Mohr-Coulomb parameters for modelling of concrete structures

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The usual procedure for modelling structures in PLAXIS v8 is to introduce plates, which are one-dimensional beam elements. This way, the results are beam deformations and cross-section forces that will allow the calculation of stresses with post-Plaxis procedures. However, the introduction of one-dimensional elements within two-dimensional soil elements requires the assumption of simplifying hypothesis. As recommended in PLAXIS v8 Reference Manual, this approach should only be used to model the behaviour of slender walls, plates or thin shells.

An alternative procedure for modelling more complex structures is to introduce these elements as clusters of the model which will be discretized in two-dimensional mesh elements. Some examples where this can be applied are plates with variable cross-sections, non slender structures or models where the structure weight has to be determined accurately. The difficulty of this procedure is to set up the material model for these clusters. This article gives an example of a calculation that was made using this approach on concrete modelled as a Mohr-Coulomb material.

Project description

The example shown in this article relate to the construction of a family house in Barcelona. The building will be constructed on a spot where the subway passes 9 m below the street level, as shown in Fig. 1. The tunnel belongs to the extension of the first line of Barcelona subway, which was made about 40 years ago. At the present, an old building exists in the same spot where the housing will be constructed, so previous demolition and excavation of the basement will be necessary. New building will have one basement and three floors. The existing construction and its neighbours are two or three floors high.
Our research is intended to determine the influence of this construction to the tensional and deformational conditions of the existing tunnel.

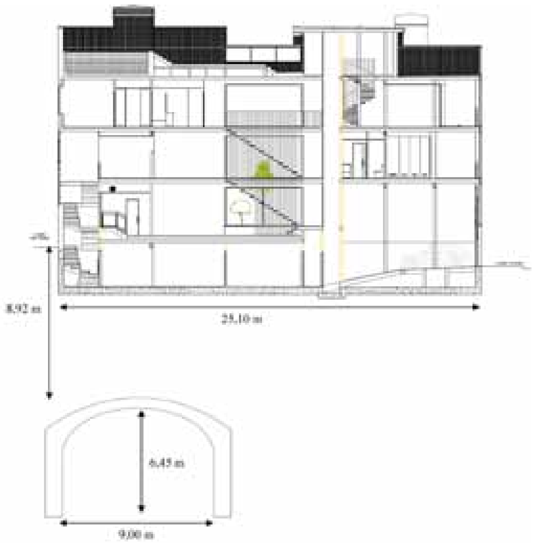

Figure 1: Project geometry

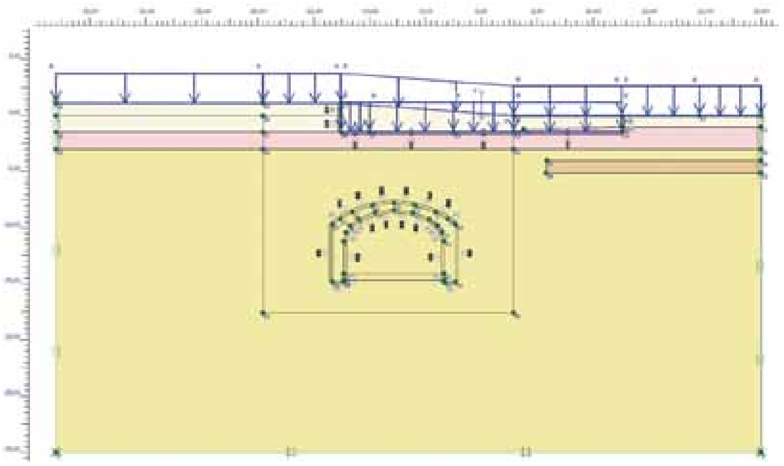

Figure 2: input of the model

FE Analysis

The stresses and displacements in the tunnel have been calculated before the construction of the housing, during the excavation and at the final situation. The calculations were performed using PLAXIS v8 with about 1200 15-noded elements. Input of the model is showed in Figure 2.
The main calculations phases are described below:

1. Construction of the tunnel. Because of the existing buildings above the tunnel, this could not be done in open-cut procedure.
2. Current situation. Uniformly distributed loads of 20 kN/m2 have been considered to take in account the weight of the existing constructions and road traffic.
3. Excavation of the parking floor and execution of the foundation slab, as retaining walls. Loads of 20 kN/m2 are applied.
4. Construction of the building. It's considered as a uniformly distributed load of 40 kN/m2.

Soil Properties

Two sets of calculations were made using two different material models on soils: the Mohr-Coulomb model and the Hardening Soil model. The soil parameters are summarized in Tables 1 and 2: Regarding the presence of water, no phreatic levels were detected during ground testing and had not been considered in calculations.

	Average depth [m]	γ [kN/m³]	E [kN/m²]	υ [-]	c [kN/m²]	φ [°]	ψ [°]
Fill	1.0	17.00	6000	0.30	0.10	22	0
Fine sand	2.1	19.00	8000	0.30	0.10	34	0
Silt	4.5	19.00	8000	0.30	5.00	29	0
Gravel and sand	12.5	20.00	40000	0.30	0.10	34	0

Table 1: Mohr-Coulomb soil parameters

	γ [kN/m³]	c [kN/m²]	φ [°]	ψ [°]	E_{50}^{ref} [kN/m²]	E_{oed}^{ref} [kN/m²]	E_{ur}^{ref} [kN/m²]	m [-]	υ_{ur} [-]	p^{ref} [kN/m²]	R_f
Fill	17.00	0.10	22	0	25912	25912	77737	0.60	0.20	100	0.90
Fine sand	19.00	0.10	34	0	23268	23268	69804	0.60	0.20	100	0.90
Silt	19.00	5.00	29	0	13242	13242	39726	0.70	0.20	100	0.90
Gravel and sand	20.00	0.10	34	0	42597	42597	127791	0.50	0.20	100	0.90

Table 2: Hardening-Soil model soil parameters

Concrete parameters

The existing tunnel was built about 1970. According to the project's history, the structure does not have a tunnel invert and the vault is constituted by mass concrete.

The concrete of the tunnel was characterized having elastoplastic behaviour using the Mohr-Coulomb drained material model.

Even if previous laboratory tests revealed that the mass concrete is considerably strong, the choice of the elastic parameters (E and υ) and strength parameters (c, ϕ, and tensile strength) of the concrete has been carried out considering several hypotheses in a conservative way.

In this sense, two hypotheses concerning the quality of the concrete were considered, given by the characteristic compressive strength: fck = 15 MPa and fck = 25 Mpa, from now on "HM-15" and "HM-25".

The elastic modulus E was determined through the formula proposed by the Spanish regulation EHE-98. According of this, the longitudinal deformation modulus relates to the compressive strength as follows:

$$\mathrm{E} = 8500 \cdot \sqrt[3]{f_{ck}} + 8[\mathrm{M}pa]$$

Two values of Poisson's ratio were considered: a value $\upsilon = 0.2$ according to EHE-98 and a value of $\upsilon = 0.0$ according to Eurocode-2 Recommendation for fissured concrete.

Regarding the plasticity parameters of Mohr-Coulomb model, these can be obtained from compressive and tensile strengths according to the representation of the yield surface as shown in Figure 3:

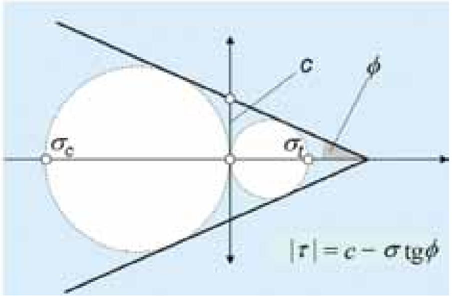

Figure 3: Deduction of Mohr-Coulomb plasticity parameters

Where σ_c and σ_t are compressive and tensile strengths. Values of these can be compared to the allowable stresses proposed by P. Jiménez Montoya (1971) for a mass concrete:

$$\sigma_c = 0.30 \cdot f_{ck}$$
$$\sigma_t = 0.03 \cdot f_{ck}$$

In addition, the EHE-98 establishes the following formula to calculate the shear resistance among concrete joints:

$$\tau_{md} \leq \beta \cdot f_{ct,d} + \frac{A_{st}}{s \cdot p} \cdot f_{ya,d} \cdot (\mu \cdot \sin\alpha + \cos\alpha)$$
$$+\mu \cdot \sigma_{cd} \leq 0.25 \cdot f_{cd}$$

Where σ_{cd} is the value of external normal stress applied to the joint plane. Considering a reinforcement steel section A_{st} equal to zero, the resulting formula has the same shape than the failure criterion of Mohr-Coulomb, with:

$$c = \beta \cdot f_{ct,d}$$
$$\mu = tg\phi$$

Where $f_{ct,d}$ is the design value of tensile strength of the concrete given by:

$$f_{ct,d} = 0.30 \cdot (fck)^{2/3} / 1.50 [MPa]$$

Where β and μ are coefficients that depend on the degree of roughness of the joint as shown in table 3.

Type of surface		
	Low roughness	High roughness
β	0.2	0.4
μ	0.6	0.9

Table 3: β and μ values according to EHE-98
Average values of β= 0.3 and μ = 0.7 were adopted.

The values of Mohr-Coulomb strength parameters can also be obtained according to the Eurocode-2. The following formula is given for the shear resistance for members not requiring design shear reinforcement:

$$V_{Rd,c} = [C_{Rd,c} k (100 \rho_{l} f_{ck})^{1/3} + k_1 \sigma_{cp}] b_w d$$

With a minimum of:

$$V_{Rd,c} = (v_{\min} + k_1 \sigma_{cp}) b_w d$$

From here on we can establish:

$\tau_{Rd,c} = V_{Rd,c} / b_w d = v_{\min} + k_1 \sigma_{cp}$, which has the form of the Mohr-Coulomb failure criterium with:

$$\tau = \tau_{Rd,c}$$
$$c = v_{\min}$$
$$tg\phi = k_1$$
$$\sigma_{cp} = \sigma'$$

where according to EC-2:

$v_{\min} = 0,035 x k^{3/2} x f_{ck}^{1/2}$, where f_{ck} is in MPa

$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$ where d is in mm

so for this structure will be k = 2,0 and k_1 recommended value is 0,15

Therefore:

$$c = 0.035 \times 2^{3/2} \times f_{ck}^{1/2} \approx \sqrt{\frac{f_{ck}}{100}} [MPa]$$

$$tg\phi = 0.15, so \phi = 9^{\circ}$$

According to P. Jiménez Montoya			
Concrete designation	Cohesion: c (kN/m^2)	Friction angle: ϕ	Tensile strength (kN/m^2)
HM-15	712	54.9°	450
HM-25	1186	54.9°	750

According to EHE-98			
Concrete designation	Cohesion: c (kN/m^2)	Friction angle: ϕ	Tensile strength (kN/m^2)
HM-15	365	35.0°	1216
HM-25	513	35.0°	1710

According to EC-2			
Concrete designation	Cohesion: c (kN/m^2)	Friction angle: ϕ	Tensile strength (kN/m^2)
HM-15	387	9°	1216
HM-25	500	9°	1710

Table 4: Mohr-Coulomb strength parameters for mass concrete according different methods

	HM-15	HM-25
γ [kN/m^3]	24	24
E [kN/m^2]	24173	27264
υ	0.2	0.2
c [kN/m^2]	365	513
ϕ [°]	35	35
Tensile strength for tension cut off [kN/m^2]	450	750

Table 5: Material properties of mass concrete

HM-15 υ=0.00	HM-15 υ=0.20	HM-25 υ=0.20
1.13 / 1.13	1.13 / 1.13	1.16 / 1.16

Table 7: Msf values of calculations. Material models for soils are [Mohr-Coulomb / Hardening-Soil]

The formula for the tensile strength from EC-2 is identical to the shown formula from EHE-98.

Table 4 summarizes the Mohr-Coulomb strength parameters according to the explained methodologies:

The final set of parameters considered to model the tunnel material are shown in Table 5:

Results of calculations

Table 6 shows synthetic results. The first values corresponds to Mohr-Coulomb and the second ones to Hardening-Soil, both models for characterizing soils. Some of the calculated stresses are shown in Figure 4.

To evaluate the obtained deformations 5 points where selected for curve representation. These are shown in Figure 5:

Displacement were reset to zero once constructed the existing tunnel and before the application of the loads. Results shows that building load counteracts previous excavation, so stresses remains similar than in the actual conditions phases. Finally, a phi-c reduction phase was done in each model to determine safety factors. Results are summarized on Table 7:

Outputs after phi-c reduction phases shows that failure mechanism is produced on soil below tunnel side walls. Some plastic points appears on the tunnel, but doesn't seem to be related to the failure, as shown in Figure 6:

Conclusions

Tunnel structure was modelled using two-dimensional elements and a Mohr-Coulomb material model was used for modelling mass concrete.Mohr-Coulomb strength parameters for concrete were estimated using two different methodologies. Concerning a mass concrete of about 15-25 MPa of characteristic compressive strength, the values obtained were: cohesion of 365-513 kN/m^2, friction angle of 35°, and tensile strength of 450-750 kN/m^2. In the example presented, many calculations were done to test parameter sensitivity. Results show that this approach gives realistic results for complex structures where the use of plate elements is not suitable.

Other methodologies for evaluating shear strength of concrete are proposed by Rui Vaz Rodrigues (2007). This article encourages Plaxis users who want to follow the same approach.

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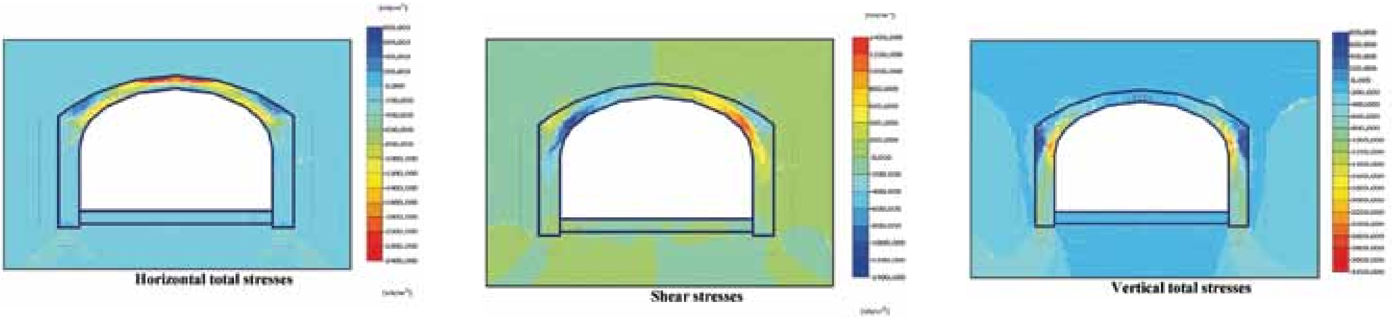

Figure 4. Stresses on the HM-25 type concrete.
These outputs are from the building loading phase and Hardening-Soil model for soils

		HM-15 $\upsilon=0.00$	HM-15 $\upsilon=0.20$	HM-25 $\upsilon=0.20$
Actual Conditions	Plastic points (%)	22.4 / 24.9	25.1 / 22.1	23.2 /25.3
	Tension cut off points (%)	0.07 / 00	0.15 / 0.00	0.00 / 0.00
	Max horiz. compressive stress [kN/m^2]	1880 / 1930	1880 / 1940	2670 / 2690
	Max vertical compressive stress [kN/m^2]	2360 / 2450	2450 / 2390	3440 / 3090
	Max shear stress [kN/m^2]	954 / 1050	915 / 1030	1250 / 1370
	Settlement on C (mm)	17 / 14	17 / 14	16 / 12
	Convergence B-D (mm)	-3 / -2	-3 / -2	-3 / -1
	Convergence A-E (mm)	6 / 6	6 / 7	4 / 6
Excavation	Plastic points (%)	4.6 / 9.7	4.9 / 9.8	4.1 / 4.4
	Tension cut off points (%)	0.00 / 0.07	0.00 / 0.22	0.00 / 0.00
	Max horiz. compressive stress [kN/m^2]	1710 / 1850	1740 / 1850	2060 / 2030
	Max vertical compressive stress [kN/m^2]	1960 / 2160	2080 / 2100	2870 / 2540
	Max shear stress [kN/m^2]	978 / 969	821 / 985	1150 / 1290
	Settlement on C (mm)	4 / 9	4 / 10	1.5 / 8
	Convergence B-D (mm)	1 / 1	1 / 1	1 / 1
	Convergence A-E (mm)	17 / 9	16 / 9	14 / 9
Building	Plastic points (%)	17.3 / 22.4	17.2 / 21.9	14.5 / 4.4
	Tension cut off points (%)	0.15 / 0.00	0.00 / 0.07	0.00 / 0.00
	Max horiz. compressive stress [kN/m^2]	1920 / 1840	1900 / 1860	2600 / 2390
	Max vertical compressive stress [kN/m^2]	2400 / 2420	2430 / 2370	3420 / 3020
	Max shear stress [kN/m^2]	966 / 1040	882 / 1030	1240 / 1360
	Settlement on C (mm)	18 / 13	18 / 14	15 / 12
	Convergence B-D (mm)	3 / 2	3 / 2	2 / 1
	Convergence A-E (mm)	14 / 7	13 / 8	11 / 7

Table 6: Results on tunnel using Mohr-Coulomb material model for concrete. Material models for soils are [Mohr-Coulomb / Hardening-Soil]

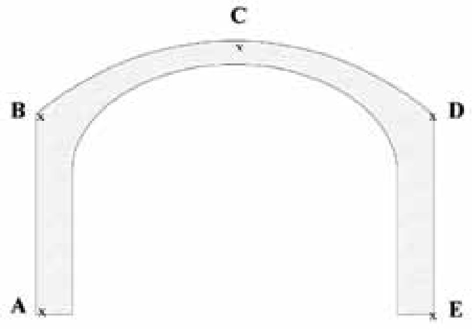

Figure 5: Points for curves

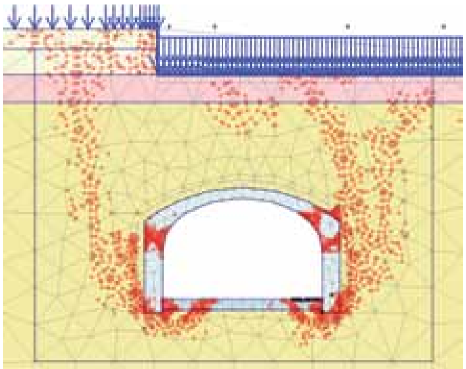

Figure 6: Plastic points on phi-c reduction phase. This shows the calculation with HM-15 $\upsilon=0.20$ concrete and Mohr-Coulomb material model for soils.

References

- Brinkgreve et al. (2004). Plaxis Reference Manual. Plaxis bv., The Netherlands.
- Comisión Permanente del Hormigón (1998). Instrucción del Hormigón Estructural. Ministerio de Fomento, Centro de Publicaciones, Madrid.
- P. Jiménez Montoya (1971). Hormigón Armado. Tomo 1. Editorial Gustavo Gili, S.A., Barcelona.
- Rui Vaz Rodrigues (2007). Shear strength of reinforced concrete bridge deck slabs. Thèse École Polytechnique Fédérale de Lausanne, no 3739, Lausanne.