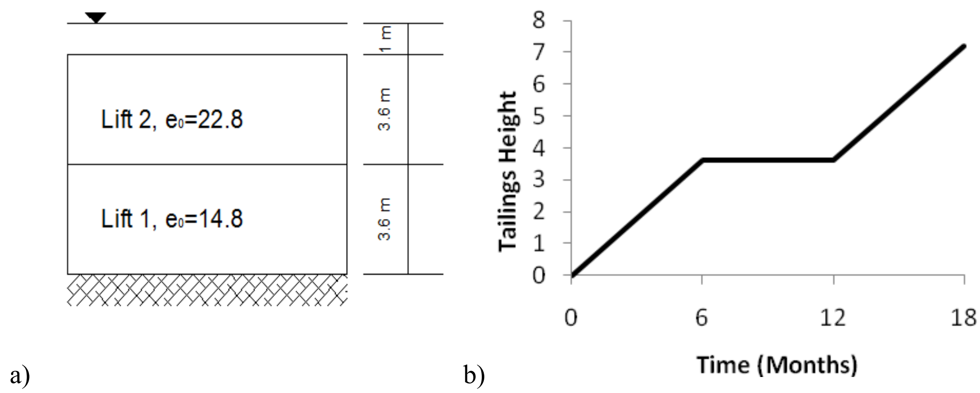


Figure 6 - Townsend's Scenario B a) Diagram and b) Filling Curve [*after* 2]

<u>MATERIAL CHARACTERISTICS</u>

The tailings slurry consists of a clay mixture commonly found in the phosphate industry. The material characteristics are provided using a power function relating effective stress to void ratio (equation 8) and another relating hydraulic conductivity and void ratio (equation 9). The relations are graphed in Figures 7a and 7b respectively.

$$e = 7.72(\sigma')^{-0.22}\,(kPa) \tag{8}$$

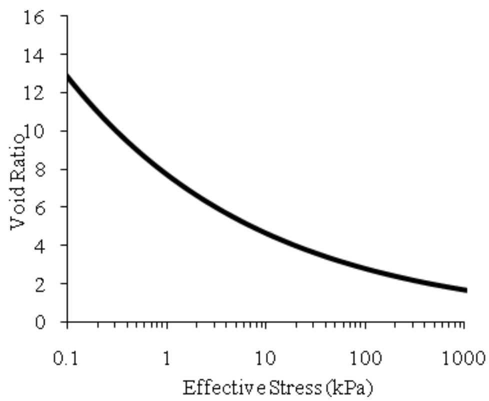
$$k = 2.5\times10^{-7} e^{4.65}\,(m/day) \tag{9}$$

Townsend's Scenario B is a challenging problem for several reasons. It occurs at high initial void ratios, meaning that even small changes in the effective stress can cause large deformations. The first lift is placed at a void ratio of 14.8, which based upon the effective stress relationship would occur at an effective stress of 0.052 kPa. The second lift is placed at a void ratio of 22.8, which corresponds to an effective stress of 0.0073 kPa. Also, since the bottom boundary is fixed, all flow occurs in the upwards direction, while the material is undergoing large deformations downwards. A Poisson's Ratio of 0.495 was chosen for this scenario as it provided the best fit to the literature results.

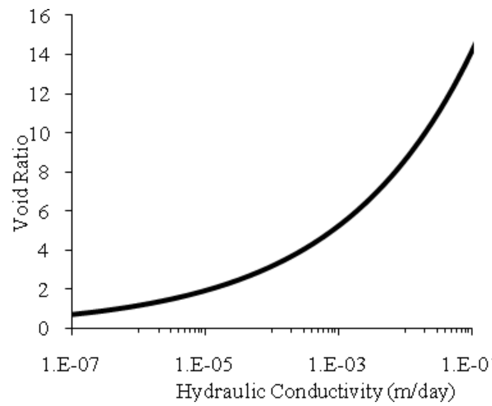
<u>RESULTS AND DISCUSSIONS</u>

The numerical models compiled within Townsend and McVay's paper covered a wide variety of large strain consolidation solution methodologies used in both academia and industry. Models varied in terms of coordinate system, dependent variables, solution technique and methodology [2]. In spite of this, fairly consistent results were obtained when they were applied to the same scenario. Two exceptions were the NWTRN and McGILL models, which are believed to be caused by a misinterpretation of filling data [2]. The WES model required an approximation to solve for the staged filling nature of the scenario. An equivalent column was devised where material at a void ratio of 18.81 was placed in two instantaneous lifts one year apart [2].

Figures 8 to 10 compare the results found by Townsend et al. (various symbols) and those obtained in SVOffice (solid line). The SVOffice results provide a very good fit of the responses provided by the various researchers. It is concluded that SVOffice can model these results very well.



a)



b)

Figure 7 – a) Effective Stress – Void Ratio Curve b) Hydraulic Conductivity - Void Ratio Relationship

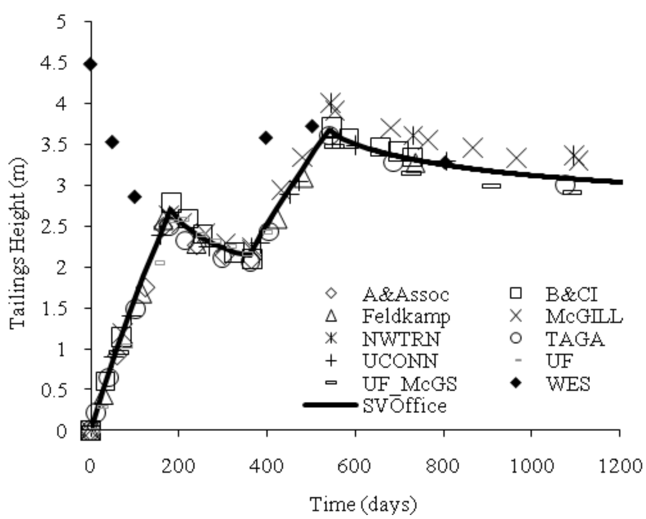


Figure 8 - Townsend B Tailings Height with Respect to Time

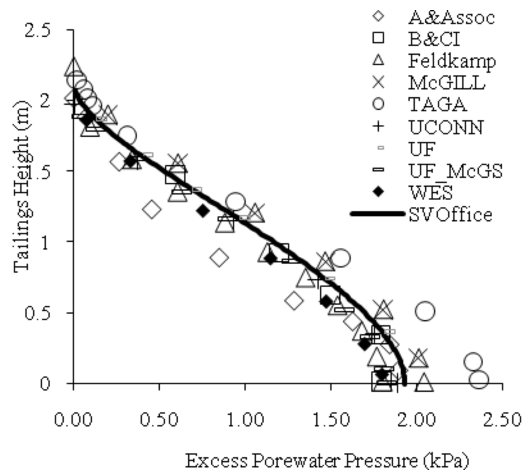


Figure 9 - Townsend B One Year Excess Porewater Pressure Profile

Long-term column test

Jeeravipoolvarn et al. (2008) present the experimental results of a long term column consolidation test on oil sands tailings [3]. They compare the experimental data to another one-dimensional finite element model.

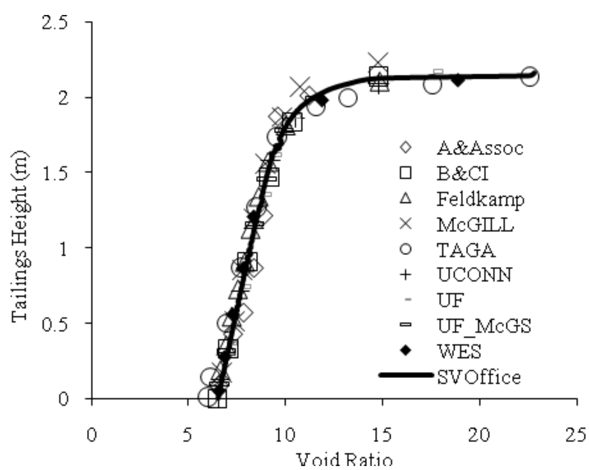


Figure 10 - Townsend B One Year Void Ratio Profiles

<u>GEOMETRY AND BOUNDARY CONDITIONS</u>

The consolidation test is done in a circular column, with a height of 10 m and diameter of 0.9 m (Figure 11). While a 1-D model is simply a series of layers, 2-D was solved as a cross-section of the column split along the center on the vertical axis. In 3-D, the full cylinder was created and solved as a mesh. It is assumed that the tailings were initially homogeneous and deposited instantaneously. The deformation boundary conditions consist of a lower boundary that is fixed in place, and an upper boundary that is free to deform. In the multi-dimensional models, the lateral boundaries are fixed in the horizontal direction, but are free to deform vertically. The flux conditions consist of a constant head of 10 m at the top boundary and a zero flux boundary at the bottom. This forces the water to flow from bottom out through the top boundary, while simulating a constant water table above the soil region.

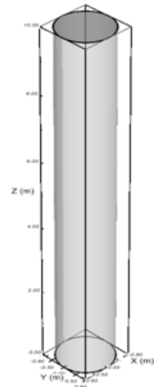


Figure 11 - Jeeravipoolbarn, 2008 Column 1 Model Geometry (3-D)

MATERIAL CHARACTERISTICS

Jeeravipoolvarn et al. use a Weibull function to relate effective stress and void ratio. It has been reported that this formulation is the most effective for representing the effective stress relationship in oil sands tailings [3]. The Weibull function is provided in equation (10). Equation (11) shows the void ratio-hydraulic conductivity relationship. These are depicted graphically in Figures 12a and 12b respectively.

The material has a specific gravity of 2.28 and an initial void ratio of 5.17. A Poisson's ratio of 0.4 was assumed.

$$e = 5.50 - 4.97 \exp(-1.03\sigma'^{-0.67})(kPa) \tag{10}$$

$$k = 6.51\times10^{-6} e^{3.824} (m/day) \tag{11}$$

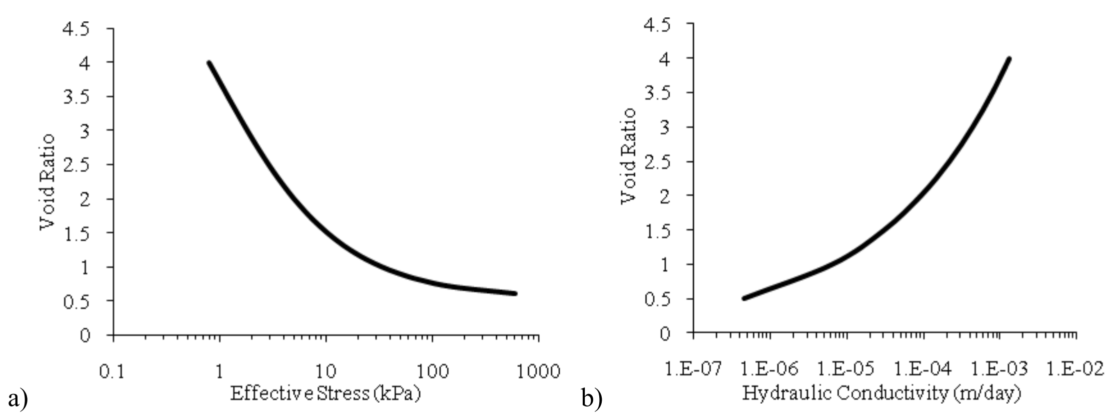


Figure 12 – a) Effective Stress – Void Ratio Curve b) Hydraulic Conductivity - Void Ratio Relationship

RESULTS AND DISCUSSIONS

The comparison between the experimental data and the predictions by Jeeravipoolvarn, et al (2008) as well as SVOffice are shown in Figure 13. There is considerable difference between the experimental data and the predictions by Jeeravipoolvarn, et al (2008). The authors indicate that the difference may be the result of thixotropy or creep behaviour of the oil sands tailings.

The SVOffice analyses were performed assuming 1-D, 2-D and 3-D conditions. The results in Figure 13 show how close the multidimensional results are to those predicted by the literature. It also shows that 2 and 3-D results are the same as those found for 1-D. With 2-D and 3-D models there is potential for more spatial variation throughout the mesh; therefore the similarity of the results is worthy of note.

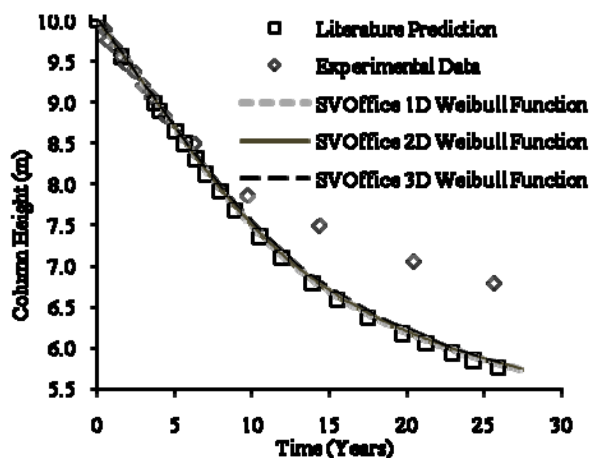


Figure 13 - SVOffice Results As Compared to Literature Prediction and Experimental Results

CONCLUSIONS AND AREAS OF FURTHER RESEARCH

The beta version of the software suite SVOffice is successful in benchmarking other analytical results. It provides similar results as CONDES0 and also fits a series of results from other models for a complicated depositional scenario of phosphate tailings. The 1-D, 2-D and 3-D analyses of a column test on oil sands also fit the results obtained previously using a different analytical model.

The modeling of 2-D and 3-D consolidation will support the optimization of designs of large tailings impoundments. It will play a major role in all aspects of dam/dike design, water balance calculations, cover system design and mine planning (in terms of tailings capacity over time). The next step in this research is to expand the benchmarking of the 1-D large strain consolidation analysis to 2-D and 3-D models.

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