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VALIDATING GEOTECHNICAL FINITE ELEMENT MODELS

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ABSTRACT: *There is a need for guidelines on validation of geotechnical finite element calculations. In Europe, different initiatives have started to work on this subject, among which the NAFEMS geotechnical committee. The author is a member of this committee and author of a reference publication on validation of numerical modelling in geotechnical engineering (Brinkgreve, 2013).This paper presents the main ideas of that publication. In addition to the reasoning behind validation of finite element calculations and a definition of related terms, it highlights the most important sources of discrepancies between a real project and its corresponding numerical model. The second part of the paper is devoted to the various methods of validation. The process of validation involves a validation of the model as a whole, as well as a validation of the various model components, but it starts with a verification of the methods and models implemented in the software. The last chapter describes some non-technical issues related with decisions, responsibilities and organizational issues to control the quality of numerical modelling as part of the geotechnical design process.*

1 INTRODUCTION

In the past decennia the Finite Element Method (FEM) has been used increasingly for the analysis of stress, deformation, structural forces, bearing capacity, stability and groundwater flow in geotechnical engineering applications. The role of the FEM has evolved from a research tool into a daily engineering tool. This has been possible because of the increase of computer power, available for every engineer, and the development of robust and user-friendly software packages. The finite element method has obtained a position next to conventional design methods, and offers significant advantages in complex situations, such as the design of tunnels and deep excavations in suburban areas. However, as with every other method, the FEM also has its limitations. These limitations are not always recognised by users of finite element software, which can lead to unreliable designs.

Despite the development of easy-to-use finite element programs, it is difficult to create a good model that enables a realistic analysis of the physical processes involved in a real project and that provides a realistic prediction of design quantities (i.e. displacements, stresses, pore pressures, structural forces, bearing capacity, safety factor, drainage capacity, pumping capacity, etc.). This is particularly true for geotechnical applications, because the highly non-linear and heterogeneous character of the soil material is difficult to capture in numerical models. The complexity of soil behaviour is caused by various phenomena on the soil particle level and leads to macroscopic observations such as stress-, stress path and strain-dependency of stiffness, critical state behaviour and failure, compaction and dilatancy, strain localisation, creep and relaxation, anisotropy, hysteresis, variability of properties, and several more. When using the finite element method, soil is modelled by means of a constitutive model (stress-strain relationship) which is formulated in a continuum framework. The choice of the constitutive model and the corresponding set of model parameters are the most important items to consider when creating a finite element model for a geotechnical project. It forms the main limitation in the numerical modelling process, since the constitutive model, no matter how complex, will always be a simplification of the real soil behaviour.

Considering the use of geotechnical finite element software, it is often the younger generation of engineers who perform the numerical modelling and produce colourful results; sometimes without fully understanding the backgrounds and limitations of the constitutive models and the numerical methods used in the software. Supervisors, i.e. project managers or senior engineers, often find it difficult to validate the outcome, especially when these do not match with what they would expect based on their experience. This leads to the conclusion that there is a need for guidelines on validation of geotechnical finite element calculations, which is the main motivation for this paper.

This paper is based on a recent NAFEMS publication on Validating numerical models in Geotechnical Engineering (Brinkgreve, 2013). After defining the terms Validation and Verification (Chapter 2), the paper summarizes the main sources of discrepancies (Chapter 3), the various methods of validation (Chapter 4) and some non-technical issues (Chapter 5). More details about these subjects as well as a checklist of discrepancy sources and questions to be asked in the validation process can be found in the original NAFEMS publication.

2 WHAT IS VALIDATION AND VERIFICATION?

The terms Validation and Verification are often used in relation to the process to control the quality of results obtained with numerical modelling software. In fact, there is a clear distinction between the two terms.

Verification is the process of determining that a computational model accurately represents the underlying mathematical model and is capable of reproducing its theoretical solution. Moreover, the process to verify that a model or method has been properly implemented in a computer program is also called verification. The aim of the verification process is to answer questions like:

- Does the computer reproduce the results that are to be expected for the implemented model or method?
- Considering some of the processes which are relevant for the practical problem and for which a solution exists, does the computer model give an answer that is acceptably close to this solution, and can any difference be explained?

A term that is often used in relation to the verification process is *benchmark*. Strictly speaking, a benchmark is an example used to compare and evaluate the performance of an entity against other entities. In the framework of the verification process, the entity is a computer model and the benchmark is generally a well-defined example problem for which a reference solution exists. The performance is the accuracy at which the reference solution is reproduced by a computer model. Schweiger (1998, 2002, 2006) uses the term *benchmarking* for a process to evaluate the variation in results from different modellers or different software packages for a well-defined example problem. Hence, he clearly includes the role of the 'user' in his definition of benchmarking.

Validation is the process of determining the degree to which a model (including the parameters selected for that model) is an accurate representation of the real world from the perspective of the intended uses of the model (NAFEMS & ASME, 2009). In other words: Validation is the process to make plausible that a computer model includes the essential features for a real situation to be analysed and the results obtained with the model are representative for the situation in reality. The aim of the validation process is to answer questions like:

- Regarding the analysis and design quantities to be considered for the real project, does the model and the selected model parameters provide an accurate representation of reality?

Both verification and validation are important to consider for developers of numerical modelling software as well as users and other persons with responsibilities to use the modelling results for the analysis and design of real projects. However, the balance in applying these two methods may be different amongst them. In general, verification is mostly (but not entirely) a task for software developers, whereas validation is mostly (but not entirely) a task for users. In any case, it is essential for users of advanced numerical tools to understand the possibilities and limitations of the models and methods they are using.

3 SOURCES OF DISCREPANCIES

Since a numerical model involves several components that may introduce approximations and errors, it is necessary to identify each of these components and their role in and contribution to the discrepancy as a whole. Identifying possible individual discrepancies gives insight and may result in an improvement of the model and a possible reduction of the overall modelling error. It may also enable a quantification of the variation of design quantities by considering parameter uncertainties and their possible value ranges. A summary of the main sources of discrepancies between reality and its numerical model is given below.

3.1 Simplifications

Finite element models are, in various ways, a simplification of reality. These simplifications should be regarded in the framework of the validation process. The following simplifications can be identified:

- Geometrical simplifications: Reality is always three-dimensional (3D), but most geotechnical models are still 2D (plane strain or axisymmetric). Other geometrical simplifications are generally made in the extent at which details are taken into account regarding the modelling of the ground surface, soil layers, structures, model boundaries, etc.
- Simplifications in material behaviour: Among all materials in the world, soil is probably one of the most complex ones. Considering the numerical modelling of geotechnical deformation problems, soil behaviour is formulated by means of a stress-strain relationship (constitutive model). In general, the more complex a model, the more model parameters it has, which all need to be determined from soil investigation data (field or lab tests). A model should at least capture the features of soil behaviour that are relevant for the engineering problem to be solved. It is important to realise that even the most complex model is still a simplification of real soil behaviour.
- Simplifications in the construction process: Geotechnical projects often involve several construction stages with different load conditions, including those that involve changes in water conditions. In principle, they all need to be considered in the

numerical analysis, since the most critical stresses, deformations or stability situation may occur in intermediate construction stages. However, the construction and loading history is generally simplified, which may lead to discrepancies between the model and reality.

3.2 Modelling errors

In addition to the aforementioned simplifications there is a variety of other sources of modelling errors. Some of these can be reduced when they are recognised; some can even be completely avoided. The process of validation can help to identify and quantify such modelling errors.

- Input errors: A finite element model requires large amounts of data to be entered in the software package. The chance that some of these data are wrong is significant. Fortunately, it is possible to avoid this source of modelling errors. The validation process can help to identify input errors. Therefore, a check on the consistency of the input data is a necessary part of the validation process. Adequate input and output features (such as tables showing an overview of crucial parameters) as well as warnings and error messages in the software package can simplify the validation process.
- Discretisation errors: The numerical modelling of a practical situation involves a division of the soil and structures into a calculation grid (mesh) with cells (elements) that contain the properties of the material. The spatial discretisation (mesh coarseness and interpolation order of the elements) defines the accuracy at which the solution is approximated. Upon mesh refinement, the numerical solution tends to converge towards the theoretical solution, but the 'true' solution is generally approached from the unsafe side (under-estimation of displacements; over-estimation of stability and bearing capacity). Apart from the spatial discretisation, a discretisation of the loading process is applied by taking finite load steps (or time steps) during the calculation. The larger the steps, the more the discretised loading procedure may deviate from the physical loading process, which is continuous in reality. In particular in transient processes (dynamics, groundwater flow), the time stepping is essential.
- Limitations in methods: Numerical methods have their possibilities and limitations. The use of a method beyond its limitations will lead to modelling errors. For example, in order to model membrane effects in a geogrid reinforcement, it is necessary that the geometry is at least updated according to the calculated displacements. Membrane effects are only obtained when large deformation effects are taken into account, for example when using the Updated Lagrange method.

3.3 Constitutive models

In addition to what is mentioned before on constitutive models, there are some particular aspects to take into account regarding the modelling of soil behaviour:

- Pseudo-elastic behaviour: Even though soil behaviour is far from elastic, the basis of a constitutive model is often elasticity theory. A true elastic model is based on an elastic potential or (complementary) strain energy function. In the case that such an elastic potential cannot be formulated, the model is a pseudo-elastic model, and may lead to energy dissipation or generation(!). The latter may lead to unreliable results. The most well-known example of a true elastic model is Hooke's law of isotropic linear elasticity. It is tempting to reformulate the stiffness in Hooke's law as a simple stressdependent power law, but this leads to a pseudo-elastic model that may suffer from inconsistencies. Fortunately, elastic formulations are never used in isolation, but they may still dominate the model behaviour, especially under unloading or reloading conditions.

- Non-associated plasticity: Plasticity theory is used to formulate the Mohr-Coulomb failure criterion into a set of yield functions to identify whether plastic strains will occur. The actual calculation of plastic strains is based on a plastic potential function, which usually has a similar shape as the yield function. A difference between both functions is denoted as non-associated plasticity. For models based on non-associated plasticity the uniqueness of the solution cannot be proven. As a result, numerical models may suffer from changing mechanisms and non-unique failure loads which are influenced by the numerical discretisation (mesh, time step). Considering the simple Mohr-Coulomb model with a dilatancy angle less than the friction angle, it should be realised that this situation may lead to the issues related to non-associated plasticity.
- Strain-softening: Regarding the use of softening models in boundary value problems, several researchers have demonstrated that the numerical results may show severe mesh-dependency. As soon as plastic strains develop, the material will locally soften, whilst material outside the plastic zone will retain its strength. The finer the mesh, the smaller the plastic zone and the larger the plastic strains, which will lead to more severe softening behaviour. This will not only affect post-peak behaviour, but also failure loads and bearing capacities in practical applications may suffer from severe mesh-dependence.
- Undrained behaviour: Modelling undrained behaviour in an effective stress approach can be very sensitive. The modelling of undrained behaviour can be simply done by adding a large bulk stiffness for the pore water, such that the soil as a whole becomes nearly incompressible. Apart from the possibly inaccurate pore pressure calculated in this way (it requires at least high-order elements), it should be realized that the actual pore pressure strongly depends on the constitutive model being used, and may be wrong for the particular type of soil being modelled. If the pore pressure is wrong, the effective stress will also be wrong and, considering effective strength properties, the resulting undrained shear strength will be wrong as well.

3.4 Uncertainties

There are many aspects in a real project that are not completely known (yet) or which cannot be measured accurately in the field. In other words, there are uncertainties as to what we need to model precisely to reflect the real construction process and the conditions that are applied to the real structure during its lifetime. Examples of uncertainties are the precise location of soil layers and the spatial variation of soil properties in the ground. All such uncertainties can lead to discrepancies between the behaviour of a model and the behaviour of a structure as observed in reality. There are various ways to deal with uncertainties in the design process. Some methods are described below:

- Global safety factor approach: Considering the calculation of geotechnical safety factors in advanced numerical models the following expression is generally used:

  Safety factor=(Available resistance)/(Minimum resistance required for equilibrium)

  In the numerical algorithm that deals with this safety factor approach, the available resistance is reduced until geotechnical failure is observed. The safety factor obtained in this way is perceived to be similar to the classical geotechnical definition of a safety factor. However, reducing the resistance of the soil material by reducing the strength properties may change the characteristics of the model behaviour, such that the model does not represent the actual material anymore. Moreover, it can be questioned what to do with other model parameters (for example the stiffness or the dilatancy angle). Finally, considering undrained materials, should the safety factor be calculated for drained or undrained conditions?

- Partial factor approach: In order to deal with differences in uncertainties, one may consider using partial factors for different parameters. The use of partial factors, such as in Eurocode 7, is based on a more elaborate probabilistic approach in which a minimum reliability level is defined for the structure as a whole. There is still no common sense on how to deal with Eurocode 7 in the finite element method. Different design approaches and calculation schemes can lead to different results. Nevertheless, it is possible to perform finite element calculations according to the various design approaches in Eurocode 7. When doing so, similar remarks as mentioned above for the global safety factor approach should be taken into account.
- Parametric analysis: To analyse the influence of uncertainties in loads, model parameters and even geometric variations, a parametric analysis may be performed. In a parametric analysis the key uncertain parameters of the numerical model are identified and their variation is characterised by means of an upper and lower bound value. From the combinations of upper and lower bound values, maximum and minimum values of various output quantities can be assembled which gives an impression of the variation (range) of results that can be expected. Although parametric analysis does not remove any of the discrepancies as mentioned in the previous sections (simplifications, modelling errors, limitations in constitutive models), some uncertainties in model parameters can at least be analysed.

3.5 Misinterpretation of results

If the modelling process has been successfully completed such that the model is an actual representation of the physical problem, it is not a guarantee that the calculation will run smoothly. Numerical ‘tricks’, tolerances and compromises may be necessary to come to a solution. It should be validated that tolerances and compromises will not lead to an unrealistic approximation of the original model. If the calculation has successfully finished and results have been obtained it is not the end of the story. It should be realised that the computer model does not directly provide the answer to the engineering problem. Therefore, a translation needs to be made from the results of the computer model towards the engineering and design issues. The translation and (mis)interpretation of results may also lead to discrepancies between the real situation and the computer model.

An example where misinterpretation might occur is in the evaluation of a safety factor from a strength reduction analysis. First of all it should be verified whether a realistic failure mechanism has developed. Absolute values of total displacements do not have a physical meaning in this case; the mechanism can best be viewed on the basis of the plastic strains or incremental displacements at failure. The safety factor itself is defined in the step where the mechanism has fully developed. When the calculation stops too early, the safety factor may be interpreted wrong (too low). Conversely, locking (depending on the type of elements used) may cause that the strength reduction process continues after a full mechanism has developed, which may lead to an overestimation of the safety factor. Special attention is needed when the model includes structural elements, which behaviour is generally assumed to be elastic. However, during the strength reduction procedure there is a transfer of loads from the soil to the structures. This requires careful inspection that the structural forces are still acceptable for the resulting safety factor.

It should also be realised that the model may not include all phenomena that are relevant for the real situation. Again, the constitutive model plays an important role in this respect, but also local mechanisms such as piping will not be found in general, since such mechanisms cannot be obtained with a standard continuum approach.

4 METHODS OF VALIDATION

In the previous chapter some sources of discrepancies between a real project and its numerical model have been identified. In order for a particular project to manage the uncertainties and reduce the discrepancies, the numerical model must be validated. Validation of the model as a whole will not be enough to make plausible that the results that are obtained from the model are representative for the real situation. In fact, discrepancies in individual components may accidentally cancel each other out if they are not validated individually. The validation process should therefore comprise the individual modelling components in addition to a validation of the integral model.

4.1 Verification

Before considering validation of a computer model for a practical situation, it is relevant to verify that the models and methods implemented in a software package are reliable. In the first place this is a responsibility of the software developers, but also users of finite element software should consider performing a verification of models and methods that are relevant for the solution of their engineering problem. Verification is done by comparing the results of computer models for typical situations with known solutions. Arbitrary practical situations are unlikely to have known solutions. However, some particular aspects or phenomena that the practical situation involves may be simplified to situations for which a known solution exists or for which a solution can be derived. In the framework of numerical modelling of geotechnical applications the following solutions can be used for verification:

- Elasticity solutions for soil continua and structures (e.g. Settlement and stress distribution below a strip footing on elastic soil; bending of beams, plates and shells subjected to different loads, considering different types of supports).
- Elastoplastic solutions (e.g. Cylindrical cavity expansion for small and large deformations)
- Bearing capacity solutions (e.g. Failure of a vertical cut in cohesive soil; bearing capacity of a rigid circular or strip footing on cohesive or frictional soil)
- Elasto-dynamic solutions (e.g. one-dimensional wave propagation; pulse load on an elastic half space (known as Lamb's problem))
- Solutions of flow or coupled problems (e.g. Darcy's solution for confined flow; Unconfined flow through a homogeneous medium with vertical faces (known as Muskat's problem); One-dimensional consolidation)

4.2 Upper and lower bound solutions (Limit Analysis)

For engineering problems involving the analysis of bearing capacity or stability and for which no analytical solution is available (which is by far for most practical situations), solutions based on upper and lower bound theorems (limit analysis) may be used to encapsulate the solution. The lower bound solution for an external failure load on a continuum is based on finding an equilibrium solution for the stress distribution in the full continuum that balances the external load and that does not violate a given failure criterion for the stress state. The upper bound solution for an external load on a continuum is based on the evaluation of all forces acting on an assumed kinematically admissible failure mechanism where the internal forces are limited by the given failure criterion for the stress state. If a solution for the external load can be found in which both the upper bound and lower bound conditions are met, the solution is considered to be the true failure load (uniqueness theorem). However, this will rarely be the case for practical situations, but it can be useful to determine an upper and lower bound solution which are sufficiently close to each other, such that they can be used to validate the numerical solution.

Much work on upper and lower bound solutions has been done by Sloan and co-workers (e.g. Lyamin & Sloan 2002a/b). Over the last decade, they developed a numerical framework based on finite elements and linear programming, which can be used for several types of geotechnical applications, including slopes, embankments, foundations and excavations. Although the method may seem to resemble the finite element method, it is based on different principles and can be used to validate stability or bearing capacity solutions obtained from true finite element models. It should be noted that the upper bound solutions are based on associated plasticity, which may introduce a difference compared to the finite element model to be validated. Nevertheless, limit analysis can be very useful to validate stability and bearing capacity solutions obtained from finite element analysis.

4.3 Validation of constitutive models and parameters

The selection of a constitutive model should be based on an evaluation of the capabilities (and limitations) that it has to describe the essential features of soil behaviour for the situation at hand. In that respect, the constitutive model provides the qualitative description of soil behaviour, whereas the parameters in the model are used to quantify the behaviour. The composition of the model plus parameters can be regarded as the 'artificial soil' that is used in the numerical model, which should be representative for the real soil behaviour in the application. Before considering the numerical model in full detail, it makes sense to evaluate the behaviour of the artificial soil (= model + parameters) separately in single stress point simulations of soil lab tests (e.g. drained and undrained triaxial tests, oedometer test, DSS test, CRS test). The results of the lab test simulations can be compared with real test data. This provides insight in the possibilities and limitations of the model to describe particular features of soil behaviour and the accuracy at which it does so. Moreover, parameters can be optimised to make a 'best fit' to the test data.

It should be noted that the stress paths, stress levels and strain levels in the real application can be significantly different than those in the soil lab tests. Hence, a good fit between the results of a simulated test and the real test data is not a guarantee that the artificial soil is a good representation of the real soil in the practical application. Nevertheless, the numerical simulation of soil lab tests is relevant to qualitatively understand the behaviour of the model and should therefore be considered in the validation process.

In contrast to soil lab tests, in-situ tests cannot be simplified to a single stress point model. However, some in-situ tests, such as a pressuremeter test, can still be modelled as a boundary value problem using a finite element model. This modelling can be useful to optimise the in-situ stiffness and strength properties.

As part of the validation of model parameters for the engineering application it might also be considered to perform a preliminary analysis on a semi one-dimensional soil column representing the ground profile at the project location. In the case that the project involves mainly vertical loading, the soil column analysis can be used to check if the calculated settlements match the expected settlements (based on engineering judgement or conventional settlement calculations). Moreover, if time-dependent behaviour (consolidation or creep) is involved, the initial soil column analysis may be used to validate the model parameters and initial conditions leading to a particular settlement rate at the start of the project. This settlement rate can be compared with measurements from the past or with what is supposed to be realistic for that location.

4.4 Validation of the mesh and model boundaries

Model boundaries are introduced to limit the extent of the numerical model. It has to be validated whether the outcome of the finite element model is not influenced by the choice of the model boundaries. This can crudely be done by redoing the numerical analysis with model boundaries taken further away and comparing the results, but that may be a time-consuming way of working. It should at least be verified after any numerical analysis that changes in stress and strain near the model boundaries are relatively small. This is not required near (vertical) symmetry boundaries, but there it should be validated that the symmetry conditions are properly applied. In the case of groundwater flow or coupled analysis, for example, there should be no flow across the symmetry boundary.

For a dynamic analysis, it should be checked that there are no spurious reflections at the model boundaries. The best way to check this is by creating an animation of the velocities in the model.

The discretisation of the model (division into elements, i.e. the mesh), in relation to the type of elements used, determines the accuracy at which the numerical solution is approximated. In general, a finer mesh gives a more accurate solution. Refinement is particularly needed in parts of the model where stress or strain concentrations will occur. This is usually the case around loads and structural elements in the mesh. Local mesh refinement (as opposed to global refinement) is a useful way to create accurate and, at the same time, efficient meshes.

In addition to the accuracy of the mesh, it needs to be validated that the quality of the elements themselves is acceptable. Very slender elements must be avoided. At least a visual inspection is needed. The ‘slenderness’ is usually expressed as the element aspect ratio, being the ratio of the radius of the maximum inner circle over the radius of the minimum outer circle, normalised at unity for the optimum element (equilateral triangle or square quadrilateral). These values are available in most commercial finite element software nowadays and should be checked.

4.5 Validation of initial conditions

It is necessary to initialise the stress in the model and validate that this stress state is in correspondence with the situation in reality. The initial situation may involve total or effective stress components, pore water pressures, pre-consolidation stress, void ratio and other state parameters, depending on the constitutive model(s) being used.

A simplified procedure to generate the initial stresses is to integrate the weight of the soil column above each stress point to calculate the vertical total stress, then subtract the pore pressure to calculate the vertical effective stress and then multiply by a given K_0-value to calculate the lateral effective stress, and assuming all shear stress components to be zero. This ‘K_0-method’ has the disadvantage that equilibrium is not guaranteed. Therefore, when such a method is used, it has to be validated that the resulting stress state is in equilibrium and that it is realistic to assume that the initial shear stresses are zero.

Alternatively, initial stresses can be calculated by loading the model with the gravitational forces associated with the self-weight of the materials. Although this ‘Gravity method’ will result in an equilibrium state at the end of the calculation, it may not properly resemble the initial stress state of the situation in reality, since the full loading history of the environment is not taken into account. Therefore, in addition to the standard gravity method, it may be necessary to perform a number of subsequent calculations to simulate the relevant loading conditions in the past that are of influence on the stress state at the start of the engineering application considered. Note that undrained soils may need to be modelled temporarily as drained soils during the initial stress calculations in order to avoid the generation of unrealistic excess pore pressures.

In an effective stress analysis, it is essential to create a realistic distribution of initial pore water pressures. Simple hydrostatic pore pressure distributions may be generated on the basis of a phreatic level, whereas more complicated situations may require a separate groundwater flow calculation to be performed. In the latter case, realistic hydraulic conductivities (permeabilities) are required, which are often difficult to obtain from soil investigation data. That is why modellers often 'abuse' the phreatic level tool to create more complicated pore pressure distributions based on non-horizontal level sections. Care has to be taken with such an approach, since in reality non-horizontal levels imply groundwater flow and possibly non-hydrostatic pore pressure distributions. A 'jump' in the phreatic level should definitely be avoided, since this would cause a similar jump in pore pressure all the way down in the model, which is highly unrealistic.

4.6 Other methods of validation

In addition to a validation of model components, there are various ways validate the integral model by comparing the results obtained from the numerical simulation with other sources.

- Measurements: Measurements taken from monitoring during the execution of a project (displacements, pore pressures, structural forces, etc.) can not only be used to validate the results of a numerical model, but also to update the parameters being used, provided the measurements are sufficiently accurate and reliable. The updated model can be used to make a better prediction of construction phases that are yet to come.
- Design charts: In the past, researchers have published various diagrams of particular engineering and design aspects in geotechnical engineering, such as settlements, stability and wall deflections. Most design charts were published in the time that computer modelling was not so common in the geotechnical engineering practice, but they are still relevant. Although design charts are quite general and may not be very precise, they can be of great help to validate numerical models in the sense that they give an indication of the order of magnitude that should be expected in certain situations.
- Other software: Other software tools, ranging from spreadsheets with simple design rules or analytical methods via conventional geotechnical analysis software to other advanced 2D or 3D numerical modelling software (other than used for the model to be validated) may be used to obtain independent solutions for comparison.

4.7 Benchmarks

A benchmark, in the framework of validation and verification, is a well-defined example problem for which a reference solution exists, whereas the term benchmarking has been defined as the process to evaluate the variation in results from different modellers or different computer software for a well-defined example problem. Most benchmarks are simplified practical problems for which no analytical solution exists; only a (numerical) reference solution. Modellers can use a benchmark to check if they obtain a similar solution. Since the solution is obtained using numerical methods, a small deviation (few percent) from the reference solution is likely to occur and is quite acceptable. Larger deviations may still be acceptable, depending on the type of problem and the level of detail that is provided with the benchmark. Published benchmarks have shown that quite large differences can occur, which underlines the need for validation of numerical models. A number of benchmark examples for geotechnical engineering have been defined and published. More details can be found in the NAFEMS publication (Brinkgreve, 2013).

In addition to published benchmarks, researchers in the past have set up instrumented full-scale tests with the purpose to investigate particular aspects of soil behaviour. Thereby, they have organised competitions for engineers to predict particular test results for which measurements would become available. The purpose of such a prediction competition is to evaluate the performance of numerical models and the modelling capabilities of engineers in general (rather than individual performances). In contrast to benchmark examples, no reference solution is available, but the reference is the measurement data that becomes available after the test has been performed. A number of such prediction competitions and their results have been documented. Similarly as for the benchmark examples, the prediction competitions show large variations in results as well as deviations from the measurements. These deviations can be reduced by knowledge and experience of the modelling engineer as well as a proper validation of the numerical model. It also underlines the importance of using an appropriate constitutive model to describe the soil behaviour.

5 NON-TECHNICAL ISSUES

In addition to the ‘technical’ issues related to the validation of numerical models for geotechnical applications, there are non-technical issues involved with the validation process.

The first issue is the availability or lack of data; in particular soil data. In practice, there is often a lack of soil investigation data. It is important to convince clients or project owners of the need of sufficient and good quality soil investigation. It does not only reduce the uncertainties in ground conditions, but it will also facilitate the validation of model parameters, thereby reducing the risk that the design is inadequate because it is based on insufficient or wrong geotechnical data.

Another non-technical issue is the distribution of responsibilities among the persons and organisations involved in the various aspects of numerical modelling. First, it is the responsibility of the *engineer* (user of finite element software) to create a computer model and to determine the required parameters such that the model accurately represents the real project and captures the phenomena that lead to the quantities that need to be determined or interpreted from the model (deformations, stresses, structural forces, flow, etc.). This responsibility includes a proper validation of the model and its components.

Second, it is the responsibility of the *supervisor* (manager) of the modelling engineer or the project manager to control that the model created and used by the project engineer is a reliable model on the basis of which the project can be properly analysed and/or designed with the required safety level. This responsibility involves a check on how and to what extent the model has been validated by asking the right questions. The responsibility of the supervisor also involves taking care of mentoring, coaching and training of the younger engineers such that they can develop the necessary skills to become experts in numerical modelling in addition to their geotechnical skills in general.

Third, it is the responsibility of the *organisation* in which numerical models are being used to create an environment in which the importance and complexity of numerical modelling is being realised on all levels. If numerical modelling is part of their activities, it should be included in their quality procedures. The organisation should be structured such that there is sufficient knowledge and room, not only to create numerical models but also to validate models and to control the process from the early stage of numerical modelling to the interpretation of the results towards the geotechnical design. Ideally, this knowledge should beshared within the organization.

Finally, it is the responsibility of the *software developer* to produce software that has been sufficiently verified and that is (ideally) free of programming errors. It is also the responsibility of the software developer to properly document the models and methods that are implemented in the software and make this documentation available to the user.

6 CONCLUSIONS

In this paper a summary is given of a recent NAFEMS publication on validating numerical models for geotechnical engineering applications. Validation, in this respect, is the process to make plausible that a numerical model includes the essential features for a real situation to be analysed and its results are representative for the situation in reality.

The first part of the paper describes the sources of discrepancies between a real project and its numerical model. Insight in the discrepancies is essential for a proper validation of the model and to reduce modelling errors. A significant part of modelling inaccuracy and uncertainty is related to the modelling of soil behaviour. A thorough understanding of the possibilities and limitations of constitutive models and underlying theories is needed to decide which model is best considering the main features of soil behaviour that are relevant for the real situation. Good soil investigation data provide the essential basis of soil modelling, parameter determination, creating initial conditions, and the validation thereof.

The second part of the paper is devoted to the various methods of validation. The process of validation involves a validation of the model as a whole, as well as a validation of the various model components, but it starts with a verification of the methods and models implemented in the software. Particular model components that need to be validated are the geometry, the model boundaries, the material behaviour, the finite element mesh, the initial conditions and the calculation phases.

In view of the validation process, it is useful to learn from situations that have been analysed in the past. In this respect the paper refers to benchmarks and prediction competitions that can be found in more detail in literature. The benchmarks and prediction competitions clearly demonstrate the need for a proper validation of numerical models, since the results from different modelling groups show significant differences.

The last chapter describes some non-technical issues related with decisions, responsibilities and organizational issues to control the quality of numerical modelling as part of the geotechnical design process.

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