

# Is There a Movement Toward Three-Dimensional Slope Stability Analyses?

By Murray D. Fredlund, PhD, PEng, Aff.M.ASCE

Over the past few years, interest in performing three-dimensional (3-D) slope stability analyses has surged. An increasing number of research papers on this topic are being published at conferences and in peer-reviewed geotechnical journals. This increased interest has resulted in a reconsideration of the fundamental issues surrounding 3-D analysis and its practical application in the geotechnical consulting industry.

Landslides and most mass movements are, in general, 3-D in character. However, it has been common practice to analyze

the sliding mass by considering static limit equilibrium conditions on a two-dimension (2-D) slice through the zone of greatest depth.

Some questions that a geotechnical engineer might ask when considering the relationship between 2-D and 3-D slope stability analyses are:

- Is there a need to perform a 3-D analysis in most situations where slope instability is a concern?
- What role do geometry and stratigraphy play in arriving at the conclusion that a 3-D slope stability analysis should be performed?
- In the case where a slope has failed, what is the relationship between performing a back-analysis and a forward-analysis, while using a 2-D analysis on a mass movement that is obviously 3-D in character?

This article attempts to address these questions while also trying to assess whether 3-D aspects should play a significant role in the engineering design, or if they are now being considered simply as the natural outgrowth of having increased computational tools.

## Background

The main observation that has emerged from comparative 2-D and 3-D studies is that 3-D factors of safety are higher. Most comparative studies have involved simple slope geometries where the ground surface extends indefinitely in the lateral directions and the underground stratigraphy remains constant. The finding that a 3-D slope stability analysis yields a factor of safety higher than the 2-D factor of safety tends to foster a certain amount of interest in performing 3-D analyses. However, the comparisons between

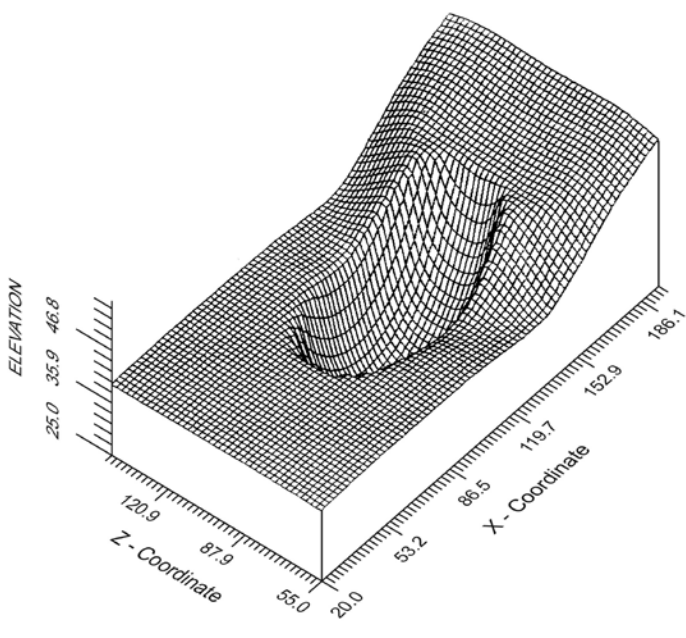


Figure 1. Example of 3-D slope failure.

2-D and 3-D slope stability analyses are not quite that simple. There are other factors related to ground surface geometry and soil conditions that need to be considered. Accounting for these other factors, a 3-D factor of safety can often be as much as 15 percent higher and occasionally up to 50 percent higher than that for a 2-D analysis.

The research literature also contains the results of many case history studies where a “back-analysis” of the failed slope has yielded a factor of safety close to 1.0, so it’s hard to dispute the fact that 2-D slope stability methods have been a good tool for geotechnical engineers. The study of slope stability analyses has been “good” in the sense that engineers have been relatively successful in predicting the stability of manmade and natural slopes.

### **Geometric and Stratigraphic Considerations**

Figure 1 shows a typical geometric ground surface. Even when the ground surface remains constant in the lateral directions, it is well known that the slip surface takes on a 3-D shape. The 3-D sliding soil mass has lateral limits that are caused by lateral variations in stratigraphy (i.e., soil conditions) and pore water pressure conditions, in addition to variations in ground surface geometry. These lateral variations commonly come under the term “end effects” on the sliding mass.

### **Back- and Forward-Analysis of Slope Stability**

The back-analysis of a slope that has failed, or appears to be near to failure, is a situation commonly encountered in geotechnical engineering practice. Generally, a geotechnical investigation of the slope is undertaken with the result that the slip surface shape is identified along with the stratigraphic soil layers. Soil samples may also be collected for geotechnical laboratory testing, and the piezometric water level in the ground is typically measured. The shear strength of the soil may be measured in the laboratory, but in any case, a back-analysis is carried out on a soil mass with a defined geometry.

The actual sliding mass in the field is 3-D in shape and should be simulated using 3-D slope stability software. However, it appears that quite often the back-analysis is undertaken using a 2-D slope stability analysis. As a result, the back-calculated shear strength parameters will now be too high. In other words, the actual sliding mass is 3-D in character and should be analyzed using a 3-D slope stability analysis in order to obtain realistic shear strength parameters. Because a 3-D analysis yields a higher factor of safety than a 2-D analysis, the 2-D shear strength parameters calculated from back-analysis will likely be higher than parameters measured in the laboratory.

Let us then suppose that the 2-D back-analysis shear strength parameters are now used in a 2-D forward-type analysis. This means that the parameters being used in forward design of remedial or other measures are actually too high. While the 2-D analysis was previously viewed as being conservative when simply performing a forward-design type of analysis, the combination of performing a back-

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analysis and a forward-analysis introduces a surprising element of unconservatism.

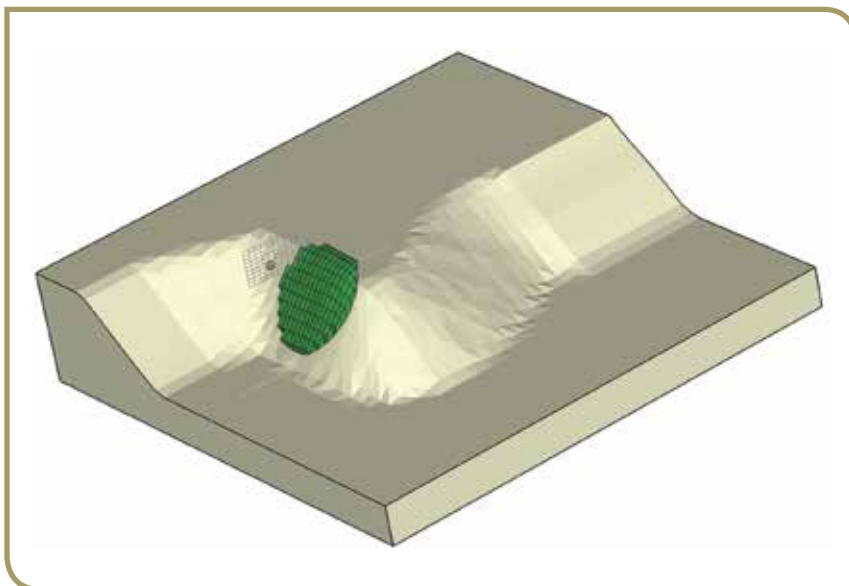
### **3-D Ground Surfaces**

Most comparative studies have been undertaken using what is referred to as a simple geometric shape. However, there are many manmade and natural slope conditions where the ground surface shape is either concave or convex; in these situations, the slope under consideration is an internal corner or an extruding corner. Concave and convex corners are particularly common to mining operations and require a 3-D slope stability simulation. For either of these conditions, it becomes extremely difficult to attempt to rely on a 2-D slope stability analysis for the assessment of stability. When the stability of a number of slopes with concave and convex-shaped corners is analyzed, we find the 3-D computed factors of safety to be 20 to 60 percent or greater than 2-D factors of safety. There are also situations such as bridge abutments, overpasses, and other manmade earth structures that only lend themselves to a 3-D analysis.

### **Influence of Slope Steepness and Unsaturated Soil Conditions**

When comparing 2-D and 3-D slope stability analyses, two other factors should be considered: slope steepness and pore pressure conditions. My colleagues and I compared the results of 2-D and a 3-D stability analysis for a simple steep slope and a simple relatively flat slope. The comparisons also assumed hydrostatic water pressure conditions above the phreatic line and either took the unsaturated shear strength into account or else neglected the negative pore water pressures.

For relatively flat slopes, the 3-D factors of safety were generally on the order of 4 to 9 percent higher than the 2-D factors of safety when negative pore water pressures were ignored. When the shear strength contribution from negative pore water pressures along with a matric suction friction angle of 15 degrees was considered, the 3-D factors of safety were 9 to 16 percent higher than the 2-D factors of safety. The difference between calculated factors of safety was slightly

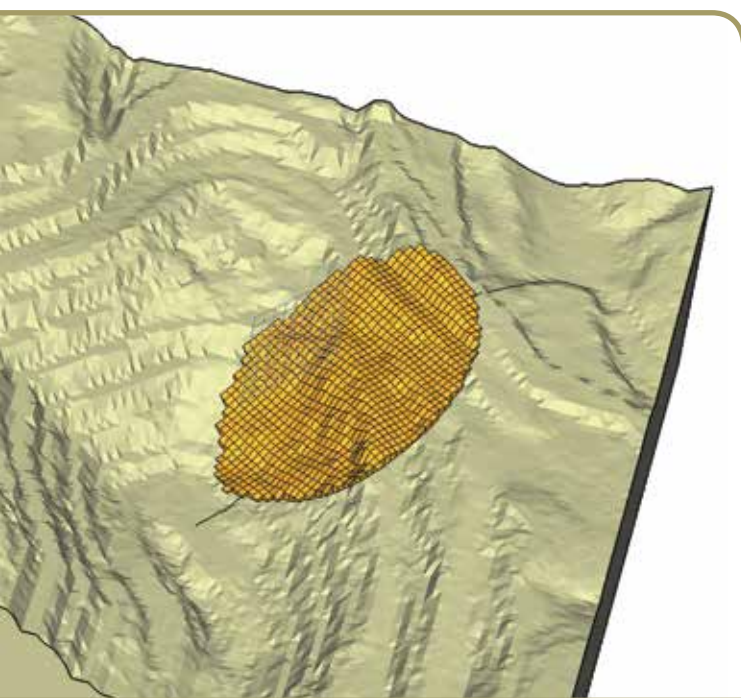


**Figure 2. Example of multi-directional 3-D analysis.**

greater, at 12 to 18 percent, for steep slopes when negative pore water pressures are considered. Negative pore water pressures generally exist in a portion of the soil profile above the groundwater table, and, as such, a 3-D analysis would appear to be most suitable, particularly in situations where a back-analysis is performed.

### Multi-directional Analysis

One historical limitation of the limit equilibrium method (LEM) is the ability to analyze a slope at any particular sliding direction. In the past, limit equilibrium 3-D solutions have



**Figure 3. Example of open-pit analysis using the method of columns in 3-D.**

been restricted to only solving problems at a single sliding direction, which is parallel to the x-axis.

The reason for this restriction in software is that all theoretical formulations of the 3-D limit equilibrium method presented in literature presume the slip direction is parallel to the x-axis of the coordinate system. This is done for the sake of convenience, as the theoretical formulations in 3-D for any arbitrary direction become complicated. However, more and more geotechnical engineers are building models in 3-D coordinate systems and subsequently are interested in analyzing slips at a variety of directions. Therefore, a comprehensive research and development project was undertaken which aimed at extending the existing abilities of the limit equilibrium

method so that any slip direction could be analyzed in a 3-D problem. The research program was a success and has resulted in the ability to analyze slips in any direction (see Lu et al., 2013; *Proceedings, 18th ISSMGE*, pp. 759-761).

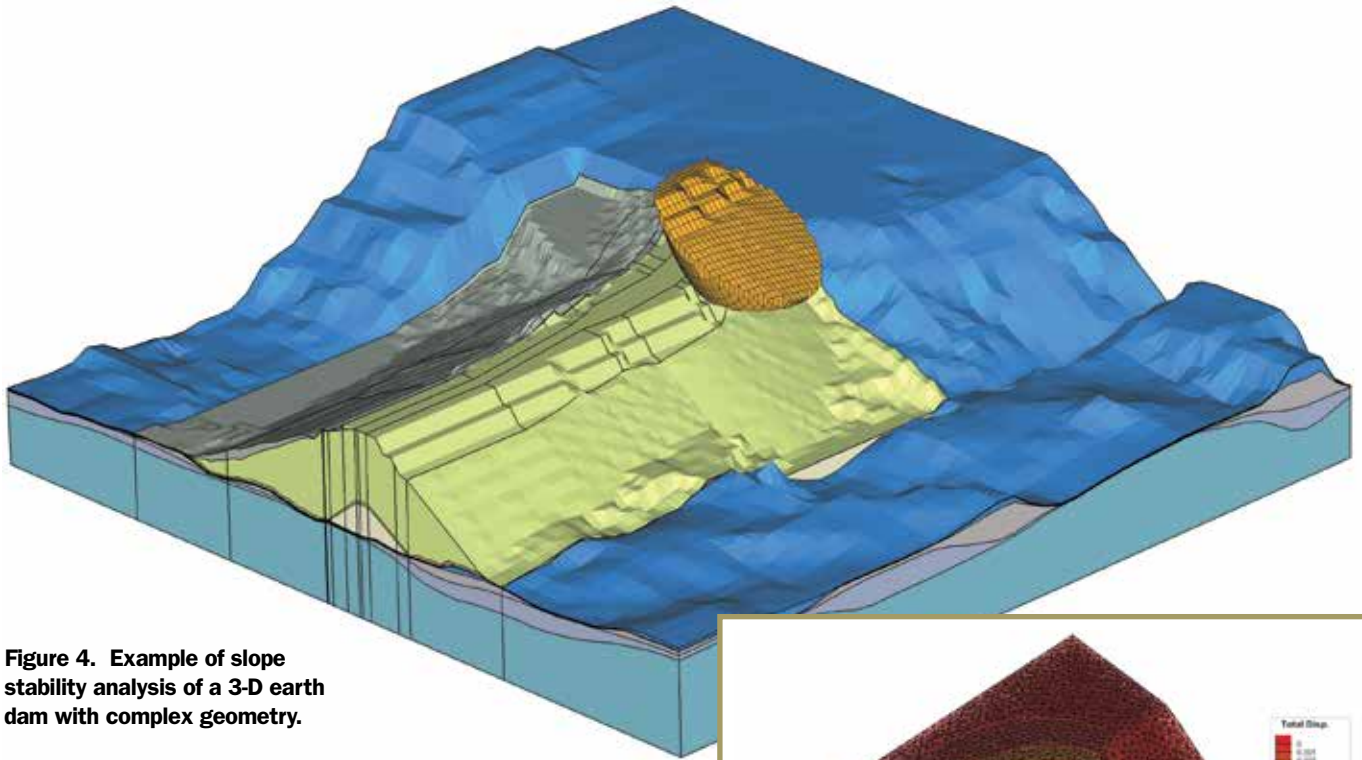
An example of a situation that lends itself to a multi-directional analysis is the development of a bluff at the edge of a city. When structural loads are placed at the top of the bluff, the prominent sliding direction may not be readily obvious. This is the case in the example shown in Figure 2. In this case, a distributed load can be placed at the top of the slope representing the placement of a proposed building. Subsequently, the direction of a number of potential failure directions can be evaluated and the most critical failure surface determined. This is particularly useful where the geology at the site is not regular and may influence the direction of sliding. Therefore, this greatly simplifies the analysis of setback distances and potential volumes of slides that may happen on a bluff overlooking a city.

### The Path Forward

It is important to learn more about the theoretical and analytical relationship between 2-D and 3-D slope stability analyses. Geotechnical engineers must be careful about taking advantage of higher 3-D factors of safety when the wealth of practical experience has been based on 2-D analyses. At the same time, there are situations where a 2-D analysis is far from reality. Also, there are limitations associated with 3-D limit equilibrium analyses. It is important, for example, to take into consideration multiple directions of movement of the sliding mass. While these may not be serious limitations, further studies are valuable.

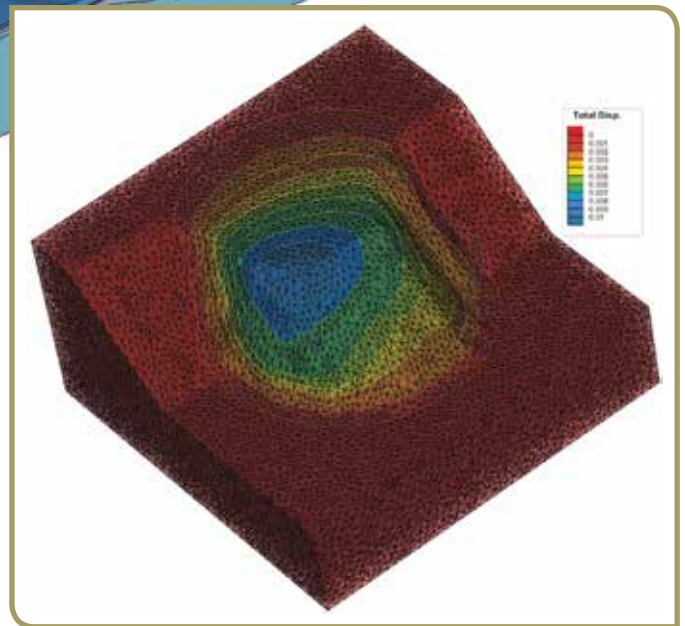
The increased abilities of software in this area have led to more advanced analyses that consider 3-D geometric, stratigraphic, and hydrogeologic regimes. Analyses of open pits (Figure 3) and complex 3-D earth structures (Figure 4) are now possible.





**Figure 4. Example of slope stability analysis of a 3-D earth dam with complex geometry.**

Computer software is available for performing 3-D slope stability analyses. The software has been tested on the commercial market, and its usage has become more commonplace in recent years. In addition to 3-D limit equilibrium methods of columns analysis, the shear strength reduction (SSR) method has also been solved for 3-D analyses (Xu et al., 2014). The SSR method makes use of a pure finite element stress/deformation analysis in which the strength parameters are reduced until a plastic slope failure is achieved. A typical 3-D shear strength reduction analysis, illustrating the reduction in shear strength with displacement, is provided in Figure 5. In general, the geotechnical engineering practice is experiencing a paradigm shift to encompass more advanced analysis methods. The industry must adopt these new analysis methods over the next few years to advance an ever-refining state of practice.



**Figure 5. Slope stability by the shear strength reduction (SSR) method in 3-D.**

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**AUTHOR**

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