SOILVISION CONSOLIDATION

Verification Manual

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Bentley Systems Incorporated

1 INTRODUCTION

The word "Verification", when used in connection with computer software can be defined as "the ability of the computer code to provide a solution consistent with the physics defined by the governing partial differential equation, PDE". There are also other factors such as initial conditions, boundary conditions, and control variables that also affect the accuracy of the code to perform as stated.

"Verification" is generally achieved by solving a series of so-called "benchmark" problems. "Benchmark" problems are problems for which there is a closed-form solution or for which the solution has become "reasonably certain" as a result of long-hand calculations that have been performed. Publication of the "benchmark" solutions in research journals or textbooks also lends credibility to the solution. There are also example problems that have been solved and published in User Manual documentation associated with other comparable software packages. While these are valuables checks to perform, it must be realized that it is possible that errors can be transferred from one's software solution to another. Consequently, care must be taken in performing the "verification" process on a particular software package. It must also be remembered there is never such a thing as complete software verification for "all" possible problems. Rather, it is an ongoing process that establishes credibility with time.

Bentley Systems takes the process of "verification" most seriously and has undertaken a wide range of steps to ensure that the software will perform as intended by the theory of saturated-unsaturated stress and deformation.

SOILVISION Consolidation couples the processes that are available separately in the SVFLUX and SVSOLID software packages.

The following models represent comparisons made to textbook solutions, hand calculations, and other software packages. We at Bentley Systems are dedicated to providing our clients with reliable and tested software. While the following list of example models is comprehensive, it does not reflect the entirety of models, which may be posed to the software. It is our recommendation that water balance checking be performed on all model runs prior to presentation of results. It is also our recommendation that the modeling process moves from simple to complex models with simpler models being verified through the use of hand calculations or simple spreadsheet calculations.

2 CONSTANT SOIL PROPERTIES (SMALL STRAIN)

This section will compare SOILVISION Consolidation to published research related to small-strain consolidation in traditional geotechnical engineering. The main goal of this section to verify the results of the SOILVISION Consolidation finite element solution engine to validated results of analytical (closed-form) and numerical solutions. The small-strain setting in SOILVISION Consolidation will be utilized. Note on Unsaturated conditions: The SVSOLID/SVFLUX consolidation formulation is based on equations that assume fully saturated conditions. Care should be taken in choosing Initial Conditions or Boundary Conditions, such that unsaturated conditions would not be imposed on the solution.

2.1 CONSOLIDATION OF HOMOGENEOUS SOIL

Reference: Verruijt (2013)

Project: Consolidation
Model: Homogeneous Soil_GT.svm

Main Factors Considered:

- Comparison to the closed-form solution of the Terzaghi's consolidation theory.
- The solution is addressed in a dimensionless form.

The Terzaghi's theory of consolidation is examined and the available closed-form solution of the theory is compared against the SOILVISION Consolidation finite element solver. The model is in 1D and it is a single drainage to the surface.

2.1.1 Model Description

The schematic of the model is shown in Figure 1 in which the base is fixed and impervious. Water is allowed to drain to the top surface as the soil consolidates due to a surcharge applied at the surface.

Equation [1] is the traditional consolidation theory (Terzaghi's equation – Terzaghi (1943)). This type of equation can be used to address many transport phenomena, such as heat conduction and chemical diffusion as well as fluid flow in porous media. It is convenient to express this equation in a dimensionless form (equation [2]).

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad [\ 1\ ]$$

where, u is the excess pore-water pressure due to an applied load (q), t is the time, z is the vertical axis, c_v is the coefficient of consolidation.

$$\frac{\partial \bar{u}}{\partial T} = \frac{\partial^2 \bar{u}}{\partial Z^2} \quad [\ 2\ ]$$

where, the dimensionless excess pore-water pressure, $\bar{u} = \frac{u}{u_o} \in [0,1]$, $u_o = q$ is the initial excess pore-water pressure, $Z = \frac{z}{h} \in [0,1]$ is the dimensionless coordinate, $T = \frac{c_v t}{h^2}$ is the dimensionless time factor.

The above dimensionless equation (equation [2]) is a Poisson's equation and its close-form solution is readily available as:

$$\bar{u} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[(2n-1)\frac{\pi}{2} Z\right] \exp\left[-(2n-1)^2 \frac{\pi^2}{4} T\right] \quad [\ 3\ ]$$

The degree of consolidation of the soil layer can be expressed as:

$$U = \int_0^1 (1-\bar{u})dZ = 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp\left[-(2n-1)^2 \frac{\pi^2}{4} T\right] \quad [\ 4\ ]$$

2.1.2 Results

The dimensionless excess pore-water pressure profiles at various time factors, T, are shown in Figure 2. This figure indicates the SOILVISION Consolidation solver matches the closed-form solution. Figure 3 shows the changes of degree of consolidation at various time factor for both SOILVISION Consolidation and the closed-form solution. The results of SOILVISION Consolidation closely match to the closed-form solution. Figure 3 also confirms that the consolidation in soil is completed at a dimensionless time factor, T = 2.

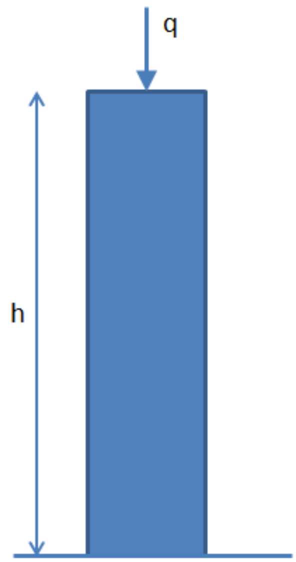

Figure 1. Geometry and boundary conditions

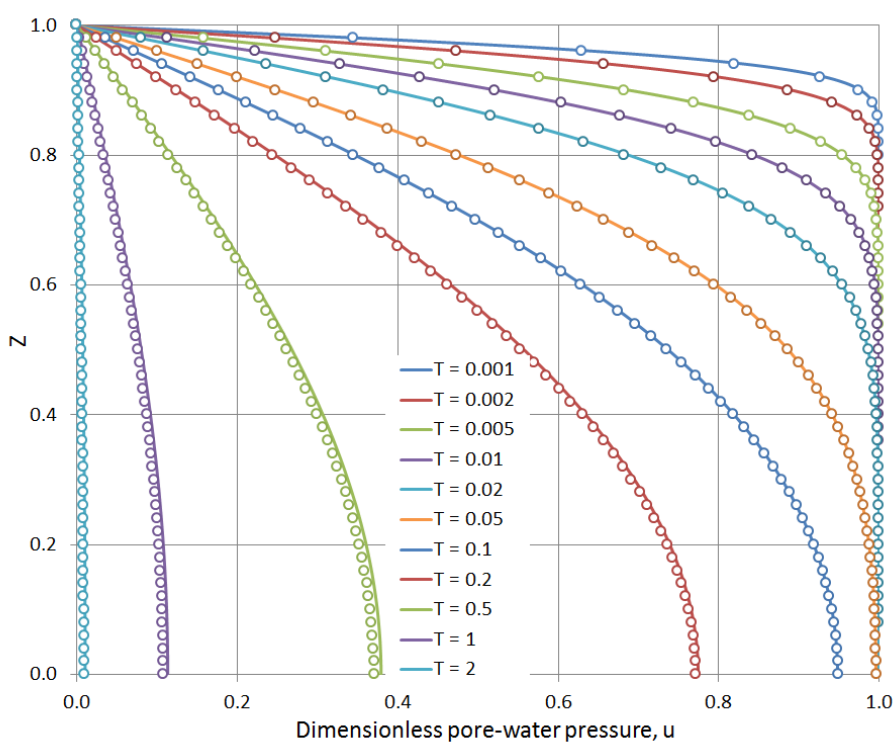

Figure 2. Dimensionless pore-water pressure profile at various dimensionless time factors, *T*. The marker points are the closed-form solution and solid lines are the SOILVISION Consolidation solution.

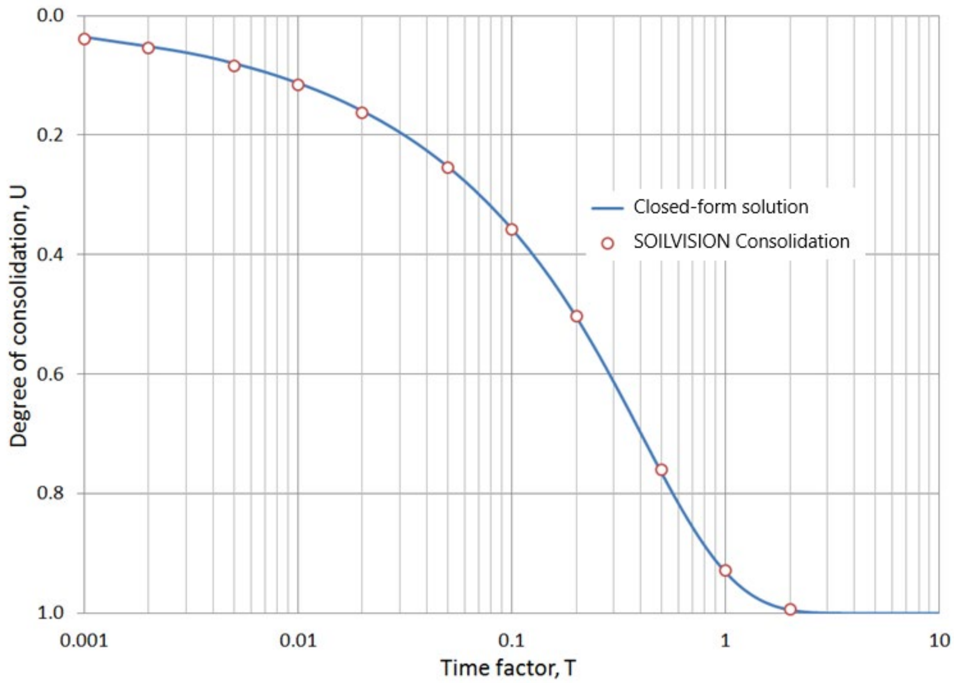

Figure 3. Degree of consolidation changes with dimensionless time factors, *T*.

2.2 CONSOLIDATION OF LAYERED SOILS

Reference: Pyrah (1996)

Project: Consolidation
Model: Layered soils Case1_GT.svm, Layered soils Case2_GT.svm, Layered soils Case3_GT.svm, Layered soils Case4_GT.svm

Main Factors Considered:

- Comparison to literature one-dimensional consolidation model of layered soils
- Validation of the linear elastic consolidation model
- Two soil layers with different layouts

The numerical solution obtained by Pyrah (1996) is used to compare with the SOILVISION Consolidation consolidation software via four consolidation scenarios for layered soils. In addition, various boundary conditions are also examined. The simulation is performed in 1D and linear elastic soil model is used.

2.2.1 Model Description

Figure 4 shows the four scenarios used in this section. The thickness of layered soils is one unit thick and the pore fluid also has a unit weight of 1. Cases 1 and 2 have an impervious base and water is allowed to drain at the top surface. Case 2 has an upper layer of higher permeability and is opposite to Case 1. Cases 3 and 4 are homogeneous with different soil types for each case and the boundary conditions are similar to the Cases 1 and 2.

Consolidation properties of soils A and B are shown in Table 1. The material properties in Table 1 were taken from Pyrah (1996) and are theoretical rather than physical property values. The linear elastic model requires a Young's modulus and Poisson's ratio as inputs. The materials are saturated and have constant coefficients of permeability.

Table 1: Parameter Inputs

Parameter	Soil A	Soil B
Hydraulic conductivity, k	1	10
Coefficient of consolidation, c_v	1	1
Coefficient of volume change, m_v	1	10
Young modulus, E	1	0.1
Unit weight of water	1	1
Poisson ratio, ν (assumed)	0	0

The Young's modulus in Table 1 was calculated via the below equations, using the K_o loading formula

For K_o loading

$$E = \frac{(1+\upsilon)(1-2\upsilon)}{m_v(1-\upsilon)} \qquad [5]$$

For Isotropic loading

$$E = \frac{3(1-2\upsilon)}{m_v} \qquad [6]$$

2.2.2 Results

Figure 5 to Figure 7 show the excess pore-water pressure profiles for various times in four cases. It is noticed that the case 3 and 4 have the same results and are plotted in Figure 7. The results show a good agreement with the results obtained by Pyrah (1996). The surface settlement rates of these cases are shown in Figure 8.

When comparing the results of cases 1 and 2, it can be clearly observed that with a higher permeability layer on top, the consolidation in this case occurs slower than that of Case 1, which has the opposite layout.

Cases 3 and 4 prove that for a homogeneous soil layer the consolidation process is not controlled by the permeability alone, but by the coefficient of consolidation (Figure 8), which is a combination of permeability and the coefficient of volume change.

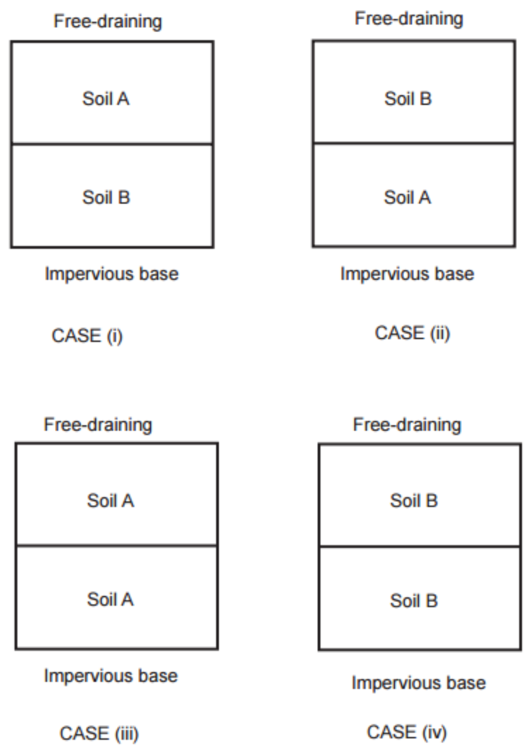

Figure 4. Geometry and boundary conditions (from Pyrah, 1996)

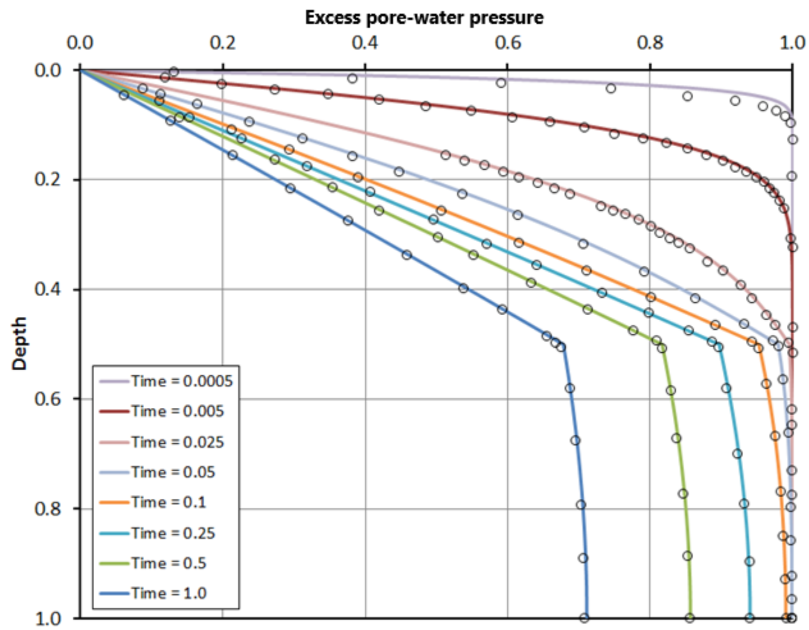

Figure 5. Excess pore-water pressure of Case 1 (data points from Pyrah, 1996)

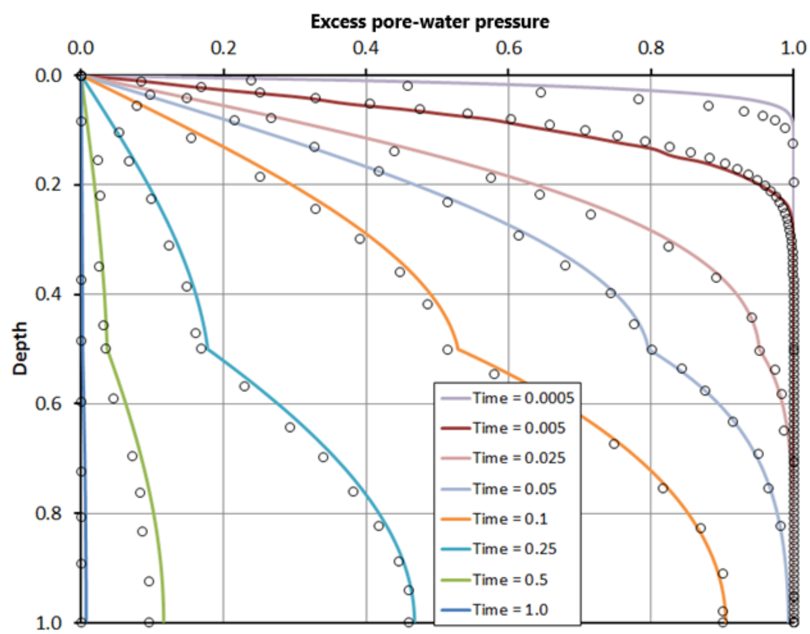

Figure 6. Excess pore-water pressure of Case 2 (data points from Pyrah, 1996)

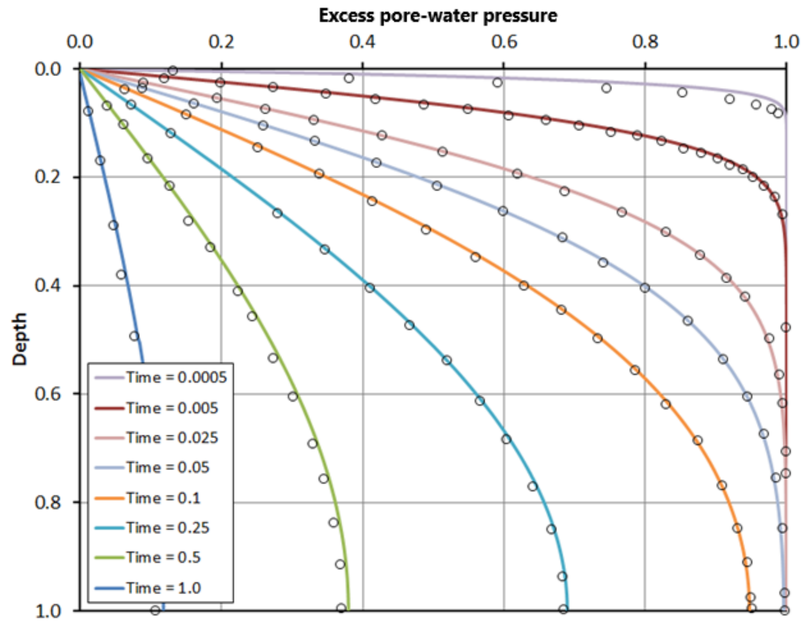

Figure 7. Excess pore-water pressure of Cases 3 and 4 (data points from Pyrah, 1996)

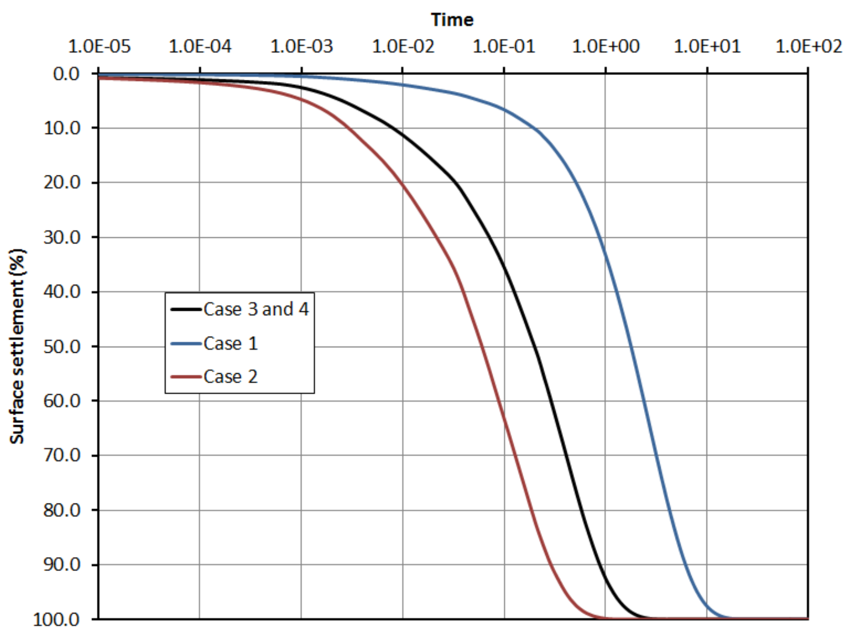

Figure 8. Surface settlement rate for the four cases.

2.3 CONSOLIDATION OF FILL LAYER IN LAGUNILLAS, VENEZUELA

Reference: Lambe and Whitman (1969)

Project: Consolidation
Model: Consolidation Lagunillas_GT.svm

Main Factors Considered:

- Comparison to literature one-dimensional consolidation model of multi-layer soils
- Validation of the linear elastic consolidation model compared against field results

This example is taken from the example 25.6 in "Soil Mechanics" by Lambe and Whitman (1969). A 15 ft (4.5 m) fill material is placed over a large area of a soil profile, including 17.5 ft (5.3 m) silt and 14 ft (4.3 m) clay. The settlement of the silt layer

is considered small compared to those of the clay layer. Therefore, the final settlement is the settlement of the clay layer. This was a field study in Lagunillas, Venezuela.

2.3.1 Model Description

Figure 9 shows the soil profile and the model geometry is shown in Figure 10. Only the clay layer is simulated and it has a thickness of 4.3 m. The applied load at the surface is equal to the weight of the fill material, which is q = 4.5 x 22 = 99 kPa. Water can drain from the clay layer in both upward and downward directions. This drainage is due to a sand layer at the base and the fact that the upper silt has a much higher coefficient of consolidation (945 m^2/year versus 1.26 m^2/year of the clay).

The geotechnical properties of the clay lauer are shown in Table 2. The linear elastic model of small strain consolidation requires Young's modulus, Poisson's ratio and saturated permeability as inputs.

Table 2: Material inputs (Lambe and Whitman, 1969)

Parameter	Clay
Hydraulic conductivity, k (m/year)	0.018
Coefficient of consolidation, c_v (m^2/year)	1.26
Coefficient of volume change, m_v (m^2/kN)	0.00153
Young modulus, E (kPa)	652
Poisson ratio, ν (assumed)	0

2.3.2 Results

Figure 11 shows a good agreement between SOILVISION Consolidation and the results by Lambe and Whitman (1969) of the surface settlement. The layer is fully consolidated in about 6 years.

The excess pore-water pressure changes with time are shown in Figure 12 from a uniform value of 99 kPa to around 2 kPa after 6 years with a double drainage boundary condition. This results in degree of consolidation are around 98 %.

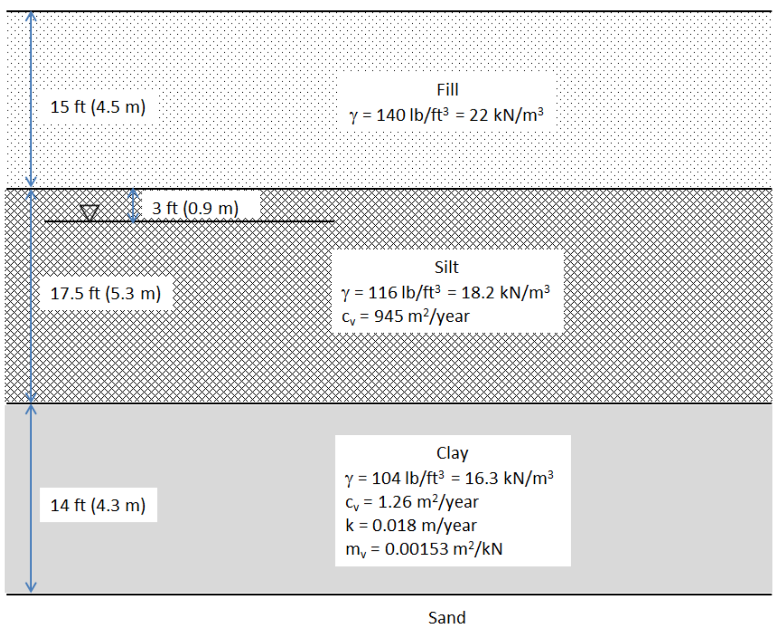

Figure 9. Soil profile (Lambe and Whitman, 1969)

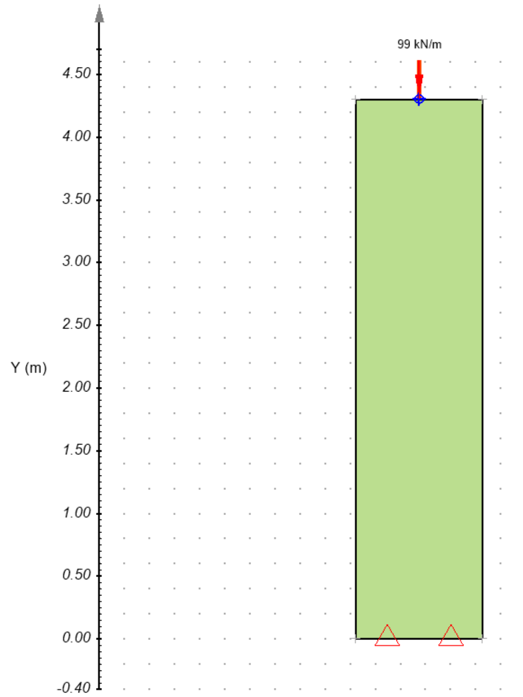

Figure 10. Geometry of model in SOILVISION Consolidation

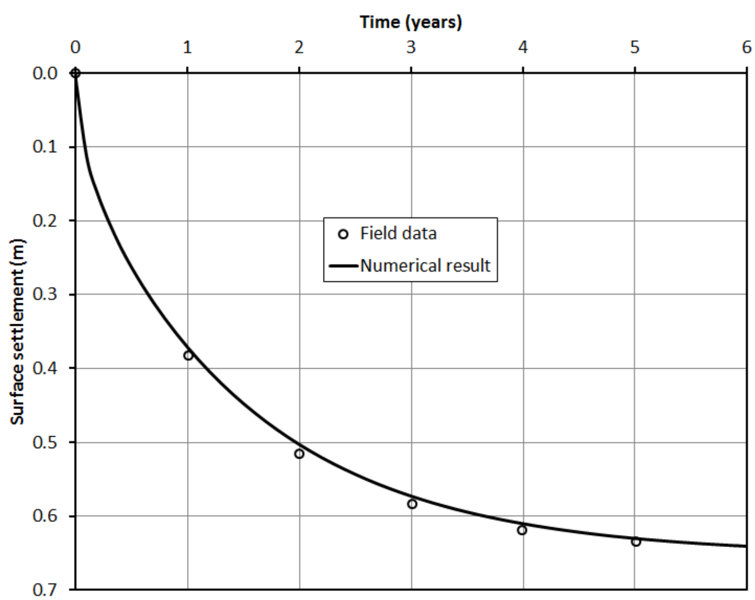

Figure 11. Settlement with time: SOILVISION Consolidation numerical result and data reported by Lambe and Whitman (1969).

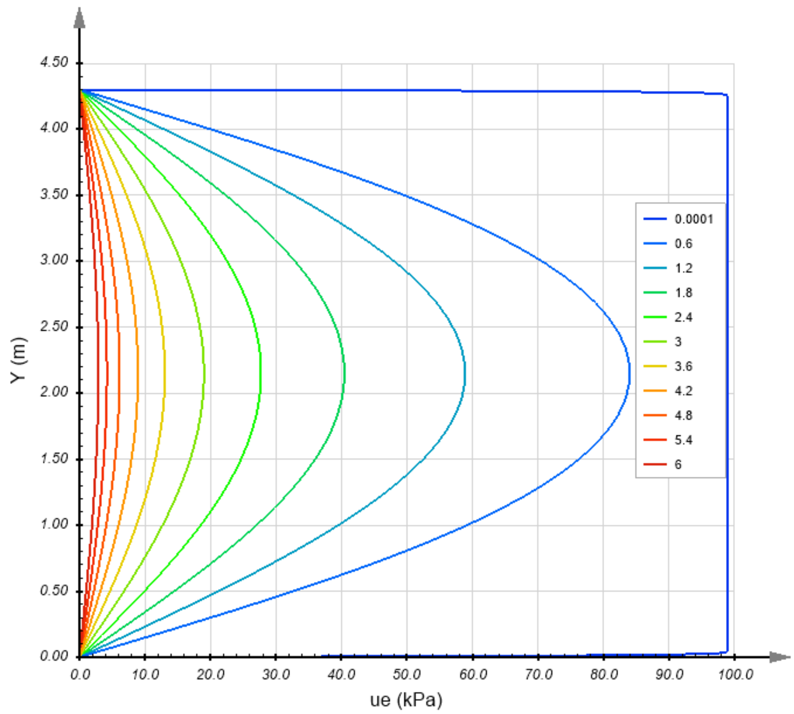

Figure 12. Excess pore-water pressure profile changes with time.

2.4 CONSOLIDATION OF MULTI-LAYERED SOILS

Reference: Lee et al. (1992)

Project: Consolidation
Model: Multi layered soils_SingleDrainage_GT.svm, Multi layered soils_DoubleDrainage_GT.svm

Main Factors Considered:

- Comparison to literature one-dimensional consolidation model of a multi-layered soil system.
- Validation of the linear elastic consolidation model with a surcharge applied at ground surface.
- Complex soil profile with single and double drainage.

This model is based on the paper by Lee et al. (1992) and it consists of a multi-layered soil system. The model is assumed to perform under 1D consolidation conditions with a uniform load, q, applied at the surface. Both single and double-drainage conditions were examined.

2.4.1 Model Description

The geometry for the model is shown in Figure 13 and includes 4 soils layers. The water table is at the surface and there are two simulation scenarios: Case 1: Single drainage in which water only drains to the surface; Case 2: Double drainage in which water is allowed to drain at both the top and bottom surfaces.

Table 3 shows the material inputs for different layers and the Young modulus was calculated via the coefficient of consolidation, c_v, and permeability. The Young modulus and Poisson's ratio were determined as the SOILVISION Consolidation solves for consolidation via Biot's consolidation theory.

Table 3: Parameter Inputs (Lee et al., 1992)

Soil name	Young modulus	Poisson ratio	Coefficient of consolidation (m^2/year)	Permeability (m/year)	Thickness (m)
Layer 1	15913.24	0	1.394	8.76×10^{-4}	0.93
Layer 2	24984.62	0	6.496	2.60×10^{-3}	1.86
Layer 3	50000.00	0	1.845	3.69×10^{-4}	2.79
Layer 4	25005.39	0	2.318	9.27×10^{-4}	1.86

$$E = \frac{(1+\nu)(1-2\nu)}{(1-\nu)\, m_v} \qquad [7]$$

$$m_v = \frac{k}{c_v \gamma_w} \qquad [8]$$

where, ν = 0 (assumed) is the Poisson's ratio, c_v is the coefficient of consolidation, γ_w is the unit weight of water, *k* is the permeability, m_v is the coefficient of volume change.

The results of consolidation analysis are presented using the dimensionless time factor, *T*, which is defined as the follow.

$$T = \frac{c_v^1\, t}{H^2} \qquad [9]$$

where, c_v^1 is the coefficient of consolidation of the Layer 1, *t* is the time in years, *H* = 7.44 m, which is the total thickness of the geometry.

2.4.2 Results

Figure 14 and Figure 15 show the results of excess pore-water pressure and settlement for the single drainage case. The results from SOILVISION Consolidation match closely to the results from Lee et al. (1992).

The results of excess pore-water pressure and settlement are shown in Figure 16 and Figure 17 in the case of double drainage simulation,. The excess pore-water pressure results from SOILVISION Consolidation are in excellent agreement with Lee et al. (1992) although there are small discrepancies in settlement (Figure 17).

With double drainage, the excess pore-water pressure drains much faster than for the single drainage case. In the case of single drainage, the time required to achieve full consolidation is *T* = 1 (dimensionless time). In contrast, the dimensionless time required for full consolidation is *T* = 0.2 for double drainage.

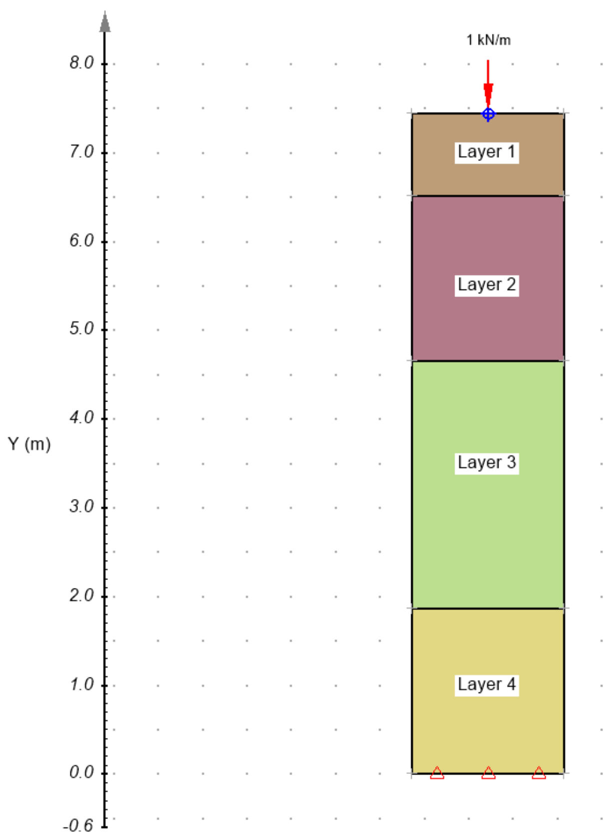

Figure 13. Geometry and boundary conditions

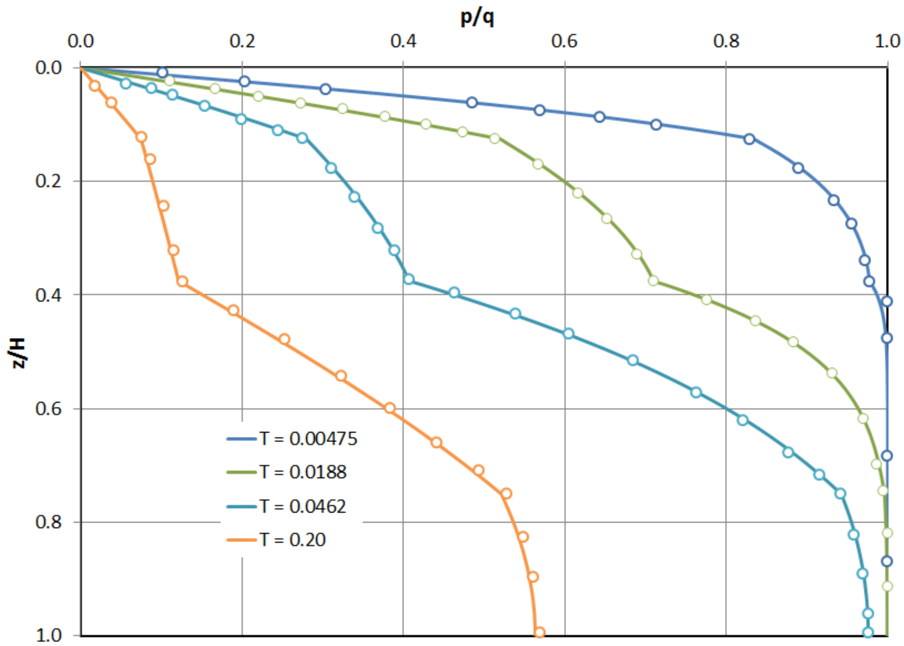

Figure 14. Excess pore-water pressure profile with depth in the single drainage case (marker points are from Lee et al., 1992).

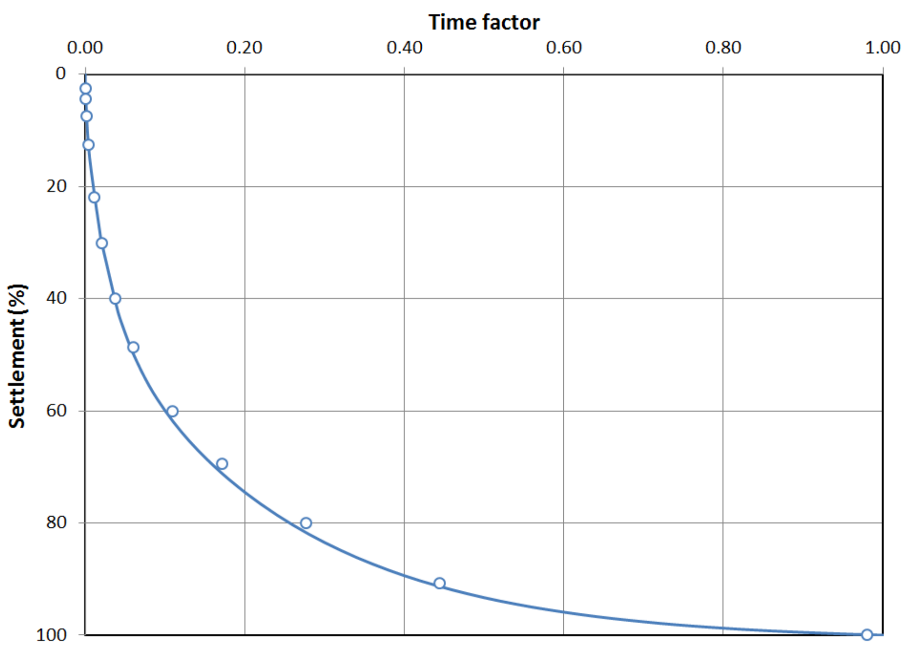

Figure 15. Settlement versus time factor in the single drainage case (marker points are from Lee et al., 1992).

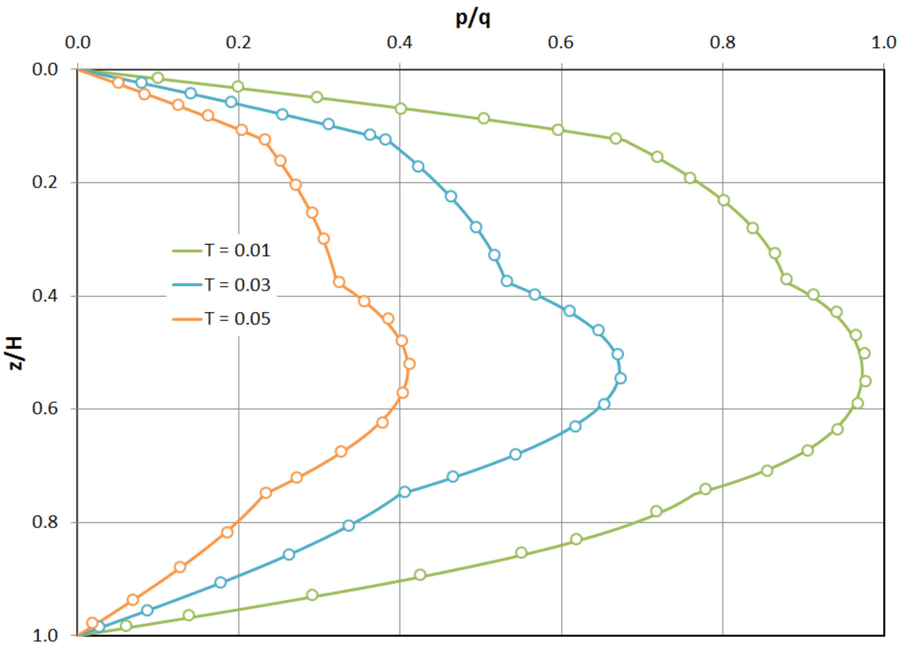

Figure 16. Excess pore-water pressure profile with depth in the double drainage case (marker points are from Lee et al., 1992).

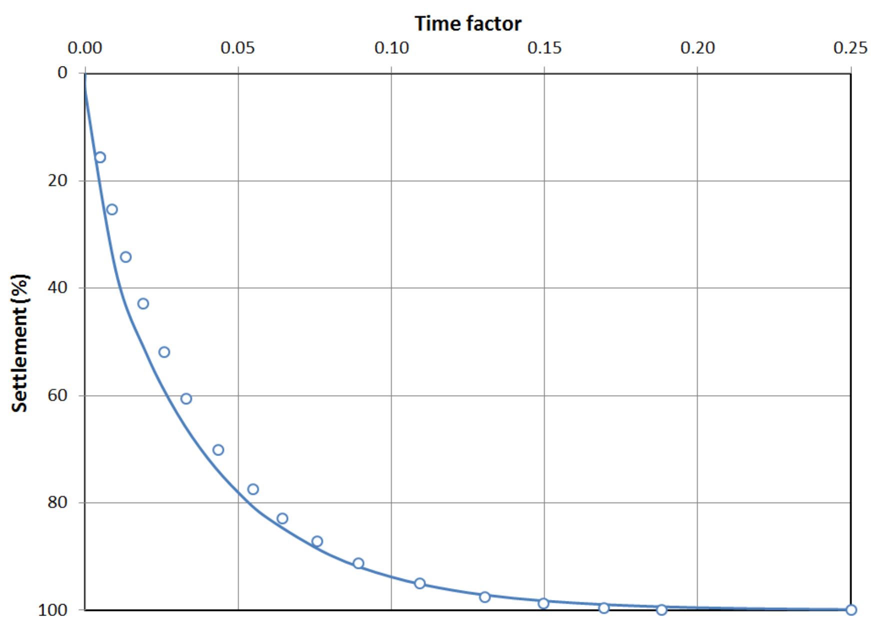

Figure 17. Settlement versus time factor in the double drainage case (marker points are from Lee et al., 1992).

2.5 2D PLANE STRAIN CONSOLIDATION BENEATH A STRIP FOOTING

Reference: Hwang et al. (1971)

Project: Consolidation
Model: Hwang_SmallStrainConsolidation_GT.svm

Main Factors Considered:
- Comparison to literature results of 2D plane strain consolidation.
- Examining the Mandel-Cryer effect in the early stage of consolidation

A 2D plane strain consolidation of excess pore-water pressure induced by a distributed load is examined in this section. The Mandel-Cryer effect is also studied as the excess pore-water pressure increases in the early consolidation stages (Mandel,

1953; Cryer, 1963). The Mandel-Cryer effect is only captured via the Biot's consolidation theory (Biot, 1941). SOILVISION Consolidation solves the Biot's consolidation equations via the finite element method.

2.5.1 Model Description

The geometry and boundary conditions used in the model are shown in Figure 18. The water drained due to the decreases in excess pore-water pressure is allowed to escape to the top surface. The base of the model is fixed and impervious. Due to the symmetry of the problem, only one half is modeled and the strip footing width is 2*a*. A finite element mesh is shown in Figure 19 with dense mesh beneath the apply load (strip footing) of 1 kPa.

The parameters used in the model are shown in Table 4 (Hwang et al., 1971). The Young's modulus shown in Table 4 correspond to an assumed Poisson's ratio = 0 using equation [5].

Table 4: Material inputs

Parameter	Soil
Hydraulic conductivity, k (m/s)	0.01
Coefficient of consolidation, c_v (m^2/s)	1
Coefficient of volume change, m_v (m^2/kN)	0.1
Poisson ratio, ν (assumed)	0
Young modulus, E (kPa)	10
Shear modulus, $G = E/(2(1+\nu))$ (kPa)	5

There are two dimensionless parameters used in this model that are calculated as a linear elastic variable and the coefficient of permeability of the soil.

The coefficient of consolidation is expressed as:

$$c_v = \frac{2\,G\,k}{\gamma_w} \quad [\,10\,]$$

where, G is the shear modulus, k is the soil permeability, γ_w is the unit weight of water
The dimensionless time is expressed as:

$$T = \frac{c_v t}{a^2} \quad [\,11\,]$$

where, t is the actual simulation time, and a is the half-width of the footing.

2.5.2 Results

The variation of excess pore-water pressure at two specific points is shown in Figure 20. The Mandel-Cryer effect is clearly captured by the SOILVISION Consolidation solver. The excess pore-water pressure initially increases before it reduces. The excess pore-water pressure results agree well with the results of Hwang at el. (1971). This figure also clearly indicates that the increasing in excess pore-water pressure is small at locations far away from the apply load location. The excess pore-water pressure reaches its peak at $T = 2 \times 10^{-2}$.

The excess pore-water pressure profile at the centre line of the model is shown in Figure 21 at $T = 0.1$. The excess pore-water pressure profile of this study matches well against the results of Hwang at el. (1971) within the depth $z/a = 2$. Below this depth, the two results shows some small discrepancies. Excess pore-water pressure contours at a dimensionless time factor, $T = 0.1$,beneath the strip footing are shown in Figure 22 and it shows the largest excess pore-water pressure is within 1 m directly beneath the footing.

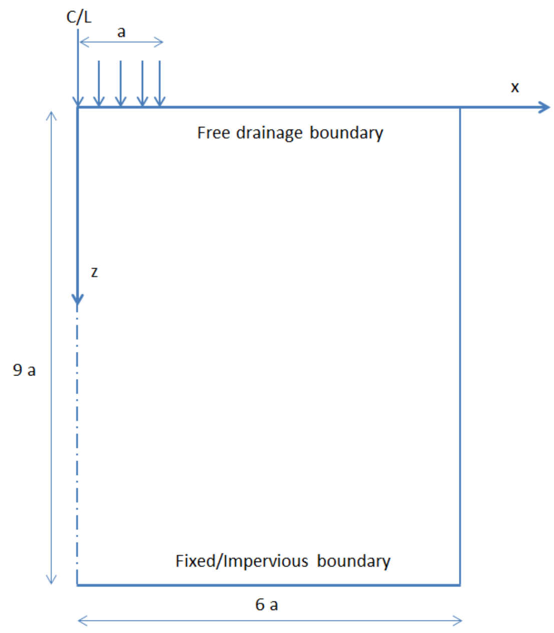

Figure 18. Geometry and boundary conditions

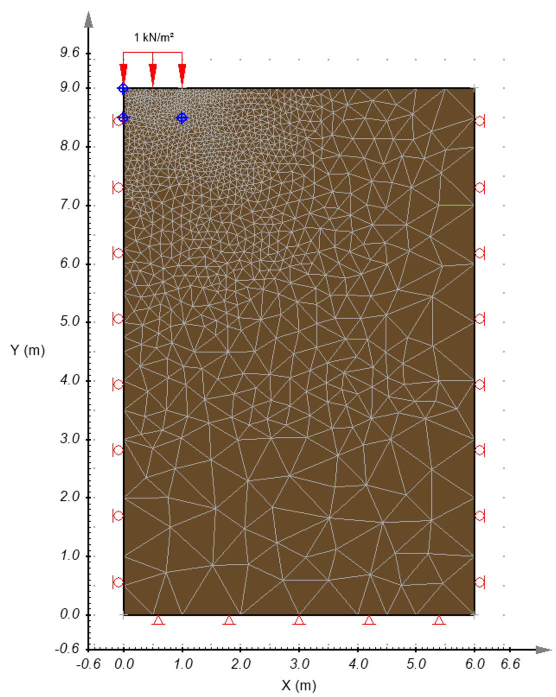

Figure 19. Finite element mesh

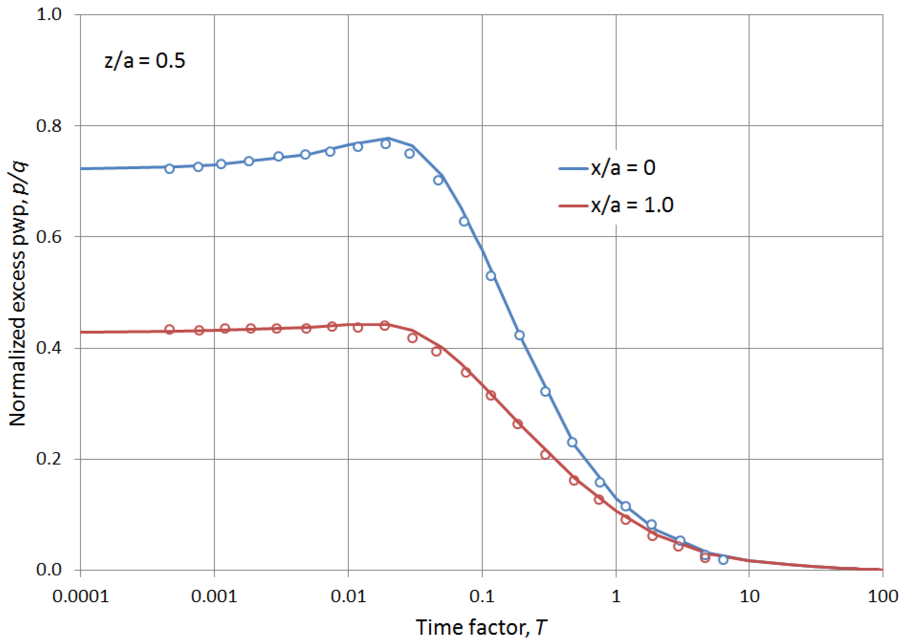

Figure 20. Variation of excess pore-water pressure with time (marker points are from Hwang et al., 1971)

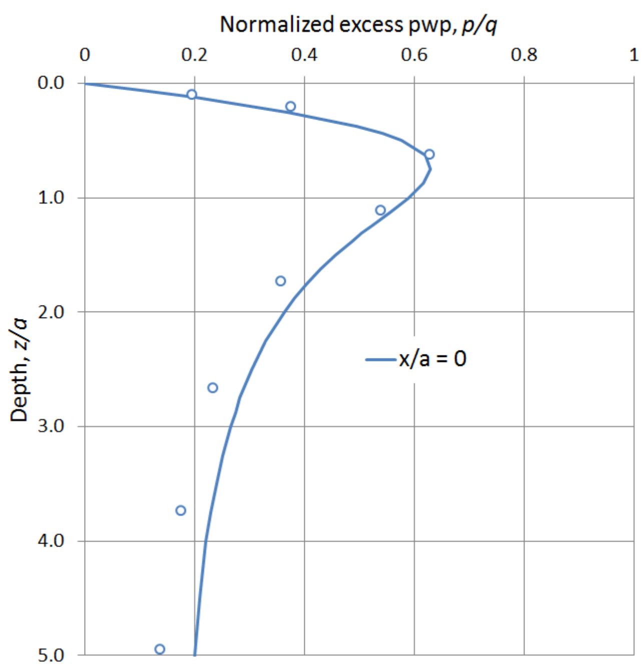

Figure 21. Excess pore-water pressure profile at $T = 0.1$ (marker points are from Hwang et al., 1971)

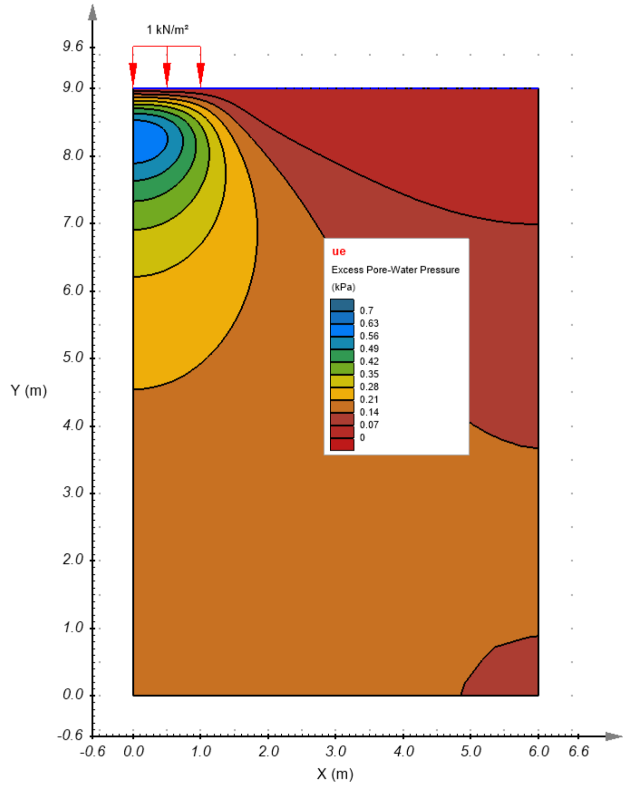

Figure 22. Excess pore-water pressure contours at a dimensionless time factor $T = 0.1$.

2.6 MANDEL-CRYER EFFECT IN 2D AND 3D CONSOLIDATION PROBLEMS

Reference: Verruijt (2013)

Project: Consolidation
Model: MandelCryer_Problem1_Poisson000_GT, MandelCryer_Problem1_Poisson025_GT
MandelCryer_Problem1_Poisson049_GT, MandelCryer_Problem2_Poisson000_GT
MandelCryer_Problem2_Poisson025_GT, MandelCryer_Problem2_Poisson049_GT
MandelCryer_Problem3_Poisson000_GT, MandelCryer_Problem3_Poisson025_GT
MandelCryer_Problem3_Poisson049_GT

Main Factors Considered:

- Comparison the consolidation results in 2D and 3D geometries.
- Examining the Mandel-Cryer for 2D and 3D problems

The Biot's theory of consolidation has unique features when compared to the consolidation theory of Terzaghi (1943). The Mandel-Cryder effect (Mandel, 1953; Cryer, 1963) shows that the excess pore-water pressure induced by an applied load initially increases to a value larger than an initial excess pore-water pressure and then decreases to the final zero value. This phenomenon is explained using the Biot's theory of consolidation (Biot, 1941).

2.6.1 Model Description

Three problems are considered in this section: Problem 1 is similar to the problem by Mandel (1953) and Problem 2 has a similar concept as suggested by Cryer (1963). Problem 3 in this section is an extension of the problem 1 to a 3D domain.

2.6.1.1 Problem 1

In this section, the 2D geometry is used to examine the Mandel-Cryder effect. A rectangular soil sample is subjected to a constant distributed load q, and the width of the soil sample is $2a$ (Figure 36). The soil sample is allowed to drain to both sides in a lateral manner. This problem is referred to as a 2D plane strain problem.

2.6.1.2 Problem 2

Figure 24 shows the geometry of the problem 2, in which left and bottom boundaries are fixed and are impermeable. Meanwhile the right and top boundaries are allowed to drain. The width and height of the geometry are referred to as *"a"* and this problem is also a plane strain analysis similar to problem 1.

2.6.1.3 Problem 3

Problem 3 is a 3D model that is made by extending the out-of-plane to have a width of *"a"* (Figure 25). The initial excess PORE-WATER PRESSURE $p_o = 0.47q$, and this result was obtained using numerical results since there is no analytical solution. Same as in problem 1, water is allowed to drain laterally and the sample is compressed from a top rigid plate of load q. Only lateral displacement is allowed on two drainage boundaries.

2.6.2 Results

The results of the problems are shown in the below sub-sections and the Mandel-Cryer's effect is illustrated by plotting the increase of excess pore-water pressure above the initial excess pore-water pressure with time prior to dissipation

2.6.2.1 Problem 1

At the initial time $t = 0$, the uniform-distributed load applied on the surface and the resulting initial excess pore-water pressure is $p_o = ½\, q$ (Verruijt, 2013). In following results, a dimensionless time T is used as shown in Eq. [12] in which c_v is the consolidation coefficient.

$$T = \frac{c_v t}{a^2} \qquad [\ 12\ ]$$

Figure 26 shows the distribution of excess pore-water pressure at the base of the sample with a Poisson's ratio of 0. At the time $T = 0.01$, the normalized pore-water pressure is larger than 1 (1.06 to be exact) and at $T = 0.1$ this normalized pore-water pressure increases further to 1.16. The normalized excess pore-water pressure is then reduced below 1 and finally becomes zero.

At higher Poisson's ratio = 0.25, the increase in normalized excess pore-water pressure is less pronounced than in the case of Poisson's ratio of 0 (Figure 27). With Poisson's ratio = 0.49, the Mandel-Cryer effect is not obvious as the normalized excess pore-water pressure is around 1.0 (Figure 28).

2.6.2.2 Problem 2

For the problem 2, the initial excess pore-water pressure induced by the applied load if $p_o = q$. The results are also expressed in term of the dimensionless time ,T, as shown in the previous section.

Figure 29, Figure 30 and Figure 31 show the changes excess pore-water pressure at the base of the model. The results are similar to those calculated in Problem 1, where the Mandel-Cryer effect is clearly captured. At a Poisson ratio of 0, the increase in excess pore-water pressure is the largest and decreases with the increase in Poisson ratio.

As the Poisson ratio approaches 0.5 (< 0.5), the increases in excess pore-water pressure above the initial excess pore-water pressure, p_o is insignificant. Figure 32 shows the changes in excess pore-water pressure at point "A" for various Poisson ratios between 0 and 0.49. It clearly shows the excess pore-water pressure reaches its peak about $T = 0.1$ and at $T = 1$ the excess pore-water pressure is about $0.1p_o$ or less.

2.6.2.3 Problem 3

The Mandel-Cryer effect in the 3D sample is shown in Figure 33, Figure 34 and Figure 35. These figures clearly show that the Mandel-Cryer's effect depends on the Poisson's ratios of the material similar to what was observed in Problems 1 and 2.

The results of Problem 3 indicate that the excess pore-water pressure reaches its peak at $T = 0.1$ and its value is about $1.17p_o$ and this value is slightly larger than the result in Problem 1 with a Poisson's ratio of 0 (Figure 26). A similar observation can be drawn from the case of Poisson's ratios of 0.25 and 0.49.

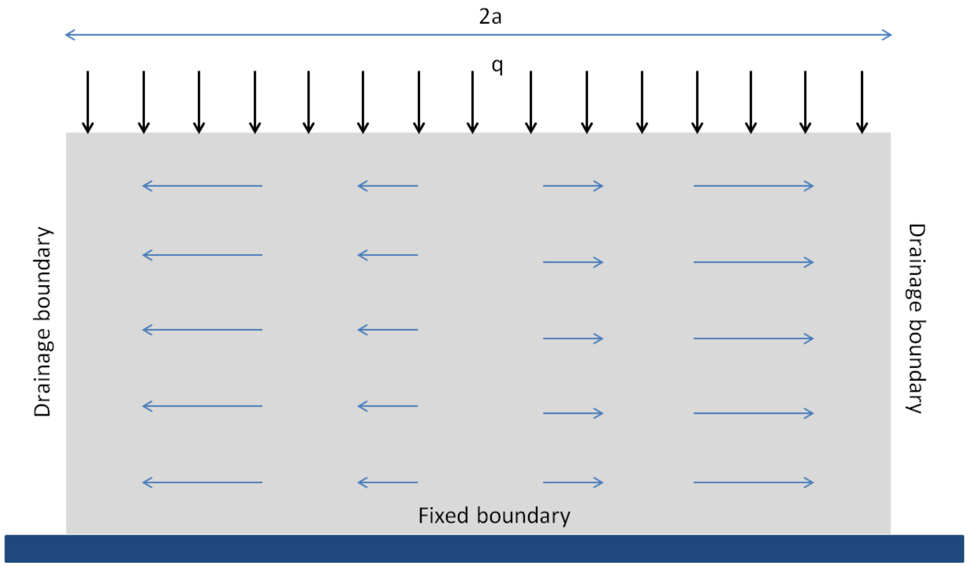

Figure 23. Geometry and boundary conditions (Problem 1)

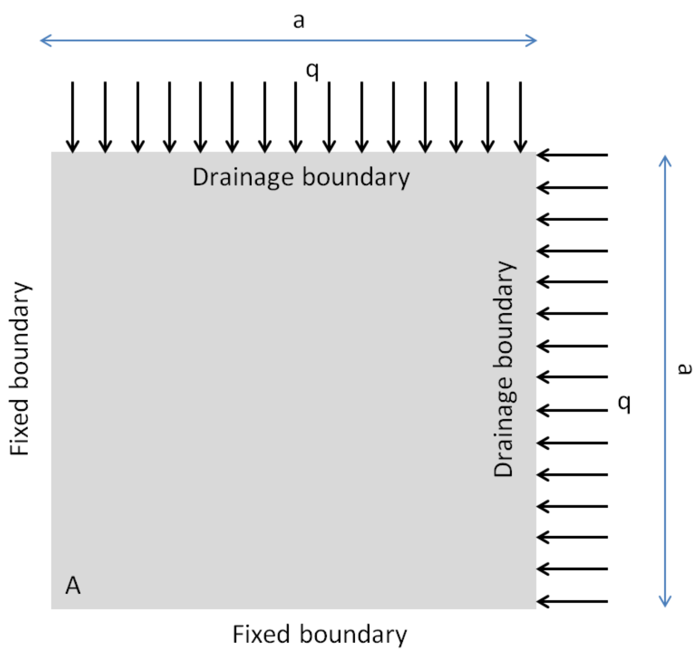

Figure 24. Geometry and boundary conditions (Problem 2)

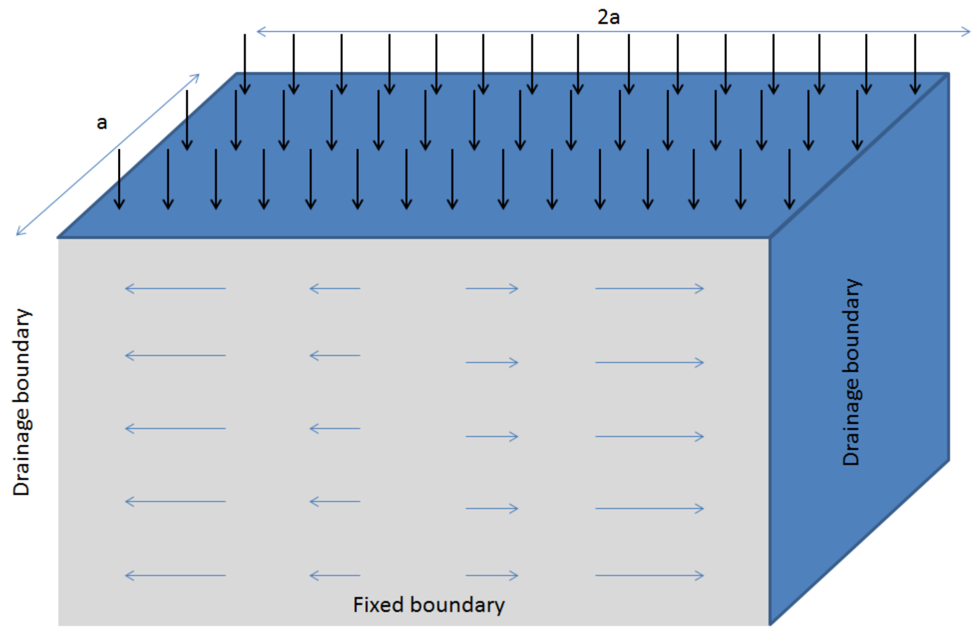

Figure 25. Geometry and boundary conditions (Problem 3)

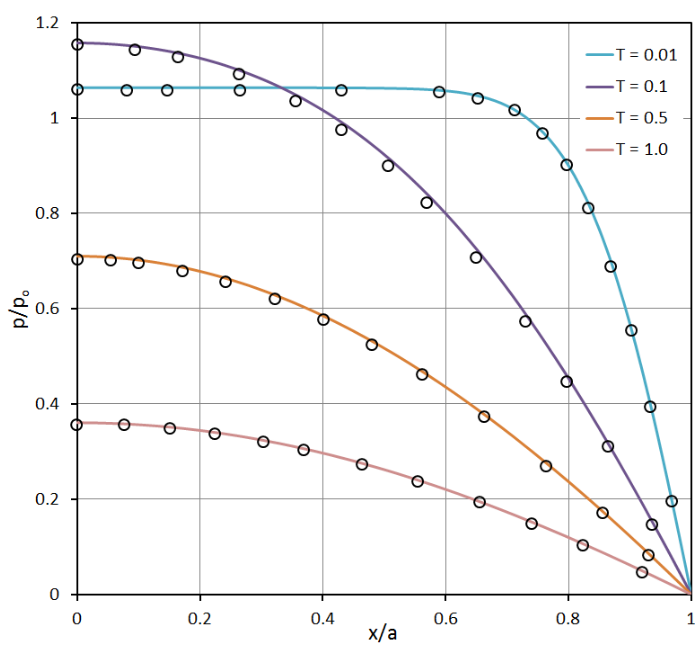

Figure 26. Normalized excess pore-water pressure along the base with Poisson ratio = 0.00 (Problem 1) (points are from Verruijt, 2013).

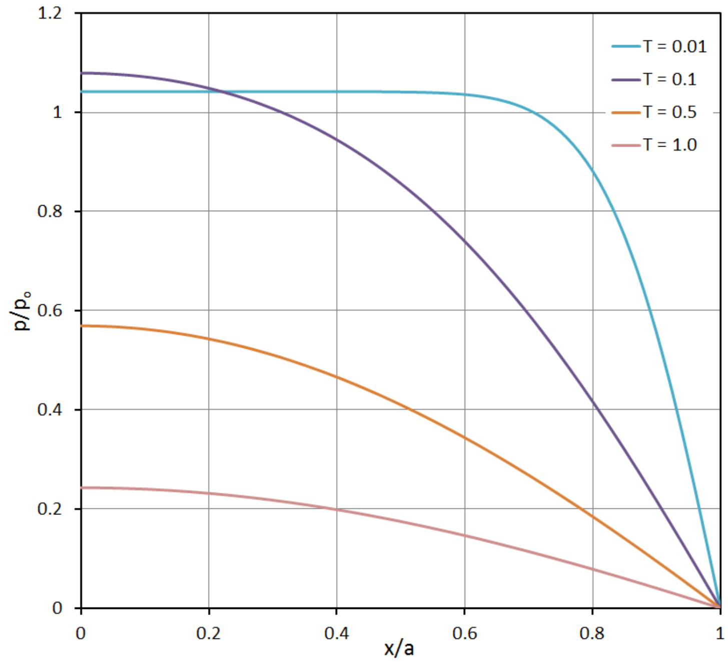

Figure 27. Normalized excess pore-water pressure along the base with Poisson ratio = 0.25 (Problem 1).

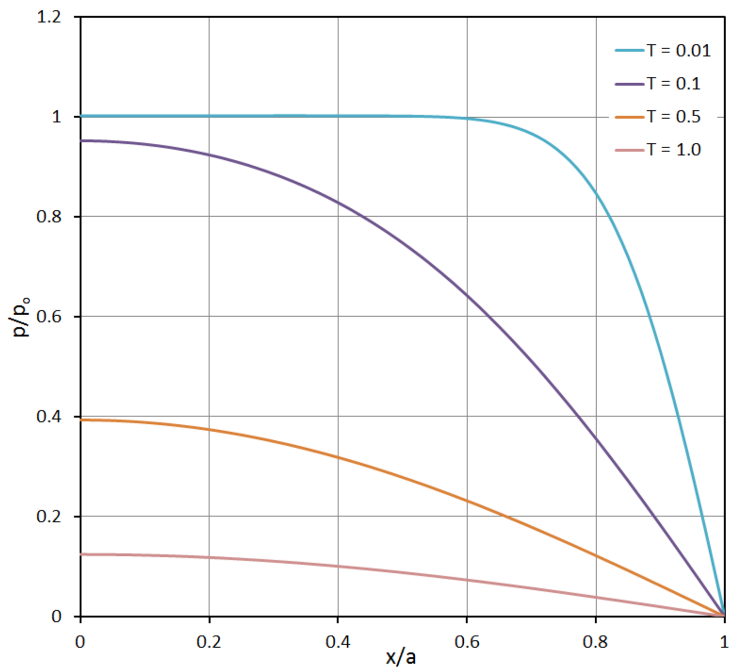

Figure 28. Normalized excess pore-water pressure along the base with Poisson ratio = 0.49 (Problem 1).

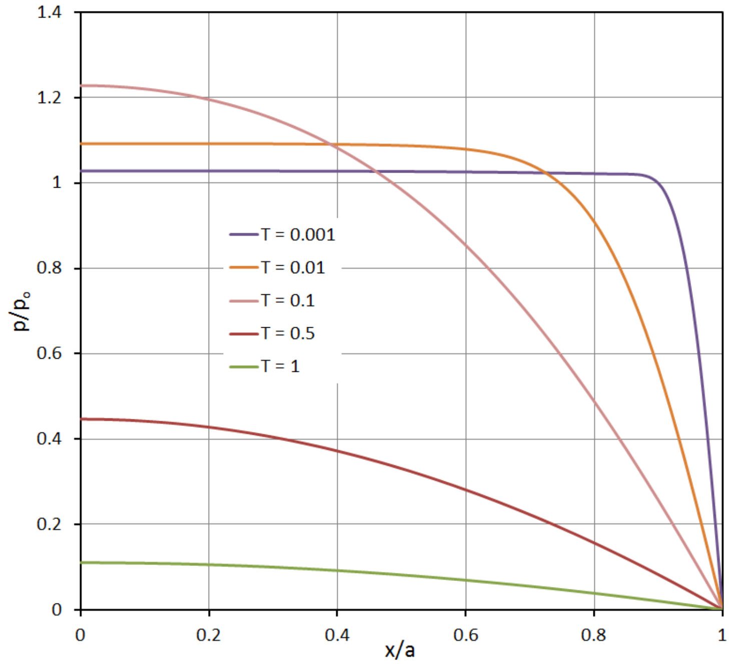

Figure 29. Normalized excess pore-water pressure along the base with Poisson ratio = 0.00 (Problem 2).

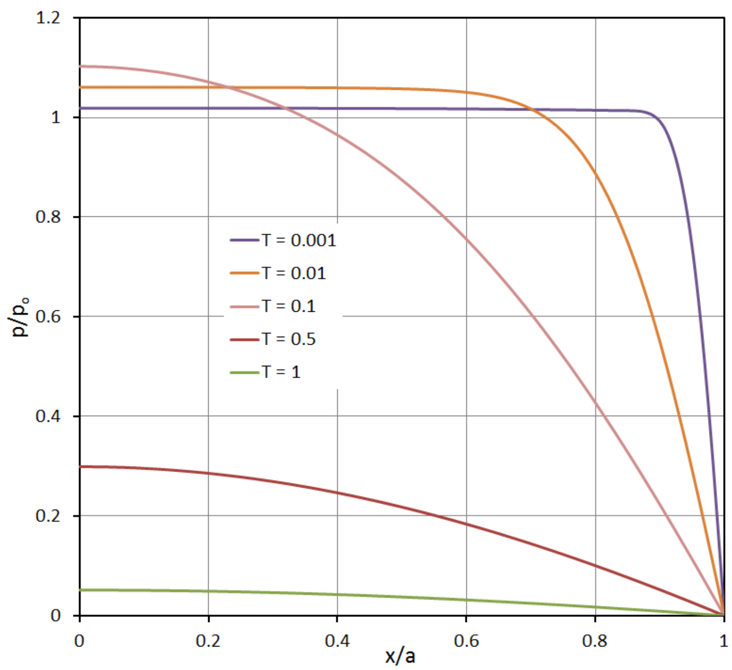

Figure 30. Normalized excess pore-water pressure along the base with Poisson ratio = 0.25 (Problem 2).

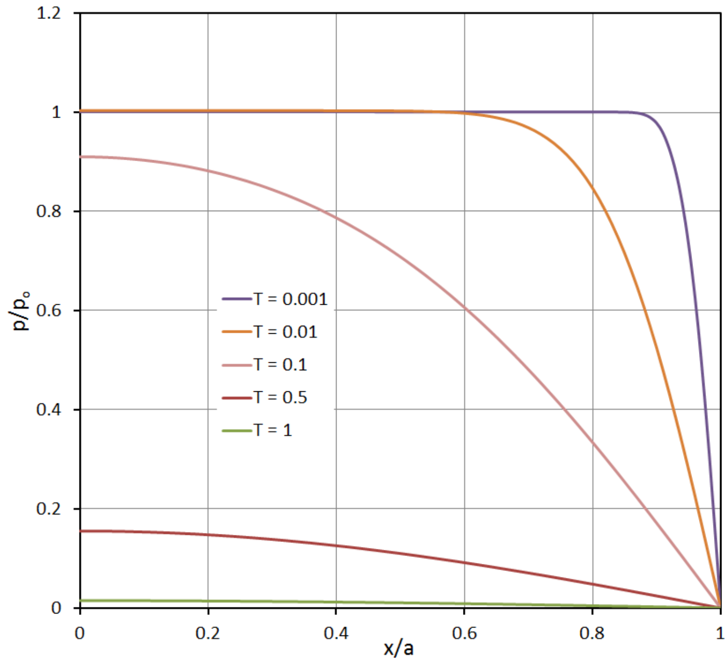

Figure 31. Normalized excess pore-water pressure along the base with Poisson ratio = 0.49 (Problem 2).

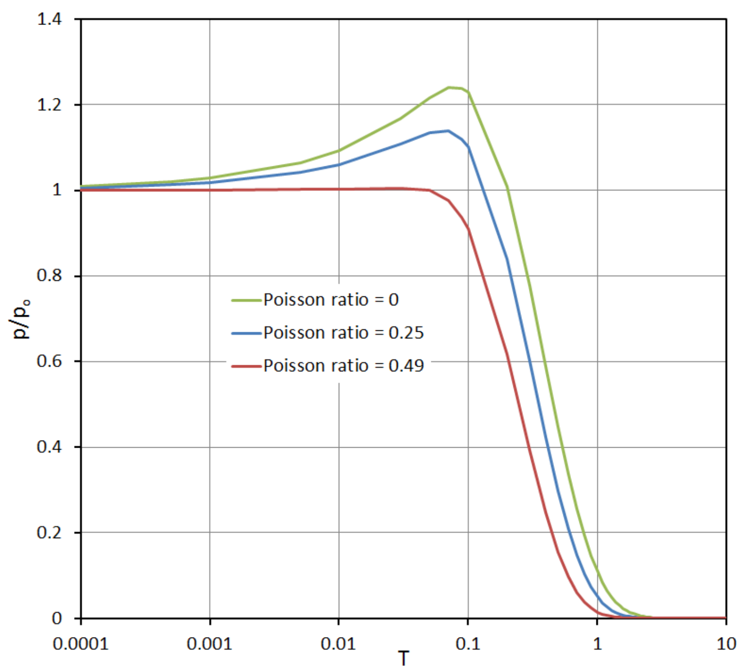

Figure 32. Normalized excess pore-water pressure changes with time factor, *T*, at point A (Figure 24) (Problem 2).

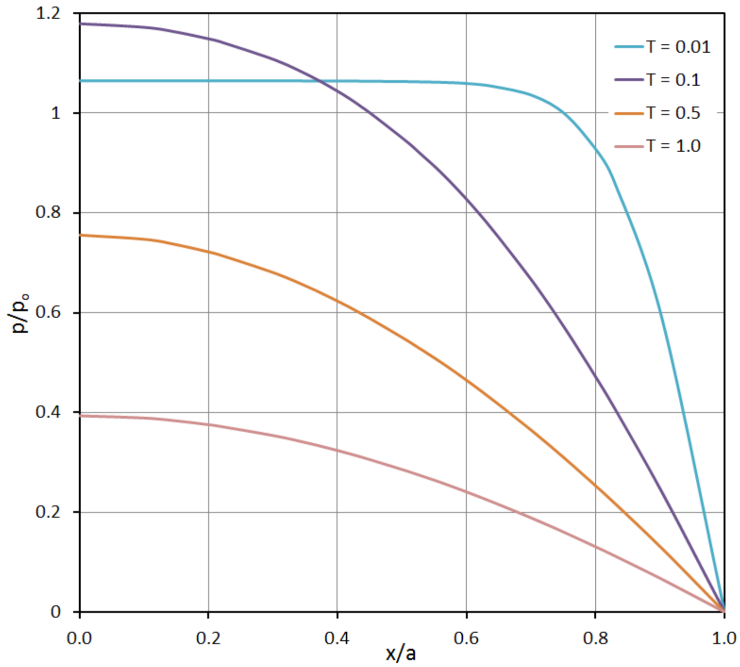

Figure 33. Normalized excess pore-water pressure centre line of the base plane with Poisson ratio = 0.00 (Problem 3).

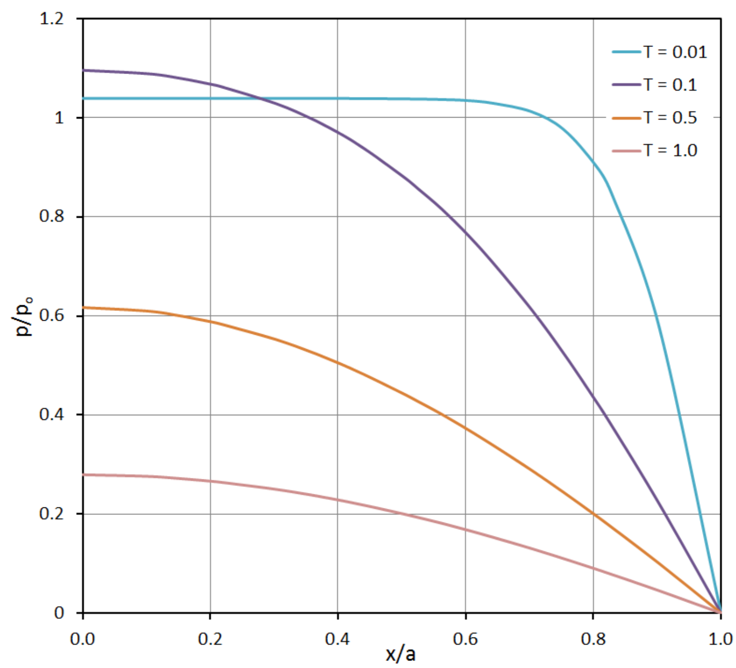

Figure 34. Normalized excess pore-water pressure centre line of the base plane with Poisson ratio = 0.25 (Problem 3).

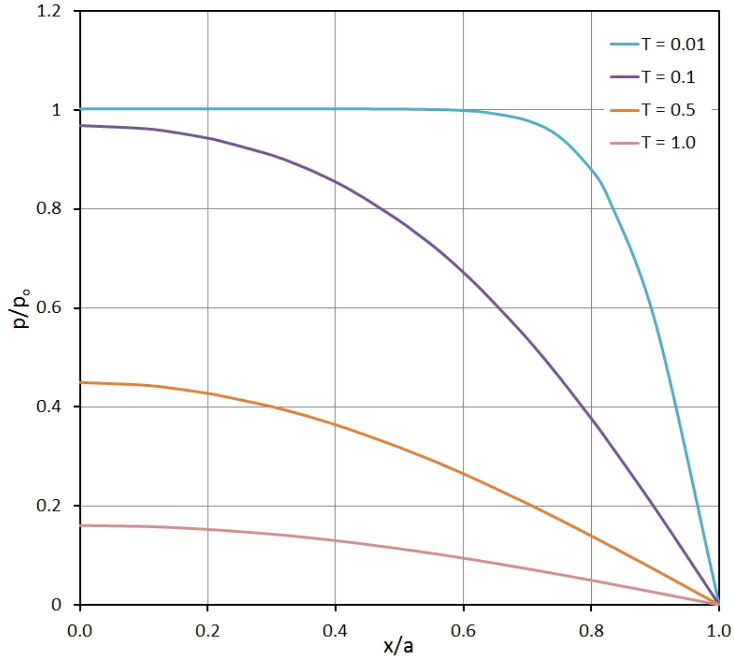

Figure 35. Normalized excess pore-water pressure centre line of the base plane with Poisson ratio = 0.49 (Problem 3).

3 NONLINEAR FUNCTIONS FOR SOIL PROPERTIES (LARGE STRAIN)

This Chapter compares SOILVISION Consolidation results to published research related to the subject of large-strain consolidation. The primary results focus of this section is the verification of the performance of the SOILVISION Consolidation finite element solution engine of nonlinear soil properties. Note on Unsaturated conditions: The SVSOLID/SVFLUX consolidation formulation is based on equations that assume fully saturated conditions. Care should be taken in choosing Initial Conditions or Boundary Conditions, such that unsaturated conditions would not be imposed on the solution.

3.1 SIDERE BENCHMARK

Reference: Bartholomeeusen et al. (2002)

Project: Consolidation
Model: Sidp1_ExtendedPowerFunction_GT, Sidp1_PowerFunction_GT

Main Factors Considered:

- 1D consolidation
- Comparison to D. Znidarcic's methodology
- Show relation to experimental and theoretical results
- Comparison of "Extended Power" and "Standard Power" functions to modeling the void ratio versus effective stress relationship

The purpose of this model is to benchmark the SOILVISION Consolidation analysis against experimental results published by Bartholomeeusen et al. (2002). Bartholomeeusen et al. correlated a number of other researcher's numerical models against experimental results. Numerous numerical models were used to back-calculate the soil properties from other calibration experiments and then to apply them to a new scenario for which the soil properties were not known. This setup was proposed using Znidarcic's formulation (Yao et al., 2002) of soil properties as his results showed the closest correlation to experimental data. Both Extended Power and Standard Power functions were utilized in this benchmark. In addition, these results were compared to the results obtained when using the CONDES0 program. It was found that Znidarcic's results could be duplicated using his formulation of the soil properties.

3.1.1 Model Description

The model consists of a 1D column, with a height of 0.565m. It is assumed that the material is initially homogeneous and is deposited instantaneously. The deformation boundary conditions consist of a lower boundary that is fixed in place and an upper boundary that is free to deform. The flux conditions consist of a constant head of 0.565m at the top boundary and a zero flux boundary at the bottom. No flow is allowed to leave through the sides of the column. The water is forced to flow from the bottom out through the top boundary, while simulating a constant water table above the soil region, as shown in Figure 36.

The material consists of sediment deposits taken from the Schelde River in Belgium. It has a specific gravity of 2.72 and an initial void ratio of 2.47. Znidarcic (1991) formulated the relationship between effective stress and void ratio as an Extended Power function (Liu and Znidarcic, 1991). The void ratio – permeability relationship is expressed as a Standard Power function. The curves are shown in Figure 37 and Figure 38 while their respective expressions are provided below (Equations [13] and [14]).

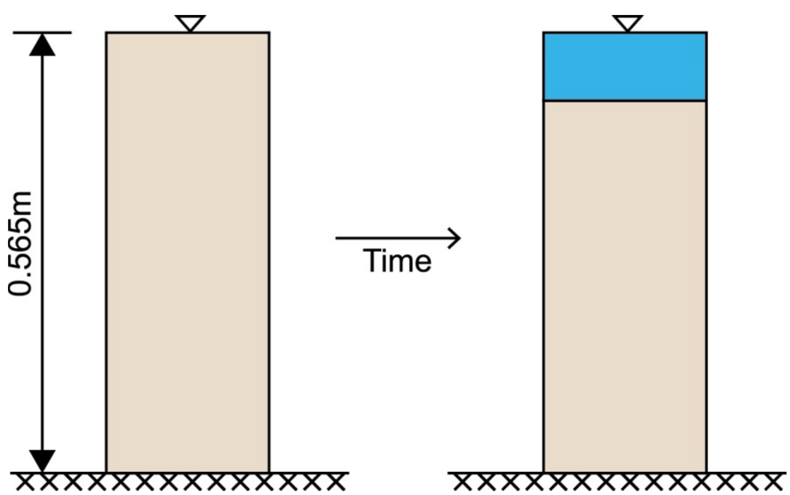

Figure 36. Geometry and boundary conditions of the problem.

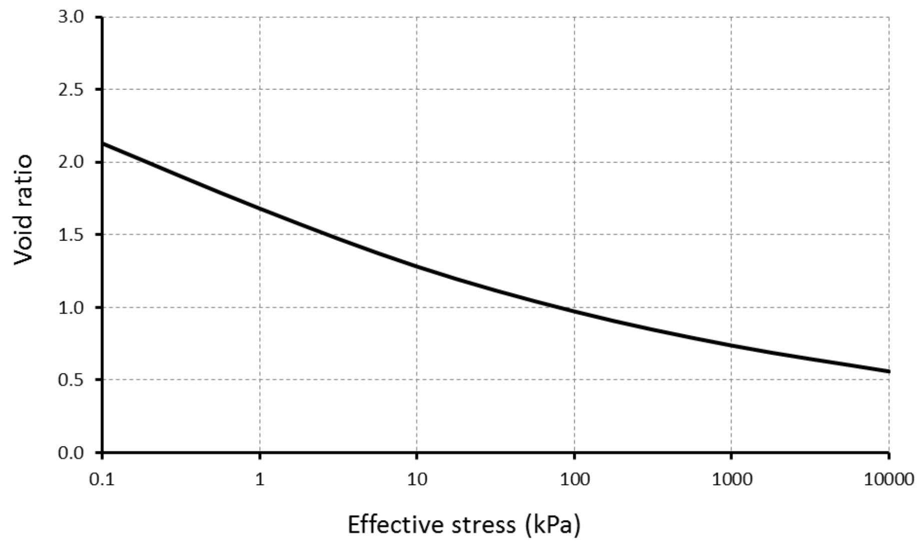

Figure 37 Sidere effective stress-void ratio relationship

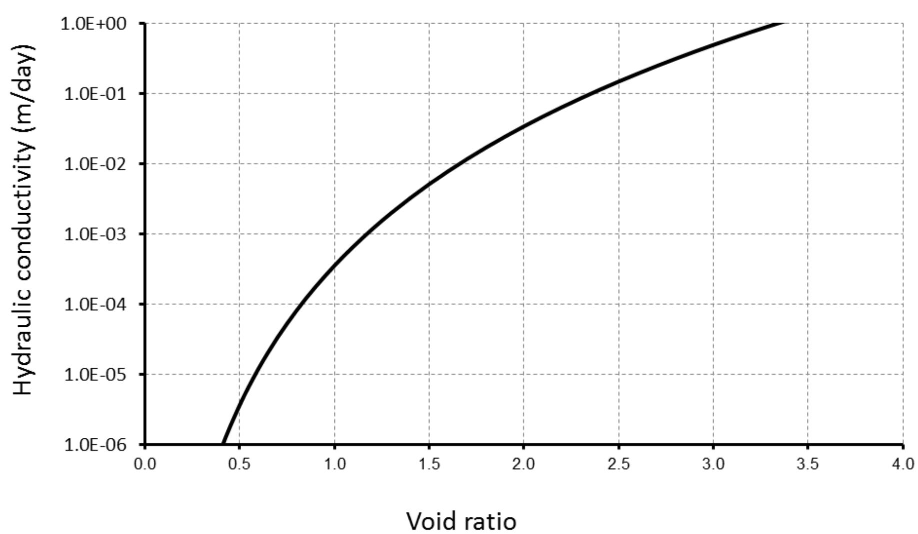

Figure 38 Sidere void ratio - permeability relationship

Extended Power function model $e = 1.69(\sigma' + 0.046)^{-0.12} (kPa)$ [13]

$k = 3.577 \times 10^{-4} e^{6.59} (m/day)$ [14]

A Standard Power Function can be used to address the effective stress – void ratio relationship as:

Standard Power function model $e = 1.69(\sigma')^{-0.12} (kPa)$ [15]

The initial condition is based on the initial void ratio. In a situation such as that tested in the laboratory, the initial condition is one of equal total stress and pore-water pressure so that the effective stress is zero. The initial void ratio can be obtained from Equation [13] as 2.45. Table 5 shows the material parameters.

3.1.2 Results

Figure 39 shows a comparison of the interface heights predicted by Znidarcic and those determined experimentally. Znidarcic's results were the closest to the experiment during the first seven days, but underestimated the amount of settlement after 8 days. It can be seen that SOILVISION Consolidation was able to duplicate Znidarcic's results using the

data points provided. The results utilizing the Extended Power function varied slightly but are extremely close to that of Znidarcic. The Standard Power function also duplicated the results of Znidarcic.

Table 5: Summary of SIDP1 model parameters

Initial Conditions:	
Model Run Time:	8 days
Maximum Time Increment:	0.1 days
Body Load Time:	0.1 days
Initial Height:	0.565 m
Initial Void Ratio:	2.47
Specific Gravity:	2.72
Initial Water Level:	0.565 m 0.565 m
Initial Stress Expression (Power Function Model):	sy0=uw0+(vr0/1.69)^(1/(-0.12)), sx0=sy0,sz0=sy0
Initial Stress Expression (Extended Power Function Model):	sy0=uw0+(vr0/1.69)^(1/(-0.12))-0.046, sx0=sy0,sz0=sy0
Initial Flux Condition:	Head Expression = 0.565 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Expression = 0.565 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress – Void Ratio:	Standard Power Function
	$e = 1.69(\sigma')^{-0.12}(kPa)$
	Extended Power Function
	$e = 1.69(\sigma'+0.046)^{-0.12}(kPa)$
Hydraulic Conductivity – Void Ratio:	$k = 3.577\times10^{-4}e^{6.59}(m/day)$
Poisson's Ratio (assumed):	0.49
Initial Stress Limit:	0.019
Unit Weight Option:	Based On Initial Void Ratio

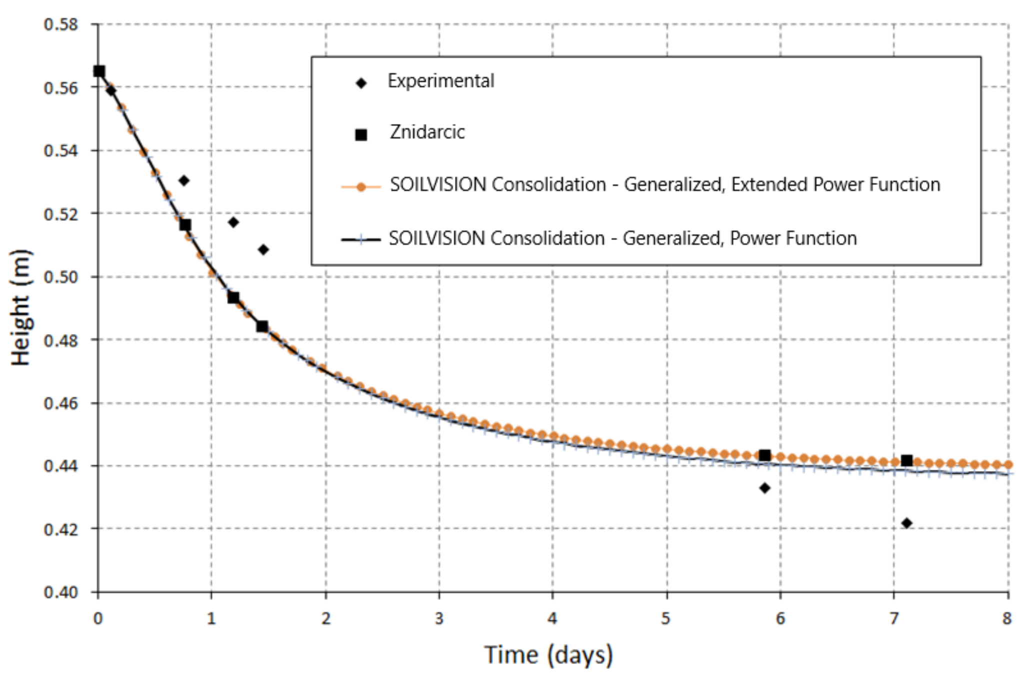

Figure 39 Sidere Results

3.2 CONDESO VALIDATION – INSTANTANEOUS FILLING

Reference: Yao and Znidarčić (1997)

Project: Consolidation
Model: CONDES0_Validation_GT

Main Factors Considered:

- One-Dimensional Instantaneous Filling Consolidation
- Comparison to CONDES0
- Validation of Extended Power Function

The purpose of this model is to benchmark the coupled SOILVISION Consolidation analysis against the program CONDES0 (Yao and Znidarčić, 1997). CONDES0 was created in 1997 to solve for 1D Consolidation of tailings materials.

3.2.1 Model Description

The model consists of a one-dimensional column, with a height of 10.0 m. The column is assumed to be instantaneously deposited and completely homogeneous at deposition. Of course, filling cannot be instantaneous in reality, but nevertheless, it can be fast enough relative to the consolidation time, to be practically instantaneous. Thus, a filling time of 0.1 day is assumed in the numerical model. The top boundary is allowed to deform, while the bottom boundary is fixed. Flow is only allowed out of the system through the top surface. The water table is held constant at a height of 10.0 m. The model parameters are as shown in Table 6.

Table 6: Summary of CONDES0 trial 1 model parameters

Initial Conditions:	
Model Run Time:	365 days
Maximum Time Increment:	5 days
Body Load Time:	2 days
Initial Height:	10 m
Initial Void Ratio:	4.16
Specific Gravity:	2.5
Initial Water Level:	10 m
Initial Stress Expression:	sy0=uw0+(vr0/4.0)^(1/(-0.08))-0.615, sx0=sy0,sz0=sy0
Initial Flux Condition:	Head Expression = 10 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Expression = 10 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress – Void Ratio:	Extended Power Function $e = 4.0(\sigma'+0.615)^{-0.08}(kPa)$
Hydraulic Conductivity – Void Ratio:	$k = 6.0\times10^{-5}e^{4.0}(m/day)$
Poisson's Ratio (assumed):	0.49
Initial Stress Limit:	0.0001
Unit Weight Option:	Based On Initial Void Ratio

The Extended Power function is the only formulation available in CONDES0; therefore an Extended Power function [16] was used to define the relationship between void ratio and effective stress. The hydraulic conductivity curve was defined using a Power function [17]. These relationships are graphically demonstrated in Figures 40 and 41, respectively.

$$e = 4.0(\sigma'+0.615)^{-0.08}(kPa) \quad [\ 16\ ]$$
$$k = 6.0\times10^{-5}e^{4.0}(m/day) \quad [\ 17\ ]$$

The initial void ratio was calculated by setting the effective stress to 0 and calculating the corresponding void ratio of 4.16.

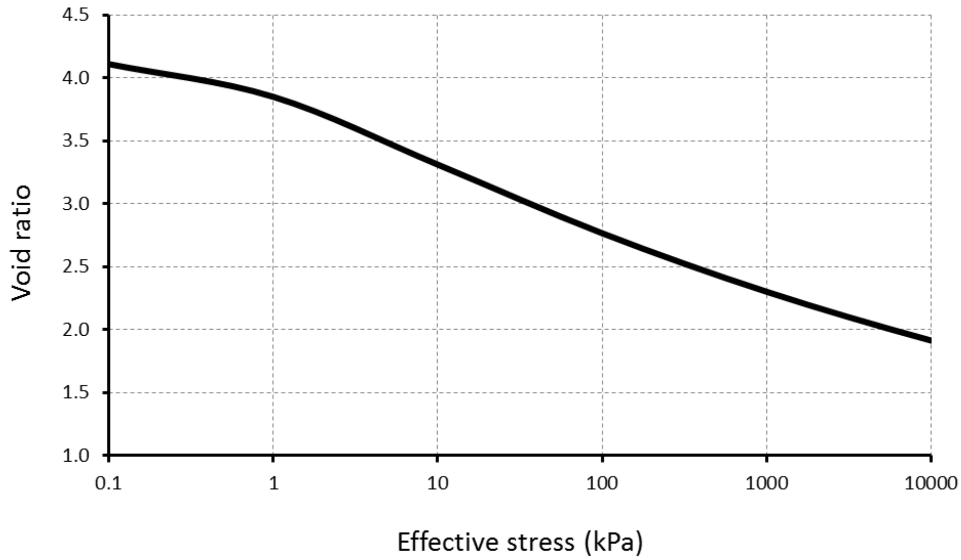

Figure 40: CONDES0 effective stress - void ratio relationship

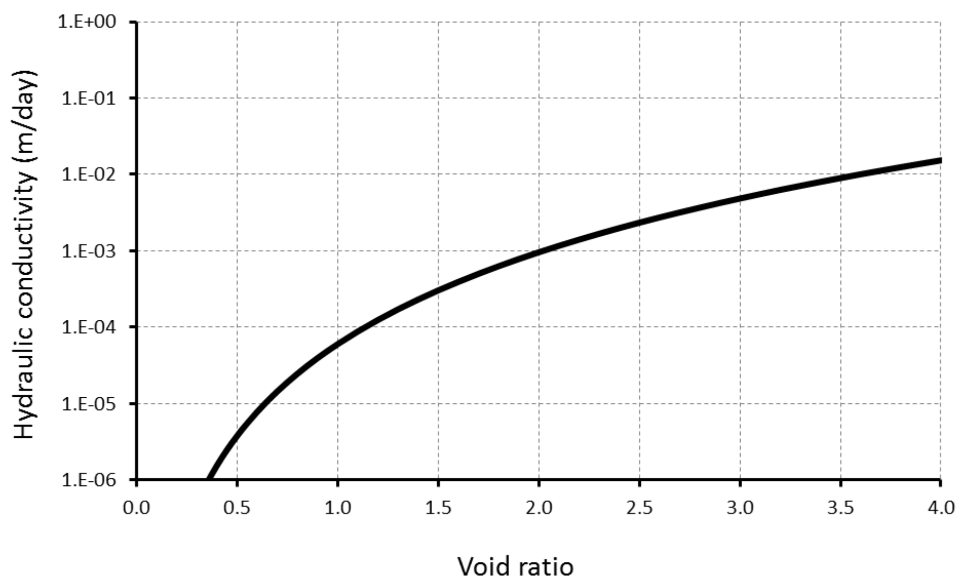

Figure 41: CONDES0 void ratio-hydraulic conductivity relationship

3.2.2 Results

It can be seen in Figure 42 that the settlement results of SOILVISION Consolidation compared well to that from CONDES0 program.

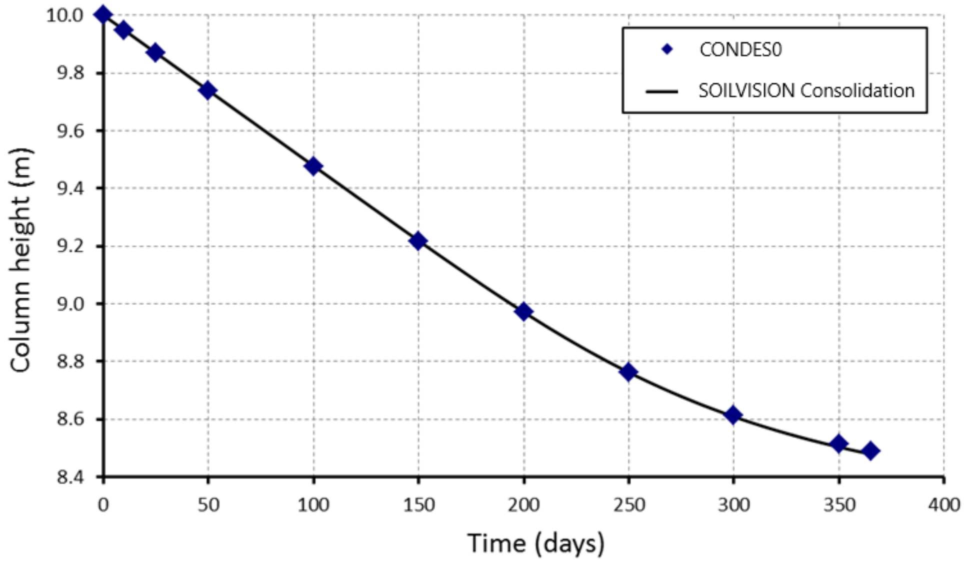

Figure 42: Column height comparison between SOILVISION Consolidation and CONDES0

3.3 OIL-SANDS TAILINGS: COLUMN 1

Reference: Jeeravipoolvarn et al. (2009)

Project: Consolidation
Model: Oil_Sands_Col_1_Power_GT, Oil_Sands_Col_1_Weibull_GT

Main Factors Considered:

- Agreement with the literature on 1D large-strain consolidation models
- Comparison of the Power and Weibull effective stress – void ratio relationsips

Jeeravipoolvarn et al. (2008; 2009) numerically analyzed the University of Alberta Standpipe 1, when the experimental results of a long term consolidation test on oil sands tailings in Standpipe 1 were available. The 1D results of SOILVISION Consolidation were compared to the experimental and predicted results in this verification exercise.

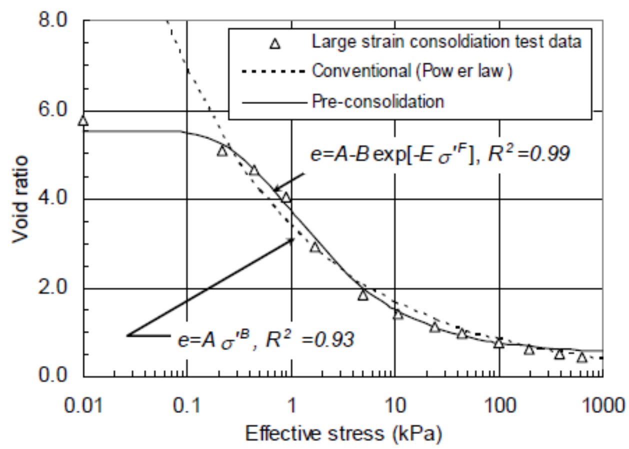

Figure 43: Comparison of Weibull Function and Power Functions (after Jeeravipoolvarn, 2010)

3.3.1 Model Description

The material used consists of oil sands tailings taken from Syncrude's Mildred Lake Tailings Impoundment in 1982. It has a specific gravity of 2.28 and an initial void ratio of 5.17. Figure 43 shows the comparison of fitting of Power and Weibull functions to the laboratory large strain consolidation test data. Although both Power and Weibull functions closely fit the measured data for the larger stresses, the Power function is unable to match the laboratory data if the effective stress is smaller than 0.2 kPa.

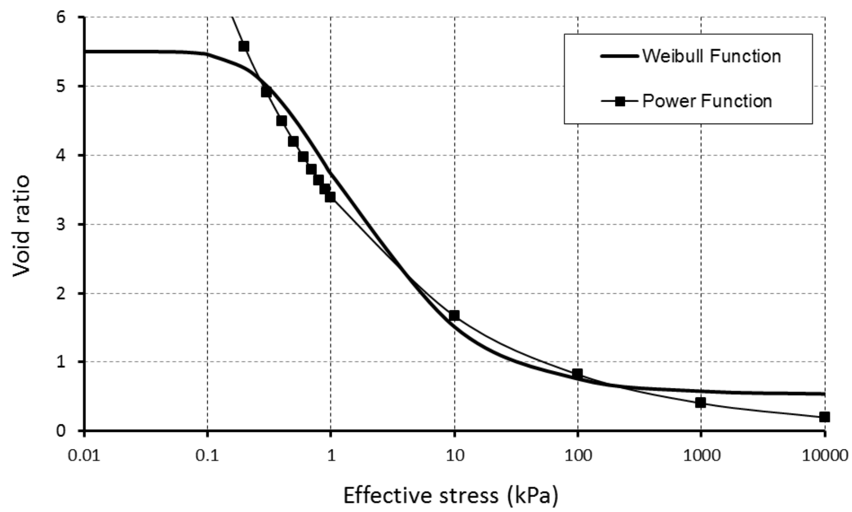

Figure 44. Weibull and Power functions utilized in the SOILVISION Consolidation Standpipe 1 numerical model for compressibility relationship

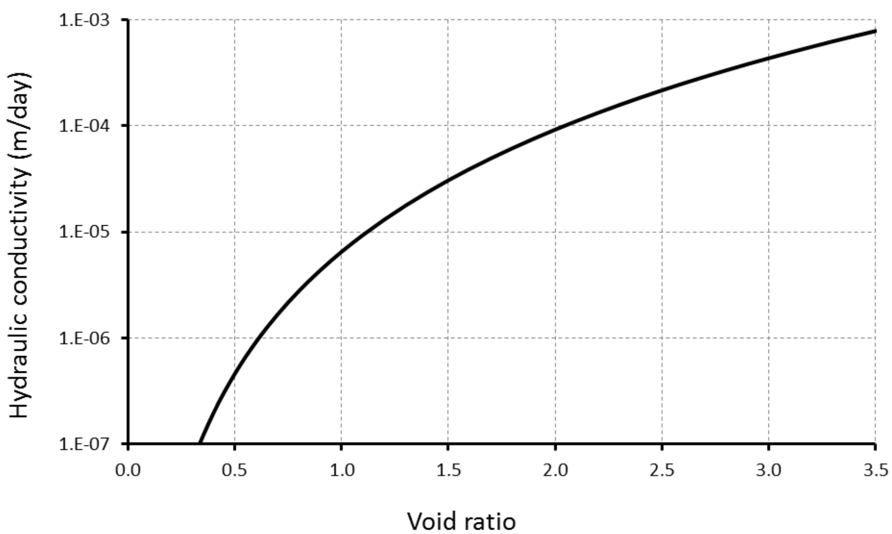

Figure 45: Column 1 void ratio - hydraulic conductivity relationship

Jeeravipoolvarn et al. (2008) used a Weibull function to relate effective stress and void ratio, whereas the SOILVISION Consolidation software has implemented a number of different functions, including the Weibull function. To compare the differences between the different functions both the Weibull and the Power functions were tested. The differences between the functions are shown in Figure 43. The Weibull function is provided in equation [18] and the Power function in equation [19]. Equation [20] shows the permeability – void ratio relationship.

$$e = 5.50 - 4.97\exp(-1.03\sigma'^{-0.67})(kPa) \quad [18]$$

$$e = 3.391(\sigma')^{-0.308}(kPa) \quad [19]$$

$$k = 6.51\times10^{-6}e^{3.824}(m/day) \quad [20]$$

where, k and σ' are in (m/day) and (kPa), respectively.

The model consists of a circular column, with a height of 10m. It is assumed that the tailings are initially homogeneous and deposited instantaneously. The deformation boundary conditions consist of a lower boundary that is fixed in place, and an

upper boundary that is free to deform. In the multi-dimensional models, the lateral boundaries are fixed in the horizontal, but are free to deform vertically. The flux conditions consist of a constant head of 10 m at the top boundary and a zero flux boundary at the bottom. This forces the water to flow from the bottom out through the top boundary, while simulating a constant water table above the soil region.

Table 7: Summary of oil-sand Column 1 model parameters

Initial Conditions:	
Model Run Time:	10000 Days
Maximum Time Increment:	1 Day
Body Load Time:	0.5 Days
Initial Height:	10 m
Initial Void Ratio:	5.17
Specific Gravity:	2.28
Initial Water Level:	10 m
Initial Stress Expression (Weibull Function Model):	sx0 = uw0 + (ln((vr0-5.50)/(-4.97))/(-1.03))^(1/(-0.67)), sy0 = sx0, sz0 = sx0
Initial Stress Expression (Power Function Model):	sx0=uw0+(vr0/3.391)^(1/(-0.308)), sy0=sx0,sz0=sx0
Initial Flux Condition:	Head Expression = 10 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Expression = 10 m
Side Boundaries:	Zero Flux
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Side Boundaries:	Fixed Horizontally
Material Properties:	
Effective Stress – Void Ratio:	Weibull function
	$e = 5.50 - 4.97\exp(-1.03\sigma'^{-0.67})(kPa)$
	Power Function
	$e = 1.69(\sigma')^{-0.12}(kPa)$
Hydraulic Conductivity – Void Ratio:	$k = 6.51\times10^{-6}e^{3.824}(m/day)$
Poisson's Ratio (assumed):	0.40
Initial Stress Limit:	0.236
Unit Weight Option:	Based On Initial Void Ratio

3.3.2 Results

Figure 46 shows a comparison of the interface heights predicted and the interface height experimentally. It can be seen that the results obtained by SOILVISION Consolidation are the same as the previous numerical model results by Jeeravipoolvarn et al. (2009). It is noted that neither numerical modeling efforts match the experimental results. Also, it can be seen that the Power function predicts slightly more consolidation than the Weibull function.

Figures 47 through 49 compare the profiles of the effective stress, excess pore-water pressure, and void ratio with those published by Jeeravipoolvarn et al. (2009). It can be seen that the results are almost identical to those obtained by using the Weibull function. There is a slight discrepancy in the results when the Power function is used for the compressibility relationship.

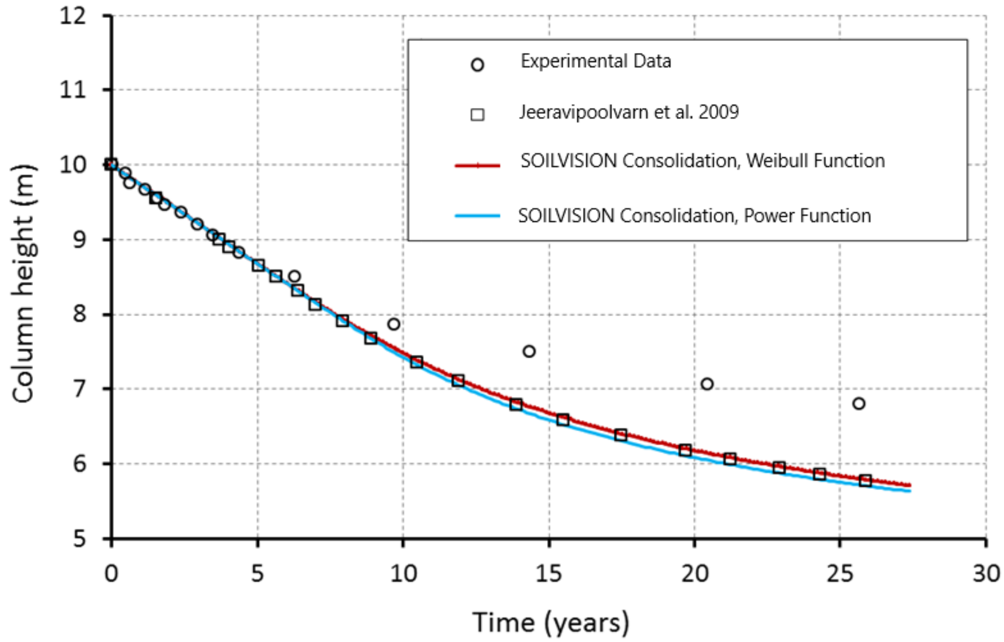

Figure 46 Column 1 settlement model comparison

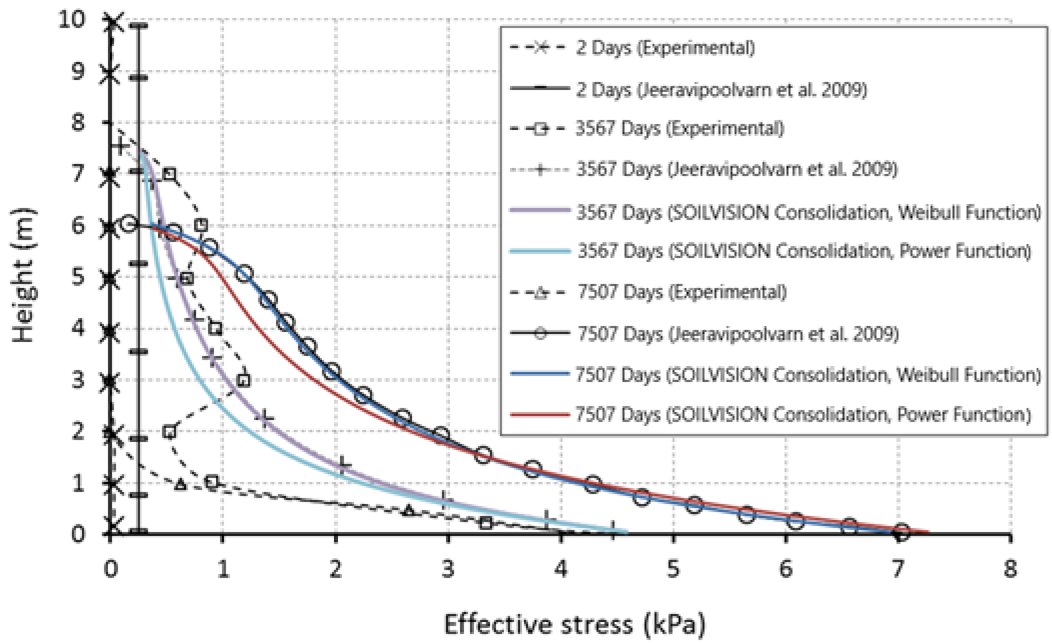

Figure 47: Column 1 effective stress profiles at various times

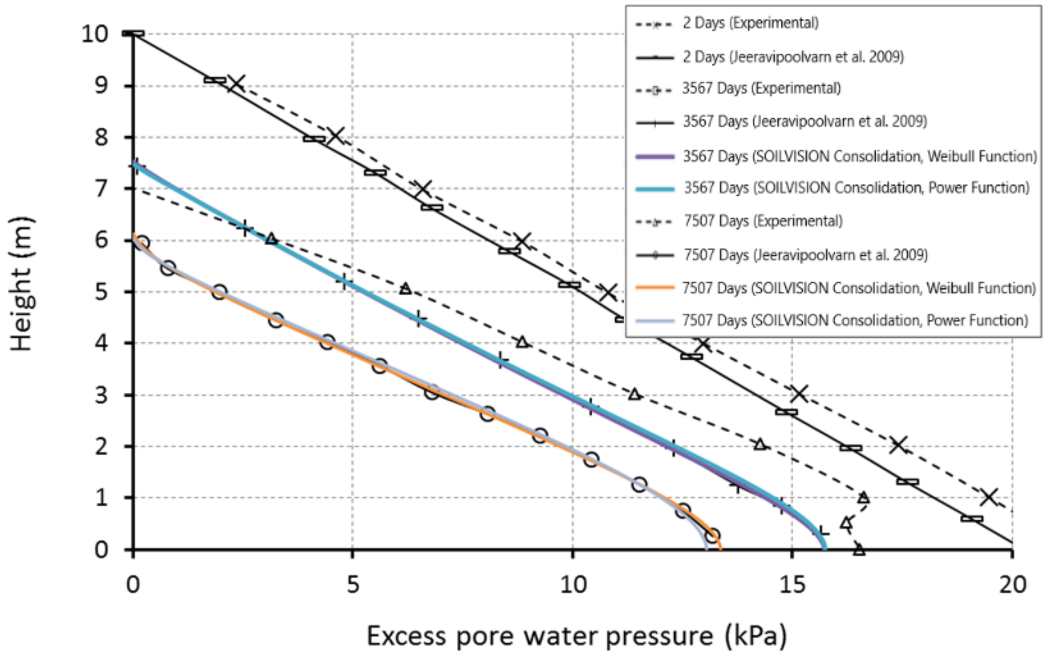

Figure 48: Column 1 excess pore-water pressure profiles at various times

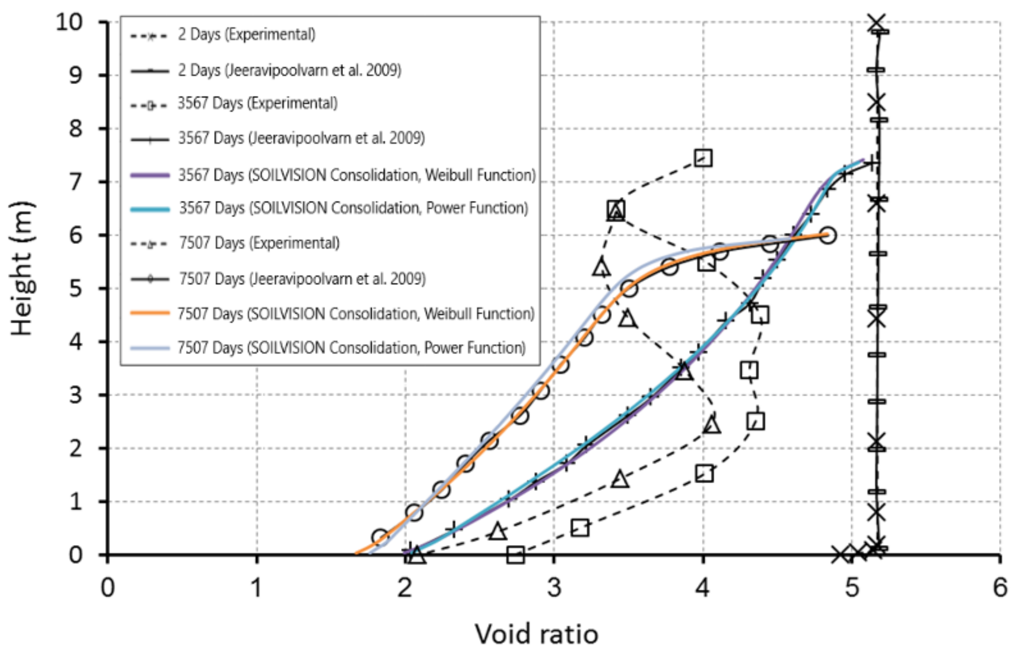

Figure 49: Column 1 void ratio profiles at various times

3.4 OIL-SANDS TAILINGS: COLUMN 3

Reference: Jeeravipoolvarn et al. (2009)

Project: Consolidation
Model: Oil_Sands_Col_3_GT

Main Factors Considered:

- Comparison to literature one-dimensional consolidation model
- Validation of the Weibull function
- Benchmarking of ability to match oil-sand tailings performance

Jeeravipoolvarn et al. (2008; 2009) numerically analyzed the University of Alberta Standpipe 3, when the experimental results of a long-term consolidation column test on oil sands tailings were available. This setup is recreated using the SOILVISION Consolidation software with the Weibull function. The Weibull function is recommended in the paper as being more ideal for oil sands tailings. This benchmark is utilized to validate the implementation of the Weibull function in the SOILVISION Consolidation software.

3.4.1 Model Description

The model consists of a column with a height of 10 m. It is assumed that the tailings are initially homogeneous and deposited instantaneously. The deformation boundary conditions consist of a lower boundary that is fixed in place, and an upper boundary that is free to deform. The flux conditions consist of a constant head of 10 m at the top boundary and a zero flux boundary on the bottom of the column. This forces the water to flow upward from bottom out through the top boundary, while simulating a constant water table above the soil region.

The material used consists of oil sands tailings taken from Syncrude's Mildred Lake Tailings Impoundment in 1982. The tailings material was mixed with cyclone tailings sand to bring the percent sand in the mixture to 82%. The tailings had a specific gravity of 2.58 and an initial void ratio of 0.87.

The effective stress-void ratio data was described using a Weibull function, while the hydraulic conductivity-void ratio data was described using a Power function. These are shown in Figures 50 and 51, respectively. The Weibull function is provided in Equation [21]. Equation [22] shows the hydraulic conductivity – void ratio relationship.

$$e = 1.08 - 0.77\exp(-1.3\sigma'^{-0.29})(kPa) \quad [\ 21\ ]$$

$$k = 2.76\times10^{-3} e^{3.824}(m/day) \quad [\ 22\ ]$$

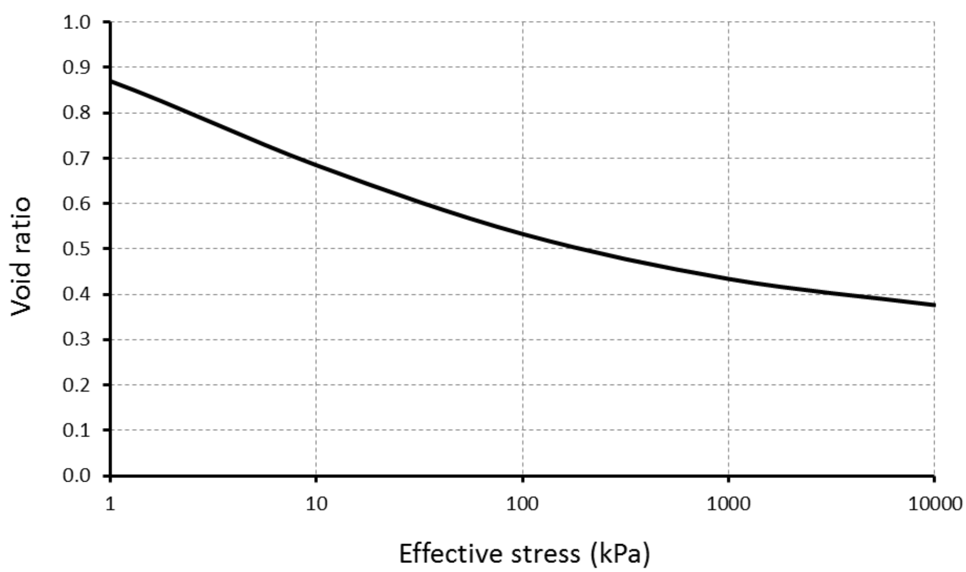

Figure 50: Column 3 void ratio-effective stress relationship

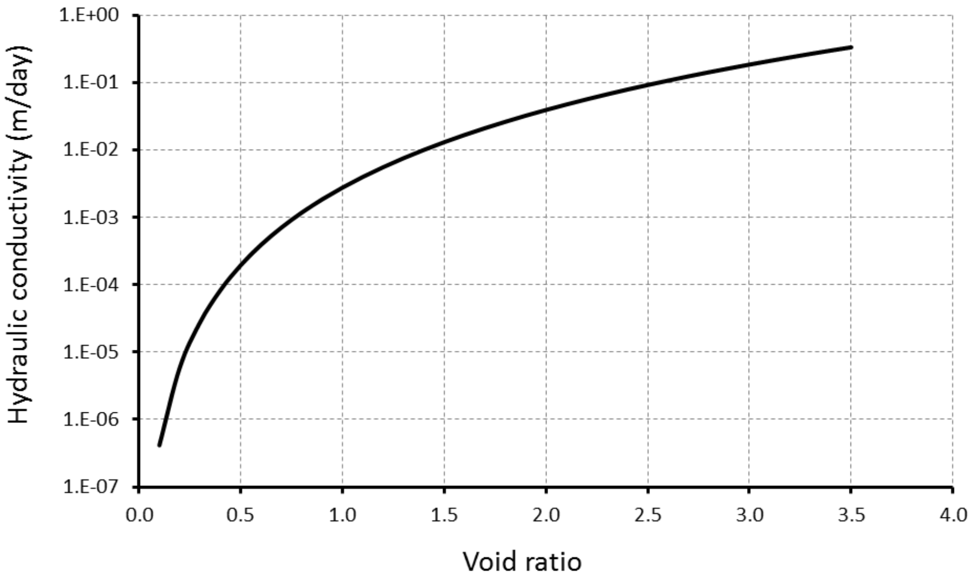

Figure 51: Column 3 void ratio-hydraulic conductivity relationship

3.4.2 Results

Figure 52 shows a comparison of the predicted interface heights and those found experimentally. It can be seen that SOILVISION Consolidation was able to duplicate the results predicted by using the Weibull function. Figures 54 through Figure 55 show the plots of effective stress, excess pore – water pressure and void ratio profiles at various elapsed times during consolidation. It can be seen that there is good agreement between the laboratory and predicted data.

Table 8: Summary of oil-sand Column 3 model parameters

Initial Conditions:	
Model Run Time:	4380 Days
Maximum Time Increment:	10 Days
Body Load Time:	5 Days
Initial Height:	10 m
Initial Void Ratio:	0.87
Specific Gravity:	2.58
Initial Water Level:	10 m
Initial Stress Expression:	sx0 = uw0 + (ln((vr0-1.08)/(-0.77))/(-1.3))^(1/(-0.29)), sy0 = sx0, sz0 = sx0
Initial Flux Condition:	Head Expression = 10 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Expression = 10 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress – Void Ratio:	Weibull Function $e = 1.08 - 0.77\exp(-1.3\sigma'^{-0.29})(kPa)$
Hydraulic Conductivity – Void Ratio:	$k = 2.76 \times 10^{-3} e^{3.824} (m/day)$
Poisson's Ratio (assumed):	0.49
Initial Stress Limit:	1.00
Unit Weight Option:	Based On Initial Void Ratio

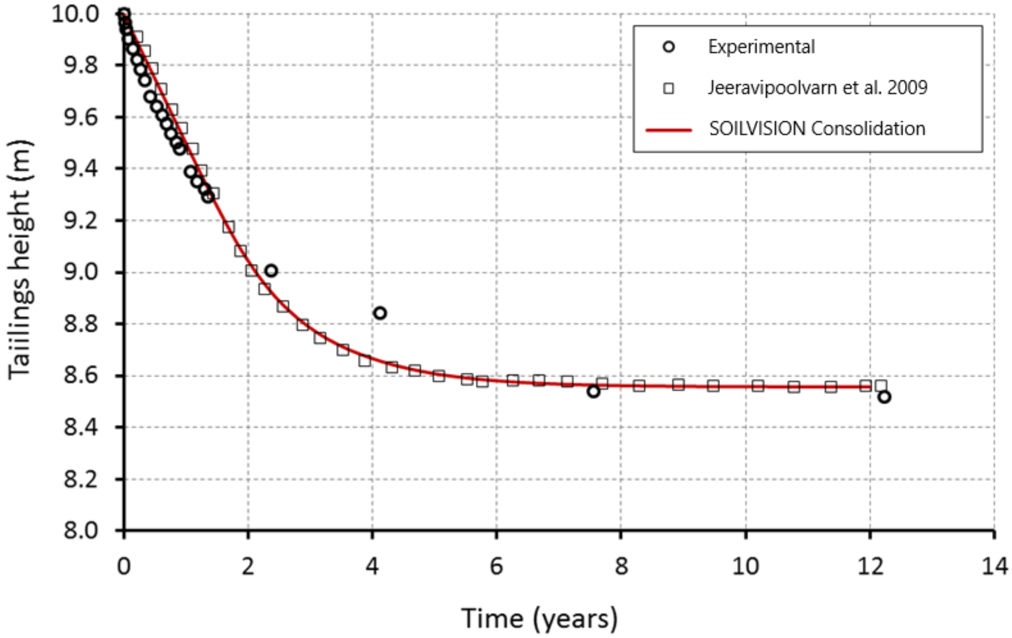

Figure 52 Column 3 settlement results

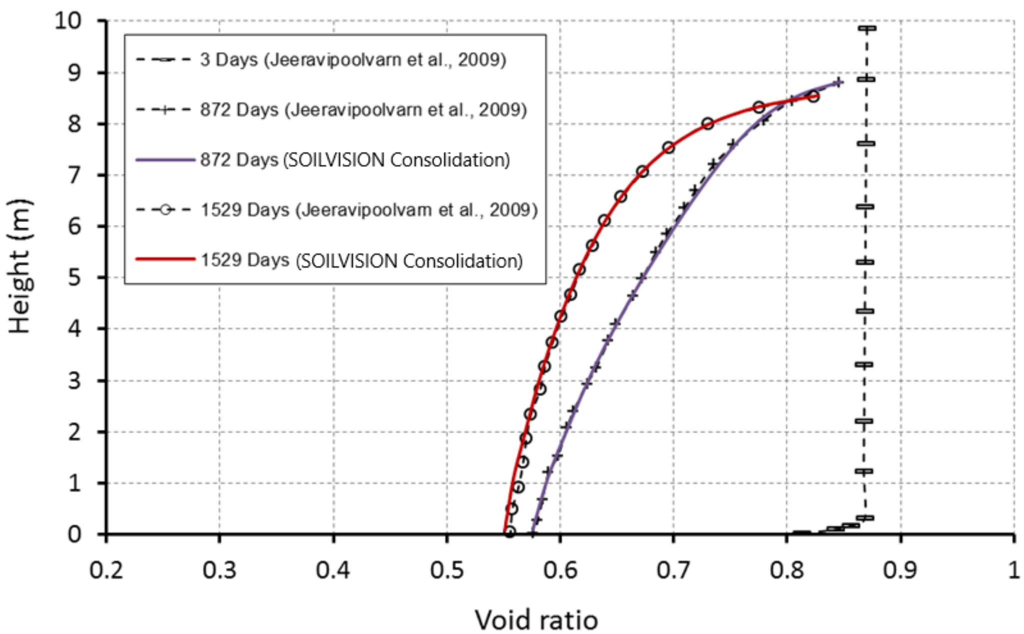

Figure 53: Column 3 void ratio profile comparison

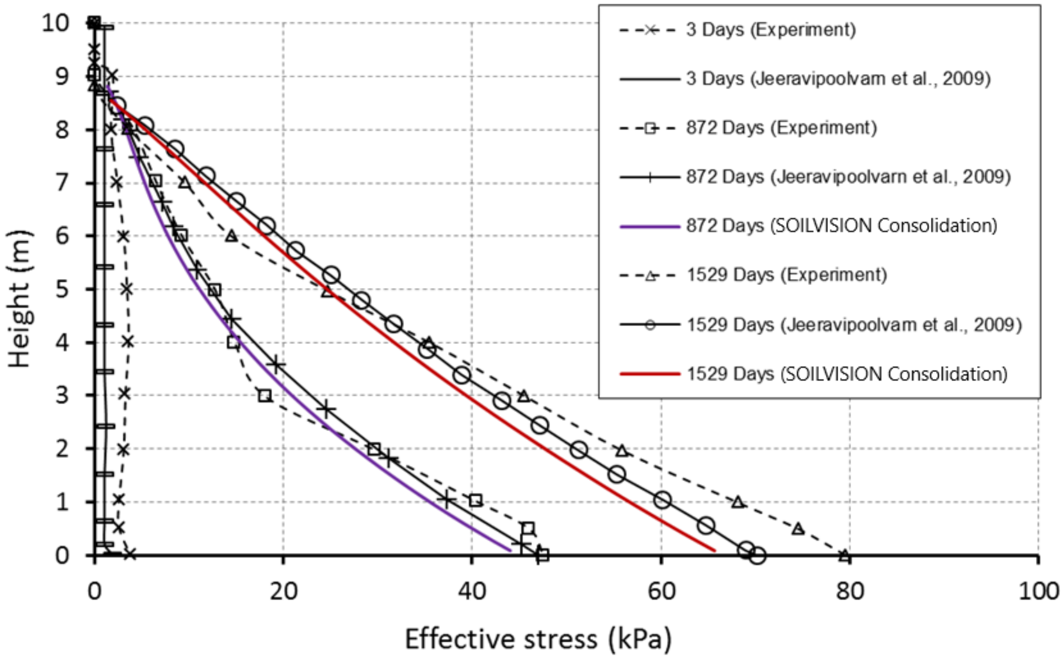

Figure 54: Column 3 effective stress comparison

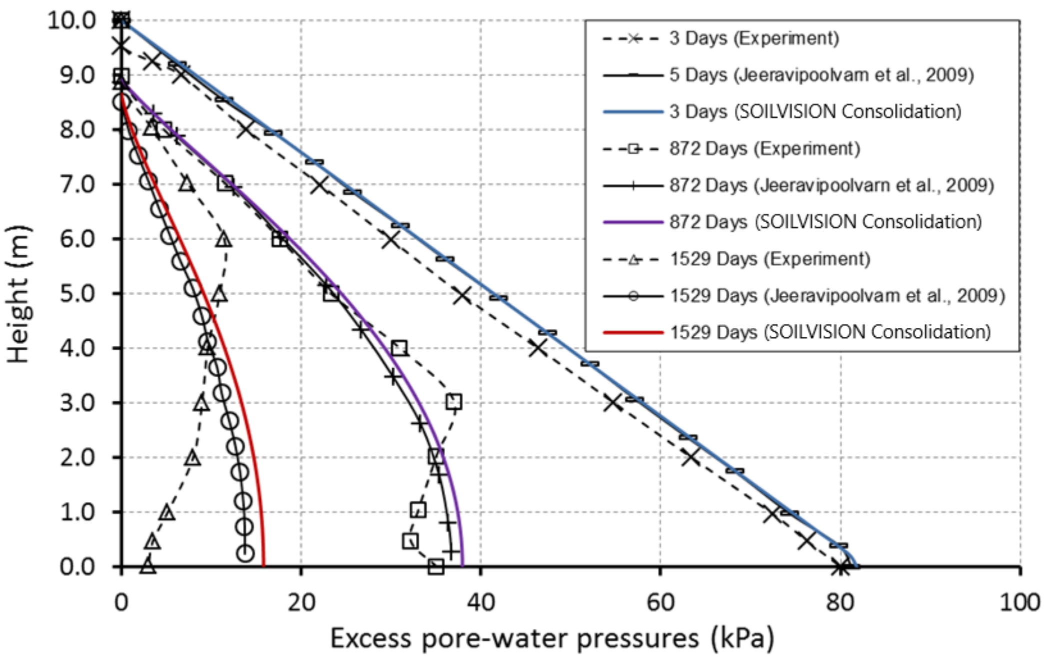

Figure 55: Column 3 excess pore-water pressure comparison

3.5 TOWNSEND SCENARIO A

Reference: Townsend and McVay, (1990)

Project: Consolidation
Model: Townsend_A_GT

Main Factors Considered:

- 1D Consolidation of a homogeneous column of tailings
- Effects of Poisson's Ratio on final settlement, as well as one year profiles of void ratio and excess pore-water pressure
- Analysis of a material with a large initial void ratio

Townsend's Scenario A (Townsend and McVay, 1990) is intended to represent waste ponds that have recently received thickened clays. These clays are allowed to consolidate under self-weight conditions. The system remains saturated with flow moving through the surface boundary. This scenario is challenging to solve since the initial void ratio is quite high (e = 14.8). Consequently, large deformations are seen over extremely small effective stress ranges.

3.5.1 Model Description

The tailings slurry consists of a clay mixture commonly found in the phosphate industry. The clay has a specific gravity of 2.82 and an initial void ratio of 14.8 (solids content, s = 16%). The material properties are provided using a power equation relating effective stress to void ratio (Equation [23]) and another relating void ratio to hydraulic conductivity (Equation [24]). These relationships are graphically shown in Figures 56 and 57, respectively.

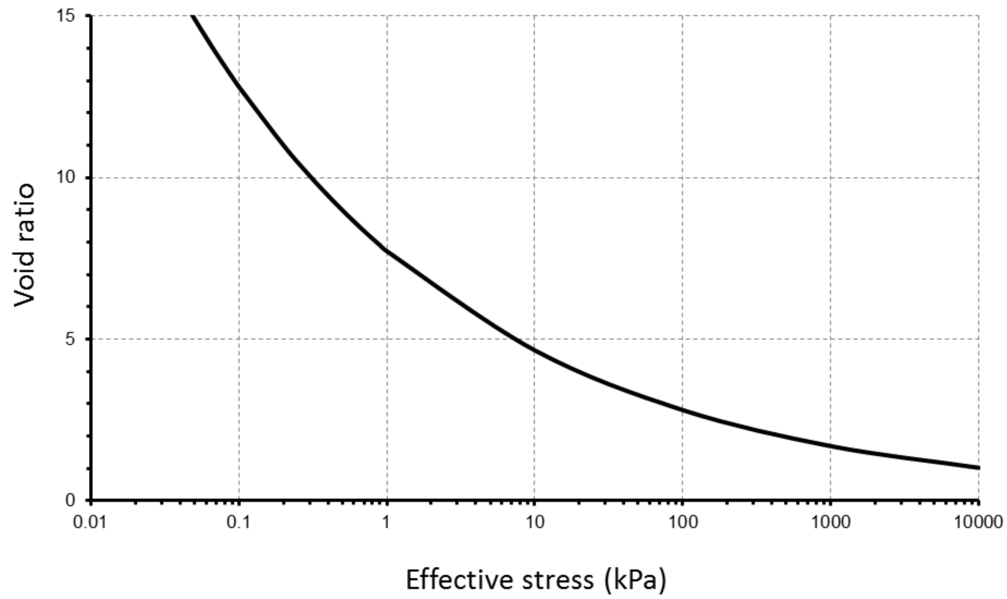

Figure 56 Townsend A: Void ratio - effective stress relationship

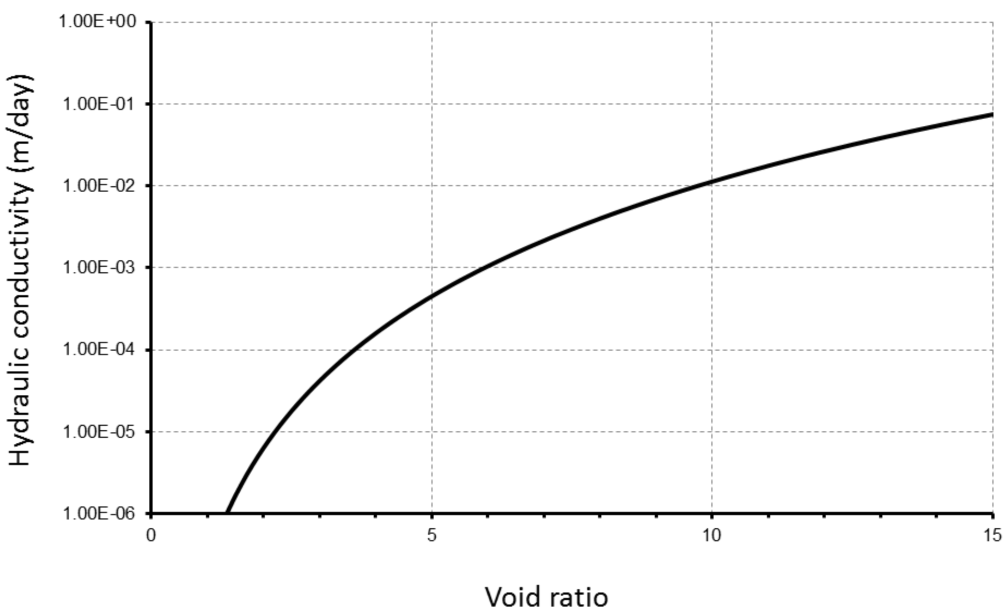

Figure 57 Townsend A: Hydraulic conductivity - void ratio relationship

The model consists of 9.6m of homogeneous tailings placed instantaneously at an initial void ratio (e_o) of 14.8 and allowed to consolidate. The column is allowed to drain freely through the top surface. The bottom boundary has a zero flux boundary

condition. Deformation is fixed on the bottom plane. Initial conditions are defined by the water table, which is maintained at 10.6m for the entire duration. The problem description is as shown in Figure 58 and the model parameters are presented in Table 9.

$$e = 7.72(\sigma')^{-0.22}(kPa) \quad [23]$$

$$k = 2.53 \times 10^{-7} e^{4.65}(m/day) \quad [24]$$

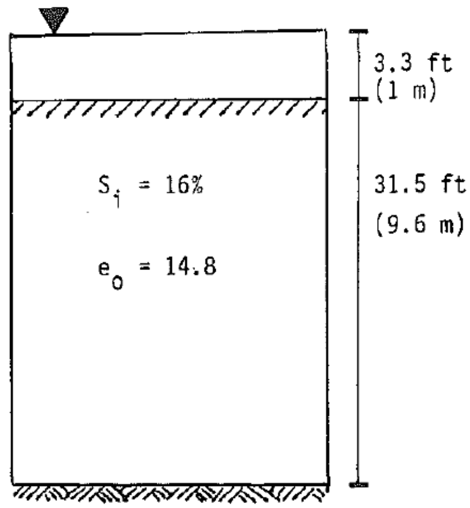

Figure 58 Townsend's Scenario A (from Townsend & McVay, 1990)

Table 9: Summary of Townsend A model parameters

Initial Conditions:	
Model Run Time:	8000 Days
Maximum Time Increment:	0.5 Days
Body Load Time:	5 Days
Initial Height:	9.6 m
Initial Void Ratio:	14.8
Specific Gravity:	2.82
Initial Water Level:	10.6 m
Initial Stress Expression:	sy0=uw0+(vr0/7.72)^(1/(-0.22)), sx0=sy0,sz0=sy0
Initial Flux Condition:	Head Expression = 10.6 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Expression = 10.6 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress – Void Ratio:	Power Function $e = 7.72(\sigma')^{-0.22}(kPa)$
Hydraulic Conductivity – Void Ratio:	$k = 0.2532 \times 10^{-6} e^{4.65}(m/day)$
Poisson's Ratio (assumed):	0.49
Initial Stress Limit:	0.0519
Unit Weight Option:	Based On Initial Void Ratio

3.5.2 Results

Figures 59 to 61 show that the numerical results from SOILVISION Consolidation show an excellent fit to the provided literature data. The literature predicts an average final tailings height of 13.66 feet (4.16 m). SOILVISION Consolidation predicts a final height of 4.07 m. The details of various predictors used are listed in Table 10 (Townsend & McVay, 1990).

Table 10. Summary of predictors and their affiliations

Abbreviation	Predictor	Affiliation	Program Description
A & ASSOC	Garlanger et al.	Ardaman & Assoc.	
B&CI	Carrier	Bromwell & Carrier, Inc.	Somogyi-based Approximate integration for magnitude
FELDKAMP	Feldkamp	-	Finite element
McGILL	Yong et al.	McGill	Piecewise linear (Yong)
N' WTRN	Trin et al.	Northwestern	Somogyi-based Piecewise linear (Olson)
TAGA	Pyke	TAGA	Piecewise linear
U-CONN	Long et al.	University of Conn.	Cargill-based
UF	McVay et al.	University of Florida	Piecewise linear (Yong) Somogyi-based Closed-form integration for magnitude
UF-McGS	UF	University of Florida	Piecewise linear (Yong)
WES	Poindexter et al.	WES	Cargill-based

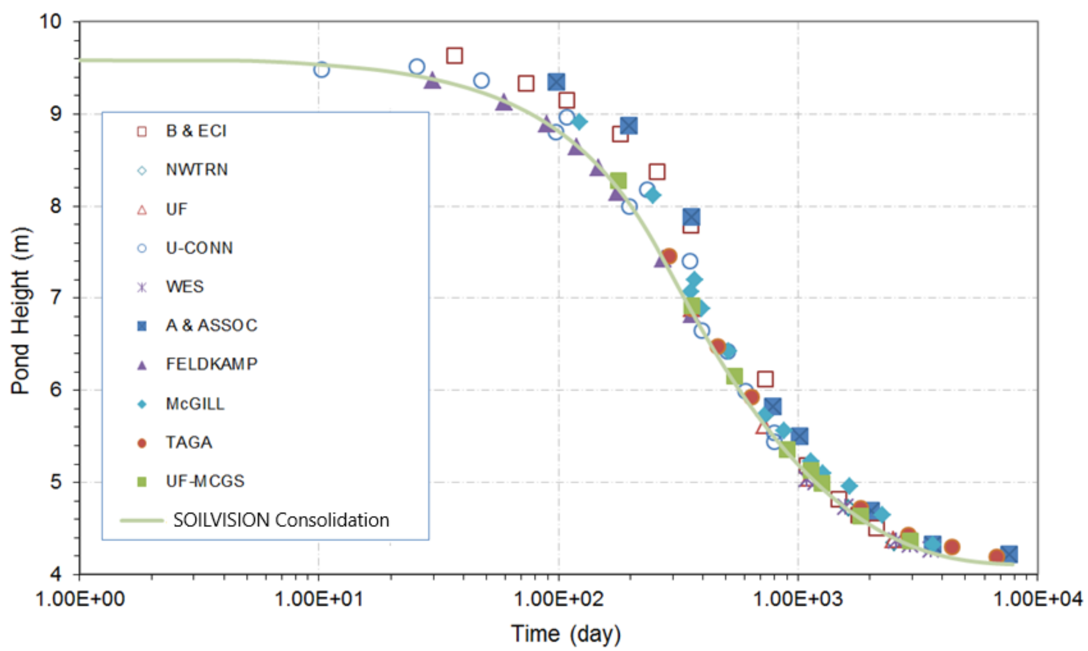

Figure 59: Townsend Model A tailings height with time

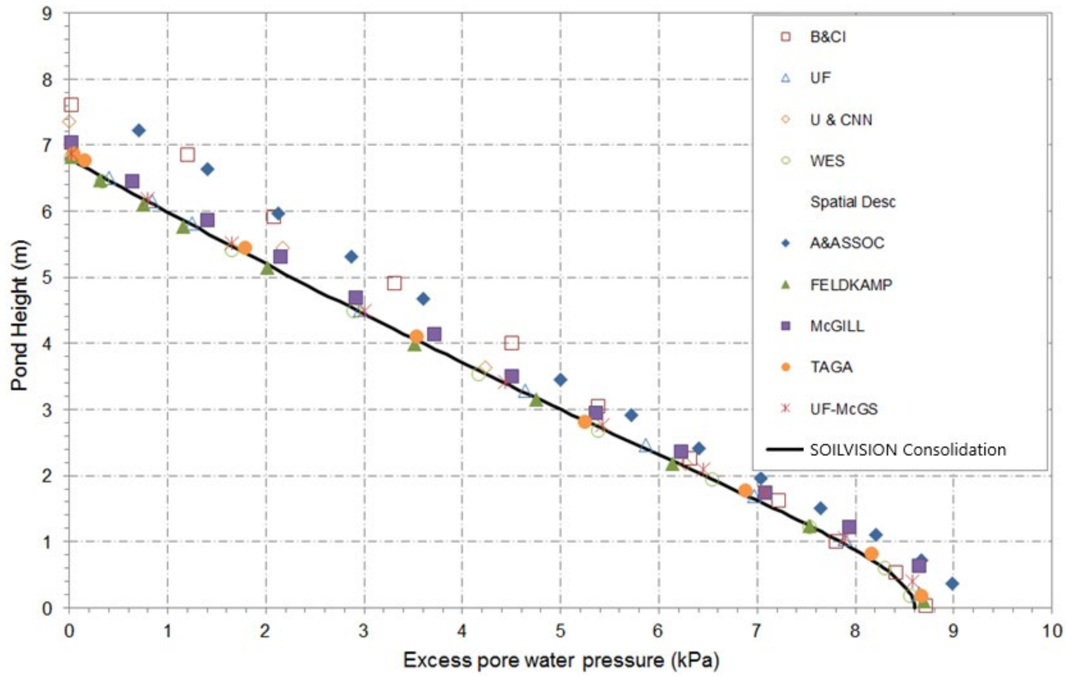

Figure 60: Townsend Model A 1-year excess pore-water pressure profile

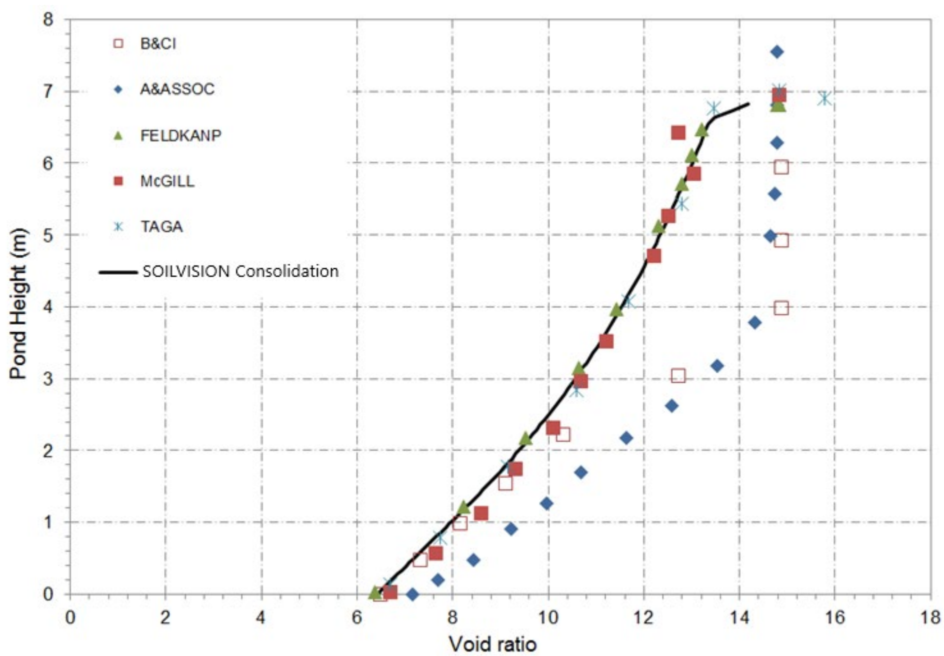

Figure 61 Townsend Model A 1-Year Void Ratio Profile

3.6 TOWNSEND SCENARIO B

Reference: Townsend and McVay, (1990)

Project: Consolidation
Model: Townsend_B_GT

Main factors considered:
- 1D Consolidation of a non-homogeneous column of tailings
- Analysis of a material with a large initial void ratio
- Staged filling

Townsend's Scenario B is intended to simulate waste clay ponds that are filled intermittently with non-uniform initial void ratio thickened clays (Townsend and McVay, 1990). These clays are allowed to consolidate under self-weight conditions. The system remains saturated with flow upward through the surface boundary. This is a relatively challenging system to solve since the initial void ratio is quite high (e = 22.8 and 14.8) and the model involves staged filling.

3.6.1 Model Description

Scenario B is a prediction for a 23.6-ft (7.2-m) deep pond filled in two stages with two different initial void ratios. The clay has a specific gravity of 2.82. The material properties are provided using a Power function relating effective stress to void ratio (Equation [23]) and another relating void ratio to hydraulic conductivity (Equation [24]). The pond will be filled in two six-month increments, separated by a six-month quiescent consolidation increment with the clay at an initial void ratio of 14.8 for the first filling increment and at a void ratio of 22.82 for the second filling increment. The filling rate is 0.0656 ft/day (0.02 m/d).

The model consists of a first layer of 3.6m of homogeneous tailings placed at an initial void ratio (e_o) of 14.8 within 6 months and allowed to consolidate 6 months. Another homogeneous second layer of tailings is then placed at an initial void ratio of 22.8 within another 6 months and allowed to consolidate. The column is allowed to drain freely through the top surface. The bottom boundary has a zero flux condition. Deformation is fixed on the bottom plane. Initial conditions are defined by the water table which is maintained at 8.2m for the entire duration. Figure 62 shows a diagram depicting the model and the model parameters are shown in Table 11.

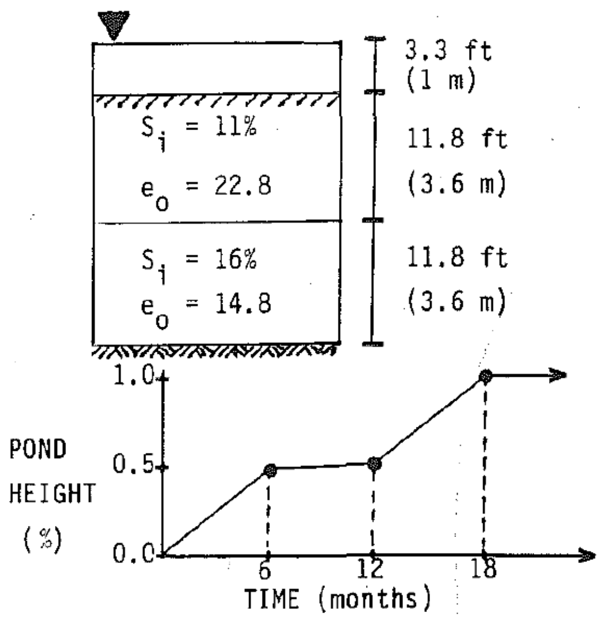

Figure 62: Townsend's Scenario B (from Townsend & McVay, 1990)

Table 11: Summary of Townsend B model parameters

Initial Conditions:	
Model Run Time:	1260 Days
Staged Filling Time:	Each layer is simulated with 18 filling stages, each stage with 10 days duration
Initial Height:	3.6 m for layer 1 and 3.6 m for layer 2
Initial Void Ratio:	14.8 and 22.8
Specific Gravity:	2.82
Initial Water Level:	8.2 m
Initial Flux Condition:	Head Constant = 8.2 m

Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Constant = 8.2 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress–Void Ratio:	Power Function $e = 7.72(\sigma')^{-0.22} \quad (\sigma' \text{ in kPa})$
Hydraulic Conductivity–Void Ratio:	$k = 0.2532 \times 10^{-6} e^{4.65} \quad \text{(m/day)}$
Poisson's Ratio (assumed):	0.40

3.6.2 Results

It can be seen in Figures 63 through Figure 65 that the numerical results from SOILVISION Consolidation show excellent agreement to the data in the published literature. The notations of predictions are given in Table 10.

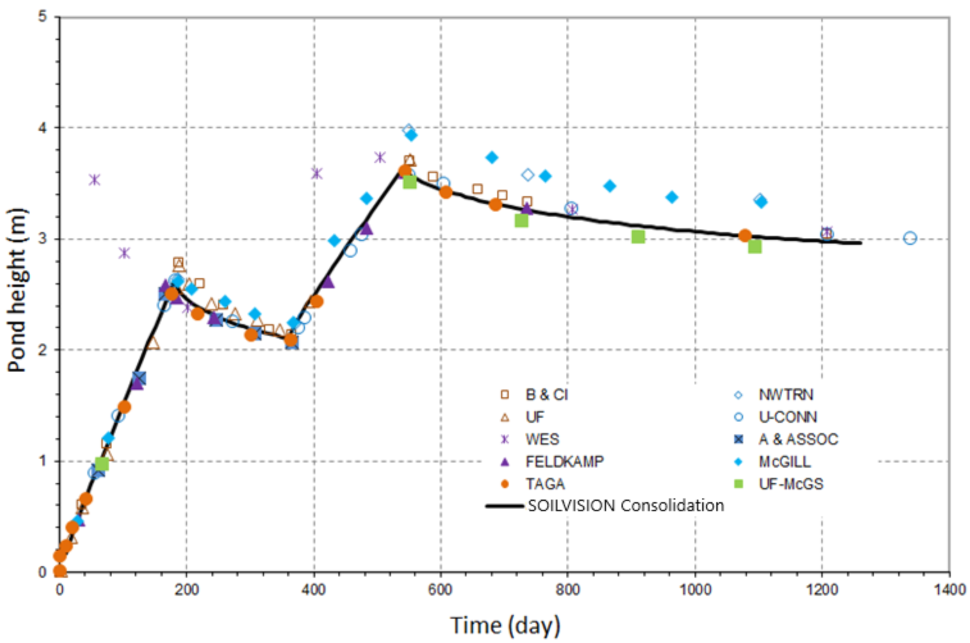

Figure 63: Townsend Model B tailings height with elapsed time

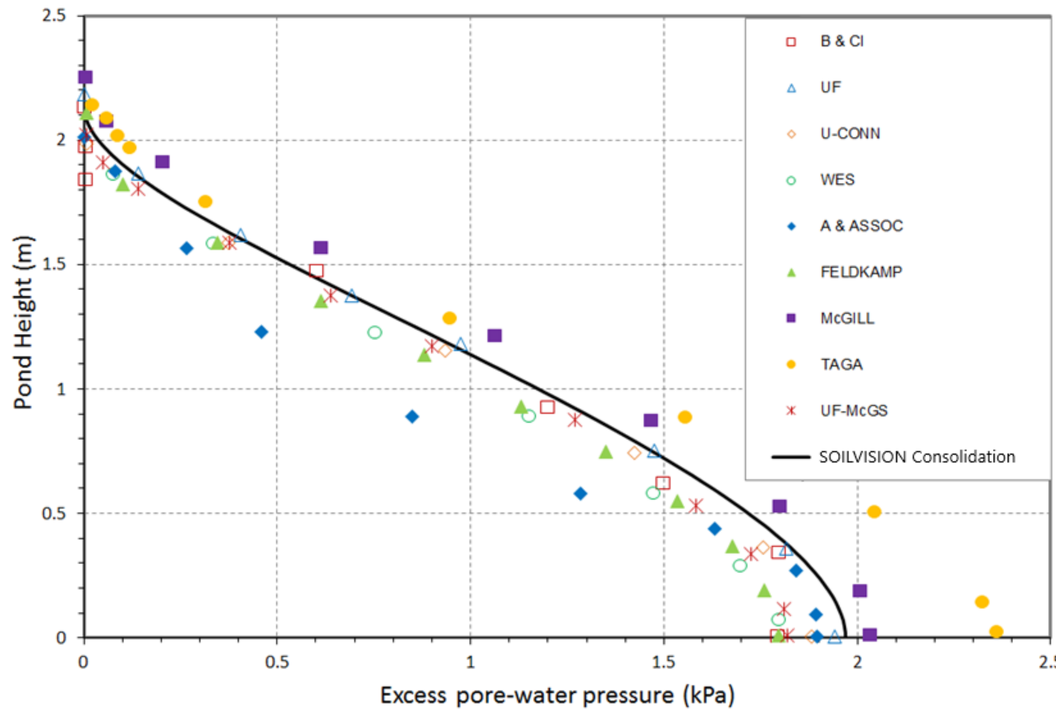

Figure 64: Townsend Model B 1-year excess pore-water pressure profile

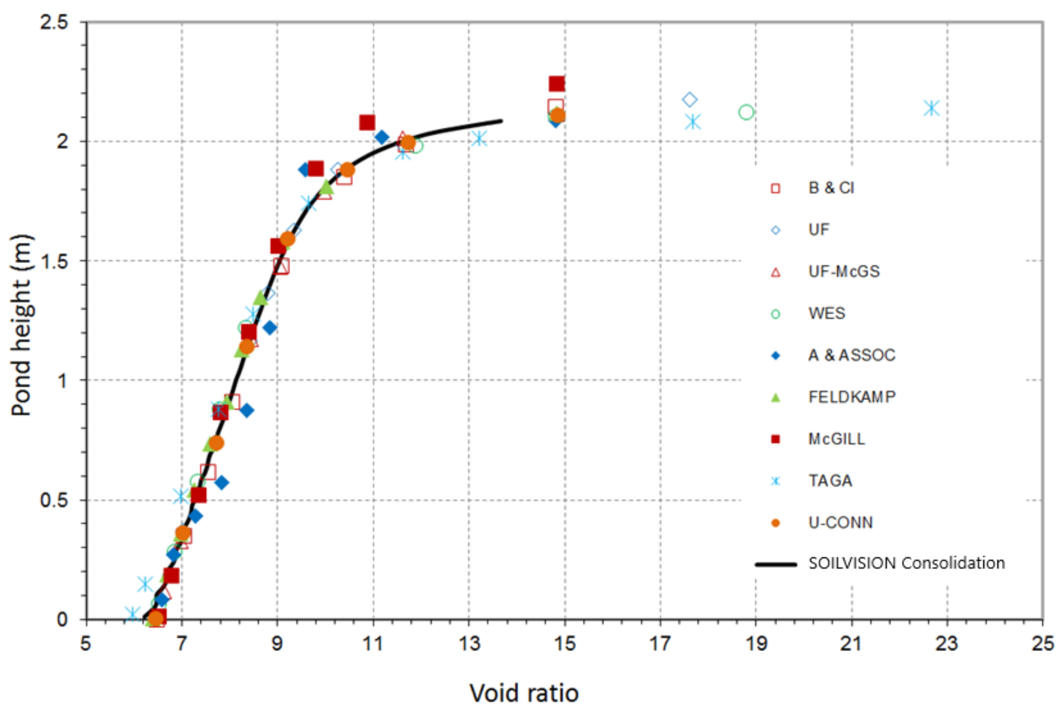

Figure 65: Townsend Model B 1-year void ratio profile

3.7 TOWNSEND SCENARIO C

Reference: Townsend and McVay, (1990)

Project: Consolidation
Model: Townsend_C_GT

Main factors considered:

- 1D Large- strain consolidation under a surcharge load

Townsend's Scenario C predicts consolidation in a homogeneous waste pond with a surcharge load applied at the surface (Townsend & McVay, 1990). The model simulates a young tailings pond with a sand cap constructed on top of the tailings.

The tailings slurry consists of a clay mixture commonly found in the phosphate industry. The material properties are provided using a Power function relating effective stress to void ratio (Equation [23]) and another relating void ratio to hydraulic conductivity (Equation [24]).

3.7.1 Model Description

The clay has a specific gravity of 2.82 and an initial void ratio of 14.8 (solids content, *s* = 16%). The material properties are provided using a Power function relating effective stress to void ratio (Equation [23]) and another relating void ratio to hydraulic conductivity (Equation [24]). The model consists of a 7.2 m deep layer of clay tailings with a surcharge load of 200 psf (q = 976.5 kg/m^2 or 9.58 kPa). The initial lift is placed at a uniform void ratio (e_o) of 14.8 and allowed to consolidate for a period of 6000 days. The column is allowed to drain freely at the surface, and there is no flux at the bottom. The deformation is completely fixed on the bottom plane and free at the surface. Initial conditions are defined by the water table which is maintained at 8.2m for the entire duration. Figure 66 shows a diagram depicting the model and model parameters presented in
Table 12.

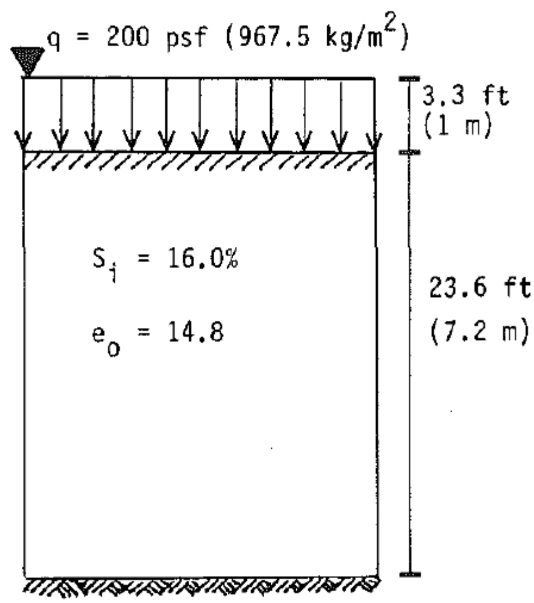

Figure 66: Townsend's Scenario C (from Townsend & McVay, 1990)

Table 12: Summary of Townsend C model parameters

Initial Conditions:	
Model Run Time:	6000 Days
Body Load Time:	5 Days
Surcharge Load Time:	5 Days
Initial Height:	7.2 m
Initial Void Ratio:	14.8
Specific Gravity:	2.82
Initial Water Level:	8.2 m
Initial Flux Condition:	Head Constant = 8.2 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Constant = 8.2 m
Deformation Boundary Conditions:	

Bottom Boundary:	Fixed
Top Boundary:	Load = 9.58 kPa
Material Properties:	
Effective Stress – Void Ratio:	Power Function $e = 7.72(\sigma')^{-0.22}$ (σ' in kPa)
Hydraulic Conductivity – Void Ratio:	$k = 0.2534 \times 10^{-6} e^{4.65}$ (m/day)
Poisson's Ratio (assumed):	0.49

3.7.2 Results

Figures 67 to 69 show the settlement-time curve, 1-year excess pore-water pressure, and void ratio curves, respectively. It was found that the deposit consolidated to a height of 2.43 meters or 34% of the initial height. The abbreviations of predictors are as given in Table 10.

Figure 69 shows the void ratio profile with depth after one year of consolidation. It is worth noting that the surficial layers have consolidated more than the underlying layers. This is due to the applied load creating a low permeability crust layer near the surface. This is an important aspect related to consolidation because the crust can impede flow from the system, meaning that the system will take longer to consolidate.

It should be noted that the information included on Figures 67 to 69 is taken from the results of the Townsend et al. (1987). The data presented by Townsend and McVay in their figures (i.e., Figures 8, 9 and 10 from Townsend and McVay (1990)) show less consolidation (i.e., final height of 55 % of initial height) than is presented in the summary table (i.e., Table 5 of Townsend and McVay, 1990) and body of the paper (with a final height of 35% of initial height).

The data that Townsend et al. (1987) presented in their paper (Townsend and McVay, 1990) matches closely with the values obtained from SOILVISION Consolidation. A comparison of these values is presented in Table 13.

Table 13: Summary of confirmatory parameters for Townsend C

	Final height (m)	1-year height (m)	1-year void ratio at column bottom	1-year excess pore-water pressure at bottom (kPa)
Townsend et al. (1987)	2.5	4.75	6.44	15.5
SOILVISION Consolidation	2.43	4.49	6.21	15.5

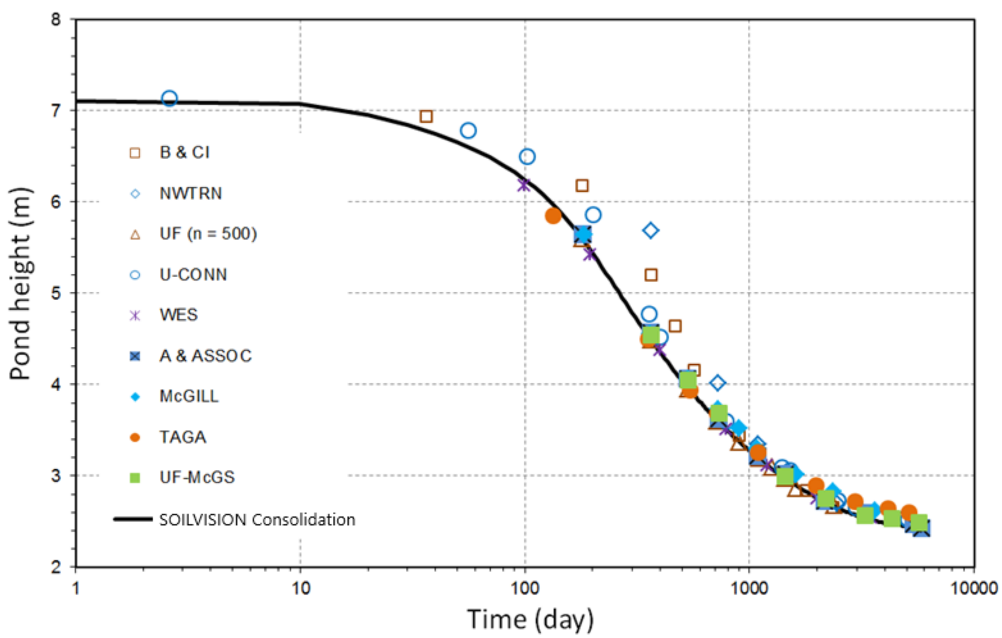

Figure 67: Townsend C settlement of tailings over time

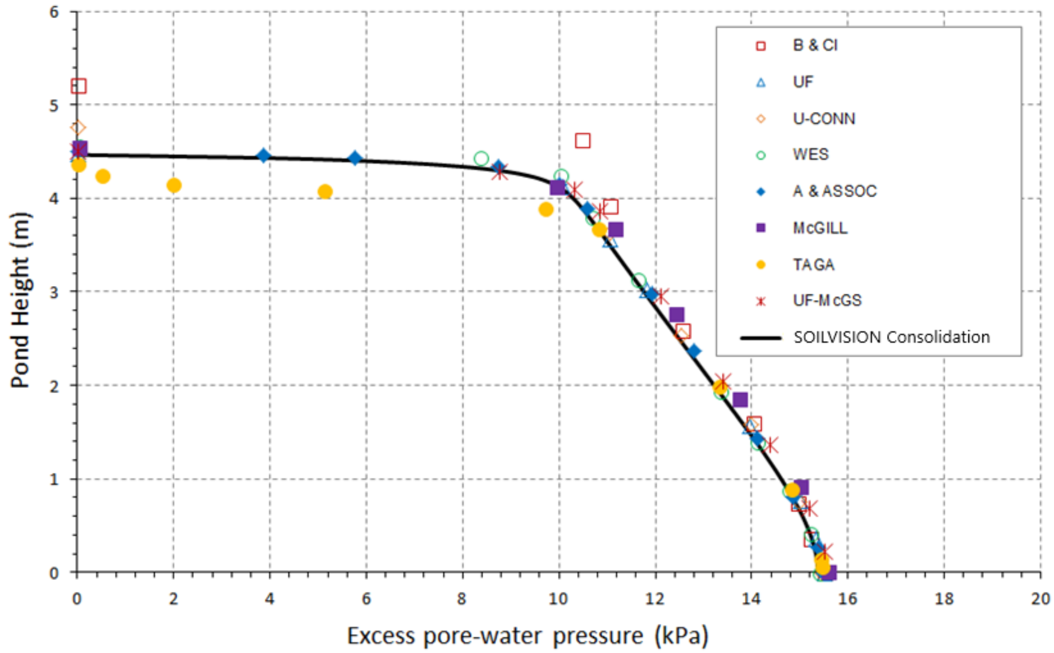

Figure 68: Townsend C 1-year excess pore-water pressure profile

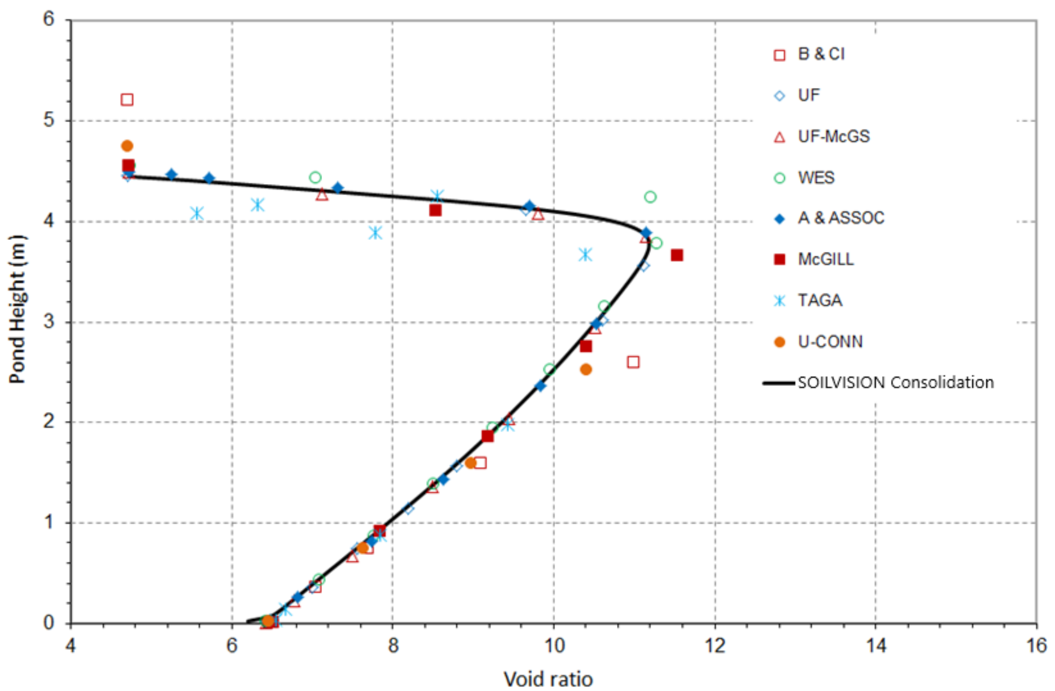

Figure 69: Townsend C 1-year void ratio profile

3.8 TOWNSEND SCENARIO D

Reference: Townsend and McVay, 1990

Project: Consolidation
Model: Townsend_D_GT, Townsend_D_SandCap_GT, Townsend_D_Surcharge_GT

Main factors considered:

- 1D Large-strain consolidation
- Effect of the addition of a capping system
- Multi-material systems

- Heterogeneous initial void ratio profiles

Townsend's Scenario D predicts the consolidation of a stable waste pond with an initial void ratio profile varying with depth and a surcharge load applied by a sandy-clay cap (Townsend and McVay, 1990). This scenario simulates mature tailings ponds that have been filled intermittently with thickened slurries and reclaimed using a capping system.

Townsend and McVay (1990) state that the models required further assumptions to be made in order to solve this problem. This was due to coding and computational limits available at that time. The relevant assumptions as stated by Townsend and McVay (1990) are as follows:

- B&CI recalculated the geometry and initial solids content so that only a single material was used. It was assumed that the sand clay cap did not consolidate significantly
- Piecewise functions were used which independently solved for the consolidation of the sand/clay cap (N'WTRN and McGill)
- Permeability and pressure gradients at the interface between the two materials were not considered (N'WTRN and McGill)
- TAGA assumed that the coefficients of permeabilities were inversely proportional to their porosities at the same effective stress.

To create a similar model, the system was modeled in SOILVISION Consolidation with the sand/clay cap in place; however, instead of applying a soil mass, a surcharge load of equal weight was applied to the top surface.

3.8.1 Model Description

The tailings slurry consists of a clay mixture commonly found in the phosphate industry. The material properties are provided using a power equation relating effective stress to void ratio and another relating void ratio to hydraulic conductivity. These formulas are provided for the clay in Equations [25] and [26] and for the Sand Cap in Equations [27] and [28]. The void ratio distribution within the deposit is shown in Figure 70. The sand/clay cap has a specific gravity of 2.82 and an initial void ratio of 2.1.

$$e = 7.72(\sigma')^{-0.22} \quad (\sigma' \text{ in kPa}) \qquad [\ 25\ ]$$

$$k = 0.2532E-6 * e^{4.65} \quad (\text{m/day}) \qquad [\ 26\ ]$$

$$e = 15.67(\sigma')^{-0.24} \quad (\sigma' \text{ in kPa}) \qquad [\ 27\ ]$$

$$k = 0.1291E-6 * e^{4.15} \quad (\text{m/day}) \qquad [\ 28\ ]$$

The model consists of four 1.8 m lifts of clay tailings covered with a 1.2 m 6:1 sandy clay cap. The bottom lift is placed at a uniform void ratio (e_o) of 6.3 while subsequent 1.8 m lifts with void ratios of 8.0, 10.6 and 14.8 were added. The column is allowed to drain freely from the surface. Deformation is fixed at the base. The sand/clay cap can be included in the model in one of two ways: (1) Include as an actual sand/clay cap layer in the model or (2) apply a surcharge equivalent to that of the sand/clay cap (6.875 kPa) to the surface. The results in section 3.8.2 are based on an actual sand/clay cap layer as part of the model. However, it should be noted that both approaches give the same results. Initial conditions are defined by the water table which is maintained at 9.4m for the entire duration. Figure 70 shows a diagram depicting the model and the model parameters are shown in Table 14**.**

Table 14: Summary of Townsend D model parameters

Initial Conditions:	
Model Run Time	8000 Days
Body Load Time	5 Days
Initial Height	8.4 m
Initial Void Ratio	and 6.3, 8.0, 10.6 and 14.8
Specific Gravity	2.82
Initial Water Level	9.4 m
Initial Flux Condition:	Head Constant = 9.4 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Constant = 9.4 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Clay Effective Stress–Void Ratio:	Power Function $e = 7.72(\sigma')^{-0.22} \quad (\sigma' \text{ in kPa})$
Clay Hydraulic Conductivity–Void Ratio:	$k = 0.2532E-6 * e^{4.65} \quad (\text{m/day})$
Sand Effective Stress–Void Ratio:	Power Function

	$e = 15.67(\sigma')^{-0.24}$ (σ' in kPa)
Sand Hydraulic Conductivity–Void Ratio:	$k = 0.1291E-6 * e^{4.15}$ (m/day)
Poisson's ratio (assumed)	0.4

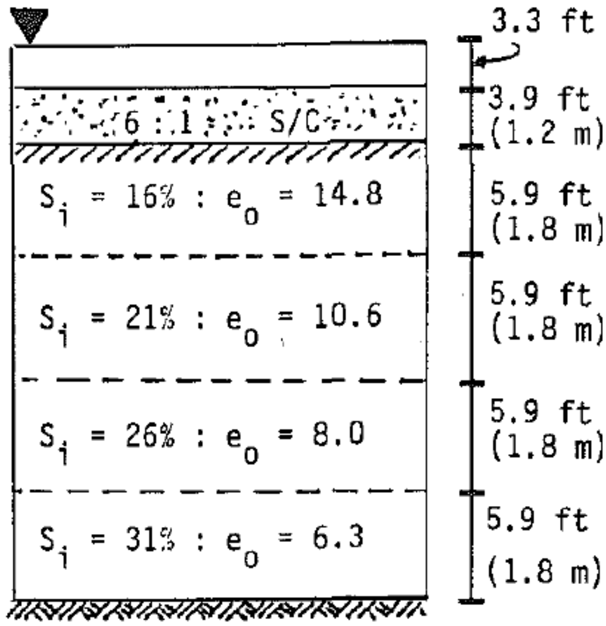

Figure 70: Townsend's Scenario D (from Townsend & McVay, 1990).

3.8.2 Results

Figure 71 shows that SOILVISION Consolidation predicts approximately the same amount of settlement as by the other predictors in the literature. The one year excess pore-water pressure and void ratio profiles are shown in Figures 72 and 73. The one year excess pore-water pressure result from SOILVISION Consolidation shows significant differences at the bottom from other results. Since different assumptions are used in model description, different results are obtained. It can be seen that the one year void ratio data from SOILVISION Consolidation is similar to the literature values, but it is not smooth. The results appear to be reasonable since at the beginning, the pond was filled with four different initial void ratio layers of tailing. It is difficult to have smooth void ratio profiles after just one year of consolidation.

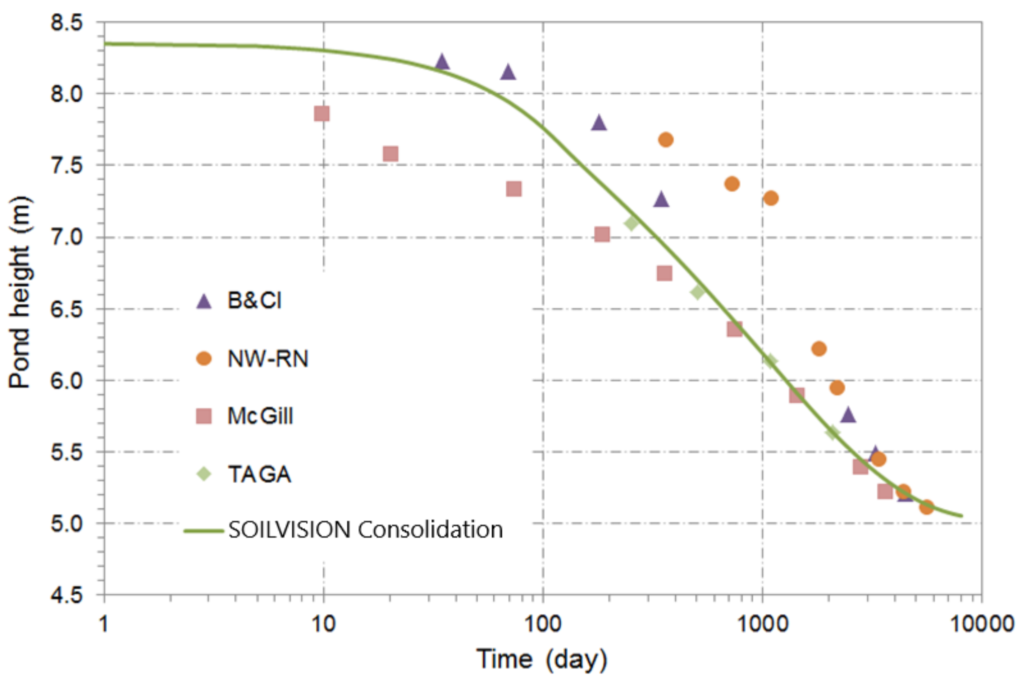

Figure 71: Townsend D column settlement over time

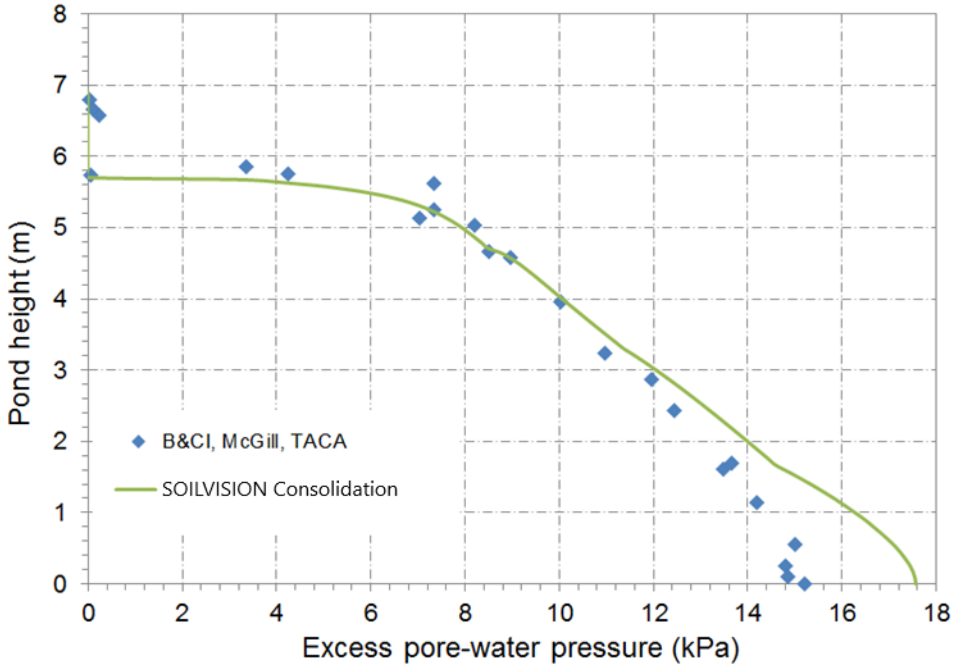

Figure 72: Townsend D 1-year excess pore-water profile

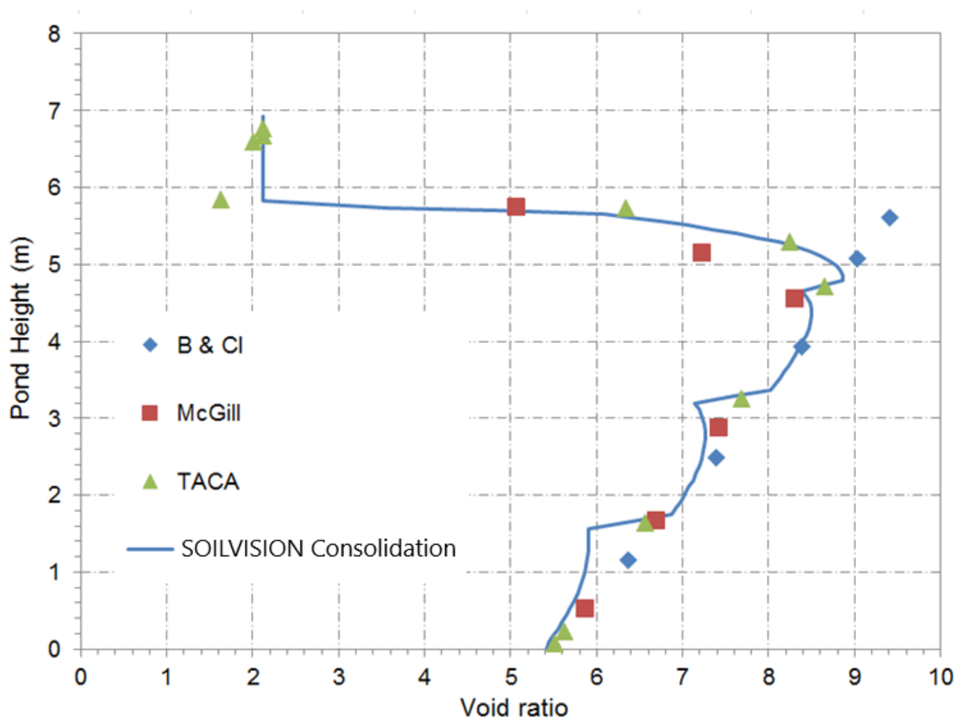

Figure 73: Townsend D 1-year void ratio profile

3.9 CONDESO - EXAMPLE 1

Reference: Yao and Znidarčić, 1997

Project: Consolidation
Model: CONDENS0_Example1_GT

Main factors considered:
- 1D instantaneous filling consolidation analysis
- Analysis of a material with a large initial void ratio

Example 1 is included in the CONDES0 user manual and is intended to represent a slurried soil is deposited into a confined facility in a short period of time (Yao & Znidarčić, 1997). The soil is allowed to consolidate under self-weight conditions. The system remains saturated with flowing upward from the system through the surface boundary.

3.9.1 Model Description

The slurry has a specific gravity of 2.71 and an initial void ratio of 32.42 (solids content, s = 7.714%). The material properties are provided using an Extended Power function relating effective stress to void ratio (Equation [29]) and another Power function relating void ratio to hydraulic conductivity (Equation [30]). These functions are shown in Figures 74 and 75.

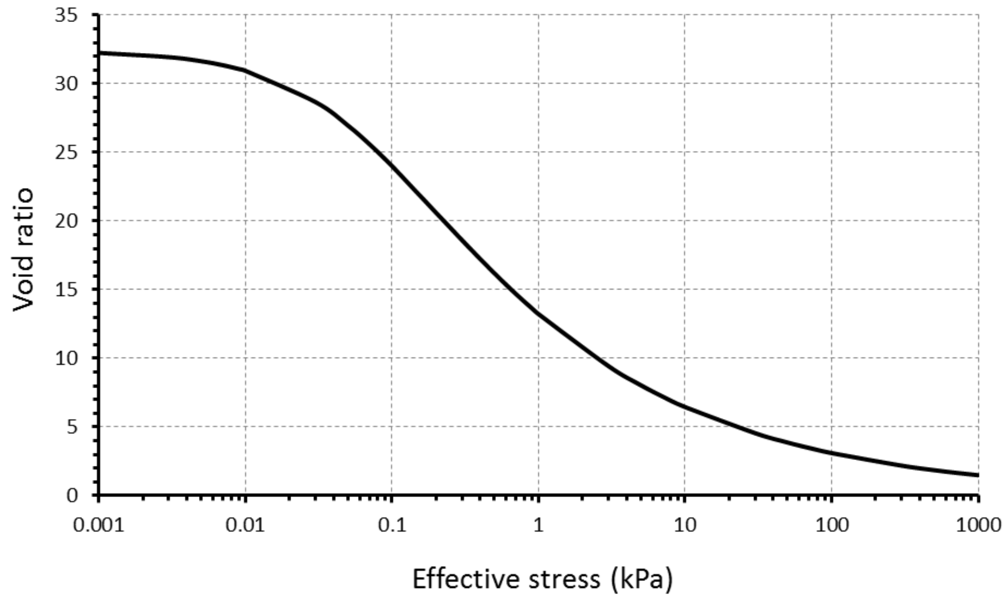

Figure 74: CONDES0 - Example 1 void ratio-effective stress relationship

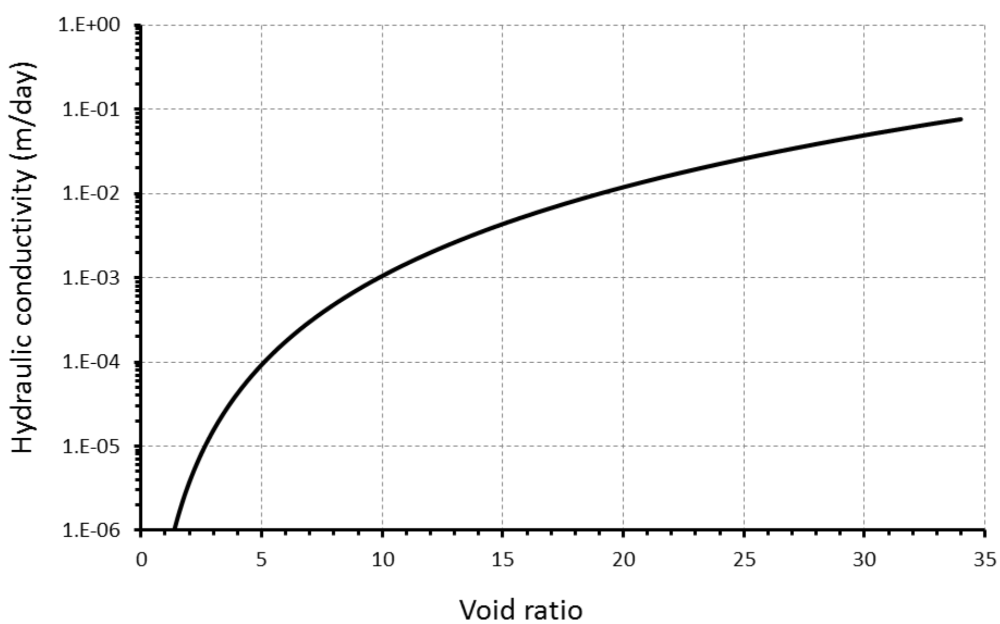

Figure 75: CONDES0 - Example 1 hydraulic conductivity-void ratio relationship

The model consists of 17.85m of homogeneous slurry placed instantaneously at an initial void ratio (e_o) of 32.42 and allowed to consolidate. The column is allowed to drain upward through the top surface. The bottom boundary has a zero flux boundary condition. Deformation is fixed on the bottom plane. Initial conditions are defined by the water table. A layer of ponding water is always on top of the soil. Table 15 lists the model parameters used in the CONDES0 - Example 1.

$$e = 13.49(\sigma' + 0.064)^{-0.319} \quad (\sigma' \text{ in kPa})$$ [29]

$$k = 3.318 \times 10^{-7} * e^{3.5} \quad \text{(m/day)}$$ [30]

Table 15: Summary of CONDES0 - example 1 model parameters

Initial Conditions:	
Model Run Time:	60 years
Body Load Time:	0.1 days
Initial Height:	17.85 m
Initial Void Ratio:	32.42
Specific Gravity:	2.71
Initial Water Level:	17.85 m
Initial Flux Condition:	Head Constant = 17.85 m
Flux Boundary Conditions:	
Bottom Boundary:	Zero Flux
Top Boundary:	Head Constant = 17.85 m
Deformation Boundary Conditions:	
Bottom Boundary:	Fixed
Top Boundary:	Free
Material Properties:	
Effective Stress – Void Ratio:	Extended Power Function $e = 13.49(\sigma' + 0.064)^{-0.319} \quad (\sigma' \text{ in kPa})$
Hydraulic Conductivity – Void Ratio:	$k = 3.318 \times 10^{-7} * e^{3.5} \quad \text{(m/day)}$
Poisson's ratio	0.3

3.9.2 Results

It can be seen in Figure 76 that the results from SOILVISION Consolidation closely match the CONDES0 results.

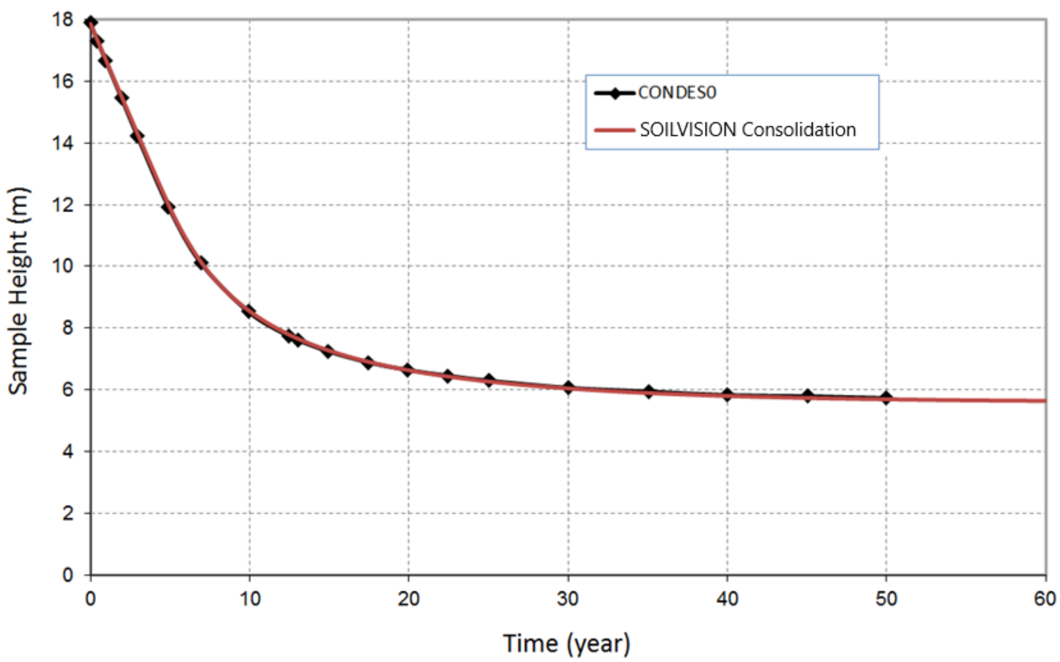

Figure 76: CONDES0 - Example 1 height with time

3.10 CONDESO - EXAMPLE 2

Reference: Yao and Znidarčić, 1997

Project: Consolidation
Model: CONDENS0_Example2_GT

Main factors considered:

- One-dimensional stage filling consolidation analysis with an impervious bottom boundary
- Analysis of a material with a large initial void ratio

Example 2 is included in the CONDES0 user manual to represent a slurried soil that is deposited into a confined facility with the stage filling schedule as shown in Figure 77. The detailed filling information is shown in Figure 78. The soil is allowed to consolidate under self-weight conditions. The system remains saturated with upward flow through the top surface boundary. The consolidation results are compared for an elapsed time of 55 years.

3.10.1 Model Description

The slurry has a specific gravity of 2.71 and an initial void ratio of 32.42 (solids content s = 7.714%). The material properties are provided using an Extended Power function relating effective stress to void ratio (Equation [29]) and another Power function relating void ratio to hydraulic conductivity (Equation [30]). These relationships are graphically shown in Figures 74 and 75.

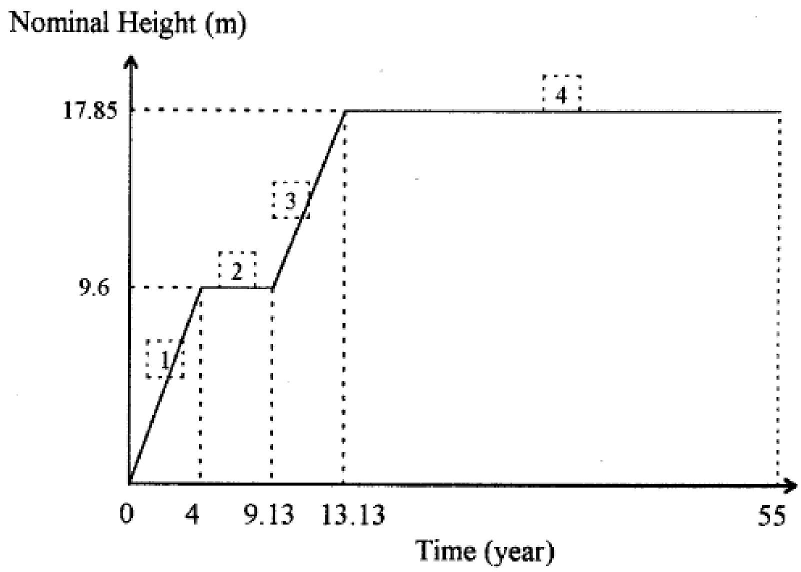

Figure 77: CONDES0 - Example 2 nominal height vs time plot (from Yao & Znidarčić, 1997)

Stage Number	Starting Time (year)	Ending Time (year)	Filling Rate (m/year)
1	0.0	4.0	2.4
2	4.0	9.13	0.0
3	9.13	13.13	2.0625
4	13.13	55	0.0

(total height : 18.75 m)

Figure 78: CONDES0 - Example 2 detailed filling information (from Yao & Znidarčić, 1997)

The model consists of 9.6m of homogeneous slurry placed at a filling rate of 2.4 m/year within 4 years starting with an initial void ratio (e_o) of 32.42 and was allowed to consolidate for 5.13 years. Then another layer of 8.25 m of homogeneous slurry was placed at a filling rate of 2.0625 m/year over another 4 years. The total nominal height was then 17.85 m. The column was allowed to drain freely through the top surface. The bottom boundary was impervious. Deformation was fixed at zero along the bottom plane. A layer of ponding water was always on top of the soil. Table 16 lists the model parameters used in modeling CONDES0 - Example 2.

Table 16: Summary of CONDES0 - Example 2 model parameters

Initial Conditions:	
Model Run Time	55 years
Body Load Time	0.1 Days
Filling Rate 1	2.4 m/year, 4 years
Filling Rate 2	2.0625 m/year, 4 years
Total Nominal Height	17.85 m
Initial Void Ratio	32.42
Specific Gravity	2.71
Initial Water Level	17.85 m

Initial Flux Condition	Head Constant = 17.85 m
Flux Boundary Conditions:	
Bottom Boundary	Zero Flux
Top Boundary	Head Constant = 17.85 m
Deformation Boundary Conditions:	
Bottom Boundary	Fixed
Top Boundary	Free
Material Properties:	
Effective Stress – Void Ratio	Extended Power Function $e = 13.49(\sigma' + 0.064)^{-0.319} \quad (\sigma' \text{ in kPa})$
Hydraulic Conductivity – Void Ratio:	$k = 3.318 \times 10^{-7} * e^{3.5} \quad (\text{m/day})$
Poisson's ratio (assumed)	0.3

3.10.2 Results

Figure 79 shows that the results from SOILVISION Consolidation match closely with the CONDES0 results. SOILVISION Consolidation predicts a slightly larger amount of consolidation settlement during the quiescent consolidation period after the filling is complete.

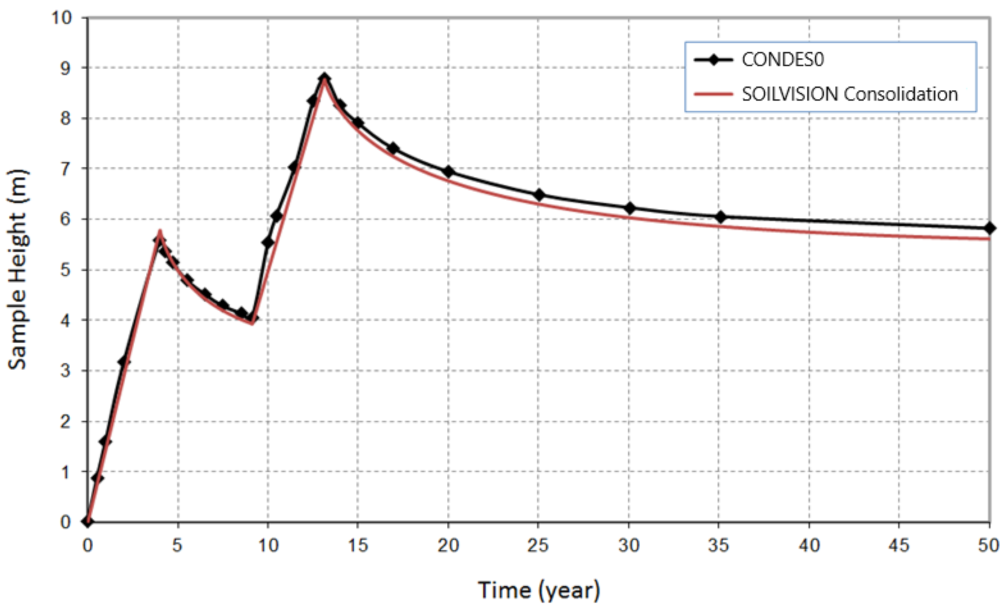

Figure 79: CONDES0 - Example 2 height with time

3.11 CONDESO- EXAMPLE 3

Reference: Yao & Znidarčić (1997)

Project: Consolidation
Model: CONDENS0_Example3_GT

Main factors considered:

- One-dimensional stage filling consolidation analysis with a pervious bottom boundary
- Analysis of a material with a large initial void ratio

Example 3 was included in the CONDES0 user manual and was intended to represent a slurried soil that was deposited into a confined facility with the stage filling schedule as shown in Figure 80. The detailed filling information is shown in Figure 81. The soil was allowed to consolidate under self-weight conditions. The system remained saturated with upward flow through both the surface boundary as well as downward flow through the bottom. The consolidation analysis was undertaken for 55 years.

3.11.1 Model Description

The slurry has a specific gravity of 2.71 and an initial void ratio of 32.42 (solids content, *s* = 7.714%). The material properties are provided using an Extended Power function relating effective stress to void ratio (Equation [29]) and another

Power function relating void ratio to hydraulic conductivity (Equation [30]). These relationships are graphically shown in Figures 74 and 75.

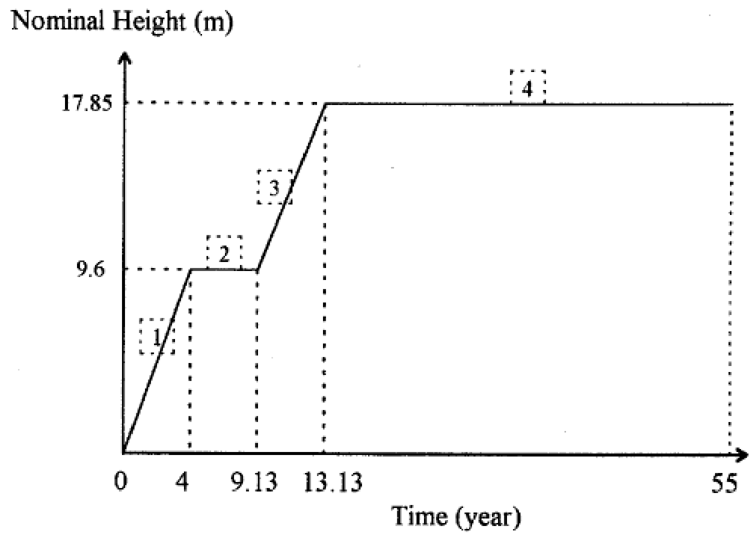

Figure 80: CONDES0 - Example 3 nominal height vs time plot (from Yao & Znidarčić, 1997)

Stage Number	Starting Time (year)	Ending Time (year)	Filling Rate (m/year)
1	0.0	4.0	2.4
2	4.0	9.13	0.0
3	9.13	13.13	2.0625
4	13.13	55	0.0

(total height : 18.75 m)

Figure 81: CONDES0 - Example 3 detailed filling information (from Yao & Znidarčić, 1997)

The model consists of 9.6 m of homogeneous slurry placed at a filling rate of 2.4 m/year over 4 years. The initial void ratio (e_o) was 32.42. The slurry was allowed to consolidate 5.13 years and then another layer of 8.25 m of homogeneous slurry placed at a filling rate of 2.0625 m/year for another 4 years. The total nominal height was then 17.85 m. The column was allowed to drain freely through both the top surface and the bottom boundaries. Deformation was fixed at the bottom plane. A ponding water layer was always on top of the soil. Table 17 lists out the model parameters used in the CONDES0 - Example 3.

Table 17: Summary of CONDES0 - Example 3 model parameters

Initial Conditions:	
Model Run Time	55 years
Body Load Time	0.1 days
Filling Rate 1	2.4 m/year, 4 years
Filling Rate 2	2.0625 m/year, 4 years
Total Nominal Height	17.85 m
Initial Void Ratio	32.42
Specific Gravity	2.71
Initial Water Level	17.85 m
Initial Flux Condition	Head Constant = 17.85 m
Flux Boundary Conditions:	
Bottom Boundary	Head Constant = 17.85 m
Top Boundary	Head Constant = 17.85 m
Deformation Boundary Conditions:	
Bottom Boundary	Fixed
Top Boundary	Free
Material Properties:	
Effective Stress – Void Ratio	Extended Power Function
	$e = 13.49(\sigma' + 0.064)^{-0.319}$ (σ' in kPa)
Hydraulic Conductivity – Void Ratio:	$k = 3.318 \times 10^{-7} * e^{3.5}$ (m/day)
Poisson's ratio (assumed):	0.3

3.11.2 Results

Figure 82 shows that the results from SOILVISION Consolidation match closely to the CONDES0 results. SOILVISION Consolidation predicts slightly larger consolidation settlement during the quiescent consolidation period after the filling was complete.

The only difference between Example 2 and Example 3 is the flux boundary condition at the bottom. In Example 3, the bottom was also allowed the flow to cross the boundary. This means the excess pore-water pressure will dissipate faster than that in Example 2 (i.e., the sample settles faster in Example 3 than that in Example 2). However, the total settlement should be the same after primary consolidation is finished since the two models have the same initial void ratio and same nominal height. These conclusions can be clearly observed by comparing Figures 81 and 82.

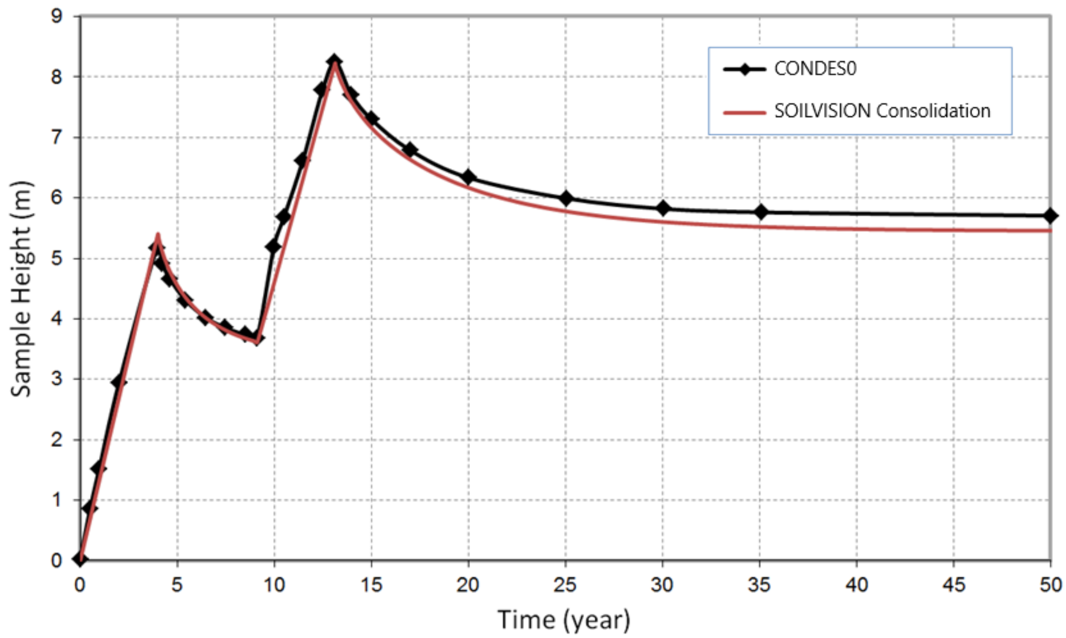

Figure 82: CONDES0 – Example 3 height with time

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