



Effect of Anisotropy on Tensile Stresses at the Bottom of a Base Course in Flexible Pavements

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The role of an aggregate base course layer in a flexible pavement system is to distribute loads to a stress level that can be sustained by the underlying subgrade. When a pavement is analyzed as a layered, isotropic elastic system, it is not uncommon to see tensile stresses at the bottom of the base course layer upon application of a wheel load. Tensile stresses cannot be sustained by unbound granular materials since they have little or no tensile strength. Because these tensile stresses are generally known to be either unrealistic or overpredicted, using such an analysis can lead to pavement designs that have lower total permanent deformation or rutting (unconservative) and higher fatigue cracking prediction in the asphalt layer (over-conservative).

➤ Studies by Tutumluer (1995) and Tutumluer and Thompson (1997) have shown that tensile stresses can be reduced or eliminated by treating the base course as anisotropic rather than isotropic. According to Kim et al. (2005) anisotropy naturally occurs in unbound materials due to preferred orientation and arrangement of aggregates as a result of their physical properties (gradation and shape) and compaction forces.

A truly anisotropic elastic material requires the specification of 21 independent parameters or elastic compliance coefficients to completely define the three-dimensional stress-strain relationships (Love 1927). Since obtaining all 21 constants is impractical and since anisotropy is generally thought to occur in more limited forms (e.g.; transverse isotropy or cross-anisotropy whereby the material possesses a vertical axis of symmetry such that its properties are independent upon rotation about that axis), the base course can be idealized to be cross-anisotropic. Love (1927) showed that the behavior of a cross-anisotropic material may be described by five parameters. However, Graham and Houlsby (1983) made a significant contribution when they developed a way to represent cross-anisotropy using only 3 parameters; i.e.; one more than for an isotropic linear elastic material.

Many studies have used anisotropic models coupled with non-linear behavior (stress-dependent stiffness) for the base course when analyzing pavements. Herein, the effects of anisotropy alone are studied assuming a constant stiffness in the base course until it yields. This was achieved using the Jointed Rock model in PLAXIS.

The Jointed Rock Model

The Jointed Rock model is a cross-anisotropic, elastic, perfectly-plastic model, especially meant to simulate the behavior of stratified and jointed

rock layers. In this model, two angles define the plane of sliding as shown in Figure 1: the dip (α_1) and strike (α_2). For a pavement system with a base course that is cross-anisotropic with a vertical axis of symmetry, $\alpha_1 = 0$ and $\alpha_2 = 90^\circ$.

Using the Jointed Rock model, the stress-strain curve for the base course is linear (stiffness not sensitive to stress level) until the yield point is reached.

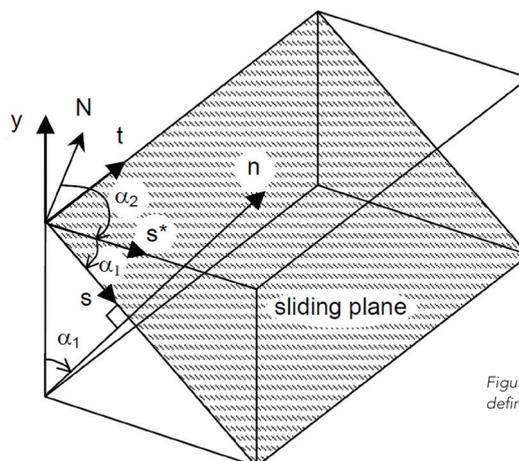


Figure 1. Dip (α_1) and strike (α_2) as defined in PLAXIS (2005)



The stress-strain behavior in the elastic range can be completely described using the following five parameters:

- E_1 = Young's modulus in the horizontal direction
- E_2 = Young's modulus in the vertical direction
- ν_1 = Poisson's ratio for straining in the horizontal direction due to stress acting also in a horizontal but orthogonal direction
- ν_2 = Poisson's ratio for straining in the horizontal direction due to stress acting in a vertical direction
- G_2 = Shear modulus in the vertical direction

For a cross-anisotropic material, strains are related to stresses through the compliance matrix as follows:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_2}{E_2} & -\frac{\nu_1}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_2}{E_2} & \frac{1}{E_2} & -\frac{\nu_2}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_1}{E_1} & -\frac{\nu_2}{E_2} & \frac{1}{E_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_1)}{E_1} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix} \quad (\text{Eq. 1})$$

In the plastic range, the Mohr-Coulomb parameters c and ϕ along with the dilatancy angle, ψ , and tensile strength govern the material's behavior.

In the Jointed Rock model, sliding is only permitted in three different directions, one of which corresponds to the direction of elastic anisotropy. In PLAXIS, a warning message is issued when the Jointed Rock model is used in an axi-symmetric analysis. This is because a dip

greater than 0 will mean that the sliding plane is conical rather than planar. However, a dip of zero can theoretically be analyzed axi-symmetrically. To verify this, two identical axi-symmetric runs were made in PLAXIS with the base course modeled as an isotropic material using both the Mohr-Coulomb and Jointed Rock

$$(E = E_1 = E_2, \nu = \nu_1 = \nu_2, G = G_2 = \frac{E}{2(1+\nu)})$$

models. Both gave identical results indicating that the Jointed Rock model can be used in an axi-symmetric analysis when the dip is 0.

The Graham and Housby Simplification for Cross-Anisotropy

By defining a new parameter, α , Graham and Housby (1983) reduced the number of constants from five to three for a cross-anisotropic material. The three parameters, modified Young's modulus, E^* , modified Poisson's ratio, ν^* , and anisotropy factor, α , are defined as follows:

$$E^* = E_2 \quad (\text{Eq. 2})$$

$$\nu^* = \nu_1 \quad (\text{Eq. 3})$$

$$\alpha^2 = \frac{E_1}{E_2} \quad (\text{Eq. 4})$$

The other three parameters E_1 , ν_2 and G_2 , are dependent on parameters E_2 , ν_2 , and α as shown below:

$$E_1 = \alpha^2 E_2 \quad (\text{Eq. 5})$$

$$\alpha^2 = \frac{E_1}{E_2} \quad (\text{Eq. 6})$$

$$G_2 = \frac{\alpha E_2}{2(1+\nu_1)} \quad (\text{Eq. 7})$$

The compliance matrix can now be re-expressed in terms of E^* , ν^* and α as follows:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} = \frac{1}{E^*} \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\nu^*}{\alpha} & -\frac{\nu^*}{\alpha^2} & 0 & 0 & 0 \\ -\frac{\nu^*}{\alpha} & 1 & -\frac{\nu^*}{\alpha} & 0 & 0 & 0 \\ -\frac{\nu^*}{\alpha^2} & -\frac{\nu^*}{\alpha} & \frac{1}{\alpha^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu^*)}{\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu^*)}{\alpha} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu^*)}{\alpha^2} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix} \quad (\text{Eq. 8})$$

For a material that is isotropic, $\alpha = 1$. For a material that is stiffer in the vertical direction, $\alpha < 1$ and $\alpha > 1$ when the material is stiffer in the horizontal direction. Since the compacted base course is likely to be stiffer in the vertical direction, it follows that $0 < \alpha < 1$.

Linear elastic isotropic materials cannot have a Poisson's ratio > 0.5 . However, Poisson's ratio for a linear, elastic, anisotropic material can exceed 0.5. In the Jointed Rock model, a value of Poisson's ratio > 0.5 cannot be input in PLAXIS. This limits the lower bound value of α that can be studied herein.

Analysis

A 3-layer axi-symmetric flexible pavement system as shown in Fig. 2 was used in this study. A single wheel load that is circular in plan with a radius of 254 mm and having a uniform pressure of 690 kPa was applied on the top of the pavement. The asphalt layer and subgrade were assumed to be homogeneous and isotropic. The asphalt layer was treated as a linear elastic material while a Mohr-Coulomb model was used to describe the subgrade. Rigid interface elements were assumed between layers.

Model Parameters	Description	Asphalt Layer	Unbound Granular Base	Subgrade
	Material Model	Linear Elastic	Jointed Rock Model	Mohr-Coulomb
	Material Type	Drained	Drained	Drained
$\gamma_{unsat} = \frac{kN}{m^3}$	Moist unit weight	24.7	16.0	14.9
E (MPa)	Young's Modulus	1,724	N/A	41.4
ν	Poisson's Ratio	0.35	N/A	0.33
E_1 (MPa)	Horizontal Young's Modulus	N/A	Calculated (Eq. 5)	N/A
ν_1	Poisson's ratio for a cross-anisotropic material	N/A	0.3	N/A
E_2 (MPa)	Vertical Young's Modulus	N/A	103 to 310	N/A
ν_2	Poisson's ratio for a cross-anisotropic material	N/A	Calculated (Eq. 6)	N/A
G_2 (MPa)	Shear Modulus for a cross-anisotropic material	N/A	Calculated (Eq. 7)	N/A
c (kPa)	Cohesion	N/A	0.01	0.01
ϕ (°)	Friction Angle	N/A	45	30
ψ (°)	Dilatancy Angle	N/A	15	0
K_0	At-rest earth pressure coefficient	1.0	0.29	0.5

Table 1: Summary of parameters in PLAXIS analysis

Table 1 summarizes the model parameters used in the analysis. A total of 45 runs were conducted using different values of E_2 (103, 207 and 310 MPa), ν_1 (0.2, 0.25 and 0.3), and α (1.0, 0.9, 0.8, 0.7 and 0.6). Only the results for $\nu_1 = 0.3$ are presented. For $\nu_1 = 0.3$, the anisotropy factor, α , according to Eq. 6 must be greater than 0.6 since ν_2 cannot exceed 0.5 in PLAXIS.

Results

When a wheel load in the form of a circular surface traction is applied at the centerline of the axi-symmetric geometry, the bottom of the base course elongates or experiences tensile strains. The maximum horizontal tensile stress ($\sigma_{x-x_{max}}$) was found to always occur directly below the edge of the circular load. Fig. 3 illustrates the variation of maximum tensile stress at the bottom of the base course with anisotropy factor (α) for different values of E_2 . It increases initially when α decreases from 1 to 0.9. Thereafter, $\sigma_{x-x_{max}}$ decreases with increasing α indicating that by considering anisotropy in the base course, tensile stresses do reduce. However, $\sigma_{x-x_{max}}$ does not reduce to zero in this set of calculations.

To reduce $\sigma_{x-x_{max}}$ further:

1. α should not be constrained to 0.6 or greater. Values of α for a variety of aggregate types and properties in the granular base course have been reported to be between 0.17 and 0.46 (Tutumluer and Thompson 1997; Masad et al. 2006); or

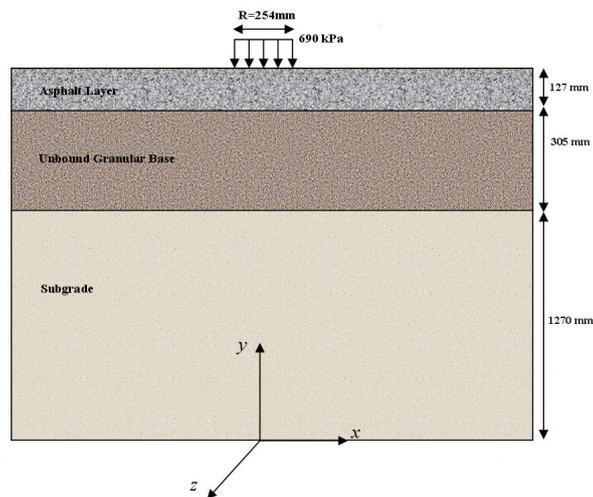


Figure 2: Flexible pavement section analyzed

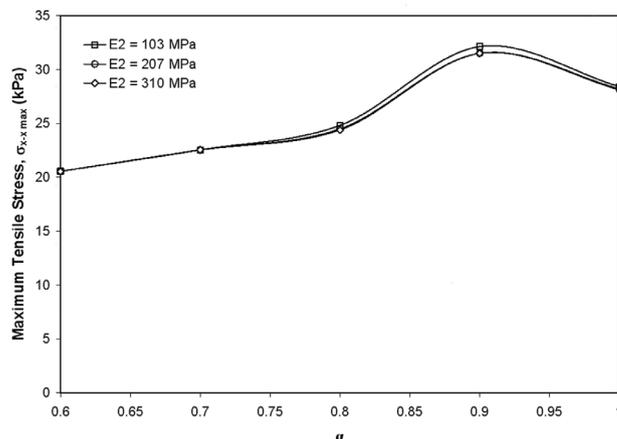


Figure 3: Variation of maximum tensile stress at the bottom of the unbound base course with respect to α assuming $\nu_1 = 0.3$

2. other factors such as stress/strain dependent modulus and the effects of overconsolidation on K_0 should be considered in pavement analysis.

Another observation of note from Fig. 3 is that the results are relatively insensitive to the value of E_2 for this set of parameters analyzed.

Summary and Conclusions

The influence of the anisotropy factor (α) on the horizontal tensile stresses generated in unbound base course layers in flexible pavements was analyzed using the Jointed Rock model in PLAXIS. A non-stress sensitive cross-anisotropic model was assumed for the base course using five constants. By defining an anisotropy factor, α , the number of elastic parameters reduces from five to three. The results show that anisotropy can lead to a reduction in the tensile stress in the unbound base layer. However, the value of α is constrained in this study because PLAXIS does not allow the user to specify a value of ν_2 greater than 0.5 when in fact, a Poisson's ratio > 0.5 is admissible with anisotropic materials. The Jointed Rock model can become more versatile if this limitation is removed in PLAXIS.

It is well known that the behavior of unbound granular layers is not linear and the resilient modulus (or Young's modulus) is highly dependent on the stress state. If stiffness nonlinearity (stress-sensitive stiffness) can be incorporated in a cross-anisotropic constitutive model, the modification can prove beneficial to both the geotechnical and pavement engineering community.

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