On Stability Analysis of Slurry–Wall Trenches

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1. Introduction

The Plaxis users at Wroclaw University focus on soil–structure interaction research, which also covers vertical excavations supported by either steel or reinforced–concrete retaining walls. The wall-construction process uses deep vertical trenches that are filled up with a bentonite suspension (Xanthakos, Hanjal). Displacement and stability analyses of the anchored walls belong to standard calculations and they are reported in many places, including the Plaxis Bulletin. In contrast, the stability analysis of the tentative trench itself, supported by the bentonite liquid, is less popular. Therefore, these aspects are the objective of this article.

The technological phase of a bentonite supported trench is – to a certain degree – a critical moment in the construction process. This is so, because the next phase, i.e. the successive replacement of the bentonite suspension with the fresh concrete, improves the stability, due to an increase of the stabilizing horizontal pressure applied to trench faces.

Geotechnical engineers have coped with the trench–stability problems for years using simple design methods (Piaskowski, Morgenstern, Washbourne, Fox, Tsai, Ng) or recently FEM–supported calculations Ng (Oblozinski). However, some questions still remain open. First of all, the slurry–wall trenches consist of sections L×B×H (say, the length L ~ 2÷8m, width B ~ 0.6÷1.2m, depth H ~ 10÷15m or more), so a true 3D stability analysis is required. Indeed, it is a well–established fact that the horizontal ground pressure is usually much less then the 2D active earth pressure yielding from the Coulomb theory. Some authors explain this behaviour making use of the silo–pressure analogy, recalling the Janssen–Terzaghi solution. Other approaches make use of more or less sophisticated limit equilibrium methods and there exists a great variety of sliding wedges of soil mass taken arbitrarily by many authors.

Clearly, layered soils can be analyzed only with difficulty within the limit equilibrium calculations. The same is true for local loads distributed on the ground surface in the trench vicinity. Eventually, no prediction of the ground surface deformation is possible if using statically determinate calculation methods. The advantages of FEM modelling become obvious here.

We used Plaxis 3D Foundation to test a very simply design method. In this context, the simplest elastic–plastic Mohr–Coulomb model seems to be relevant.

2. Deterministic methods

Stability evaluations of slurry–supported trenches use generally 3D models in two versions which are based on:

- the force equilibrium for the sliding soil mass (wedge),
- simulations of developing displacements of one (or a few) points selected on the trench face.

2.1. Limit equilibrium methods

As a first approximation within the limit equilibrium analysis, the 2D solution for triangular wedge and the infinite trench length can be applied [Nash and Johns], in particular using the Coulomb critical angle of sliding $\theta_{cr} = \pi/4 + \phi/2$, Fig.1a. This way, the earth pressure is overestimated and more realistic shapes of the wedge are of interest, Fig.1c–e.

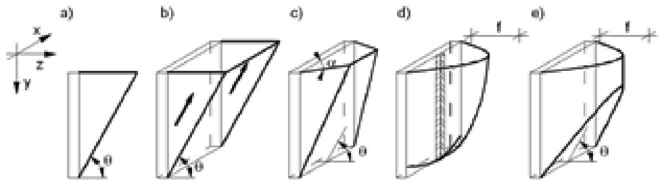

Figure 1: Shapes of the sliding wedges studied by: a) Nash and Johns (2D); b) Morgernstern and Amir–Tahmasseb; c) Washbourne; d) Tsai and Chang; e) Piaskowski and Kowalewski.

The simplest transition from 2D to 3D solution in Fig.1b bases on taking into account shear forces on all sides of the sliding wedge (Morgernstern and Amir–Tahmasseb). Washbourne modified the shape of rigid block assuming the angle $\alpha = \pi/4+\phi/2$ between slide surface and face of the trench, Fig.1c. FEM simulations made by the authors indicate that such a value of the angle α seems to be underestimated.

The latest 3D solutions by Tsai and Chang employ more realistic – smooth and convex – shear surface. The method uses vertical columns as a generalization of standard 2D slices. The Piaskowski and Kowalewski solution, proposed as early as in the mid-sixties, uses a vertical elliptic cylinder cut by a critical plane. The approach has a profound justification in terms of elliptic compression arches observed in rock mechanics (though in vertical planes, not the horizontal one).

From our experience and many tests performed, we could recommend the situation presented in Fig.1b which reconciles simplicity and accuracy.

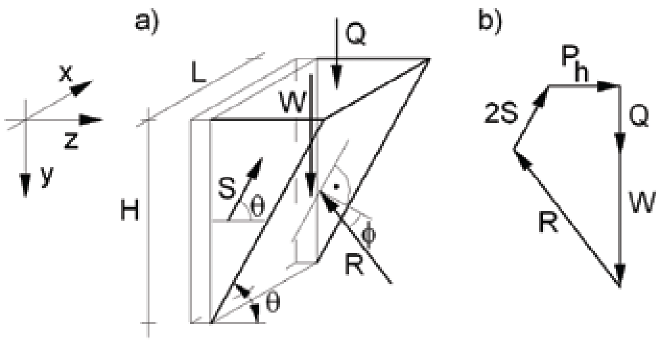

Figure 2a: The 3D-view of the sliding block; b) the polygon of acting forces in the plane of symmetry.

Introduce the acting forces [kN]: W – bulk effective weight of a wedge, R, S – soil reactions, Q active load in line of symmetry (Q = 0 hereafter), P_s – hydrostatic horizontal slurry pressure on the vertical face L × H of the trench, P_h – hydrostatic horizontal ground-water pressure on the face L × (H-h_w) of the trench. Note that the slurry table is kept on the ground level and the water table is situated h_w meters under the ground

level. Both P_s and P_w do not depend on the angle θ which is to be found. The reaction S is calculated by integrating horizontal stresses over the triangle and the horizontal stresses are, by assumption, proportional to effective vertical ones. Testing calculations with Plaxis 3D Foundation did not confirm large values of such coefficient of lateral pressure K which could be expected due to arching effects. The values situated between K_a and K_o were generally observed, so $K = K_a$ can be assumed as a safe approximation, $K_a = tg^2(\pi/4+\phi/2)$.

For simplification, it is also assumed that there is no hydraulic contact between ground water and the slurry - no filtration is considered. To be more realistic, such contacts occur in noncohesive soils but they are of a specific character. The filtration of slurry suspension takes place towards the soil mass thus increases safety margins. It is also reported (Elson, Filz), that the penetration of the slurry suspension has a very limited scale and a skin-contact colmatation is observed – called "filter cake". Such a behavior is not obvious in coarse-grain soils.
The governing equations for cohesionless soils follow the standard Coulomb approach with the discussed modifications, Fig.2:

$$\begin{cases} \sum F_z = 0 \\ \sum F_y = 0 \\ R_z = R_y \cdot tg(\theta - \phi) \end{cases} \Rightarrow \begin{cases} P_h = R_z - 2 \cdot S_z \\ W = R_y + 2 \cdot S_y - Q \\ R_z = R_y \cdot tg(\theta - \phi) \end{cases} \qquad (1)$$

Critical failure plane θ_{cr} can be found such that it maximizes the value of P_h.
Note that in 3D, for realistic values of L/H, the critical angles θ_{cr} are usually some 10% greater than $\pi/4+\phi/2$. Such a behaviour is governed by the stabilizing forces S applied to the lateral triangular surfaces. Clearly, the critical angles θ_c tend to $\pi/4+\phi/2$ for L>>H, i.e. if the relative contribution of the forces S becomes small.

The limit equilibrium in terms of horizontal forces can be expressed as $P_s - P_h - P_w = 0$ thus also as $FS_1 = P_s/(P_h + P_w) = 1$ or as $FS_2 = (P_s - P_w)/P_h = 1$. Due to a lack of uniqueness ($FS_1 \neq FS_2$ for $FS_i > 1$, i=1,2), and bearing in mind a comparison of results with Plaxis calculations, the authors define factor of safety FS in the standard way:

$$FS = \frac{\tan \phi}{\tan \phi_{red}} \qquad (2)$$

where the limit equilibrium $FS_i = 1$ must be reached for ϕ_{red}.
Clearly, the factor of safety has a global character, as the one using resultant forces, so it can be less useful when a local loss of stability can happen.

Example 1.
Consider the depth of the trench H = 10m and the water table which can change: h_w = 1m, 2m, 3m, respectively. The material parameters are presented in Table 1.

	γ	γ'	K_a	ϕ	c
	kN/m³	kN/m³	–	°	kPa
Fine sand	18.5	9.0	0.31	32.0	0

Table 1: Parameters of a homogeneous soil used in (1),(2).

The results in Fig.3a confirm that short sections of the trench are more safe. Therefore, the static analysis in direction perpendicular to the trench width B is out of considerations.

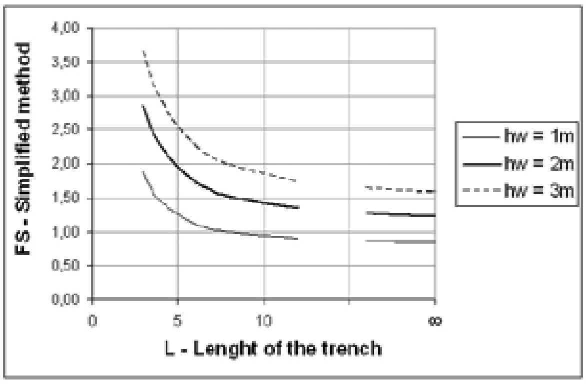

Figure 3a: Plots of FS versus section length L (symbol ∞ stands for the 2D case), H = 10m.

The role of the slurry density can be presented as follows.

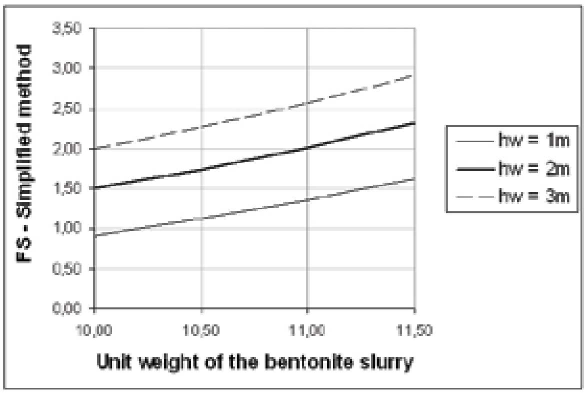

Figure 3b: Plots of FS versus slurry unit weight for L = 6m, H = 10m.

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2.2. The FEM–based testing using Plaxis 3DFoundation

The trench dimensions are 6×1×10m (L×B×H) but two axes of symmetry reduce it to a quarter 3×0.5×10m. The soil spreads within a bounded block 12×14×15m which vertical boundaries are fixed for horizontal displacements.

Excavation process was performed by successive removing 1m-thick ground layers at each calculation phase. Also at each phase, the slurry pressure was increased by application of external loads on trench faces (linearly increasing with depths, starting from the ground level) as well as on the bottom of the trench. The slurry unit weight was 10.5kN/m^3. For the water table h_w = 2m was assumed.

The standard ϕ-c reduction technique was used to determine values of the factor of safety FS thus the methodology coincides with the one presented by the expression (2).

The material parameters are as follows.

	γ	γ'	K_0	ϕ	c	ψ	E	ν
	kN/m^3	kN/m^3	–	°	kPa	°	MPa	–
Fine sand	18.5	9.0	0.47	32.0	0	0	70.0	0.25

Table 2: Parameters of a homogeneous soil analyzed by Plaxis

Example 2.

When the values of FS start to stabilize during the reduction of ϕ, the maximal 3D displacements are close to 20mm (Fig.4a), on the axis of symmetry the sliding wedge develops almost linearly, the angle Θ_{cr} is close to $\pi/4+\phi/2$ and the sliding wedge is relatively large. For engineering purposes, most of the 3D models presented in Fig.1 can be used to model the shape of the wedge.

Clearly, some settlements far from the trench can be also observed – caused by the elastic soil behavior, unloading first of all.

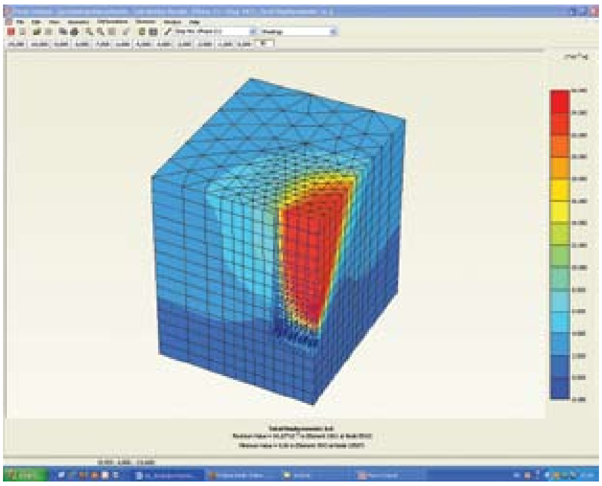

Figure 4a: The 3D total displacements (at failure).

Focusing on horizontal displacements, it can be observed that the failure initiates in the lower part of the trench, Fig.4b. The same conclusion holds for incremental displacements. The uniform red color in Fig.4a confirms an almost vertical kinematics of the wedge.

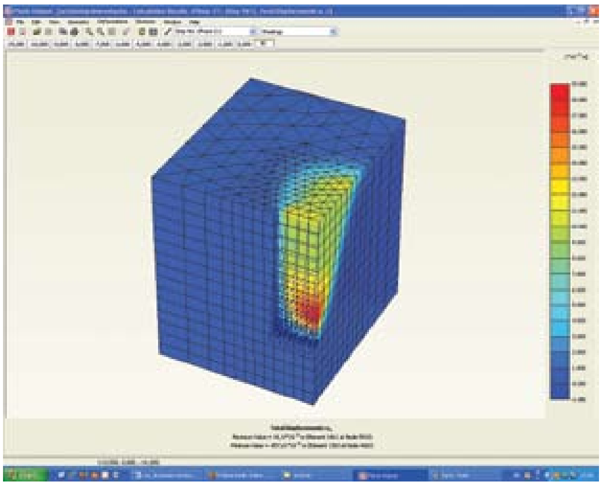

Figure 4b: The horizontal displacements of soil towards the trench (at failure).

Example 3.

Assume the section length of the trench L = 6m and the water table that can change: h_w = 1m, 2m, 3m, respectively. Fig.5 presents the decreasing of the factors of safety FS when the excavation proceeds. Although based on very different assumptions, both methods coincide.

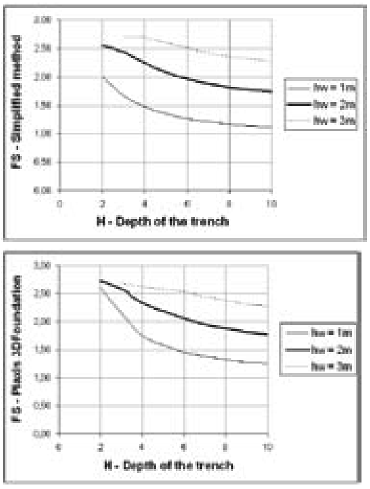

Figure 5: Comparison of two calculation methods in term of the factors of safety FS.

To get a more complete comparison of results, a wider spectrum of numerical examples for H and h_w is presented in Fig.6. Generally, the limit equilibrium method seems to be more conservative. Significant differences, up to 20-25%, can be observed but only for high water table h_w = 1m; the influence of the trench depth H is less evident. On the other hand, the differences are located in the range of small values of FS. In our opinion, just the small values of FS are the general reason of the differences, not the high water table itself. This happens due to the simplified wedge shape that can be more decisive for small values of FS.

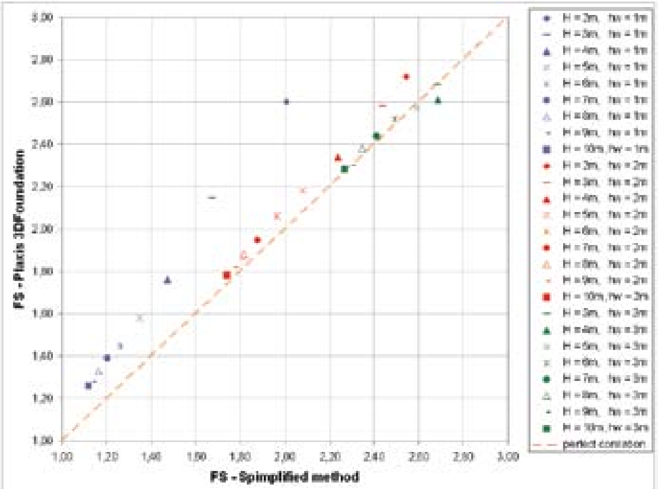

Figure 6a: set of points FS versus FS for the same geoengineering data (the dashed line would mean a perfect correlation of results).

2.3. Further examples calculated using Plaxis 3D Foundation

In addition to the presented material, consider a little weaker 1m-thick sublayer situated at the depth of 4-5m.

	γ	γ'	K_0	ϕ	c	ψ	E	ν
	kN/m³	kN/m³	–	°	kPa	°	MPa	–
Fine sand	18.5	9.0	0.47	32.0	0	0	70.0	0.25
Weaker layer	22.0	12.0	1.00	0	15.0	0	32.0	0.30

Table 3: Parameters of a layered soil analyzed by Plaxis

Example 4.

Fig.7 correspond to Fig.4, respectively. Note that the differences in kinematics are not so much significant as expected.

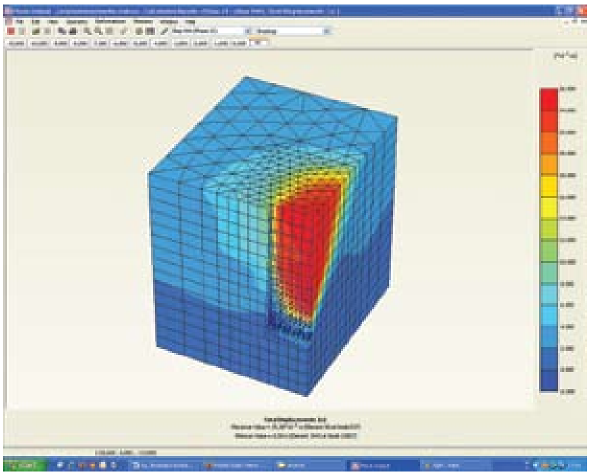

Figure 7a: The 3D soil displacements (at failure).

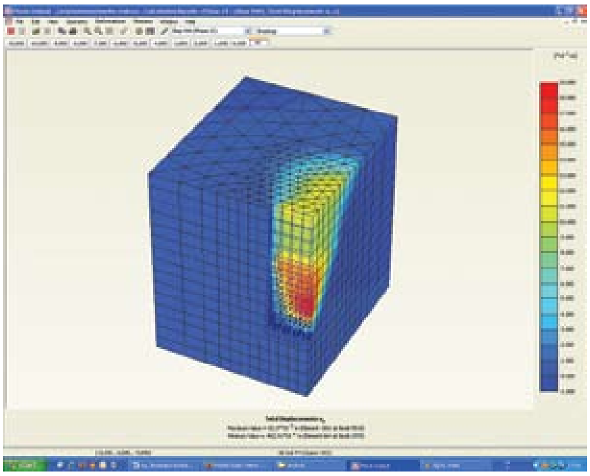

Figure 7b: The horizontal displacements of soil towards the trench (at failure).

3. A probabilistic method

Another safety analysis can be based on a probabilistic methodology (Brzakala and Gorska), following the method of the so-called design point (see Thoft-Christensen and Baker, Baecher and Christian).

Consider two uncorrelated random variables:

- the water table h_w, with the expected value $E\{h_w\}$= 2m and the standard deviation σ_h = 1m,
- the friction angle ϕ, with the expected value $E\{\phi\}$= 32°, and the standard deviation $\sigma\phi$ = 3.2°.

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Note that only two moments of the random variables are required and the probability distributions are not specified in this method (second-order distribution-free approach). Other deterministic data follow from the previous section (a homogeneous soil).

In terms of the dimensionless coordinates

$$z_1 = \frac{h_w - E\{h_w\}}{\sigma_{h_w}} \quad \text{and} \quad z_2 = \frac{\phi - E\{\phi\}}{\sigma_\phi}, \qquad (3)$$

Hasofer and Lind (see Thoft-Christensen and Baker) introduced a measure of safety – called the safety index $\beta = \min\sqrt{z_1^2 + z_2^2}$ – which means the shortest distance from the beginning of coordinate system (expected values of the considered random variables) to a failure surface.

So, first the failure surface can be found making use of Plaxis 3D Foundation assuming a limit displacement. For two considered random variables, the failure surface reduces to a curve, almost linear one in Fig.8. It is composed of all points (z_1, z_2) for which the displacement limit condition is reached (25mm in this case). In detail, successive values of h_w were fixed and the limit state in terms of the displacement was reached by reducing the angle of friction.

As the second step, the shortest distance β has to be found and the design point for which this distance is reached.

Clearly, less attention is paid to points and the shape of the failure surface in regions situated far from the design point.

Analysis of a greater number of random variables is in principle the same, making use of the same two steps. However, for practical applications, Thoft-Christensen and Baker recommend to focus on the most significant variables. “Significant”means here both a large parameter-sensitivity of the model and large randomness (standard deviaton) of the parameter. Neither slurry density nor soil density fulfil this requirements but the water table and the soil strength do.

Finally, note that the obtained value of $\beta = 1.4$ is relatively low - in random conditions we would recommend a value $\beta > 2$.

The direct comparison with Plaxis safety evaluation is not easy because of completely different background. Assuming the mean values as a reference level, so the deterministic parameters $h_w = 2$m and $\phi = 32°$, the FS yielding from the ϕ-c reduction method in Plaxis is however similar: FS = 1.7.

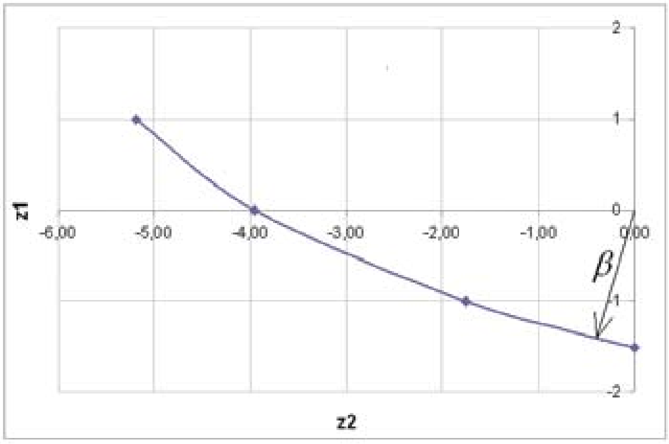

Figure 8: The Hasofer and Lind safety index β = 1.4.

4. Conclusions

1. Plaxis 3D Foundation appeared to be a useful numerical tool for testing a simplified design method of stability analysis.
2. For 3D analysis of stability, a significant reduction of the resultant soil pressure P_h can be observed, especially for small L/H that can increase the trench safety to required levels.
3. The trench depth H in the numerical examples was limited to 10m but the results can be representative also for deeper trenches. The calculations reveal that the failure initiates mainly within the upper 10m, event for H >> 10m. Such a conclusion is in agreement with other models (Piaskowski and Kowalewski). There is also a coincidence with the geoengineering practice, though probably many other factors support such a practical conclusion (suspension weight increasing with depth, soil orthotropy, soil parameters changing with depth, etc.).
4. In contrast to calculations using Plaxis 3D Foundation, more complex studies (displacements, local inhomogeneities, local loadings, etc.) are far beyond the scope of the limit equilibrium methods.

Acknowledgement

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References

- Baecher G.B., Christian J.T. (2003), Reliability and Statistics in Geotechnical Engineering. John Wiley & Sons.

- Brzakala W., Gorska K.: On safety of slurry–wall trenches, Studia Geotechnica et Mechanica, 2008, XXX, No.1–2, 199–206

- Elson W. K.: An Experimental Investigation of the Stability of Slurry Trenches, Geotechnique, 1968, 18, 37–49

- Filz G. M., Adams T., Davidson R. R.: Stability of long trenches in sand supported by bentonite–water slurry, Journal of Geotechnical and Geoenviromental Engineering, 2004, 130(9), 915–921

- Hanjal I., Marton J., Regele Z. (1984), Construction of slurry walls. Budapest, Akad. Kiado.

- Morgenstern N.R., Amir-Tahmasseb J. (1965), The stability of a slurry trench in cohesionless soils. Geotechnique, 15(4), 387–395.

- Nash J.K.T., Jones G.K. (1963), The support of trenches using fluid mud. Grouts and Drilling Muds in Engineering Practice. London, 177–180.
- Ng C.W.W., Lings M.L., Simpson B., Nash D.F.T. (1995), An approximate analysis of the three–dimensional effects of diaphragm wall installation. Geotechnique, 45(3), 497–507.

- Oblozinski P., Ugai K., Katagiri M, Saitoh K., Ishii T., Masuda T., Kuwabara K.: A design method for slurry trench wall stability in sandy ground based on the elasto–plastic FEM, Computers and Geotechnics, 2001, 28(2), 145–159

- Piaskowski A., Kowalewski Z. (1965), Application of tixotropic clay suspensions for stability of vertical sides of deep trenches without strutting. 6th Int.Conf.SMFE, Montreal, Vol.III, 526–529.

- Thoft-Christensen P., Baker M.J. (1982), Structural Reliability Theory and its Applications. Berlin, Springer-Verlag.

- Tsai J.S., Chang J.C. (1996), Three–dimensional stability analysis for slurry trench wall in cohesionless soil. Canadian Geotechnical Journal, 33, 798–808,

- Washbourne J. (1984), The three dimensional stability analysis of diaphragm wall excavation. Ground Engineering, 17(4), 24–29.

- Xanthakos P.P. (1979), Slurry wall as structural system. New York, McGraw–Hill.