

# Extending slope stability analysis to multi-plane 2D and 3D limit equilibrium approaches

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## ABSTRACT

Slope stability numerical modeling, typically by the 2D limit equilibrium method, has become common for geotechnical engineers. There are, however, a number of limitations associated with always using 2D limit equilibrium methodologies. The shape of the assumed slip surface is often not realistic and 3D terrain and stratigraphy are not accurately modeled. The shear stresses on the ends of the 2D cylindrical slip surface are also not considered. There are also practical challenges encountered when applying the method to a large engineering structure due to the difficulty of selecting a "representative" 2D slice to analyze.

This paper examines the extension of 2D limit equilibrium methodologies to other emerging technologies, namely, 2D multi-plane analysis (MPA) and 3D limit equilibrium analysis. The 2D MPA analysis allows for the analysis of many planes at multiple orientations. A rigorous 2D analysis provides the geotechnical engineer with additional information on a geotechnical structure but also raises additional questions on how to properly select analysis orientation and how to interpret methodology output properly. The application of the MPA methodology is examined. The comparison between a 2D and a 3D orientation analysis for such typical use-cases will also be examined. The paper also provides guidelines for the application of 3D slope stability methodologies. Approaches for the successful application of 3D analysis techniques are suggested for engineering practice.

La modélisation numérique de la stabilité de la pente, typiquement selon la méthode d'équilibre limite 2D, est devenue courante pour les ingénieurs géotechniques. Cependant, il existe un certain nombre de limitations associées à l'utilisation constante de méthodologies d'équilibre limite 2D. La forme de la surface de glissement supposée n'est souvent pas réaliste et le relief 3D et la stratigraphie ne sont pas modélisés avec précision. Les contraintes de cisaillement sur les extrémités de la surface de glissement cylindrique 2D ne sont pas prises en considération. Il existe également des difficultés pratiques lors de l'application de la méthode à une grande structure d'ingénierie en raison de la difficulté de sélectionner une tranche 2D "représentative" à analyser.

Cet article examine l'extension des méthodologies d'équilibre limite 2D à d'autres technologies émergentes, à savoir l'analyse multi-plans 2D (MPA) et l'analyse d'équilibre limite 3D. L'analyse MPD 2D permet d'analyser de nombreux plans à plusieurs orientations. Une analyse rigoureuse en 2D fournit à l'ingénieur géotechnique des informations supplémentaires sur une structure géotechnique, mais soulève également des questions supplémentaires sur la façon de sélectionner correctement l'orientation de l'analyse et comment interpréter correctement la sortie de la méthodologie. L'application de la méthodologie MPA est examinée. La comparaison entre une analyse d'orientation 2D et 3D pour de tels cas d'utilisation typiques sera également examinée. Le document fournit également des lignes directrices pour l'application des méthodologies 3D de stabilité des pentes. Des approches pour l'application réussie de techniques d'analyse en 3D sont suggérées pour la pratique d'ingénierie.

## 1 INTRODUCTION/BACKGROUND

Slope stability analyses are commonly performed as part of geotechnical engineering practice. The circular failure arc concept appears to have been first solved using statics by Petterson, (1915) and then later solved using the method of slices by Fellenius, (1936).

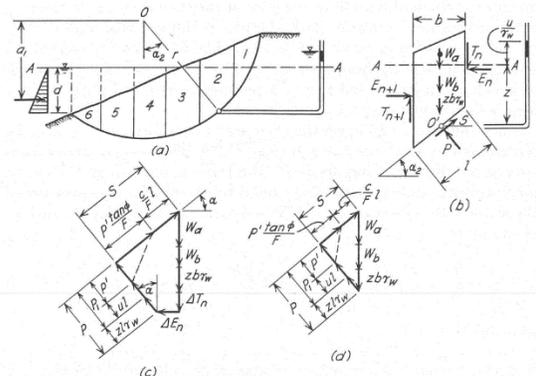


Figure 1. Example of early method of slices force diagrams (After Bishop, 1955)

The concept of solving both force and moment equilibrium was introduced by Bishop (1953). Modern methods solving comprehensive force and moment equilibrium followed (Spencer, 1967; Morgenstern-Price, 1965, Fredlund/GLE-Fredlund and Krahn 1977, Fredlund et al. 1981).

The implementation of the method of slices in various computer codes has resulted in its dominance as a practical method for slope stability analysis in the geotechnical engineering practice. Early mainframe computer codes were difficult to use and have been replaced by sophisticated CAD-based user interfaces which make the design of 2D profile cross-sections easy for users. A limit equilibrium method (LEM) analysis is generally considered to be faster to complete than a comparable finite element-based slope stability analysis.

Geotechnical engineers have become somewhat complacent with respect to using 2D slope stability as it is easy to perform. However, 2D analyses suffers from fundamental limitations foremost of which are; i) the slip shape is most often assumed to be cylindrical, and ii) the slope geometry is assumed to be unchanged in the third dimension. Such assumptions have generally shown 2D analysis to be conservative with respect to 3D factor of safety analyses in amounts between 10 to 100% (Domingos, 2016). If unsaturated aspects of slope stability are taken into consideration then the difference can be even higher (Lulu, 2015). There is an opportunity to refine designs through the application of 3D slope stability analysis.

It should be noted that the difference between a 2D and a calculated 3D factor of safety can be different in each situation. In other words, it is impossible to assume a specific 2D/3D difference for a particular scenario.

One of the issues in geotechnical consulting firm relates to how the 3D site geometry is handled. A geotechnical consulting firm may have been bought by another corporations and in the past the workplace organization may have centered around having a technical department managing the site data on a different software platform. The geotechnical engineer must specify to the data management department that a particular 2D cross-section is required. The results of topography, borehole, and piezometric data are summarized and a 2D cross-section is produced a number of days later. The geotechnical engineer then analyzes the cross-section and may find that an alternative cross-section is desired which may be more critical in terms of slope stability. This is particularly true if there are 3D effects in the foundation stratigraphy or the topology.

A better path forward would be to optimize and integrate; i) the development of the site geometry and ii) the analysis of the slope stability of the site. This paper will present methods of analysis using SVDESIGNER™ and SVSLOPE® in order to illustrate how a more comprehensive analysis can be undertaken by working with a 3D visualization of a site and integrating both 2D and 3D analysis. The application of software tools to analyze slope stability in 3D has traditionally been limited. Recent developments allow for the easy application of 3D

stability analysis for typical problems (Fredlund, 2015; Reyes, 2014).

### 1.1 What are we actually doing in a 2D analysis?

It is helpful to first understand what we are doing with a 2D slope stability analysis. Geotechnical engineers generally conclude that it is conservative and therefore safe to undertake a 2D analysis from a design standpoint. A 2D analysis actually undertakes the modeling of a 3D cylinder of infinite width as shown in Figure 2. In this model it is important to realize that what the modeler is actually representing is the solution of the slipping of a cylindrical slip surface for which the shear resistance on each vertical end is not considered. The anchor considered in this scenario must have its strength reduced because theoretically in the analysis it is represented as a steel plate.

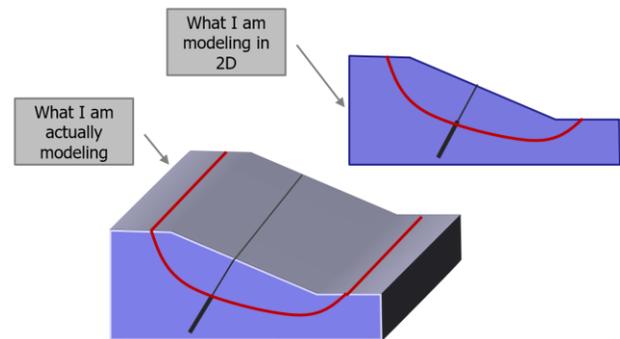


Figure 2. Illustration of a 3D equivalent of 2D limit equilibrium model

It is difficult to solve for the true factor of safety when the topology varies in the 3<sup>rd</sup> dimension. A simple varying topology example is shown in Figure 3 (Jiang et. al., 2003) where a relatively simple slope is presented. It is difficult to calculate the true factor of safety using a 2D plane strain analysis.

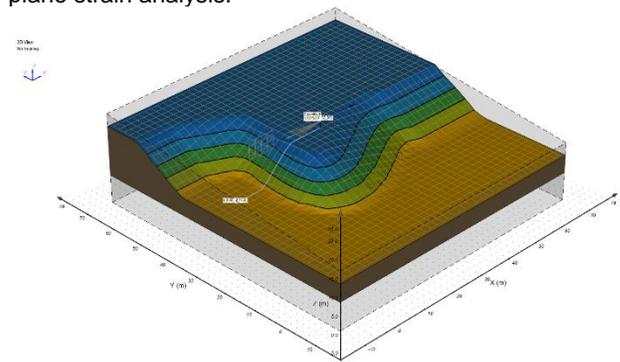


Figure 3. Simple slope that is difficult to analyze for the factor of safety using a plane strain 2D analysis

### 1.2 Is 3D analysis new?

3D slope stability analysis have been studied for several decades. Hovland (1979) was the first to analyze a three-dimensional slope using the method of columns.

The Hovland method is an extension of the assumptions associated with the two-dimensional Ordinary method. That is, all intercolumn forces acting on the sides of the columns are ignored. The normal and shear forces acting at the base of each column are derived as the components of the weight of the column. Hovland (1979) observed that the 3D analysis resulted in a smaller factor of safety than from the 2D analysis for some situations.

Hungr (1987) proposed a method that was an extension of Bishop's (1955) Simplified method to three-dimensional analysis without any additional assumptions involved. The vertical inter-column shear forces acting on both lateral and longitudinal faces of each column were neglected. The results of the Hungr study indicated that the ratio of  $FOS_{3D}/FOS_{2D}$  was greater than 1.0 for all cases. Hungr (1987) suggested that the extended 3D method would be intuitively expected to exhibit similar performance to the original Bishop method for 2D analysis.

Hungr et al. (1989) compared three-dimensional methods that were extensions of the assumptions in Bishop's (1955) simplified and Janbu's simplified two-dimensional methods for a number of solutions. Favourable results from Bishop's method were obtained for rotational and symmetric sliding surfaces. Bishop's simplified method was found to be conservative when used for some nonrotational and asymmetric surfaces, because it neglects internal strength.

Lam and Fredlund (1993) presented the theory and implementation of a more generalized three-dimensional slope stability model extending the two-dimensional general limit equilibrium (GLE) formulation (Fredlund and Krahn 1977, Fredlund et al. 1981). The dominant intercolumn force functions were found for the vertical shear forces acting on each faces of the column. The effect of other force functions was negligible for the studied geometries.

A review of limit equilibrium methods used for three-dimensional slope stability analysis was presented by Kalatehjari and Ali (2013).

- (i) Simplified Bishop method (Bishop 1955)
- (ii) Spencer's method (Spencer 1967)
- (iii) Morgenstern-Price method (Morgenstern and Price 1965)
- (iv) Generalized limit equilibrium method (Fredlund et al. 1981)

### 1.3 How does 3D fit with 1.3 or 1.5 designs?

A question often encountered with respect to 3D analysis is "Can I design a slope to a factor of safety of 1.3 or 1.5 using a 3D analysis?". In order to answer this question it is imperative to first understand the existing geotechnical engineer's thought process on this matter. . When a

geotechnical engineer designs a slope to 1.3 they essentially know that, according to their 2D analysis, they have a 30% over-design. They also know that a 2D analysis is conservative, therefore they have an unknown extra factor of safety,  $F_{3D-2D}$ , that is the difference between a 3D analysis and a 2D analysis. The geotechnical engineer has traditionally relied on this "extra" factor of safety to assist in the design providing confidence without a clear assessment of the magnitude of safety that has been invoked (Figure 4).

The difficulty with this logic is that  $F_{3D-2D}$  has been proven through academic studies to be anywhere between 5% and 60% (Zhang, 2015). Therefore the engineer may, in reality, have a slope that either truly only has a 30% over-design or they may have a slope that is 90%+ over-designed. Either end of the scale has implications that the client should be aware of.

Performing a 3D analysis can be related to determining the ratio of  $F_{3D-2D}$  and being able to make professional decisions based on the obtained information. A 3D analysis is increasingly becoming an essential part of most slope stability studies. In other words, it is suggested that both a 2D and a 3D stability analysis should be undertaken. The reason for performing both a 2D and a 3D analysis is that the slip shape of most slides is always 3D in nature.

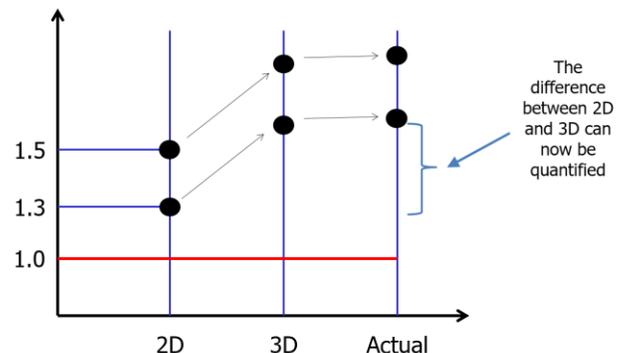


Figure 4. Illustration of the difference between 2D and 3D slope stability analysis

### 3.1 Continuity between 2D and 3D LEM

It is important to know the difference between the slip shape analyzed in a 2D analysis and a 3D analysis. In a 2D analysis the slip shape is assumed to be extended infinitely in the 3<sup>rd</sup> dimension and the shear forces on the end of the slip surface is ignored. An example 2D analysis is presented in Figure 1. The equivalent 3D analysis is shown in Figure 6 where there is no shear strength applied to the vertical ends of the slip surface.

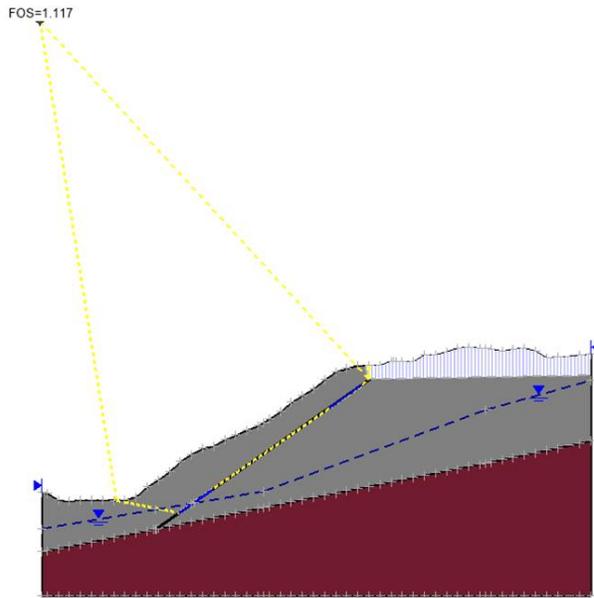


Figure 5. Example 2D stability analysis

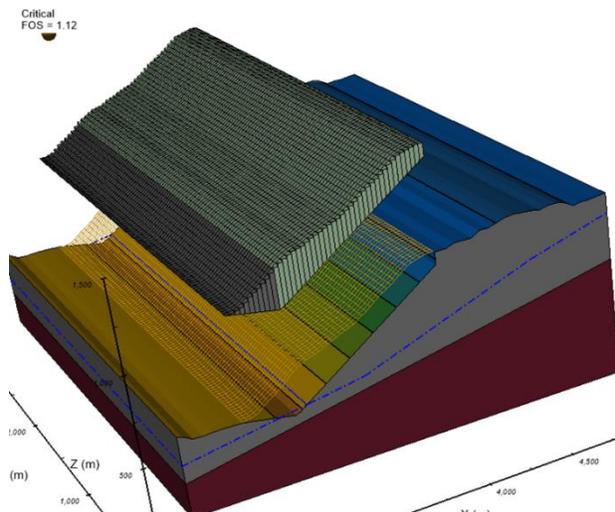


Figure 6. Equivalent 3D analysis (slip surface exploded out of the slope)

A simple test of a 3D analysis can be performed by creating a 3D model of a 2D extruded slip surface and applying zero shear strength to the end-walls of the slip surface. This test can be used to verify the 3D equivalent scenario of any 2D analysis. This test also highlights the fundamental limitation of a 2D analysis which; i) considers the slip surface to be of infinite length in the 3<sup>rd</sup> dimension and, ii) does not consider the influence of shear forces on the end surfaces.

A simple procedure to check for the difference that slip surface shape makes is to analyze a slip surface using a 2D analysis and then, extrude it to 3D space, and perform a 3D analysis using an elliptical slip surface. This simple test provides the geotechnical engineer with the potential increase in the factor of safety brought about simply by

changing a slip surface shape from a barrel with no shear on the sides in a 2D analysis to a 3D elliptical shape analysis.

#### 1.4 What is the suggested slip surface analysis path forward?

There are a number of recommendations that are suggested for transitioning between a 2D and a 3D analysis. The recommendations can be summarized in the following points.

1. **Run both a 2D and 3D analysis:** The difference between a 2D and a 3D analysis is not constant and varies based on a number of factors for each site studied. Therefore, it is recommended that geotechnical engineers run both a 2D and 3D analysis in order that differences can be understood and a proper professional judgement applied.
2. **Extrude 2D models to 3D:** The first and easy step is to extrude 2D models to 3D and understand the differences in factor of safety brought about by the slip surface shape.
3. **Build full 3D models:** The next suggested step is to build full 3D numerical models such that the influence of topology and geo-strata can be determined.
4. **Run finite element model comparisons:** It is relatively easy to run a comparison between the limit equilibrium analysis and a finite element shear strength reduction (SSR) analysis. This exercise ensures that the user has found the most critical slip surface. Models can be transferred to SVSOLID and easily run using the SSR technique (Lu et al., 2014)

The above-mentioned analysis steps are relatively easy to perform using currently available software and increase the analysis information available to the geotechnical engineer for practical engineering decisions.

## 4 3D ANALYSIS

Moving to a 3D analysis from a 2D plane analysis is relatively straight-forward but requires consideration of a few additional aspects. The 3D geometry of the site must first be compiled and developed based on the collected topographic, piezometer, and borehole data. Once the 3D conceptual model of the site is built the focus is on performing a 3D analysis. Such an analysis brings in the following additional points for consideration:

1. **Slip direction must be determined:** The orientation of a potential slip becomes an additional searching parameter and must be determined.
2. **Ellipsoid aspect ratio must be determined:** Many slips are primarily dish-shaped and the 2D circular slip surface converts the shape to a 3D ellipsoid. The ellipsoid aspect ratio must

be evaluated through use of a searching parameter.

3. **Block search extents must be determined:** In the case of a block search, the combination of planes forming a block search must be determined through a multi-plane type of searching mechanism.

Application of these concepts is relatively easy using modern available software and results in expanded possibilities for analyzing complex 3D geometries. The analysis of 3D characteristics significantly extends the ability of the geotechnical engineer to undertake complex engineering designs (Figure 7).

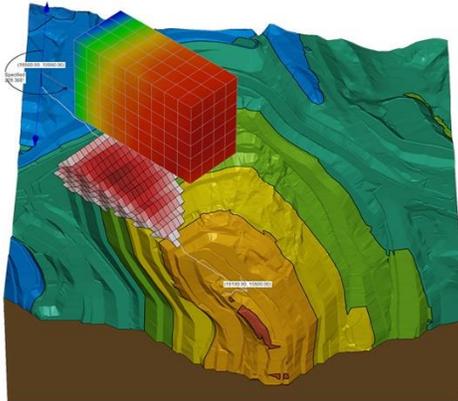


Figure 7. Analysis of an open pit using a block searching method with 'Grid and Tangent' searching

## 5 2D/3D MULTI-PLANE ANALYSIS (MPA)

The analysis of an engineering sites often involves assessing the larger question of the location of the critical slip surfaces that need to be given attention. A multi-plane analysis (MPA) provides the ability to search over large areas in 2D and 3D in order to identify areas of potential risk.

Multi-Plane Analysis consists of three core aspects emulating from a 3D SVSLOPE model of the site:

1. Defining the positions and directions (i.e., planes) of the model to analyze;
2. Performing many concurrent analyses based on the defined planes;
3. Combining the outputs of the analyses into one set of results that is easy to visually interpret.

### 1.5 Defining planes

MPA focuses on rapidly defining and performing many slope stability analyses. Each analysis is geometrically defined by a point, which is required to lie somewhere along the slope of interest, and a direction that specifies the central sliding direction at that position. Each of these 'point and direction' pairings may be thought of as a plane. MPA presents users with several powerful tools for

creating these planes such that coverage of the entire model may be achieved with minimal time and effort.

The most powerful plane definition method requires the user to simply click on a slope of interest. The system then follows the slope laterally in both directions, with planes being created at user-specified intervals until the end of the slope is reached. Each plane created using this process has its direction automatically set to be the dominant downhill direction around that region of the slope. Multiple planes with slightly varying angles may optionally be created through each point. Similarly, another tool allows the user to draw a polyline laterally along a slope in the region of interest, upon which the system will create planes along the line in a manner similar to the previous method. The user may also draw planes on the model manually through graphical or text inputs.

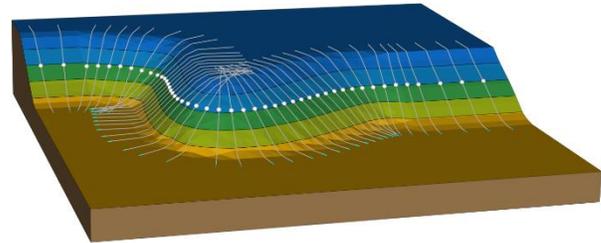


Figure 8. Definition of planes along a curved slope

### 1.6 Performing analysis

For each plane defined, the system creates a new model to be analyzed by the limit equilibrium method. It is possible to perform the analysis in either 3D or 2D. In a 3D analysis, each model contains a copy of the original model geometry using a slip surface trial search method that is automatically specified based on the plane definition. In a 2D analysis, a 2D slice of the original 3D geometry is located and analyzed for each plane. In either case, the new models are analyzed in parallel, making full use of the available computational power of the system hardware.

### 1.7 Visualizing results

After computations are complete, the results of each analysis are collected for visualization in the original 3D model. The factors of safety for the critical slip surfaces in each plane are reported and overlaid spatially. The shape of each slip surface can also be displayed graphically on the model, with various filters being available to limit the number of trials to be displayed. For 3D analysis, the model may be overlaid with a texture representing the minimum factor of safety of all slip surfaces that pass through the ground surface throughout the model.

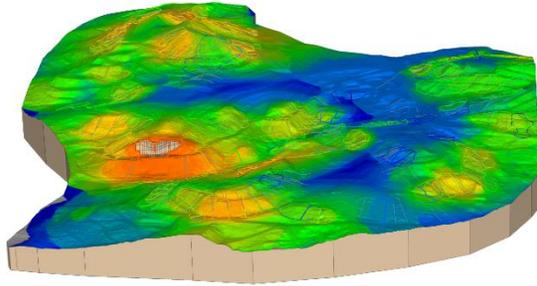


Figure 9. Visualization of 3D analysis results using a factor of safety texture

## 6 APPLICATIONS

In order to illustrate the application of these new techniques to practical engineering scenarios few examples are presented. All examples are based on real-world scenarios encountered by clients of SoilVision Systems Ltd. The examples have been created for illustration purposes only and should not be considered an analysis of any particular real-world site.

### 1.8 Tailings Dam Analysis

The analysis of tailings dams for stability is of particular concern given the recent failures at Mt. Polley in B.C., Canada and Fundao in Brazil. The analysis of a single section of a tailings dam is costly and traditionally involves a complex slope stability 2D analysis of a particular section. Questions arising when using this approach are as follows:

- The geotechnical engineer may not know if the section of concern is the most critical (i.e., weakest) section of the tailings dam.
- A 2D analysis is conservative and may calculate a factor of safety (FOS) that is low (Lulu et. al., 2015).
- The 3D orientation of topology and stratigraphy is not taken into consideration in the traditional analysis.

The advantage of using the MPA approach is that the analysis of the tailings dam is largely automated; therefore making the analysis more comprehensive than a traditional analysis and easier to perform. The suggested approach also provides the engineer with a more comprehensive analytical visualization (Figure 10). Particular zones of low factor of safety may readily be identified and targeted for further detailed 2D or 3D analysis.

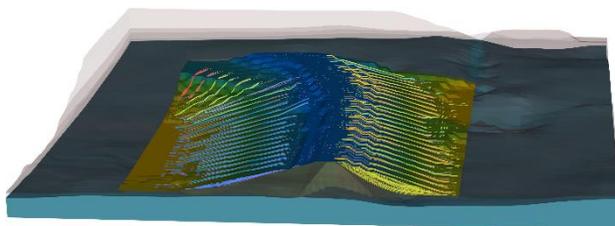


Figure 10. Visualization of 2D MPA analysis of a complex tailings dam

### 1.9 Riverbank Analysis

The analysis of riverbank slope stability remains an item of continued concern in municipal planning. The analysis of a single section of a riverbank is costly and traditionally involves a complex slope stability 2D analysis of a particular section of concern. This approach are the same limitations with respect to other 2D analyses mentioned in this paper.

Using a MPA analysis is particularly well-suited to this type of study since the entire riverbank analysis can be setup and performed in a single step. Aerial photos of the surrounding area can also be overlaid on the system to help identify the locations of potential weakness in relation to municipal structures. An example of a typical riverbank analysis is shown in Figure 11. The geological structure can first be built using conceptual modeling software. Then the material properties and analysis methods can be selected. Building loads, anchors and other features can also be accounted for in the analysis. Zones of potential weakness as a function of distance along the riverbank can be identified. This allows the engineer to obtain a better understanding of the zones that should be used for further detailed analysis.

The MPA can be performed in 2D mode or in 3D mode.

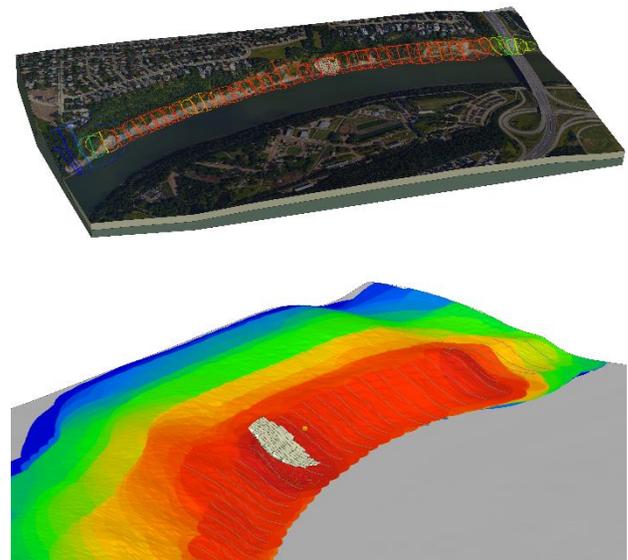


Figure 11. Visualization of 3D MPA analysis of a riverbank

### 1.10 Municipal Stability Analysis

City planning engineers require information related to the slope stability of various areas around potential new developments as well as existing developed areas in order to make appropriate planning decisions. As cities expand there is often pressure to expand developments onto hillsides with ideal views but where slope stability

may be of increased concern. The placement of large structures on the top of hillslopes or the potential leakage from underground pipes can affect the stability of the slopes. There may also be 3D effects in the topology and stratigraphy that need to be taken into consideration.

Analysis of an example location in western United States is shown in Figure 11. It would be potentially time-consuming to analyze the area shown using a traditional 2D slice methodology. Hundreds or thousands of analysis can be set up when using the MPA method and analyzed in a relatively short time. The resulting map of contoured 3D factors of safety can be seen in Figure 11. Such visualization allows city planners to quickly identify zones of potential weakness for further subsequent studies.

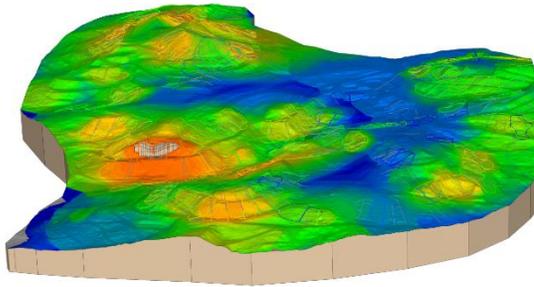


Figure 11. Visualization of 3D MPA analysis of a municipal area

## 2 CONCLUSIONS

Traditional 2D profile analysis is somewhat limited in light of many scenarios encountered in present-day geotechnical engineering practice. New engineering scenarios suggest that a more rigorous approach can be adopted that involves an extension of existing 2D methodologies and allows a reasonable transition to 3D analysis. Methodologies are suggested for the transition to 3D analysis while applying new computational methods to the analysis. The end result gives further insight into the assessment of the stability of slopes on complex real-world sites.

Methods of searching for the proper orientation or shapes of slip surfaces are available and can be applied in a time-efficient manner. Further techniques such as MPA allow the application of 2D and 3D slope stability technologies over larger areas. The application of slope stability analysis to riverbanks, municipal areas, and tailings/earth dams is expedited and advanced through use of the MPA procedures.

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