Estimation of unsaturated soil property functions for geotechnical engineering applications

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ABSTRACT
This paper illustrates the calculations necessary for the estimation of hydraulic unsaturated soil property functions for geotechnical engineering applications. A single set of soil properties are used to illustrate the importance of separating volume change behaviour and desaturation behavior of a soil when estimating unsaturated soil property functions. The improved estimation procedures are based on the measurement of gravimetric water content versus soil suction curve along with the shrinkage curve of a soil. The recently released software package called *SVSOILS*^{TM} is used to illustrate the variety of possible volume-mass graphs that can be computed and used to illustrate the interpretation of the laboratory results. The analysis leads to the estimation of the hydraulic conductivity function for the unsaturated soil.

RÉSUMÉ
Cet article illustre les calculs nécessaires pour l'évaluation des fonctions insaturées hydrauliques de propriété de sol, USPFs, pour des applications géotechniques d'ingénierie. Une série unique depropriétés de sol sont employées pour illustrer l'importance du comportement de changement devolume de séparation et du comportement dedésaturation d'un sol en estimant USPFs. Les procédures améliorées d'évaluation sont baséessur la mesure de la teneur en eau gravimétriquecontre la courbe d'aspiration de sol (*w*-SWCC)avec la courbe de rétrécissement, (SC), d'un sol. Le progiciel récemment libéré *SVSOILS*^{TM} appelé est employé pour illustrer la variété de graphiquespossibles de la volume-masse qui peuvent être calculés, tracés et employés pour illustrer l'interprétation des résultats de laboratoire. L'analyse mène à l'évaluation de la function hydraulique de conductivité pour le sol insaturé.

1 INTRODUCTION

The application of unsaturated soil mechanics in geotechnical engineering practice requires considerably more computational effort than is involved for the application of saturated soil mechanics. Unsaturated soil mechanics formulations are nonlinear because the unsaturated soil property functions are nonlinear. Consequently, an iterative computational procedure is required to obtain a convergence and "correct" solution. Calculations for the estimation of unsaturated soil property functions, USPFs, are based on an understanding of the soil-water characteristic curve, *w*-SWCC, and the shrinkage curve, SC. The estimation of USPFs requires the use of integration and differentiation mathematical techniques. These mathematical procedures can be expedited through use of the computer software program called *SVSOILS*™ developed and maintained by *SoilVision Systems Ltd*.

The objective of this paper is to illustrate the calculation of the unsaturated hydraulic properties for a compressible soil. Refinements in the estimation procedure along with the measurement of the shrinkage curve are required when the soil undergoes volume change as soil suction is changed (See Figure 1).

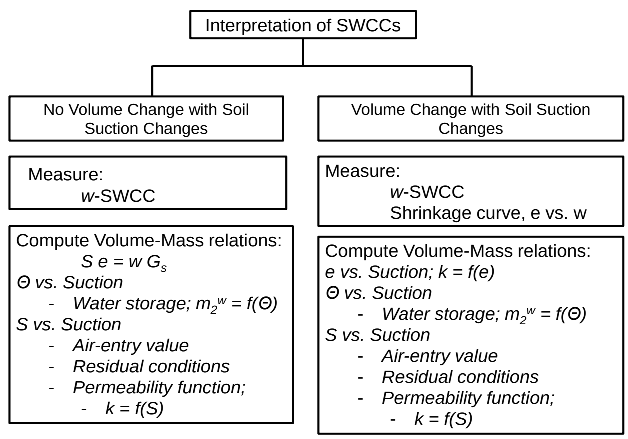

Figure 1. Measurements required for the calculation of USPFs for "volume change" and "no volume change" soils as soil suction is increased

The calculation of the USPFs are based on the saturated soil properties and two additional laboratory tests; namely, i.) the SWCC, and ii.) the shrinkage curve, SC.

The scope of the paper is limited to the calculation of the unsaturated hydraulic properties of one soil. The regression analyses presented in this paper are limited to use of the Fredlund-Xing (1994) SWCC equation and the M. Fredlund (2002) shrinkage curve equation. The calculation of the permeability functions is limited to the use of the Fredlund et al., (1994) integration equation.

2 SOIL TESTING PROGRAM

The soil analyzed is an artificial compressible clayey silt. The soil changes volume as soil suction is applied and slowly desaturated towards residual suction conditions. The overall volume change of the soil upon drying is in the order of 20%. The specific gravity of the soil is 2.75 and the initial gravimetric water content is 40%.

2.1 Laboratory Data for SWCC and SC

Figure 2 shows the gravimetric water content versus matric suction, (w-SWCC), up to 800 kPa for the clayey silt along with the best-fit curve through the data points. Three total suction data points are shown for total suctions between 4000 kPa and 400,000 kPa. The Fredlund-Xing (1994) SWCC equation can be applied to the water content versus soil suction data.

$$w(\psi) = \frac{w_s(1 - \ln(1 + \psi/\psi_r)/\ln(1 + 10^6/\psi_r))}{\left\{ln\left[\exp(1) + (\psi/a_f)^{n_f}\right]\right\}^{m_f}} \qquad [1]$$

where: $w(\psi)$ = gravimetric water content at any soil suction, ψ,
w_s = saturated gravimetric water content,
a_f = fitting parameter near the inflection point on the w-SWCC,
n_f = fitting parameter related to the maximum rate of gravimetric water content change,
m_f = fitting parameter related to the curvature near residual gravimetric water content conditions,
ψ_r = suction near residual conditions of the soil

The best-fit parameters for the Fredlund-Xing (1994) w-SWCC are as follows: af = 65.9 kPa, nf = 1.07, and mf = 0.76. The residual suction value was estimated to be 1531 kPa. The calculated degree of saturation at the start of the test was 96.4%.

A shrinkage curve, SC, equation proposed by M. Fredlund, (2002) can be used to best-fit the void ratio versus gravimetric water content drying curve.

$$e(w) = a_{sh}\left(\left(\frac{w}{b_{sh}}\right)^{c_{sh}} + 1\right)^{1/c_{sh}} \qquad [2]$$

where: a_{sh} = minimum void ratio upon complete drying,
b_{sh} = variable related to the slope of the drying curve calculated as: $b_{sh} = (a_{sh} \times S_0)/ G_s$, and
c_{sh} = sharpness of curvature as the soil desaturates

Figure 3 presents the SC for the clayey silt along with the best-fit curve through the data points which are as follows: a_{sh} = 0.6061, b_{sh} = 0.2137 and c_{sh} = 4.713.

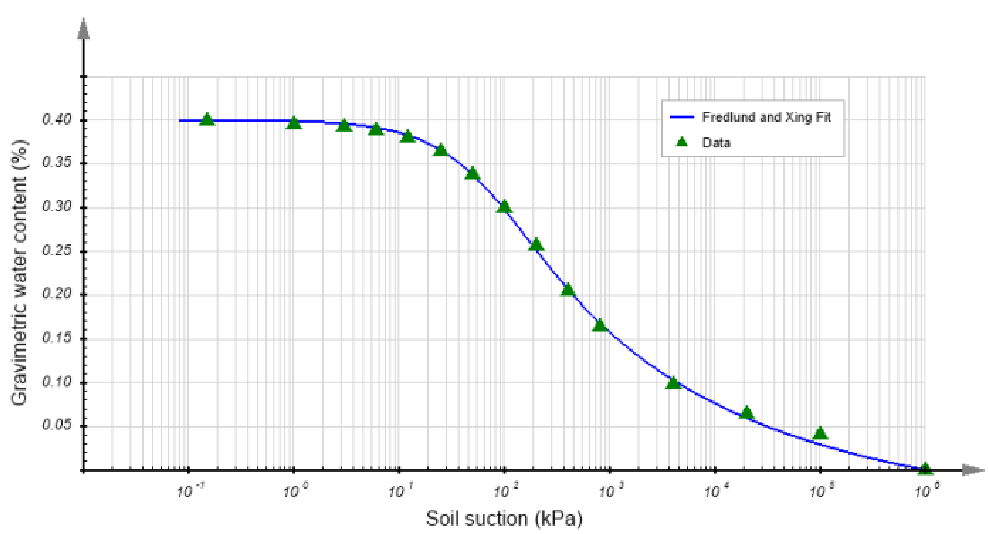

Figure 2. Gravimetric water content versus soil suction for artificial clayey silt

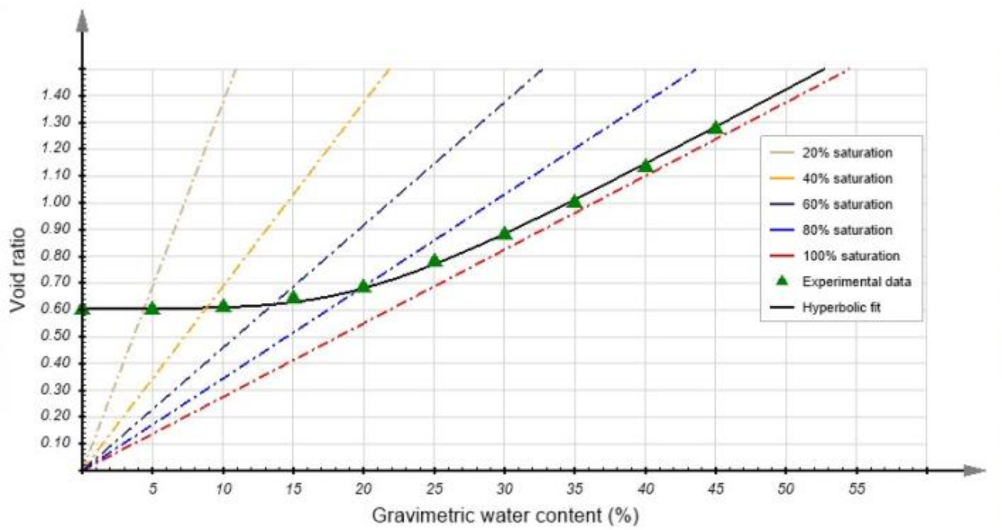

Figure 3. Shrinkage curve for the artificial clayey silt

3 OTHER VOLUME-MASS SWCCS FOR ARTIFICIAL CLAYEY SILT

The independent components defining volume-mass relations can be used to more accurately estimate the unsaturated soil property functions. Each of the commonly used volume-mass variables (i.e., void ratio, volumetric water content, and degree of saturation) can be plotted versus soil suction. The label, SWCC, can be attached to each of the volume-mass variables.

The properties of the artificial clayey silt are such that volume change occurs as soil suction is increased. The components of volume change and degree of saturation change can be separated through use of the shrinkage curve in conjunction with the w-SWCC. This separation of processes is important since volume change and ... different ways.

3.1 Void Ratio SWCC (Clayey Silt)

The initial void ratio of the clayey silt was 1.15. The void ratio corresponding to various gravimetric water contents is calculated from the shrinkage curve. The desorption SWCC is similar to the drying process associated with the shrinkage curve. Consequently, it is possible to plot void ratio versus soil suction, (e-SWCC). Figure 4 shows the amount of volume change that occurs prior to the start of the desaturation process.

3.2 Volumetric Water Content, θ_w (Clayey Silt)

The "instantaneous" volumetric water contents can be calculated based on the w-SWCC and the shrinkage curve, SC, as shown in equation [4].

$$\theta_w(\psi) = \frac{G_s w(\psi)}{1 + e(w(\psi))} \quad [3]$$

where: $w(\psi)$ = gravimetric water content written as a function of soil suction (w-SWCC), and

$e(w(\psi))$ = void ratio written as a function of soil suction, ψ.

The initial volumetric water content is 51.2%. Figure 5 shows that there is a decrease in volumetric water content at the same suction value as shown on the gravimetric water content SWCC (i.e., 5 kPa).

The volumetric water content SWCC (θ_w-SWCC), is used to calculate the water storage function, m_2^w.

3.3 Water Storage Functions

The water storage function, m_2^w, for the artificial clayey silt soil is shown in Figure 6. The water storage function is ... volumetric water content SWCC (θ_w-SWCC) and it is not necessary to separate the effects of volume change from degree of saturation changes when determining the water storage modulus.

3.4 Degree of Saturation versus Soil Suction (Clayey Silt)

The degree of saturation function can be calculated through use of the w-SWCC and the shrinkage curve, SC, while satisfying the following basic volume-mass relationship.

$$S(\psi) = \frac{G_s w(\psi)}{e(w(\psi))} \quad [4]$$

where: $w(\psi)$ = Fredlund-Xing (1994) equation for the w-SWCC Note: any other equation that best-fits the laboratory data can also be used), and

$e(w(\psi))$ = void ratio written as a function of soil suction, ψ.

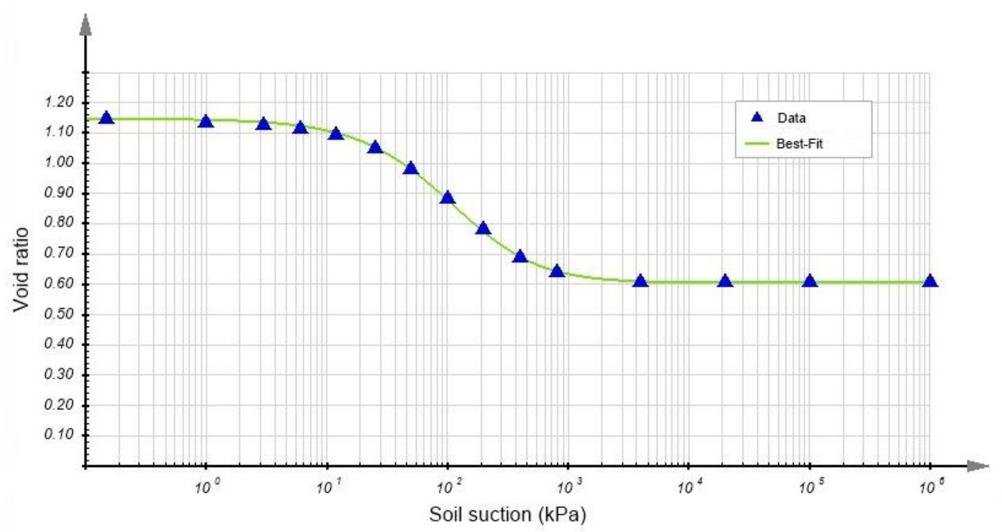

Figure 4. Void ratio SWCC for the artificial Clayey Silt

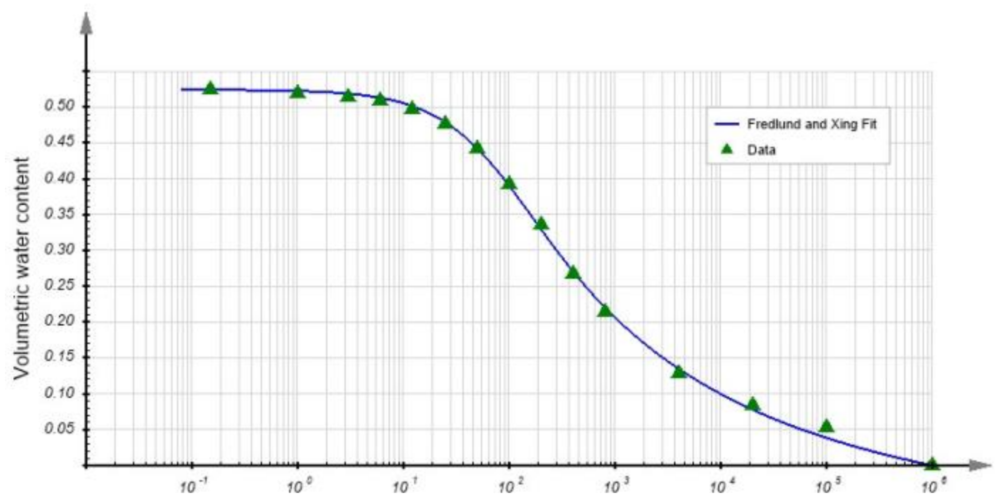

Figure 5. Volumetric water content SWCC for the Artificial Clayey Silt

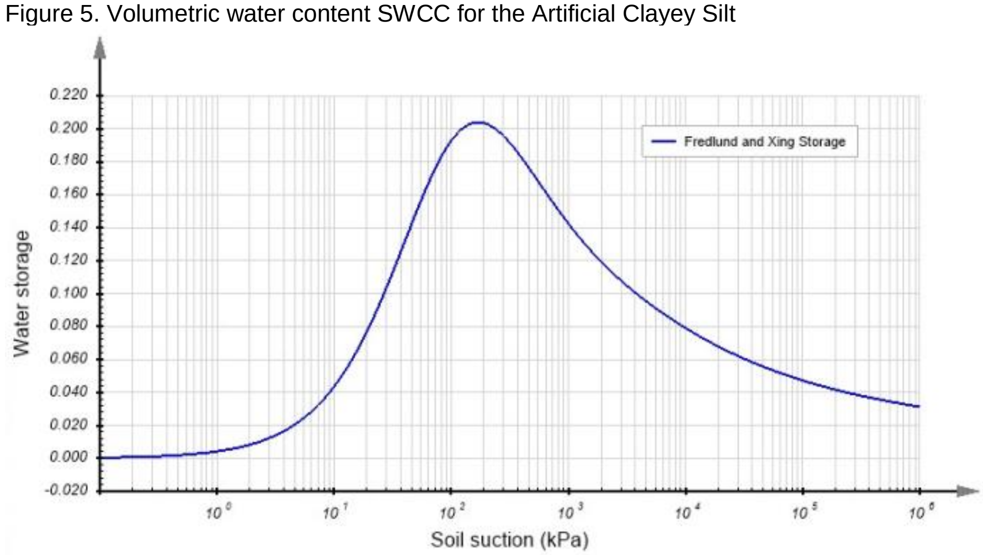

Figure 6. Water storage function for the artificial clayey silt

The "true" air-entry value for the soil is determined using the degree of saturation SWCC. Data points can be generated at regular spacing along the log suction axis of the degree of saturation SWCC. These data points can then be best-fit with the Fredlund-Xing (1994) SWCC equation to yield new fitting parameters for the S-SWCC. The fitted parameters for the clayey silt are: af = 345.9 kPa; nf = 1.605; mf = 0.519 and the residual suction is estimated to be 18,000 kPa. The degree of saturation SWCC (S-SWCC), for the clayey silt is shown in Figure 7.

The S-SWCC can be re-plotted using a transformed suction axis and the construction procedure proposed by Zhang and Fredlund (2015). The inflection point along the transformed degree of saturation plot can be used to compute the "true" air-entry value of the soil. The sole purpose for using the transformed suction scale is to determine the first and second derivative of the S-SWCC equation. These derivatives are used, in turn, to determine the inflection point and then pass a tangential line through the inflection point, extended back to the point of near saturation (i.e., initial degree of saturation).

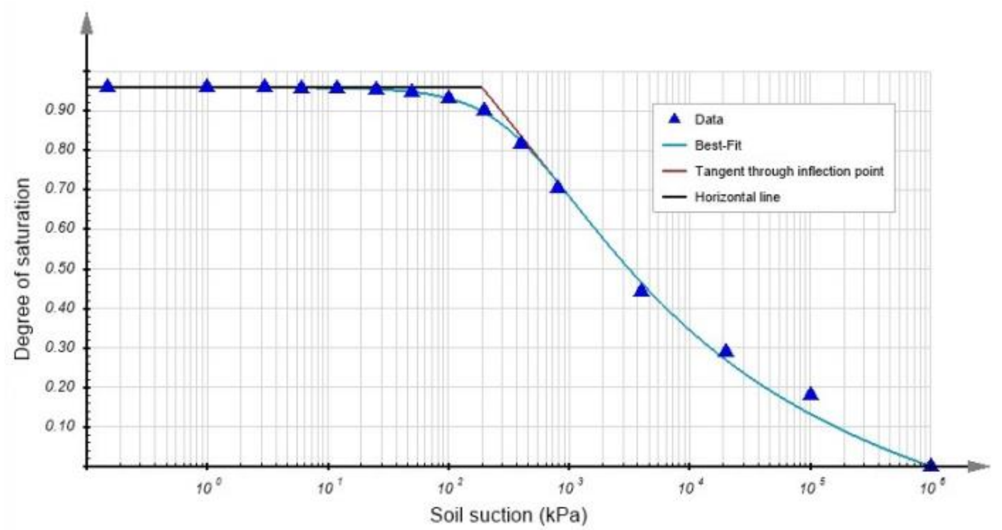

Figure 7. Degree of saturation SWCC for the artificial clayey silt

It should be noted that the downward bend in the degree of saturation SWCC (i.e., the bend away from near saturation), is quite different from that previously observed on the w-SWCC. The air-entry value calculated on the S-SWCC (using the transformed suction scale) produces the correct or "true" air-entry value which in this case is 188 kPa whereas the downward bend observed on the w-SWCC was approximately 15 kPa.

4 PERMEABILITY FUNCTIONS

The water permeability functions are obtained through integration along the degree of saturation SWCC, starting the integration from the "true" air-entry value. The effects of volume change (or void ratio change), can be taken into consideration by using the Kozeny-Carman approximation that assumes that the saturated coefficient of permeability varies according to void ratio, e, cubed divided by 1 plus the void ratio.

4.1 Permeability Function as the Artificial Clayey Silt Desaturates

The Fredlund et al (1994) procedure can be used to calculate the permeability function associated with the desaturation of the clayey silt. Integration of the Kozeny-Carman equation by the Simpson method starts from the "true" air-entry value along the degree of saturation SWCC (S-SWCC) shown in Figure 7. Figure 8 shows the relative permeability function which can be used in conjunction with any starting saturated coefficient of permeability to generate the permeability function for the clayey silt soil.

There are numerous other graphical representations that could be used present the hydraulic properties of a soil that undergoes volume change and desaturation as soil suction is changed; however, space does not permit those presentations in this paper.

5 CONCLUSIONS

The following conclusions have been illustrated with respect to estimating the hydraulic conductivity functions for a soil that undergoes volume change and desaturation as soil suction is increased.

1.) Estimations of the hydraulic conductivity functions for modeling saturated-unsaturated seepage modeling may be calculated based on the laboratory measurement of the gravimetric water content SWCC (w-SWCC) and the shrinkage curve, SC.

2.) Calculations during increasing suction must account for the appropriate volume-mass SWCC, particularly when the soil undergoes volume change as soil suction is increased.

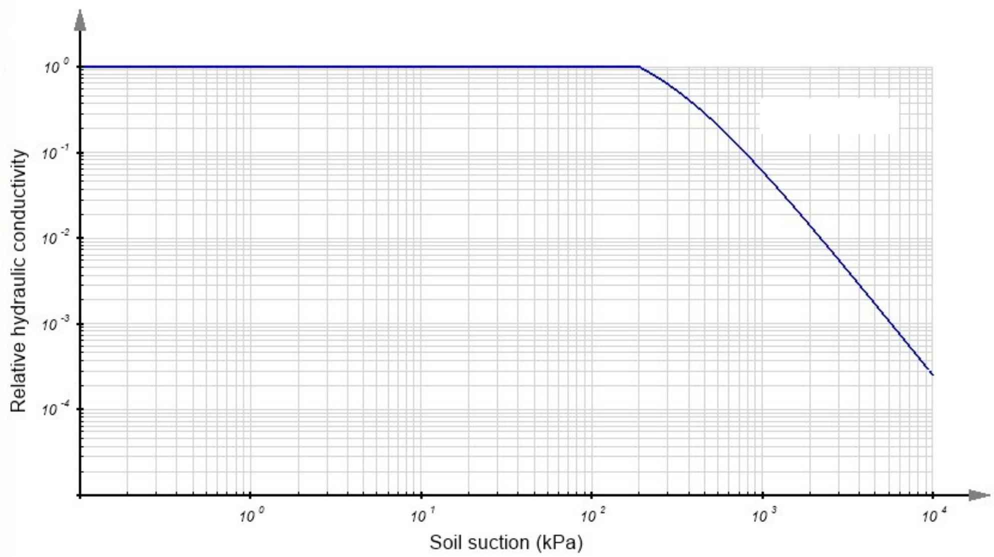

Figure 8. Permeability function for the artificial clayey silt

3.) The "true" air-entry value of the soil can be determined using the degree of saturation SWCC (S-SWCC).

4.) The coefficient of permeability function can be calculated using the degree of saturation soil-water characteristic curve in accordance with the Fredlund et al. (1994) integration procedure, starting from the air-entry value of the soil.

5.) The effects of volume change can be taken into consideration using the void ratio SWCC (e-SWCC).

6.) The water storage function must be calculated using the volumetric water content SWCC (θw-SWCC).

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