

The calculation of actual evaporation from an unsaturated soil surface

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ABSTRACT

The calculation of Actual Evaporation, *AE*, is required when calculating the net moisture flux across the ground surface. The Wilson-Penman (1990) equation appears to provide reasonable estimations of actual evaporation, *AE*, from saturated clayey soils but tend to over-estimate actual evaporation from coarse-grained soils. Two distinct approaches have emerged in recent years for the calculation of actual evaporation from unsaturated coarse-grained surfaces. Both approaches are based on the concept that evaporation tends to “shut-off” as the natural water content approaches residual water content conditions. The first approach to calculating *AE* from unsaturated soils has involved the adjustment of calculated total suction at the soil surface. The second approach determines an “evaporation-rate reduction point” from the drying soil-water characteristic curve and then calculates a vapor pressure reduction factor when calculating *AE*. Both approaches constitute an attempt to take “surface resistance” to evaporation into consideration. The paper compares the theoretical context for both approaches (i.e., total suction adjustment and vapor pressure adjustment), for the calculation of actual evaporation.

Keywords: Actual evaporation, potential evaporation, total suction, surface resistance, vapor pressure.

INTRODUCTION

The primary components associated with the determination of water balance (or net infiltration, *I*) at ground surface are shown in Eq. [1].

$$\text{Infiltration } (I) = \text{Precipitation } (P) - \text{Actual Evaporation } (AE) - \text{Transpiration } (T) - \text{Runoff } (R) \quad [1]$$

Evaporation models can first be classified on the basis of whether Potential Evaporation, *PE*, or Actual Evaporation, *AE*, is calculated (See Figure 1). Most methods used in geotechnical engineering practice are based on energy balance and aerodynamic considerations. It has become apparent that “surface resistance” plays an important role in the computation of actual evaporation, *AE*. Several evaporation models that have been used in geotechnical engineering practice until the late 1990s did not take the “surface resistance” of the soil into consideration. Examples of these models are shown in Figure 2.

The calculation of actual evaporation has proven to be one of the most complex analyses associated with unsaturated soil mechanics. Models that have been proposed and used in geotechnical engineering for calculation of evaporative flux can be divided into two broad categories; namely, those that take “surface resistance” into consideration and those that “do not take surface resistance” into consideration (See Figure 3). “Surface resistance” has proven to be a significant factor in calculating “actual evaporation”.

Figure 3 shows that the Wilson (1990) and Wilson et al., (1997) models for actual evaporation have been incorporated into numerical modeling codes in two different ways; namely, where consideration is given to “surface resistance” (e.g., SVFlux 2009), and where consideration has not been given to “surface resistance” (e.g., SoilCover, 1994 and Vadose/W, 2008). The word “modified” has been added to the description identifiers for each method where “surface resistance” has been taken into consideration.

dry, coarse soils in arid regions. Wilson (1990) noted that the Wilson-Penman (1990) method would over-predict actual evaporation from soil surfaces in dry regions. It now appears that the primary reason for the over-prediction of actual evaporation was the omission of the “surface resistance” term. Wilson (1990) also pointed out that osmotic suction effects were not taken into consideration in the Wilson-Penman method. This paper does not address the issues related to osmotic suction but focuses on clarifying the formulations that attempt to address “surface resistance”.

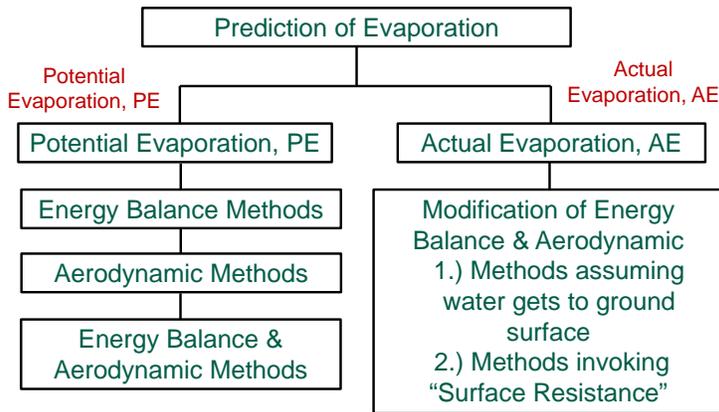


Figure 1. Evaporation models classified in terms of whether potential evaporation, PE, or actual evaporation, AE, is being calculated.

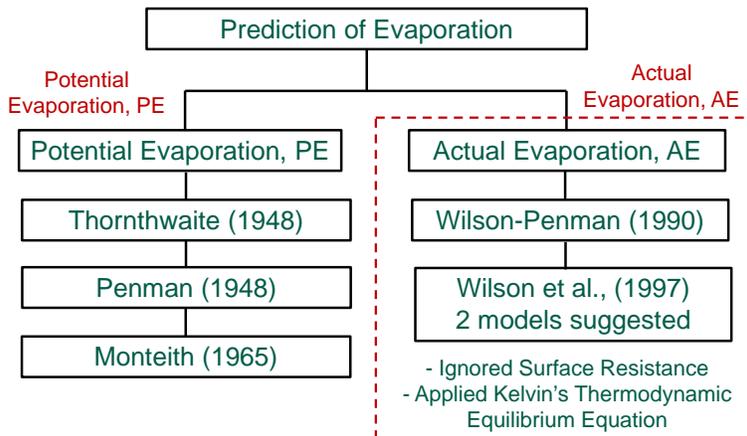


Figure 2 Evaporation models that did not take “surface resistance” of the soil into consideration.

Fredlund et al., (2012) suggested that an adjustment should be made to the total suction calculated at the soil surface to account for non-equilibrium conditions. The adjusted total suction was incorporated into the computer code of SVFlux (2009). The proposed procedure was verified using datasets found in the research literature. Actual evaporation models were solved using both “coupled” and “uncoupled” methodologies. The revised procedures to calculate actual evaporation were referred to as: i.) the *modified* Wilson-Penman (1990) method, ii.) the *modified* “Limiting Function” method (Wilson et al., 1997), and iii.) the *modified* “Experimental Function” method (Wilson et al., 1997).

Tran (2013) re-visited the Penman-Monteith (1965) formulation that incorporated the concept of a “canopy cover” (i.e., a type of “surface resistance”) from vegetated surfaces and found that the formulation could also be applied to evaporation from unsaturated soil surfaces. In this case an empirical methodology was used to adjust the calculated vapor pressure at the soil surface. The calculated actual evaporations from the revised Penman-Monteith methodology were compared to measurements of evaporation made on several sand column tests. Consequently, there were now two methodologies available for the calculation of “actual evaporation” from unsaturated soil surfaces.

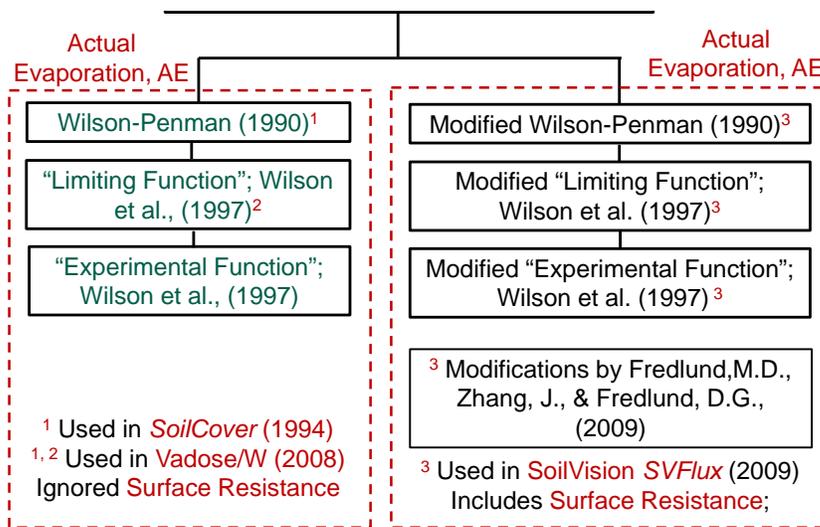


Figure 3 Broad classification of models based on modifications of the original Penman (1948) equation

The objective of this paper is to provide a summary (and clarification) of the various methods that have been proposed within geotechnical engineering for the calculation of actual evaporation, *AE*. The scope of this paper is limited to a presentation of the theories involved and reference is made to the datasets from laboratory column test measurements that have been used to verify the theoretical formulations.

Evaporation Models Based on Thermodynamic Equilibrium at the Evaporating Surface

Figure 4 identifies some of the variations of the Penman (1948) method that have been proposed for the calculation of actual evaporation. These models did not initially take “surface resistance” into consideration but were later *modified* by Fredlund et al., (2012) to account for “surface resistance”.

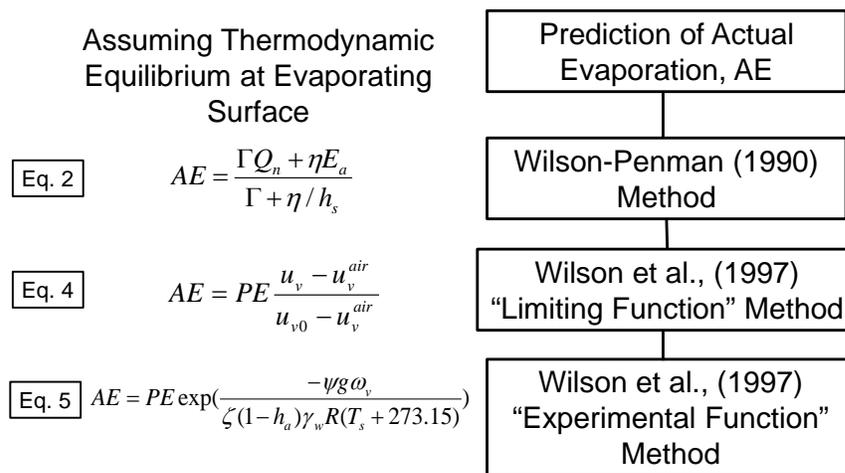


Figure 4. Actual evaporation equations based on thermodynamic equilibrium at the soil surface.

The Wilson (1990) and the Wilson et al., (1997) methods for calculating actual evaporation are assumed to be driven by a vapor pressure gradient that satisfy thermal equilibrium at the ground surface. The water in the soil at ground surface can be negative (i.e., corresponding to an equivalent total suction). Soil suction (i.e., total suction) at ground surface was assumed to reduce the vapor pressure and thereby reduce evaporation from ground surface. The Wilson (1990) and the Wilson et al., (1997) equations appears to provide reasonable calculations of actual evaporation when the clay soils at ground surface remain near saturation but appear to over-estimate actual evaporation from unsaturated, coarse-grained soils.

the overlying air as the primary driving mechanisms that influenced the vapor pressure gradient; namely, net radiation and wind speed (i.e., mixing of air) at the ground surface. The properties of the soil did not come into the analysis and it was assumed that water was available for evaporation at ground surface.

Wilson-Penman (1990) Model

Wilson (1990) made one modification to the Penman (1948) equation which then became known as the Wilson-Penman (1990) equation as shown in Figure 4:

$$AE = \frac{\Gamma Q_n + \eta E_a}{\Gamma + \eta / h_s} \quad [2]$$

where: AE = actual evaporation rate from a soil surface in mm/day; Γ = slope of saturation vapor pressure versus temperature curve, $kPa/^\circ C$, Q_n = net radiation at the water (or saturated ground) surface, mm/day , η = psychrometric constant, $kPa/^\circ C$, $E_a = 2.625(1 + 0.146W_w)(u_{v0}^{air} - u_v^{air})$, mm/day , W_w = wind speed at 2 m, km/hr , h_s = relative humidity at the soil surface (i.e., $h_s = u_v^{soil}/u_{v0}^{soil}$, where u_{v0}^{soil} = saturated vapor pressure in the soil at ground surface, kPa). The inclusion of the relative humidity term in the denominator took into consideration the affinity (or holding power) of the soil for water at ground surface. While net radiation (and wind) attempted to remove water vapor away from the soil surface, the soil attempted to retain water. The Wilson-Penman (1990) model appeared to provide reasonable results in situations where the soil at ground surface remained essentially saturated and the permeability of the near ground surface soil was such that water could be transmitted to the soil surface.

Actual evaporation was calculated in an uncoupled manner when using the Wilson-Penman (1990) method. The assumption was made that the ground surface thermal flux was zero. The soil surface temperature was computed using the following empirical relationship.

$$T_{soil} = T_a + \frac{1000(R_n - R_g) - L_v AE}{C_f \eta f(u) L_v} \quad [3]$$

where: T_{soil} = soil temperature at soil surface, $^\circ C$, T_a = air temperature, $^\circ C$, C_f = conversion factor, (i.e., $1 kPa = 0.00750 mHg$), η = psychrometric constant, $0.06733 kPa/^\circ C$, $f(u)$ = function depending speed, $f(u) = 0.35(1.0 + 0.146 W_w)$, W_w = wind speed, km/hr , R_n = net radiation, $J/m^2/day$, R_g = ground surface thermal flux, $J/m^2/day$, and L_v = volumetric latent heat of vaporization, J/m^3 .

Wilson-Fredlund-Barbour (1997) “Limiting Function” Model

Wilson et al., (1997) suggested that actual evaporation could be written as a “Limiting Function” between actual evaporation and potential evaporation (See Figure 4). This model assumed that the temperature at the soil surface was the same as the air temperature. Therefore, the vapor pressure (and relative humidity) at the ground surface was the same as the relative humidity in the air above the ground surface.

$$AE^{[4]} = PE \frac{u_v - u_v^{air}}{u_{v0} - u_v^{air}}$$

where: u_v = actual vapour pressure at the soil surface, kPa; u_{v0} = saturated vapour pressure at the soil surface temperature, kPa; u_v^{air} = vapour pressure in the air above the soil surface, kPa. Calculations using the Wilson-Fredlund-Barbour (1997) “Limiting Function” model were essentially the same as those from the Wilson-Penman (1990) model. Once again, the model over-predicted actual evaporation for dry, coarse-grained soils in arid regions.

A series of drying tests were undertaken in the laboratory on thin layers of sand, silt and clay soils (i.e., about 1 mm thick). It was observed that all three soils with different grain sizes gave similar ratios between actual and potential evaporation as shown in Figure 5.

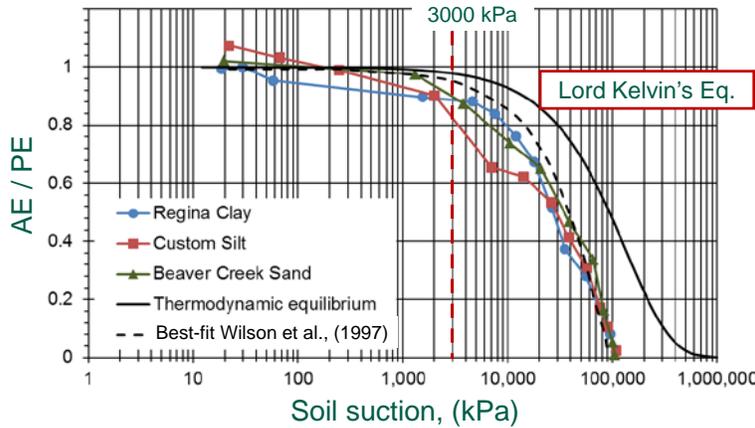


Figure 5 Ratio of actual evaporation to potential evaporation for thin soil layers (Wilson, 1990).

Also shown on Figure 5 is Lord Kelvin's equation. The laboratory data has a similar shape to the Lord Kelvin equation; however, it is translated to the left by approximately 70% of a log cycle. The dashed line in Figure 5 is the result of incorporating an empirical factor of 0.7 into Lord Kelvin's equation.

$$AE / PE = \exp\left(\frac{-\psi g \omega_v}{\zeta (1 - h_a) \gamma_w R (T_s + 273.15)}\right) \quad [5]$$

where: ζ = a dimensional empirical parameter with a suggested value of 0.7; h_a = relative humidity of overlying air; ψ = total suction (i.e., matric suction plus osmotic suction), kPa; ω_v = molecular weight of water, 0.018 kg/mol; γ_w = unit weight of water, 9.807 kN/m³; g = gravity acceleration, m/s²; R = universal gas constant, 8.314 J/(mol.K); and T_s = soil surface temperature, °C. Equation [5] was based on, and is applicable for evaporation from thin soil layers. The air and soil temperatures were assumed to be the same.

Limitations of Evaporation Models Based on Thermodynamic Equilibrium at the Evaporating Surface

Numerous attempts have been made to apply Lord Kelvin thermodynamic equilibrium equation at the ground surface (McCumber and Pielke, 1981; Camillo et al., 1983; Wilson, 1990). However, attempts to apply thermodynamic equilibrium at ground surface failed to take into consideration the resistance to water movement at the soil-atmosphere interface. Researchers began to realize that Lord Kelvin equation was invalid close to the soil surface (Wetzel and Chang, 1987; Kondo et al., 1990 and Lee and Pielke, 1992).

Figure 6 compares relative evaporation, RE, and relative humidity to soil suction for a sand column test performed by Wilson (1990). The results show that RE dropped to approximately 40% at a suction of 20 kPa while the relative humidity began to reduce at soil suctions in excess of 3000 kPa. The results suggest that another physical mechanism is involved in evaporation near-ground-surface for drying of sand.

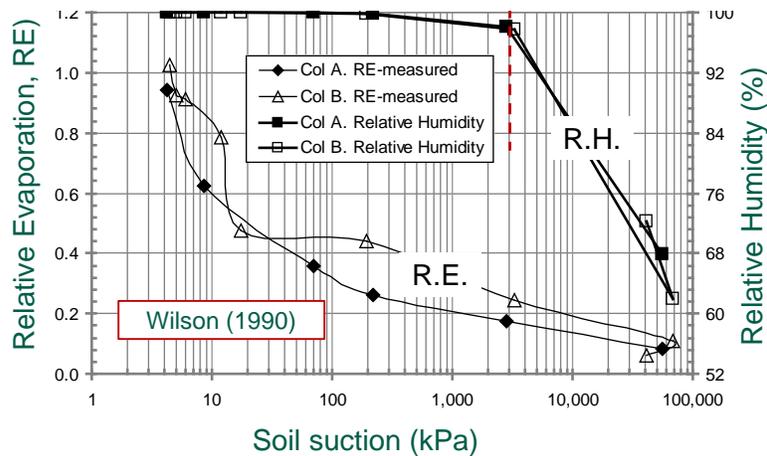


Figure 6. Measured relative evaporation, RE, and relative humidity, RH, from a sand column tested by Wilson (1990).

Similar results from a column evaporation test on sand have been presented by Bruch (1993) and Yanful and Choo (1997). The observed deficiencies in applying the thermodynamic equilibrium equation have led to the development of two types of models that attempt to take “surface resistance” into consideration.

Actual Evaporation Models that take Surface Resistance into Consideration

There are two main classes of models that provide a more reliable calculation of the relative humidity (or vapor pressure) at the soil surface. Both classes of models have been proposed to more accurately calculate actual evaporation from unsaturated and dry soil surfaces. For the first class of models, the relative humidity at ground surface is calculated through use of a modified thermodynamic relationship as suggested by numerous researchers (Alvenas and Jansson, 1997; Bittelli et al., 2008; Fredlund et al., 2011; and recently Dunmola, 2012). A complete model has been formulated by Fredlund et al., (2012) and is referred to as the “total suction adjustment” approach. The corresponding model has been implemented in SVFux (2009) software from *SoilVision Systems*. The total suction at the soil surface depends on residual suction conditions defined by the drying soil-water characteristic curve, SWCC. The model is referred to as the Fredlund-Zhang-Fredlund method. Details of the method have been published in the SVFux (2009) User’s Manual and by Fredlund et al., (2012).

The second class of models utilizes an empirical method of modifying the relative humidity at the soil surface humidity based on the findings of several researchers (Kondo et al., 1990; Lee and Pielke, 1992; Tran, 2013). The formulation of the model is based on the original Penman-Monteith (1965) equation that incorporates the “canopy concept” associated with evaporation from vegetated surfaces. Tran (2013) formulated a model utilizing the concept of “surface resistance”. The model is based on evaporation-rate reduction observed when drying soils near residual water content conditions. The procedure is referred to as the “vapor pressure adjustment” procedures.

The relationship between the “total suction adjustment” procedure and the “vapor pressure adjustment” procedure is illustrated in Figure 7. The evaporation of moisture near ground surface has been shown to be quite complex. As a result it is not possible to simply apply the thermodynamic equilibrium equation at the soil surface without applying an adjustment to either total suction or relative humidity.

Equilibrium at Evaporating Surface

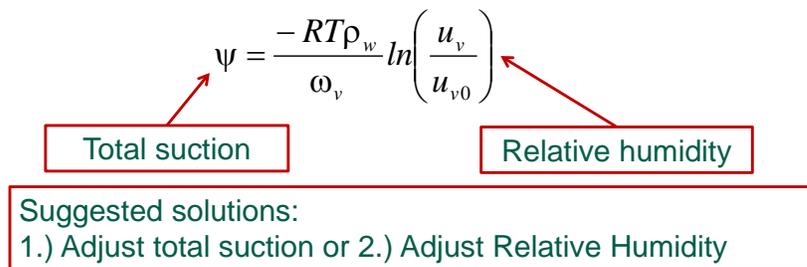


Figure 7. Relationship between adjustments applied to either total suction or relative humidity to accommodate “surface resistance”.

There are two distinctly different classes of actual evaporation models that take the concept of “surface resistance” into consideration (See Figure 8). While the formulation of the two models is different, both models appear to capture the importance of significantly reduced evaporation rates from dry, coarse-grained soils. Column evaporation datasets have been used to show that both models provide reasonable predictions of actual evaporation under dry conditions (Fredlund et al, 2012; Tran, 2013).

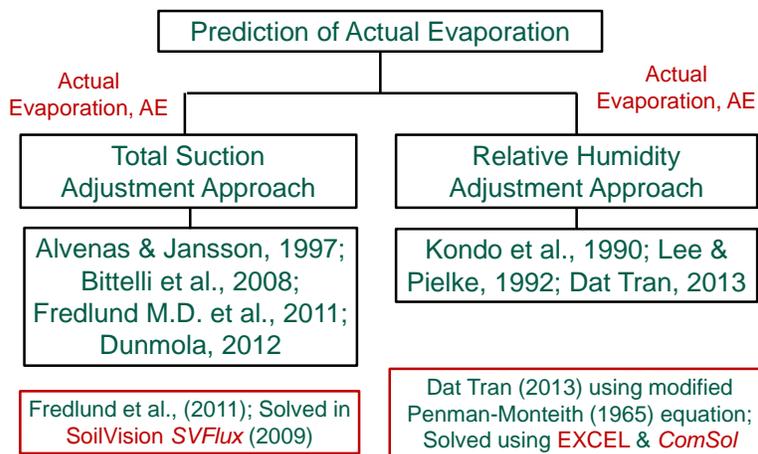


Figure 8. Classification of models taking the concept of “surface resistance” into consideration.

Definition of “Surface Resistance”

“Surface resistance” can be defined as the resistance to water vapor diffusion from near the soil surface (Aluwihare and Watanabe, 2003). The original usage of the term was in connection with evaporation from leaves and vegetated ground surfaces (i.e., the canopy effect) (Monteith, 1965). “Surface resistance” was illustrated by Aluwihare and Watanabe, (2003) and is reproduced in Figure 9.

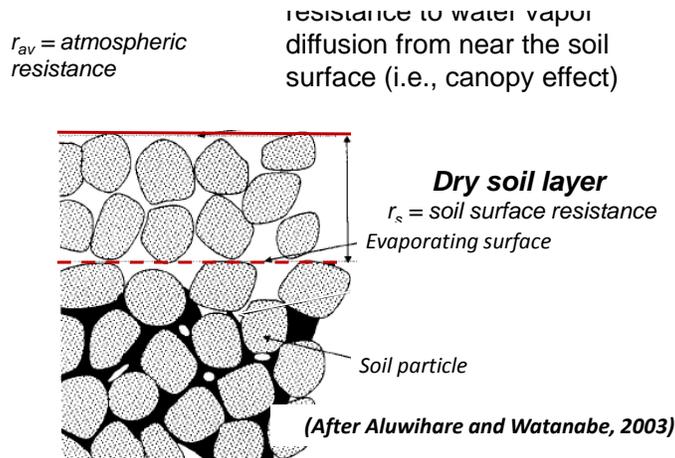


Figure 9. Illustration of “surface resistance” components (after Aluwihare and Watanabe, 2003).

Several researchers have attempted to quantify “surface resistance” and have written its magnitude in terms of the inverse of hydraulic conductivity (Fen Shu, 1982; Carmillo & Gurney, 1986; and van de Griend & Owe, 1994). Evaporation from a soil surface is based on molecular diffusion through a thin layer at the soil surface and turbulent diffusion in the air.

Fredlund-Zhang-Fredlund (2009) Formulation Using the “Total Suction Adjustment” Approach

Several researchers have attempted to adjust the total suction at the surface of a drying soil in order to more accurately simulate actual evaporation (Alvenas & Jansson, 1997; Bittelli et al., 2008; Fredlund, M.D. et al., 2011; Dunmola, 2012). The formulations take the form of a further modification to the Penman (1948) equation and makes use of an adjustment to total suction. The formulation is called the “total suction adjustment” approach. Details of this approach were proposed by Fredlund, M.D. et al., (2011) and were also implemented into the SVFlux (2009) computer model (Fredlund et al., 2012). The equation for relative humidity can be used to describe the ratio between AE/PE ; however, it is necessary to apply an adjustment factor, δ , to the total suction values as shown in equation [6]:

$$h_s = \exp\left(\frac{10^\delta \psi g \omega_v}{\gamma_w R(273.15 + T_{soil})}\right) \quad [6]$$

where: δ = a dimensionless total suction adjustment factor generally varied from 0 to 3.

The Total Suction Adjustment Factor

The empirical total suction adjustment factor, (written in terms of negative pore-water pressures, u_{wa}), is shown in equation [7].

$$u_{wa} = u_{wo} 10^\delta \quad [7]$$

where: u_{wa} = adjusted negative pore-water pressure, u_{wo} = original pore-water pressure from water phase partial differential equation, and δ = empirical adjustment factor. The adjustment of the negative pore-water pressure must be made when using the Wilson (1990) and Wilson et al., (1997) formulations for actual evaporation. An adjustment factor of 1.8 closely simulated the evaporation rate from columns of sand tested by Wilson (1990). The adjustment factor varies for different soils with the largest values being applicable for coarse-grained soils. The adjustment of the total suction applies for coupled and uncoupled solutions. The adjustment factor should remain equal to zero

attempts to simulate surface resistance near ground surface.

Figure 10 illustrate the rationale behind the calculation for the δ adjustment factor. The residual suction from the drying SWCC, is used as the reference point for calculating the δ adjustment factor. The empirical adjustment factor, in essence, translates the SWCC over to the 3000 kPa point on Lord Kelvin’s curve. Typical values of the δ adjustment factor for various residual suction values are illustrated in Figure 11. The maximum δ adjustment factor for coarse sand soils is 3.48 while no adjustment is required for clayey soils with a high air-entry value.

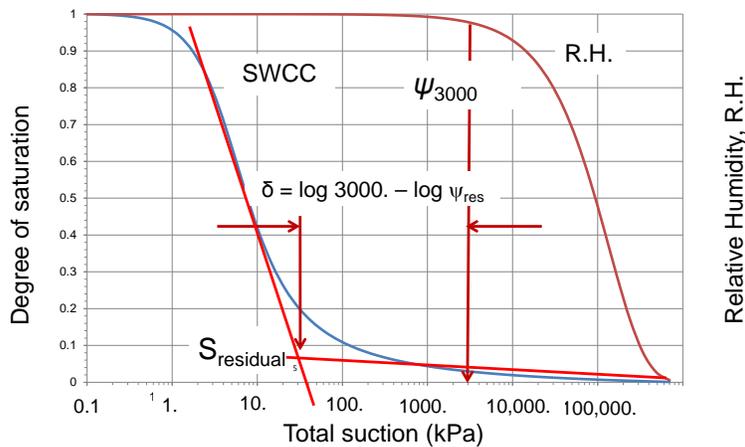


Figure 10. Illustration of the construction procedure to obtain the δ adjustment factor.

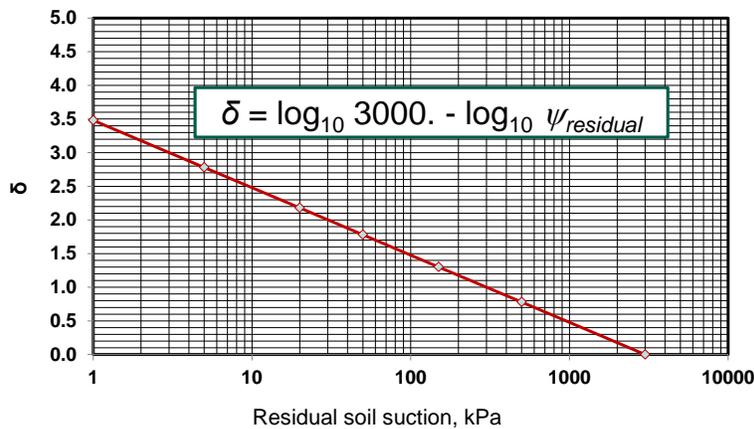


Figure 11. The δ adjustment factors for various residual suction values.

Fredlund-Zhang-Fredlund (2009) Model Calculations of Actual Evaporation from Sand Columns

Measurements by Wilson (1990) of evaporation from a sand soil column can be compared with numerical model calculations using the Fredlund-Zhang-Fredlund (2009) model in the SVFlux and SVHeat software (*SoilVision*, 2009). Both “coupled” and “uncoupled” solutions provided a reasonable simulation of actual evaporation from a sand soil. The numerical simulation results used a δ value of 1.8 and were published in Fredlund et al., (2011) and Tran et al., (2014). Space does not permit repeating the results of the simulation in this paper.

Tran (2013) Evaporation-Rate Reduction Model utilizing the Penman-Monteith (1965) equation

humidity (Kondo et al., 1970; Lee and Pielke, 1992; Tran, 2013). The complete formulation of the model is based on the Penman-Monteith (1965) model with “surface resistance”.

Evaporation-Rate Reduction Point

A reduction in actual evaporation rate has been observed from the data collected on a series of soil columns tests in the research literature. The evaporation-rate starts to reduce from potential evaporation-rate when the suction is somewhere between the air-entry value and residual suction.

$$\Psi_R = \begin{cases} \Psi_{aev} & \text{if } a=0 \\ \Psi_{res} & \text{if } a=1 \\ \Psi_{res}^a \times \Psi_{aev}^{1-a} & \text{if } 0 < a < 1 \end{cases} \quad [8]$$

where: Ψ_R = suction at evaporation-rate reduction point, kPa; Ψ_{aev} = air-entry value based on the SWCC, kPa; Ψ_{res} = residual suction from the SWCC, kPa; a = an empirical factor which varies between 0 and 1. Published results show that the best-fit value of “ a ” for sand and silt are 0.60 and 0.75, respectively.

Proposed Methods for Predicting Relative Humidity

Predicting the relative humidity at the soil surface is required in the evaporation model. Lee and Pielke (1992) used the soil moisture availability term, β , to designate the evaporation-rate reduction point, θ_R .

$$\beta = \begin{cases} \frac{1}{4} \left[1 - \cos \left(\frac{\theta}{\theta_R} \pi \right) \right]^2 & \theta < \theta_R \\ 1 & \theta \geq \theta_R \end{cases} \quad [9]$$

where: β = coefficient representing the surface moisture availability; θ_R = volumetric water content at evaporation-rate reduction point; θ = soil volumetric water content of the top soil layer.

The actual water vapour pressure can be computed from the soil moisture availability factor.

$$u_v = \beta \times u_v^{sat} + (1 - \beta) \times u_v^{air} \quad [10]$$

where: u_v = actual vapor pressure at the soil surface, kPa; u_v^{sat} = saturated vapor pressure at the soil surface, kPa; u_v^{air} = actual air pressure immediately above the soil surface, kPa.

Calculation of Soil Surface Resistance

Several researchers introduced equations to estimate “surface resistance” including Fen Shu (1982), Camillo and Gurney (1986) and van de Griend and Owe (1994). Soil surface resistance varies from zero at the wet soil surface to several thousand at the dry soil surface. The van de Griend and Owe's (1994) equation for soil surface resistance is shown and is written as follows:

$$r_s = 10 \times e^{0.3563(\theta_{min} - \theta_{top})} \quad [11]$$

where: r_s = soil surface resistance at top 0 – 1 cm, s/m; q_{top} = volumetric water content of the top 1 cm layer, (%); q_{min} = an empirical minimum above which the soil is able to deliver vapor at a potential rate, (%). The original model sets $q_{min} = 15\%$ as a fixed value for all soils.

Tran's (2013) Modified Penman-Monteith (1965) Equation

The Penman-Monteith's equation, (i.e., combination of energy balance and mass balance) was used to derive a soil-atmospheric equation that considers the effect of “surface resistance” on actual evaporation.

$$10 \quad AE = \frac{\Gamma Q + \eta \frac{f'(u)}{f(u)} E_a}{\Gamma + \eta A \frac{f'(u)}{f(u)}} \quad [12]$$

where: AE = transpiration rate from a soil, mm/day; E'_a = aerodynamic evaporative term, $E'_a = f(u) (u_{ao} - u_a)$, mm/day/Pa; Q_n = heat budget, mm/day; u_{ao} = saturation vapor pressure of the mean air temperature, kPa; u_a = vapor pressure of the air above the surface, kPa; $f(u)$ and $f'(u)$ = the transmission functions for mass and heat, respectively. Further details related to the transmission functions, $f(u)$ and $f'(u)$, can be found in Tran (2013).

Ability of the Tran (2013) Model to Simulate Actual Evaporation from Sand Soils

Soil column drying tests for the Beaver Creek sand (Wilson, 1990) and Processed Silt (Bruch, 1993), were re-analyzed using the Tran (2013) model. The actual evaporation rate obtained using the numerical model (i.e., using ComSol Metaphysics software and Excel); along with measured water contents and temperatures at the soil surface showed close agreement was observed between all the laboratory results and the computed results. Comparisons between the numerical model results and laboratory test results can be found in Tran (2013) and Tran et al., (2014).

The proposed soil-atmosphere flux equation describing evaporation from a soil surface takes into consideration net radiation, wind speed, relative humidity of the air and soil surface, and “soil surface resistance”. The evaporation rate is shown to decrease during the drying process mainly as a result of an increase in soil surface resistance.

Concluding Remarks on Modeling Actual Evaporation

Two approaches were examined that attempted to incorporate “surface resistance” into the formulation of the models. One approach was referred to as the “total suction adjustment” approach and the other was referred to as the “relative humidity adjustment” approach. The application of the modified actual evaporation models have been compared to measured evaporation rates from several sand column tests. The comparative simulations showed that both the total suction adjustment approach and the relative humidity adjustment approach produced greatly improve the simulation of actual evaporation.

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