Effect Of Mesh Resolution On Long-Term Water Balance Calculations

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ABSTRACT: The quantification of the performance of earth covers has been largely determined through the use of test plots as well as numerical modeling. Such numerical models are run with the use of complex climatological data and unsaturated soil property functions. The input information is then used by the finite element solvers to determine vertical flow rates and, ultimately, long-term percolation rates through the earth cover. Calculations are generally complicated by the fact that the infiltration into a dry soil is one of the more complex types of numerical modeling scenarios. The challenge occurs largely because of the non-linearities present in the unsaturated soil properties. These non-linearities becomes part of the numerical model which must run for 1, 10, or 50 years. This paper examines the numerical difficulties associated with these calculations and examines the impact of small numerical issues over long time periods. In particular, the role of mesh density as it is related to the accuracy of water balance calculations is examined.

1 INTRODUCTION

Numerical models are increasingly being used for the analysis of the long-term performance of earth covers. While numerical models offer an excellent way to solve the partial differential equations associated with moisture flow, there are some issues with this type of analysis that need to be addressed. In a typical analysis, climate data is collected from somewhere between 5 and 100 years and a numerical model (typically a 1D model) is run to determine the amount of precipitation that ends up as net percolation. The primary unknown in these numerical models is the actual evaporation (AE). AE may be calculated as a fraction of potential evaporation (PE) through use of the Wilson-Penman equation. It should also be noted that runoff can have an influence during high-intensity storms.

Significant effort has been devoted to the solution of the climatic coupling (Wilson, 1997; Shackelford, C., 2005). However, the numerical model inherently poses a problem which is fundamentally difficult to solve. The problem of infiltration of precipitation into a dry soil is a well-documented “problem” case (Haverkamp, 1977). The reason for the difficulty with this type of model is that the gradients in the numerical model may be high when a precipitation event hits a soil. The soil may be dry and, therefore, has an extremely low hydraulic conductivity.

The movement of a wetting front (or increased saturation levels) into a dry soil is a classic seepage problem that has previously received attention. Two previous models, which have previously solved this problem, are worth mentioning.

The Haverkamp (1977) model involves infiltration into a 1D column of material. A series of infiltration experiments were performed by Haverkamp in the laboratory using a plexiglass column uniformly packed with sand to verify the numerical results. The model was originally solved using 1D finite elements and the 1D finite difference solution methods. Time-steps used in the analysis were varied in the original work to determine their effect on the solution. The best solution presented occurred with small time-steps (10 seconds) and a dense grid.

The material properties used in Haverkamp’s analysis used equations defined in terms of elevation head rather than soil suction. The model was initially set up in SVFlux and then minor modifications were made to the FlexPDE finite element script file to duplicate the solution exactly. The script file presenting a precise comparison of results can be provided upon request.

The results of the comparison can be seen in Figure 1. The automatic time-stepping feature in the SVFlux / FlexPDE software is used to choose ideal timesteps. It can be seen that the mesh and time-steps selected by SVFlux automatically duplicates the best results presented by the Haverkamp (1977) solution.

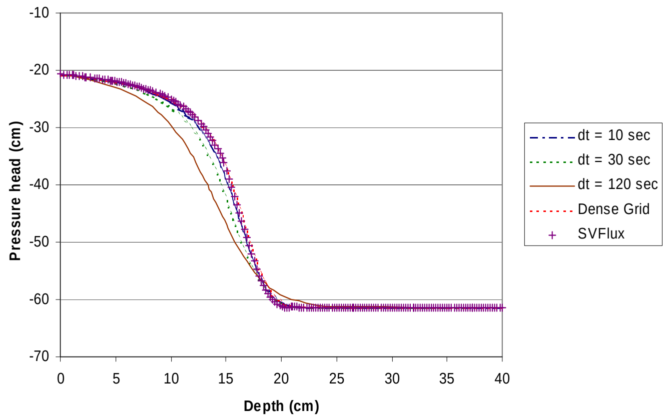

Figure 1 Comparison between SVFLUX and Haverkamp (1977) as presented by Celia (1990)

Celia (1990) performed comparisons of 1D solvers by varying the time-steps and the solution methods (finite difference or finite element). The results were considered classic solutions and are commonly used to benchmark the validity of 1D infiltration models. The solution presented by Celia used the *h*-based formulation of Richard's equation and a Newton-Raphson iterative method.

A replica of Celia's model was set up using the SVFlux software. Celia presented the material properties for the model as van Genuchten's equation for the soil-water characteristic curve and as van Genuchten and Mualem's equation for representing the unsaturated hydraulic conductivity curve. Since both methods are implemented in the SVFlux software the parameters used for the material could be input directly.

The results of the comparison can be seen in Figure 2. As in the previous model it can be seen that the automatic mesh generation and automatic time-step refinement allow quick convergence to the correct solution.

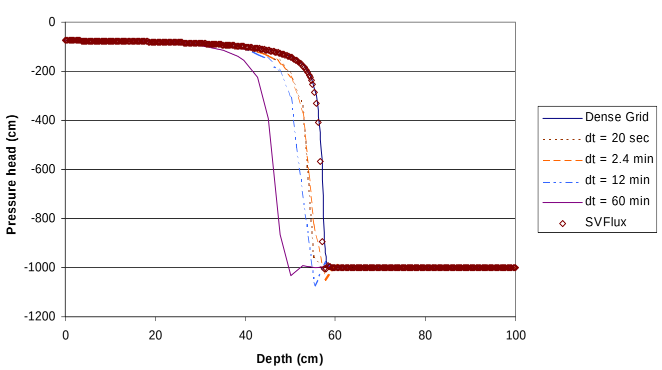

Figure 2 Comparison between SVFLUX and results presented by Celia (1990)

It should be noted that significant effort has previously been invested in ensuring that a single case of infiltration is properly represented in a numerical model. In today's cover models there are typically thousands of precipitation events that may be introduced into the numerical model over a period of 10, 20, 50 or 100 years. Each event may be difficult to solve as a single event. It is extremely challenging to have a numerical model which handles all applied precipitation events with satisfactory accuracy.

This paper examines the use of both automatic time-stepping and mesh refinement to improve solutions for the calculation of vertical flow. A few example models are presented which illustrate the limits of numerical analysis in earth cover evaluation. These examples show the benefits of using automatic mesh refinement to extend the limits of possible solutions.

2 ANALYSIS

It should be noted that the ability of a software program to handle infiltration into a dry soil is somewhat dependant on the ratio of the intensity of the applied precipitation event to the saturated hydraulic conductivity of the soil layer at the surface. To date separate studies have shown the ability of the SVFlux / FlexPDE software to handle applied precipitation events of up to 10 times the saturated coefficient of permeability (Fredlund et al, 2006).

2.1 Example No. 1

The mesh density of a numerical model is one of the primary sources of error (Yeh, 2000). In order to demonstrate this issue a simple numerical model is set up in the SVFlux software. The numerical model is a simple fixed-mesh model consisting of a vertical column of soil with unsaturated soil properties defined. A few random precipitation events are then applied to the top of the soil column and the impact of these precipitation events is then tracked in the model. The soil-water characteristic curve is first then steepened until the results of the numerical model become erroneous (

Table 1). The unsaturated hydraulic conductivity was estimated using the Modified Campbell method. Once the numerical model is brought to failure, the mesh density in the numerical model is increased until the model again reaches the correct answer. The results of this experiment may be seen in Figure 3.

Table 1 Fredlund & Xing soil-water characteristic curve parameters

	Fredlund & Xing			
ID	af	nf	mf	hr
1	1.24	3.83	0.18	34.9
2	1.24	4.5	0.3	34.9
3	1.24	5.5	0.4	34.9
4	1.24	6.6	0.6	34.9
5	1.24	8	1.5	34.9

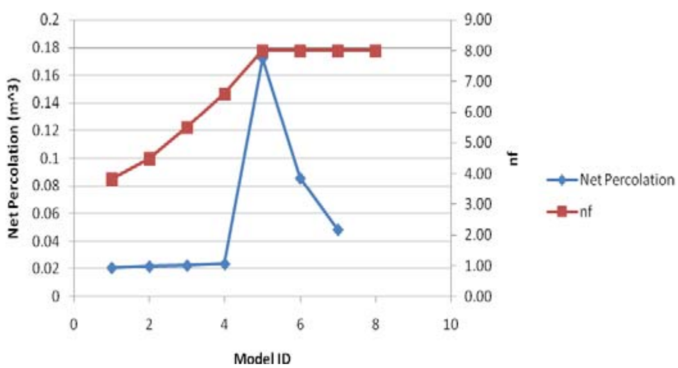

Figure 3 Influence of mesh density on solution validity

It can be seen in Figure 3 that the influence of mesh density on infiltration models is significant.

2.2 Example No. 2

A second experiment was then set up as a numerical model. In the numerical model the total modeling time is set to 7 days. In each of these days a progressively more intense storm is introduced to the numerical model. The particular storm events are presented in the following table (Table 2). The model is: i. first run with automatic mesh refinement turned on. Then the model is run with a default static and unrefined mesh. The static unrefined mesh yields large volume-mass errors in the calculations. The mesh density is then increased in the static-mesh scenario until a static mesh yields a theoretically correct answer.

Table 2 Precipitation events applied to the numerical model

Time (days)	Precipitation (m^3/day/m^2)
1	0.0001
2	0.0005
3	0.001
4	0.005
5	0.01
6	0.05
7	0.1
8	0.5
9	1
10	5

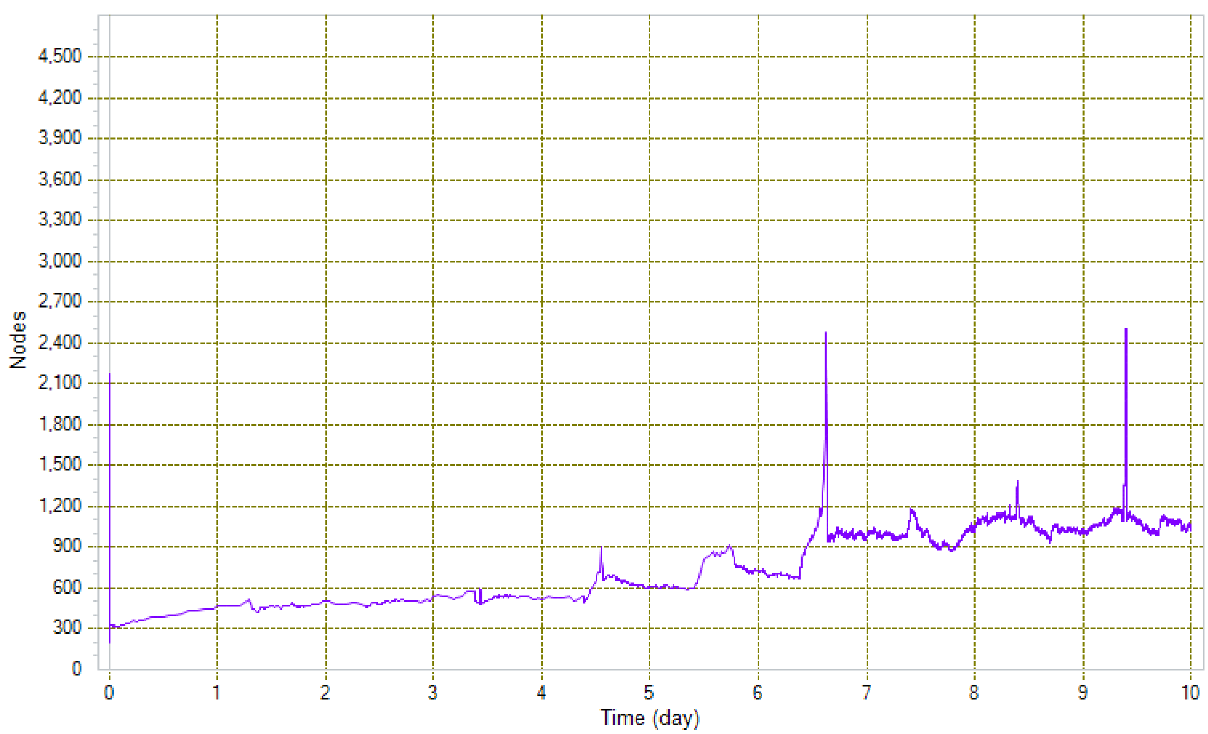

Figure 4 Number of nodes in the 1D model versus time

It can be seen from the scenario with the automatic mesh refinement turned on that the mesh refinement detects cases where further mesh refinement is needed. This is illustrated in Figure 4 which shows spikes in the number of nodes in the model which correspond to the applications of the storm events.

It should be noted that the static mesh requires a dense mesh for the entire modeling duration. This then yields excessive model run-times.

A practical application of mesh refinement in 2D can be seen in infiltration into a pile of waste rock. This is illustrated in the following snapshots in time. It can be graphically seen how the mesh refinement responds to the high degree of non-linearity encountered on the seepage front.

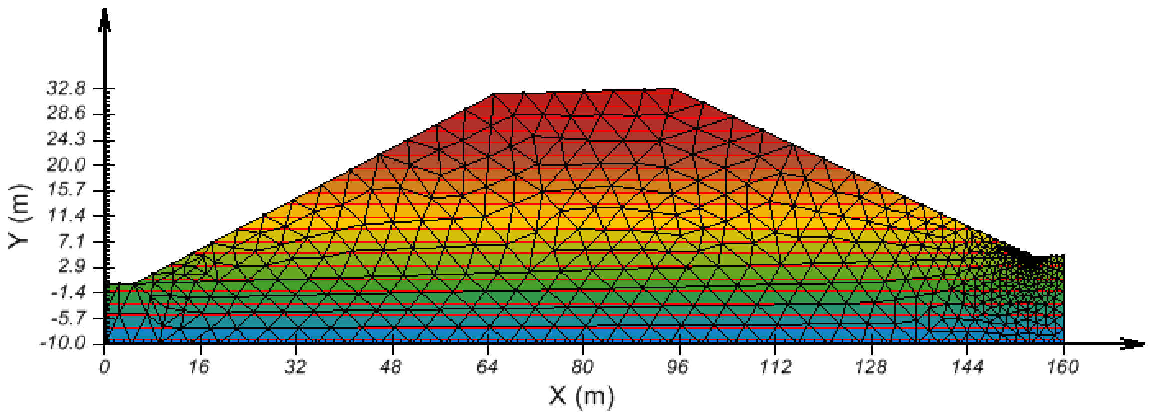

Figure 5 Time = 0 days

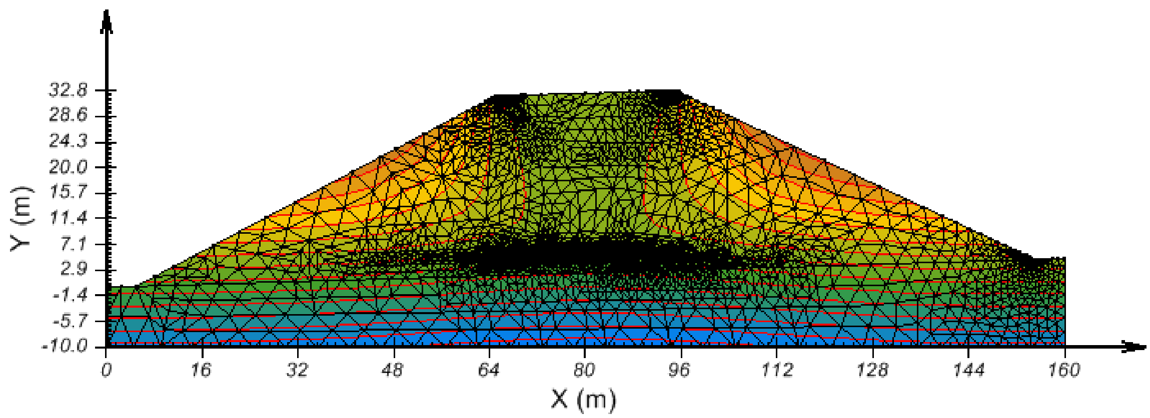

Figure 6 Time = 5 days

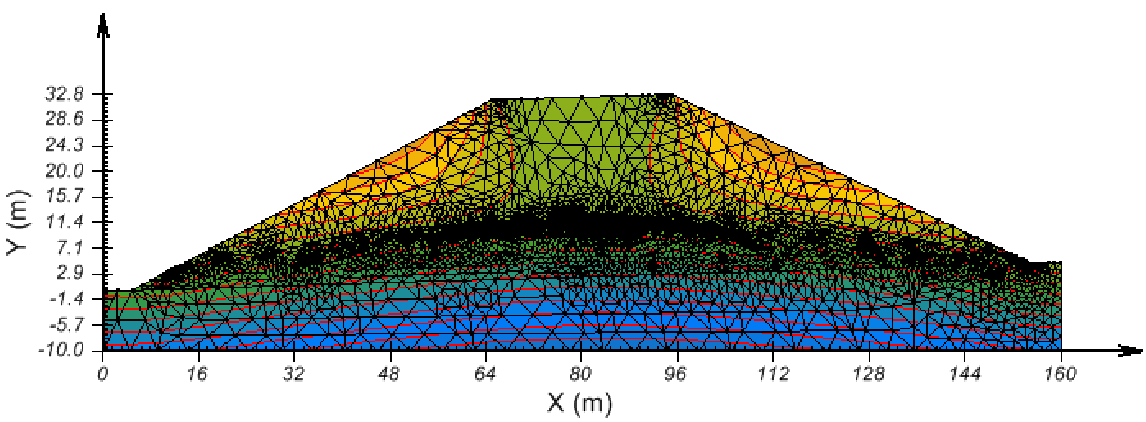

Figure 7 Time = 10 days

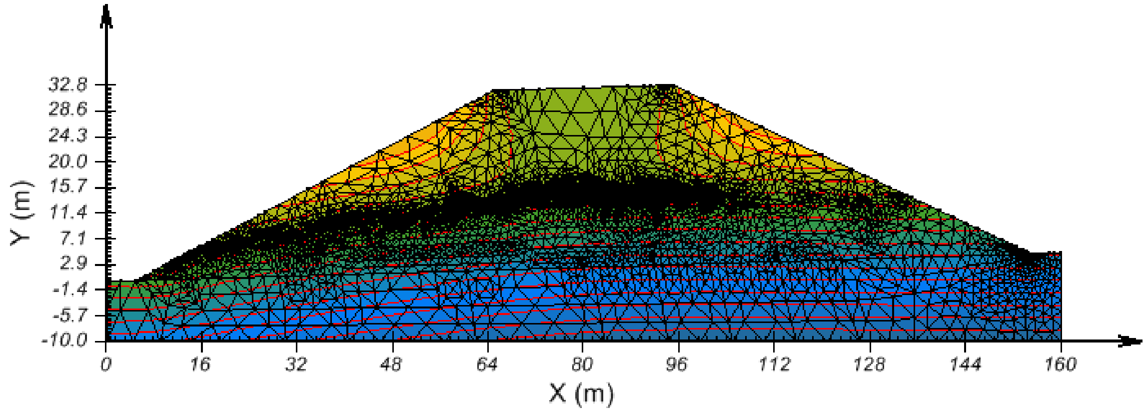

Figure 8 Time = 20 days

3 CONCLUSIONS

From the presented examples it can be seen that the use of automatic time-step refinement and automatic mesh refinement significantly enhances the ability of software to solve difficult infiltration models. This technology is especially applicable to the evaluation of earth covers. Automatic mesh refinement produces a definable improvement in the ability to handle steep unsaturated soil properties such as those of sands and gravels.

It can be seen with this study that the numerical modeling time required by numerical models performing cover modeling has the potential to be reduced through the selective use of additional nodes only in times of peak intensity storms.

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