Remarks on site response analysis by using Plaxis dynamic module

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Introduction

Dynamic FE analyses can be considered the most complete available instrument for the prediction of the seismic response of a geotechnical system, since they can give detailed indication of both the soil stress distribution and deformation. However, they require at least a proper soil constitutive model, an adequate soil characterization by means of in situ and laboratory tests, a proper definition of the seismic input.

This article discusses how to calibrate a finite element model in order to obtain a realistic response of the given system subjected to seismic loading. Plaxis 2D v.8.2 (Brinkgreve, 2002) that includes the dynamic module was used in this research. A series of dynamic analyses of vertical propagation of S-waves in a homogeneous elastic layer was carried out. This scheme was chosen because a theoretical solution of the problem is available in literature and some comparisons can be easily done. The influences on the response of boundaries conditions, mesh dimensions, input signal filtering and damping parameters was investigated.

The information obtained in this preliminary calibration process can be used thereafter for the analysis of any geotechnical system subjected to seismic loadings.

Figure 1. Amplification function of an elastic layer over rigid bedrock

PHYSICAL PROPERTIES			STIFFNESS PARAMETERS					
γ	*RAYLEIGH DAMPING*		*E*	*υ*	*G*	*Eoed*	*V_S*	*V_P*
[kN/m³]	α	β	*[kN/m²]*		*[kN/m²]*	*[kN/m²]*	*[m/s]*	*[m/s]*
141	-	-	4.889x10⁵	3	1.88x10⁵	6.581x10⁵	3615	6763

Table 1. Material properties of the Linear Elastic layer

Reference theoretical solution

Vertical one-dimensional propagation of shear waves in a visco-elastic homogeneous layer that lies on rigid bedrock can be described in the frequency domain by its amplification function. The latter is defined as the modulus of the transfer function that is the ratio of the Fourier spectrum of the free surface motion to the corresponding component of the bedrock motion. Therefore, for a given visco-elastic stratum and a given seismic motion acting at the rigid bedrock the motion at the free surface can be easily obtained. First, the Fourier spectrum of the input signal is computed. Then, this function is multiplied by the amplification function and after that the motion is given by the inverse Fourier transform of the previous product.

If the properties of the medium (density, ρ or total unit weight of soil, γ; shear wave velocity, V_s; material damping, D) and its geometry (layer thickness, H) are known, the amplification function is uniquely defined.

For a soil layer on rigid bedrock with the following parameters:

H = 16 m; γ = 14.1 kN/m³; ρ = 1.44 kg/m³; V_S = 361.5 m/s; D = 2 %

the amplification function (Roesset, 1970) is:

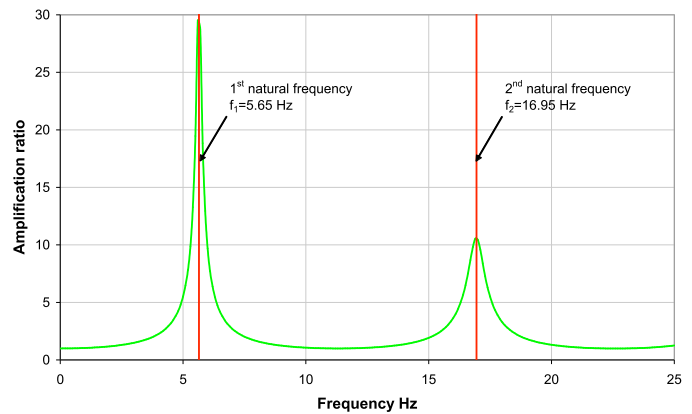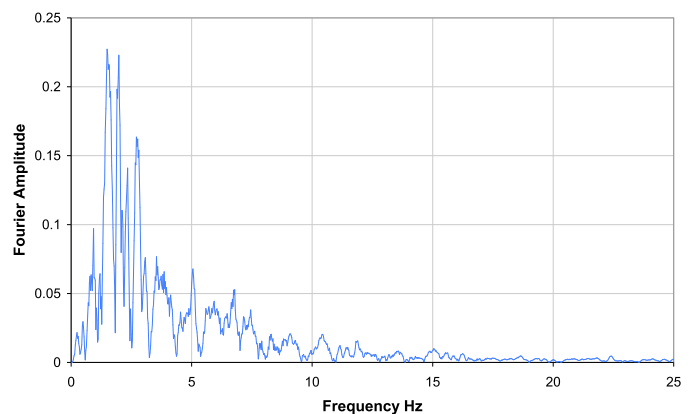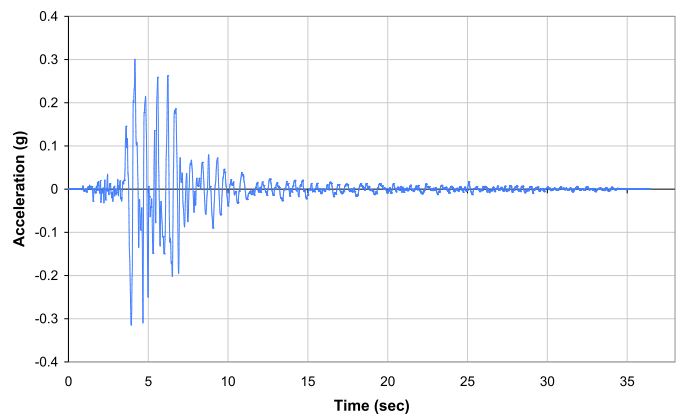$$A(f) = \frac{1}{\sqrt{\cos^2\left(2\pi \dfrac{H}{V_s} f\right) + \left(2\pi \dfrac{HD}{V_s} f\right)^2}}$$

Figure 2. Seismic input: a) accelerations time-history; b) Fourier spectrum

Figure 1 shows its graphical representation in the amplification ratio-frequency plane. Here, and in the following similar figures, two vertical red lines indicate the first and the second natural frequency of the system. In the previous indicated hypotheses, the nth natural frequencies f_n of the layer are:

$$f_n = \frac{\omega_n}{2\pi} = \frac{V_S}{4H}(2n + 1)$$

Numerical modeling

3.1 Input signal

In numerical computation, the earthquake loading was often imposed as an acceleration time-history at the base of the model.

Here, the input signal chosen for numerical analyses is the accelerometer registration of Tolmezzo Station (Friuli Earthquake, Italy, May 6th, 1976). The sampling frequency is 200 Hz, the duration is 36.39 s and the peak acceleration is 0.315 g.

Accelerations time-history and Fourier Spectrum of the signal are reported in Figure 2.

3.2 Finite element model

The finite element model is plotted in Figure 3. It is constituted by a rectangular domain 80 m wide and 16 m high and two additional similar lateral domains, in order to place far enough the lateral boundaries (total width 240 m). This should help minimizing the influence of the boundaries on the obtained results, even though no clear indications exist in literature on this aspect. Recently, Amorosi et al. (2007) have shown a case of site response analysis in which they have extended the width of the mesh eight times its height, in order to obtain acceptable results.

The medium is schematized as a Linear Elastic layer that is implemented in the Plaxis code. Its parameters are indicated in Table 1.

The initial stress generation was obtained by the k_0-procedure in which the value of the earth pressure at rest, k_0 was chosen by means of the well-known formula for the elastic medium:

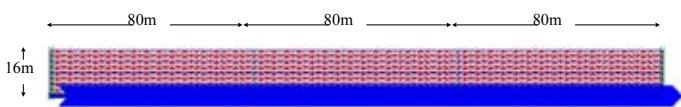$$k_0 = \frac{\upsilon}{1 - \upsilon} = 0.429$$

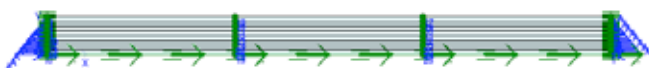Figure 3. Finite element model utilized in the dynamic analyses

Figure 4. Free Horizontal Displacements (FHD) condition on lateral boundaries of FE Model

The mesh generation in Plaxis is fully automatic and based on a robust triangulation procedure, which results in an "unstructured" mesh. In the meshes used in the present analyses, the basic type of element is the 15-node triangular element. The dimensions of any triangle can be controlled by local element size. By subdividing the homogeneous layer in sub-layers with a fixed thickness and by using the local element size, it is possible to assign to the triangles a maximum size.

An average dimension that is representative for refinement degree of the mesh is the "Average Element Size" (AES) that represents an average length of the side of the elements employed.

Every time a numerical analysis is performed, the mesh influence must be tested. Kuhlmeyer & Lysmer (1973) suggested to assume a size of element not larger than $\lambda/8$, where λ is the wavelength corresponding to the maximum frequency f of interest. In this case $\lambda/8 = V_S/8 f = 1.81$ m, being $V_S = 361.5$ m/s and f = 25 Hz. In the analyses of the present work an AES=1.58 m was used.

Results of dynamic analyses

Christian et al. (1977) have shown that the right lateral boundaries conditions for S-waves polarized in horizontal plane and propagating vertically are the vertical fixities. Horizontal displacements must be allowed. In order to equilibrate the horizontal litho static stresses acting on lateral boundaries, it is suitable to introduce load distributions at the left-hand and right-hand vertical boundaries. In this manner, the amplification function of all points placed on the free surface of the model is the same. Figure 4 plots the graphical lateral boundaries condition utilized in Plaxis.

The use of such boundary conditions instead of adopting lateral dampers as suggested by Kuhlmeyer & Lysmer (1973) permits to calibrate the damping parameters of the system with more accuracy.

In numerical calculations two types of damping exist: numerical damping, due to finite element formulation, and material damping, due to viscous properties, friction and development of plasticity.

In Plaxis (and in most dynamic FE codes), the material damping is simulated with the well-known Rayleigh formulation. The damping matrix C is assumed to be proportional to mass matrix M and stiffness matrix K by means two coefficients, α_R and β_R according to:

$$C = \alpha_R M + \beta_R K$$

Different criteria exist to evaluate the Rayleigh coefficients (see for instance Lanzo et al., 2004; Park & Hashash, 2004; Amorosi et al., 2007). In terms of frequency, the dynamic response of a system is affected by the choice of these parameters to a large extent.

In the numerical implementation of dynamic problems, the formulation of the time integration constitutes an important factor for stability and accuracy of the calculation process. Explicit and implicit integration are two commonly used time integration schemes. In Plaxis, the Newmark type implicit time integration scheme is implemented. With this method, the displacement and the velocity at the point in time t+Δt are expressed respectively as:

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$$u^{t+\Delta t} = u^t + \dot{u}^t\,\Delta t + \left[\left(\frac{1}{2} - \alpha_N\right)\ddot{u}^t + \alpha_N\ddot{u}^{t+\Delta t}\right]\Delta t^2$$

$$\dot{u}^{t+\Delta t} = \dot{u}^t + \left[(1-\beta_N)\,\ddot{u}^t + \beta_N\ddot{u}^{t+\Delta t}\right]\Delta t$$

The coefficients α_N and β_N, which should not be confused with Rayleigh coefficients, determine the accuracy of numerical time integration. For determining these parameters, different suggestions are proposed, too. Typical values are (Barrios et al., 2005):

a) $\alpha_N = 1/6$ and $\beta_N = 1/2$, which lead to a linear acceleration approximation (conditionally stable scheme);

b) $\alpha_N = 1/4$ and $\beta_N = 1/2$, which lead to a constant average acceleration (unconditionally stable scheme);

c) $\alpha_N = 1/12$ and $\beta_N = 1/2$, the Fox-Goodwin method, which is fourth order accurate (conditionally stable scheme);

In order to keep a second order accurate scheme and to introduce numerical dissipation, a modification of the initial Newmark scheme was proposed by Hilber et al. (LUSAS, 2000), introducing a new parameter γ (α in the notation of the author), which is a numerical dissipation parameter. The original Newmark scheme becomes the α-method or Newmark HHT modification. The α-method leads to an unconditionally stable integration time scheme and the new Newmark parameters are expressed as a function of the parameter γ, according to:

$$\alpha_N = \frac{(1+\gamma)^2}{4} \qquad \beta_N = \frac{1}{2} + \gamma$$

where the value of γ belongs to the interval [0, 1/3]. By assuming $\gamma=0$ the modified Newmark methods coincides with the original Newmark method with constant average acceleration.

Moreover, in order to obtain a stable solution, the following condition must apply in the Plaxis code:

$$\alpha_N \geq \frac{1}{4}\left(\frac{1}{2} + \beta_N\right)^2$$

Neither the linear acceleration approximation or the Fox-Goodwin method does meet such requirement.

If no damping, material and/or numerical, is introduced in a dynamic analysis, the model reaches the resonant conditions at the natural frequencies of the system with a corresponding theoretically infinite amplification ratio. Figure 5 shows the response at a control point on the free-surface obtained for an undamped analysis ($\alpha_N = \beta_N = \alpha_R = \beta_R = 0$) in terms of the acceleration time-history and the Fourier spectrum as a result of the input signal shown in figure 2. The numerical results are very close to the expected theoretical values.
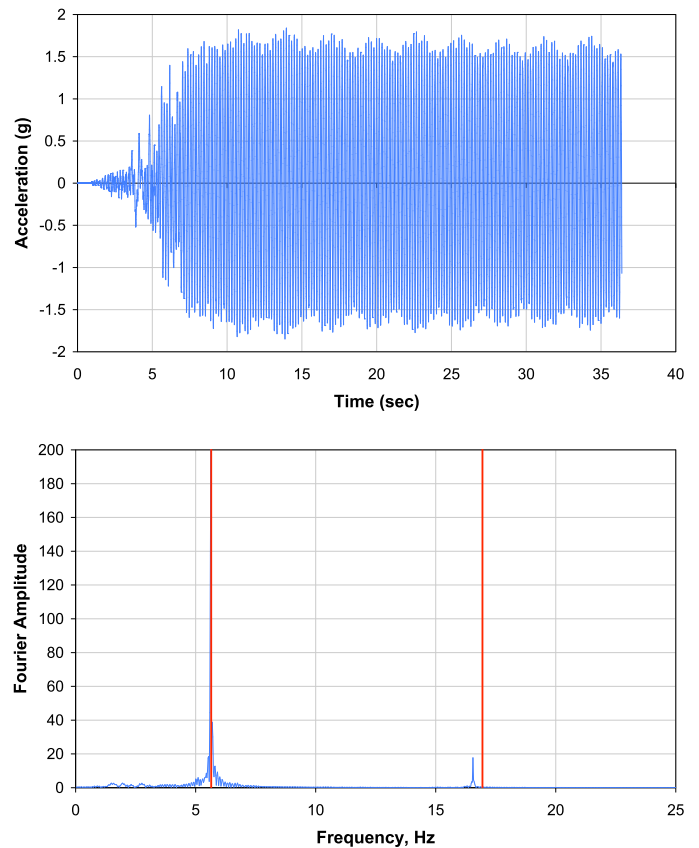

Figure 5. Signal at surface of undamped analysis: a) accelerations time-history; b) Fourier Spectrum
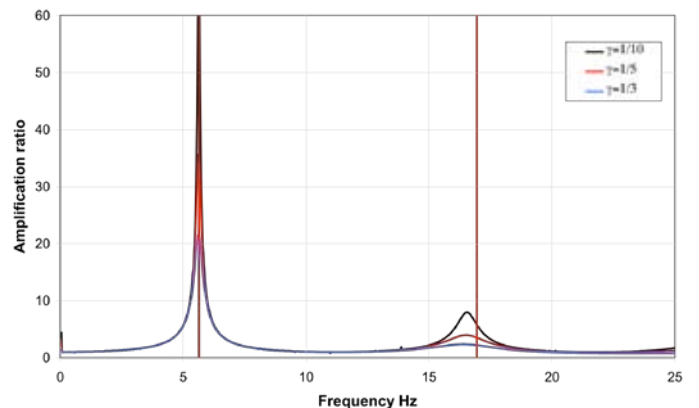

Figure 6. Influence of Newmark numerical damping coefficients on amplification function of the model

The standard setting of Plaxis is the damped Newmark scheme with $\alpha_N = 0.3025$ and $\beta_N = 0.6$, that correspond to $\gamma = 0.1$.

Figure 6 explains the results of numerical analyses for three different values of γ. Rayleigh coefficients were put equal to zero. When γ increases, the peaks amplification at the natural frequencies of the layer decrease. However, the shape of amplification function is not essentially modified. The numerical damping coefficients chosen by default in Plaxis (black curve in Figure 6) conduct to an amplification ratio (A=7.97 at f=16.55 Hz) smaller than the theoretical one (A=10.54 at f =16.95 Hz, see Figure 1) in correspondence to the second natural frequency of the layer.

Note also that the value of second natural frequency of the stratum is underestimated by the time domain analyses. This is due to the finite element formulation with lumped masses instead of consistent mass matrices (Roesset, 1977). The natural frequencies with a lumped masses formulation, which is implemented in Plaxis, are always smaller than the true frequencies. Consistent mass matrices overestimate them. The accuracy of the results decreases with the number of vibration modes.

Numerical damping has a great influence on the dynamic response of a geotechnical system and this issue should be particularly considered when an earthquake signal needs to be preliminarily processed. In fact, to reduce the calculation time, filtered signals at the frequency of interest (i.e., accelerograms with a reduced number of registration points) are often used for the input motion. In this case, users should be aware that the analysis needs an adequate calibration of Newmark coefficients, in such a manner to avoid the loss of important frequency contents of the signal. A comparison of the system response to a complete signal and a 25 Hz filtered signal is represented in Figure 7.

Figure 8 shows the different amplification functions for three values of Rayleigh damping coefficient α_R. The coefficient β_R is given equal to zero for avoiding excessive damping of the motion at high frequencies. The results are referred to a numerical damping of $\gamma = 0.055$. This value has been worked out to obtain a good agreement between numerical and theoretical values of the amplification ratio that correspond to the second natural frequency of the layer as shown in Fig. 10.

The solution with free horizontal displacements (FHD) on lateral boundaries is only reasonable for non-plastic material and when local site response is the objective of the study. If a 2-D configuration of the problem should be examined, horizontal fixities on the left and on the right hand of the model must to be applied. In these conditions, silent boundaries are often used to simulate infinite media.

Different methods exist to apply a silent boundary (Ross, 2004). In Plaxis, viscous adsorbent boundaries can be introduced, which are based on the method described by Lysmer & Kuhlmeyer (1969). By default, relaxation coefficients c_1 and c_2 are set to 1.0 and 0.25, respectively.

By placing the lateral boundaries sufficiently far from the central zone, the effects due to the reflection of waves on boundaries can be neglected.

A comparison of the results with Standard Earthquake Boundaries SEB (Fig. 3) and Free Horizontal Displacements FHD (Fig. 4) on lateral boundaries is presented in Figure 10, by using default values for c_1 and c_2. It seems to suggest that better results are obtained by using FHD rather than SEB.
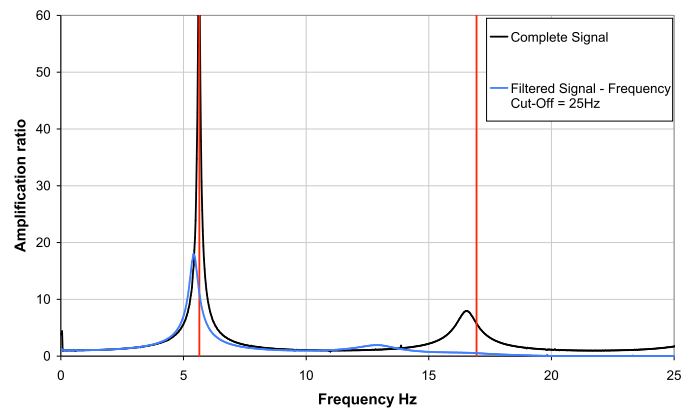

Figure 7. Influence of input signal filtering on amplification function of the model ($\gamma = 0.1$)
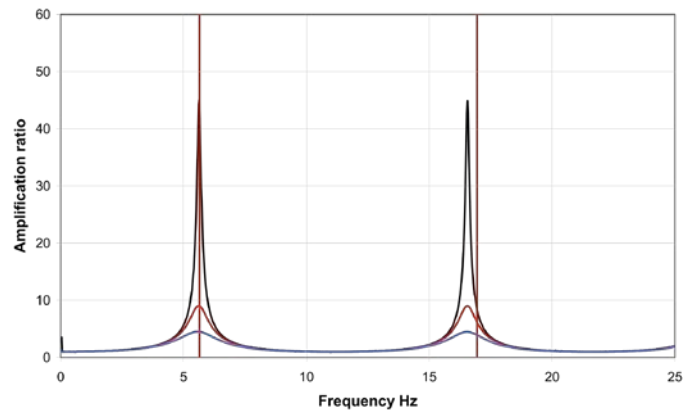

Figure 8. Influence of Rayleigh material damping coefficients on amplification function of the model ($\gamma = 0.055$)
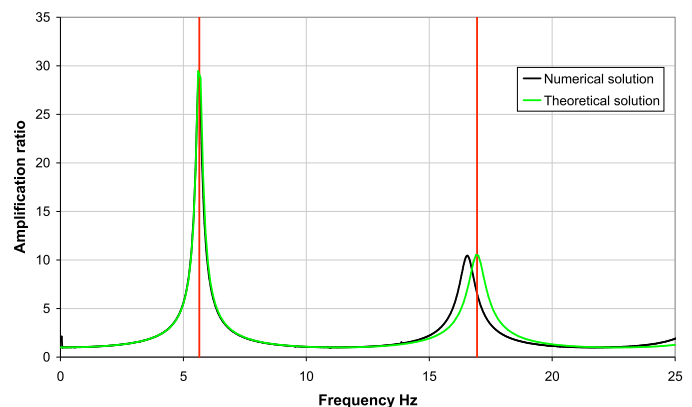

Figure 9. Comparison between numerical and theoretical solution for D=2%

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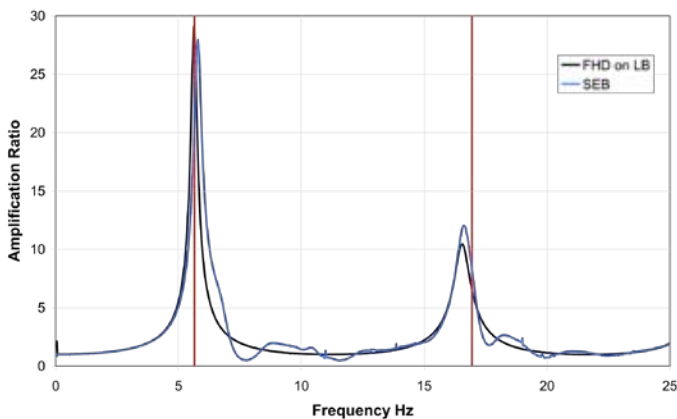
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Figure 10. Comparison between SEB and FHD on lateral boundaries solutions

Conclusions

The use of dynamic analyses to calculate the seismic response of a geotechnical system is dependent on advanced site characterization and numerical knowledge.

It is necessary a good calibration of the numerical model before conducting a dynamic analysis for any type of 2-D problem. Some parameters (equivalent stiffness, numerical and material damping, etc.) can be chosen by comparing the dynamic response of model under vertical shear waves propagation to the theoretical solutions. In the present article, an example of procedure to calibrate the finite element model parameters has been presented in order to control the system damping.

Material damping is often modelled by Rayleigh formulation. Moreover, numerical damping is also needed in order to attain a stable calculation. This leads to some difficulty to control the actual damping of the numerical model. A possible choice in order to limit such uncertainty is to set the minimum value for Newmark γ which allows stability, then fit the theoretical solution. This can be achieved by assuming Rayleigh $\beta=0$ and changing Rayleigh α only, in order to model the material damping with reasonable approximation in the desired range of frequencies. The best-fit criterion can be, for instance, reproducing the amplification of the seismic signal over the first and second natural frequency of the system. Modelling lateral boundaries and filtering input signal need to be carefully considered when performing such calibration.

The proposed approach was preliminarily used for the analysis of some geotechnical earthquake problems as the seismic response of flexible earth retaining structures (Visone & Santucci de Magistris, 2007) and the transverse section of a circular tunnel in soft ground (Bilotta et al., 2007).

Acknowledgments

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