Open pit 3D limit equilibrium numerical modelling of complex geostructures

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In the mining industry, the majority of the open pit slope design work is currently undertaken using 2D limit equilibrium (LE) slope stability methods. These 2D methods, which have their origins in the method of slices originally developed for classical soil mechanics, have been extensively improved over the years and offer both simplicity and speed of analysis using existing user-friendly commercial software. However, when applied to analyse pits with complex geometry and geology, these 2D methods can result in conservative designs as they do not account for 3D spatial variation or non-cylindrical slip surfaces. The potential advantage of being able to analyse and design open pit slopes using 3D limit equilibrium methods is therefore apparent from the safety, accuracy, mining optimization, and economic viewpoints.

This paper describes some of the state-of-the-art techniques that have been developed by SoilVision Systems to capatalize on the potential benefits of analysing open pits using 3D limit equilibrium methods. These techniques include the 3D represention of complex geostrata, faults, anisotropic materials, as well as complex trial surfaces and new methods for calculating factors of safety in 3D in multiple directions. Research work is continuing in order to further validate and refine the implementation of these methods for 3D LE analyses.

Introduction

In the mining industry, the majority of the open pit slope design work is currently undertaken using 2D limit equilibrium (LE) slope stability methods. These 2D methods, which have their origins in the method of slices originally developed for classical soil mechanics, have been extensively improved over the years and offer both simplicity and speed of analysis using existing user-friendly commercial software. However, when applied to analyse pits with complex geometry and geology, these 2D methods can result in conservative designs. This is because the 2D slice approach simplifies 3D complexity into a consecutive series of plane-strain analyses which requires the use of conservative assumptions to account for 3D spatial variation. The potential advantage of being able to analyse and design open pit slopes using 3D LE methods is therefore apparent from the safety, accuracy, mining optimisation and economic viewpoints.

The benchmarking and theoretical correctness of 3D solutions has been historically established and documented by Fredlund (2011). 3D LE analyses requires advancements in 3D LE theory as well as methodologies for the representation of complex 3D geostrata for use in the analyses. The use of new 3D methodologies also implies the use of more sophisticated searching techniques for the determination of the critical slip surface.

This paper describes some of the state-of-the-art techniques that have been developed to capatalzse on the potential benefits of analysing open pits using 3D LE methods (3LE). These techniques include the 3D represention of complex geo-strata, faults, and anisotropic materials as well as complex trial surfaces and new methods for calculating factors of safety in 3D.

The paper begins by presenting new techniques for building complex geostrata from triangulated irregular networks (TINs) obtained from existing mining software packages. The use of TINs greatly speeds geotechnical model development and the building of a 3D model. Searching techniques which combine multiple methodologies such as ellipsoidal and wedge-type failures that can also be combined are then covered. The use of multi-directional analysis and anisotropic strength constitutive models is also discussed. The combination of these technologies provides the opportunity to build 3D numerical models using the LE method, which provides more detail and rigor in the modelling process. Such models have shown to more accurately quantify the stability of pit slopes Methods are presented for slicing 3D geological and geotechnical models into a 2D models so that stability assessment results from both approaches can be evaluated and compared. Finally, a 3D analysis of an open pit slope is provided as an example.

Geometry building

The mining industry typically uses commercial 3D geological modelling, visualization, and optimization software to provide ore reserve estimates for open pit operations. Optimization methods such as Whittle 4X apply economic and physical constraints, such as overall slope angles, to provide the pit shell required to demonstrate technical and economic viability. The pit shells are then designed and analysed to the required levels of accuracy for the level of feasibility study or final design being undertaken. This is usually an iterative process. The economic pit shells, together with associated geology, are made available for geotechnical design by exporting the data out of the visualization and optimization software and importing into the geotechnical analysis software. Continuous improvement in resource modelling software has enabled the industry to model and optimize highly complex topology, geology, orebodies, and open pit shapes as illustrated in Figure 1 and Figure 2.

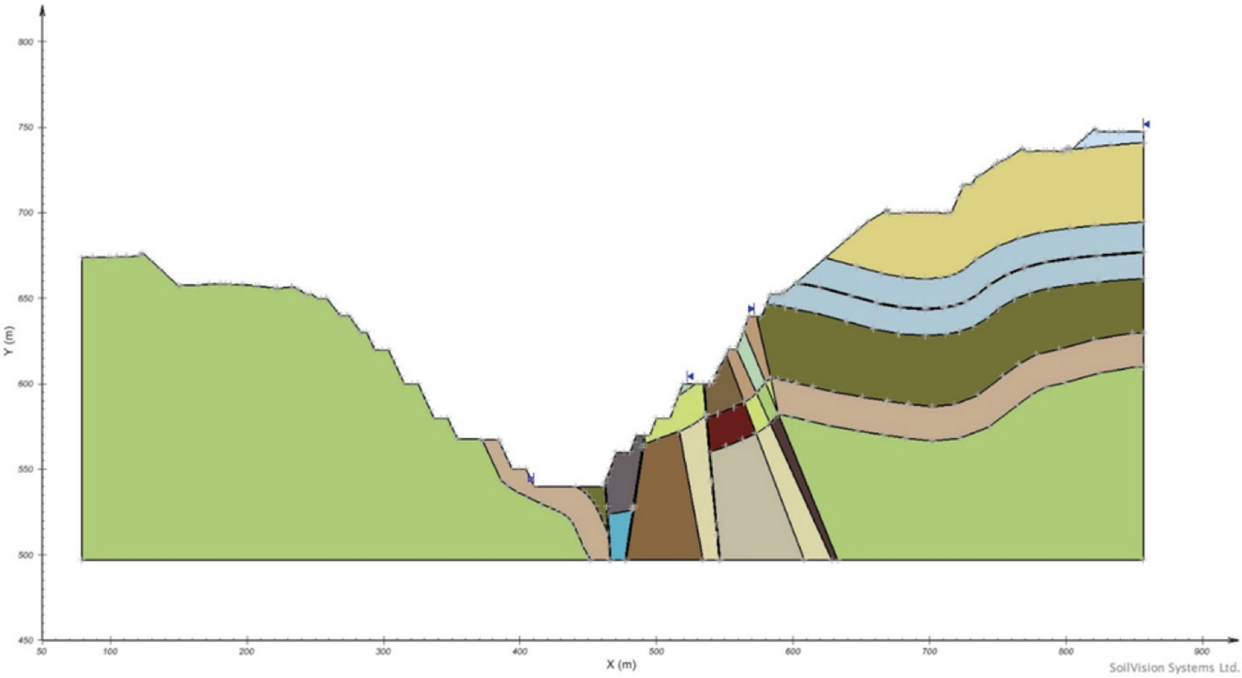

Figure 1 – Illustration of a complex but typical 2D cross-section of an open pit slope stability analysis

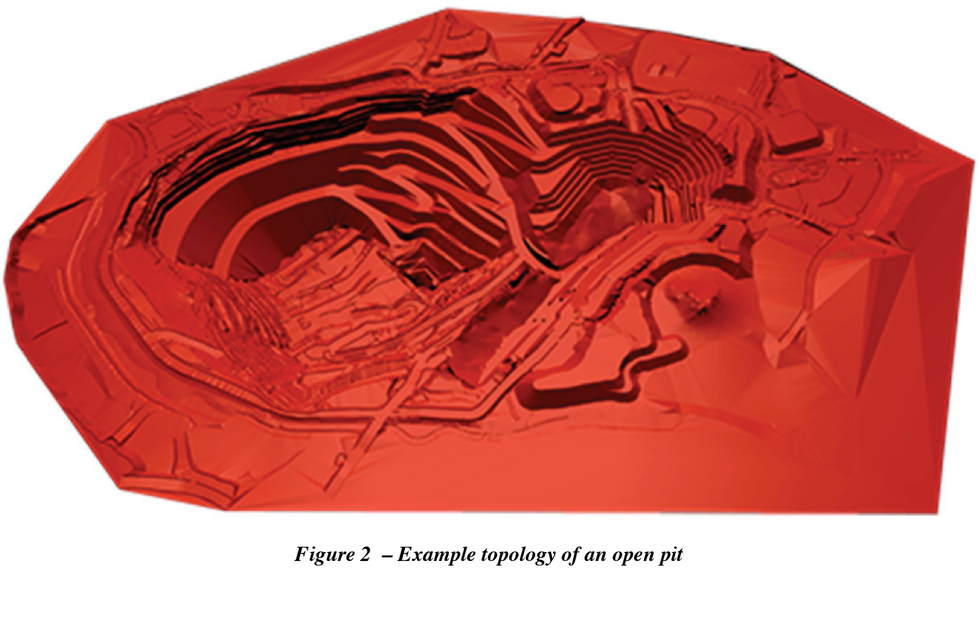

Figure 2 – Example topology of an open pit

Grids vs. triangulated irregular networks (TINs)

The initial method to representing geometry in 3D slope stability models that SoilVision Systems Ltd. (SVS) considered was the use of ’grids’ where each ’cell’ of the grid is represented by a rectangular plane. Early experimentation with the numerical modelling of open pits by SVS showed that grids are problematic for the representation of open pit geostrata simply because (i) it is difficult to provide enough resolution for an accurate analysis, (ii) they do not represent vertical or near-vertical surfaces well, and (iii) pinch-out zones between surfaces are not accurately defined.

A typical topology of an open pit using a grid is demonstrated in Figure 3. The lack of resolution is clearly evident when compared to the surface represented by a TIN or ’wireframe’ as shown in Figure 4. It is possible to increase the resolution of the grid by increasing the number of vertices, but in the case of an open pit the vertices must be increased to a very high number to achieve reasonable resolution and despite this, the problems with improperly defined pinch-out zones may still exist.

Representation of surfaces by TINs offers a number of benefits compared to the representation of open pit geometry using grids in that it increases the accuracy of surface representation as well as the speed at which model-building from a geological model can proceed. It is a relatively simple task to export TINs from any geological modeling software and then subsequently import them and build geometry within the SVDESIGNER modelling tool.

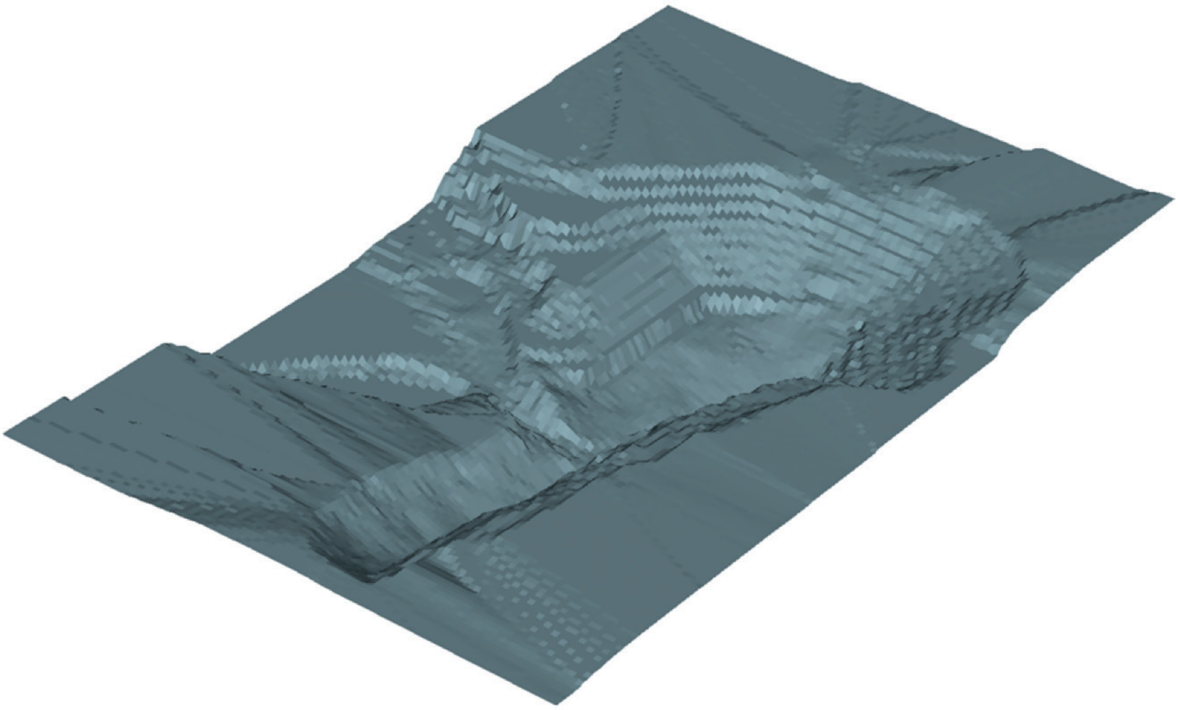

Figure 3 – Open pit surface converted to a 181 x 101 resolution grid

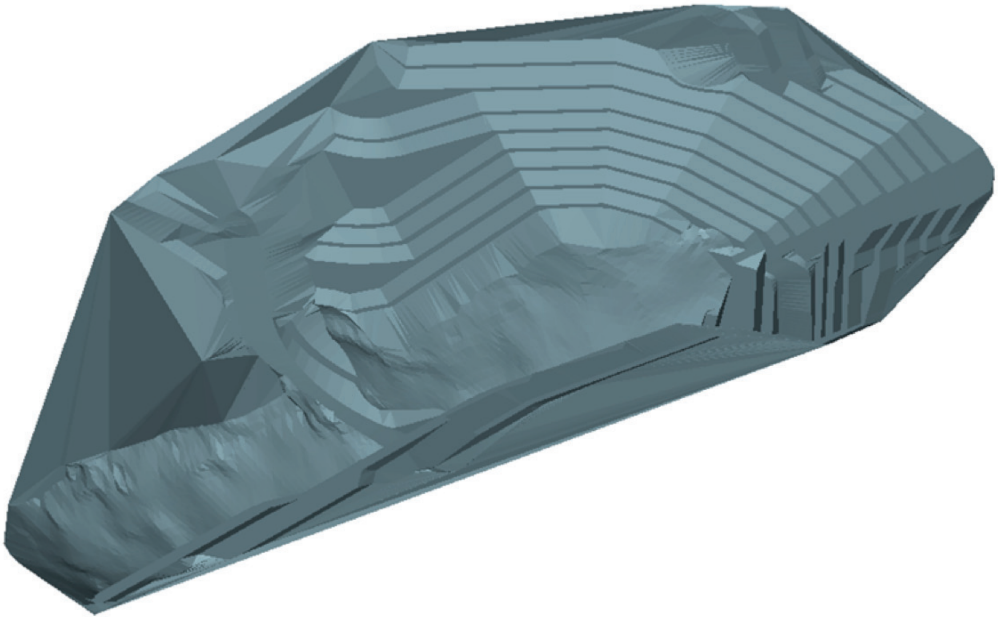

Figure 4 – Open pit surface represented by a TIN

Intersections

The primary difficulty internally at SVS with utilizing TINs was how to handle intersections. A library of algorithms that allowed management of surface grids had previously been developed. Similar intersection algorithms were required to be altered to manage intersections between surface TIN meshes. For this, a research project was instituted by SVS together with the Department of Computer Science at the University of Saskatchewan to develop mesh intersection algorithms for TINs. A key aspect of the research was also to optimize the computational time, which required that the algorithms had to complete in a reasonable time (*i.e.*, less than a minute) when analysing the intersections between several TINs, each of which might have hundreds of thousands of nodes. In addition, the mesh density along intersection lines could not be allowed to become overly dense, which would have adversely affected the solution times to establish the intersections. The intersection of two typical grids is shown in Figure 5.

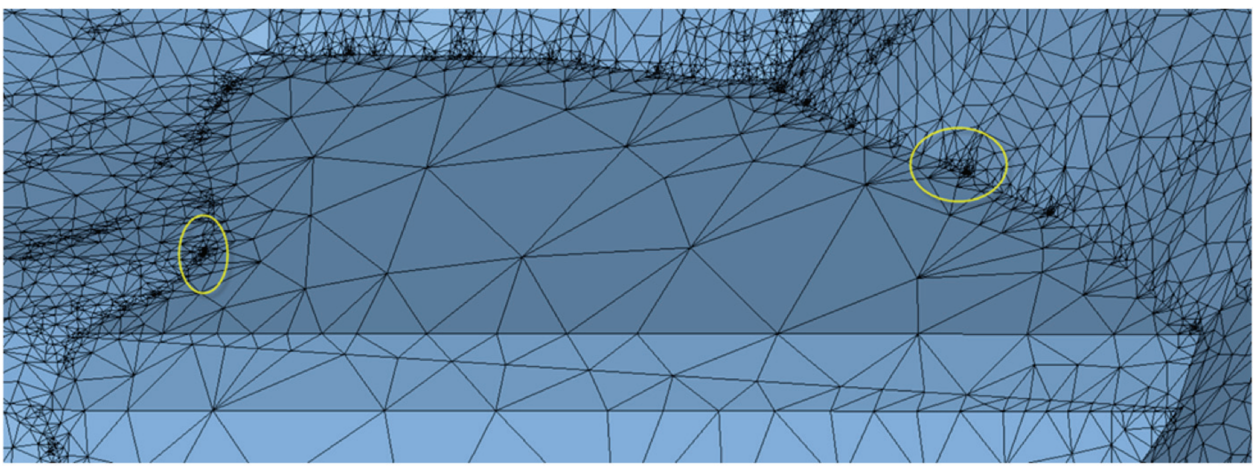

Figure 5 – Example of typical intersection mesh between two TINs

The research resulted in reasonable TIN intersection algorithms that were subsequently rigorously tested and validated in multiple open pit scenarios. The algorithms have since been further optimized several times in order to continue to reduce execution speed and improve accuracy and efficency.

Numerical solver considerations

Modifications were required to the 3D LE solution engine in order to accommodate the representation of geometry using TINs. When the LE method-of-columns is utilized, the intersection of each column with each potential geostrata TIN must be first be determined before analyses are undertaken. A searching algorithm must be called potentially multiple times for each column in every trial slip surface generated for the analysis. If hundreds or thousands of trial slip surfaces are utilized in the analysis, the resulting computational time can become excessive. During initial development, searching for slip surface intersections rapidly became excessive, typically requiring hours of computational time with only reasonably complex geostrata. This required further research in order to dramatically improve efficiencies. An enhanced searching algorithm was eventually developed and the search times for surface intersections for each analysis were typically reduced from hours to minutes.

Multi-directional analysis

Historical 3D LE factor of safety (FOS) solutions have generally been restricted to solving problems at a single sliding direction parallel to the x-axis. This limitation of the sliding direction arises out of the historical development of the LE equations. The definition of the 2D shear force mobilized along any slice for a saturated material may be seen in Equation [1]. The typical 2D geometry definition for a circular slip surface may be seen in Figure 6.

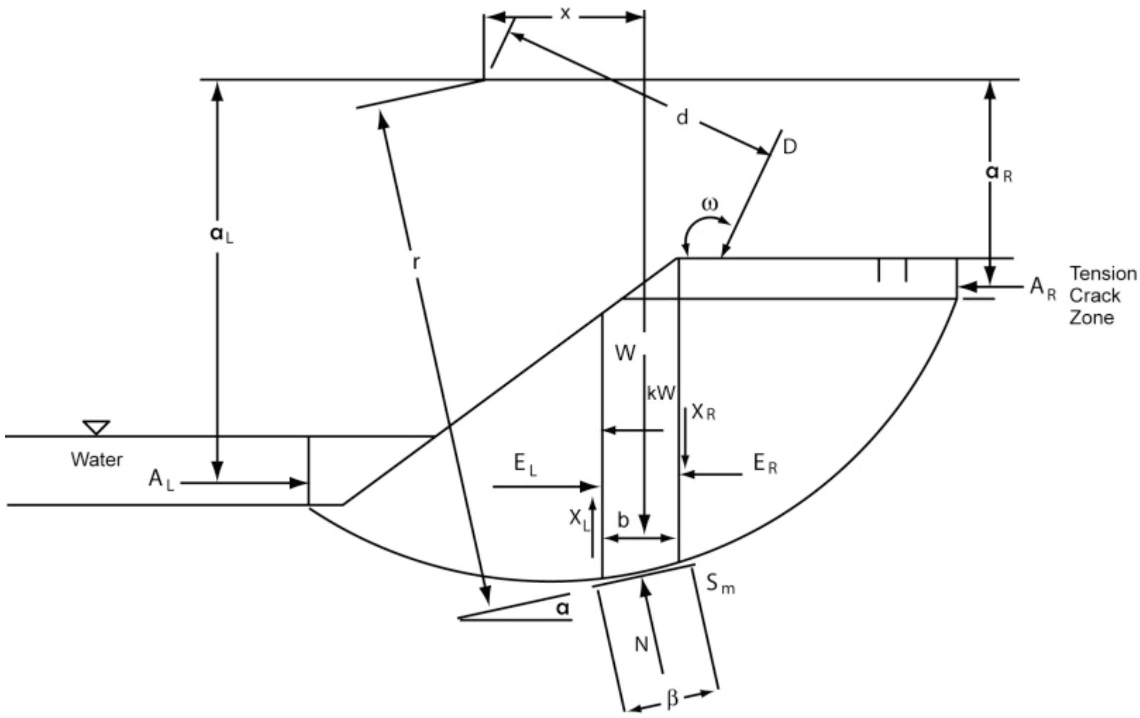

Figure 6 – Definition of forces and slope geometry variables for a circular slip surface

$$S_m = \frac{c'\beta}{F_s} + \frac{[(\sigma_n - u_w)\tan\phi']\beta}{F_s} \quad [1]$$

where

c' = effective cohesion

ϕ' = effective angle of internal friction

u_W = pore-water pressure at the base of a slice

β = length along the base of a slice

σn = normal stress acting on the base of a slice

F_s = overall factor of safety

S_m = shear force mobilized at the base of a slice

S_{soil} = shear strength of the soil expressed as a force (*i.e.* $S_{soil} = \tau\beta$)

S_c = cohesion strength expressed as a force (*i.e.*, S_c = c'b)

S_f = frictional strength expressed as a force (*i.e.*, $S_f = [(\sigma_n - u_w)\tan\phi']\beta$).

The factor of safety equation with respect to moment equilibrium is:

$$\left(F_s\right)_m = \frac{\sum \left[c'\beta r \cos a + \left[W + \left(X_L - X_R\right) + \left[D \sin \omega\right] + \left[D_r \sin \theta\right] - u_w \beta \cos a\right] \tan \phi'\right] / m_a}{\pm Aa + \sum Wx - \sum Pf + \sum kWe \pm \left[Dd\right] \pm \left[D_r d_r\right]} \quad [2]$$

The factor of safety equation with respect to horizontal force equilibrium is:

$$\left(F_s\right)_f = \frac{\sum \left[c'\beta + \left(\frac{W}{\cos\alpha} + \frac{\left(X_L - X_R\right)}{\cos\alpha} + \frac{\left[D\sin\omega\right]}{\cos\alpha} + \frac{\left[D_r \sin\theta\right]}{\cos\alpha} - u_w\beta\right)\right] / m_\alpha}{\pm A + \sum \left\{W + \left(X_L - X_R\right) + \left[D\sin\omega\right] + \left[D_r \sin\theta\right]\right\}\tan\alpha + \sum kW - \left[D\cos\omega\right] - \left[D_r \cos\theta\right]} \quad [3]$$

where

c' = effective cohesion of the material at the base of a slice

u_w = pore-water pressure

N = normal force on the base of a slice

W = self-weight of each slice

D = load (line load and/or distributed load)

D_r = reinforcement load,

A = hydrostatic water force acting at the left, L, or right, R, extremities of the slip surface

r = radius (or moment arm) associated with the mobilized shear force, S_m

x = horizontal distance from the centroid of each slice to the center of rotation (or to the axis point)

f = perpendicular offset of the normal force from the centre of rotation (or from the axis point)

d = perpendicular distance from a line load to the centtr of rotation (or the axis point)

d_r = perpendicular distance from a reinforcement load to the centtr of rotation (or the axis point)

X_R = vertical interslice shear forces on the right side of a slice

E_R = horizontal interslice normal forces on the right side of a slice

ω = angle of the line load from the horizontal. This angle is measured counter-clockwise from the positive x-axis

l = lambda value representing the percentage of the interslice force used in the analysis

f' = effective angle of internal friction of the material at the base of a slice *b* = length along the base of a slice

q = angle of the reinforcement load from the horizontal. This angle is measured counter-clockwise from the positive x-axis

a = angle between the tangent to the center of the base of each slice and the horizontal.

The consideration of perfect alignment of each column with the x,y,z coordinate space is the easiest scenario to consider when the 2D method of slices is extended to the 3D method of columns. Basic parameter definitions for the 3D method of columns are shown in Figure 7. Introducing potential rotation of each column away from being aligned with the coordinate system complicates the theoretical development significantly and a comprehensive development of such equations is not present in the research literature.

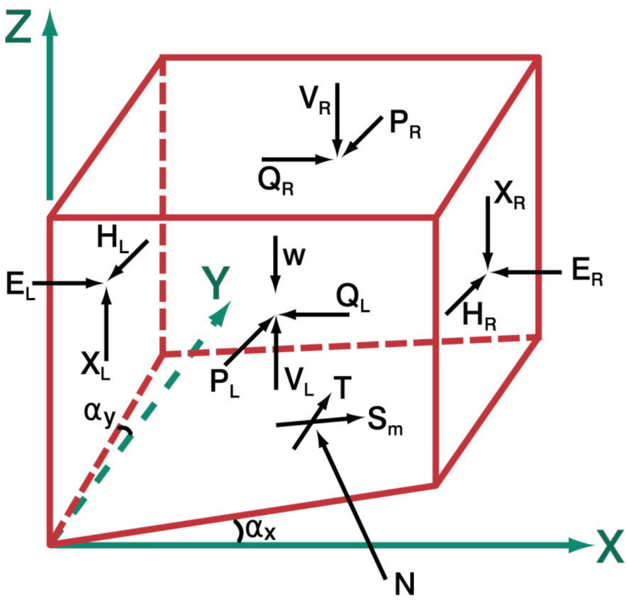

Figure 7 – Free body diagram of a single column (sliding direction: from right to left)

It was noted by SVS that an increased number of consulting firms are building geometry, seepage, and stress/deformation models in 3D and are subsequently interested in analysing multi-directional slope stability slip sufaces. Consequently, internal research undertaken at SVS has been aimed at extending the existing abilities of the 3D unidirectional LE methods such that any slip direction can be analysed in a 3D problem. This research ultimately enabled a mutidirectional method to be successfully developed, details of which are expounded below.

The method requires the user to first enter an estimate of the primary direction of sliding. All other analysis parameters remain the same as in an analysis parallel to the x-axis. In addition to specifying the primary sliding direction, the software provides an option to specify a range of sliding directions relative to the primary sliding direction to analyse. The analysis of any arbitrary sliding direction within the range specified currently works with all existing 2D methods of searching for the critical slip surface. These methods include the grid and tangent method, the entry and exit method, as well as wedge slides and arbitrary surfaces which are fully explained in existing commercial LE software packages such as SVSlope. In the 3D LE analyses (SoilVision Systems, 2015), ellipsoidal, combined wedge and surface failures, and arbitrary slip surface failures are supported with the new sliding direction specification. An optimization algorithm to determine the most critical sliding direction was alos developed.

An example of a classic 3D slope stability analysis problem is presented in Figure 8. It should be noted with this problem that (i) it is impossible to analyse in 2D and (ii) the most likely sliding direction is part of the analysis. Therefore this example is an ideal test case for multidirectional sliding analysis. This particular model can be analysed using either grids or TINs. Grids are used for analysis in this particular example model.

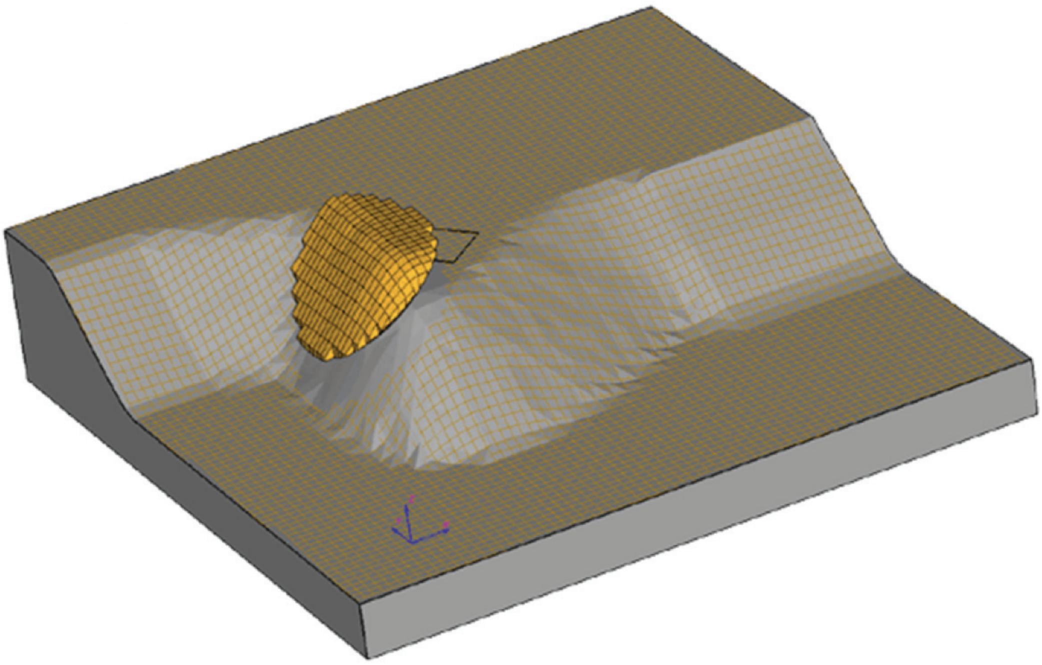

Figure 8 – Analysis of a slip surface where the slip direction becomes a searching parameter

Once the sliding surface direction is specified, the grid of columns is set to be parallel and perpendicular to the specified sliding direction. The user does not need to perform any modifications to their existing 3D model geometry. This is useful, as the existing model can then be made easily compatible with the GIS coordinates of the existing site. This makes the representation of the results as plotted over a GIS location very amenable.

Once the user has specified the primary sliding direction and the range of different sliding directions to analyse, the software places the grids over the new geometry and determines the locations of all the intersections of each column in the analysis. The analysis then proceeds with very few other changes in comparison with 3D unidirectional LE analyses, and the results are evaluated in a similar manner.

Anisotropic shear strength models

In relation to the research and development into modelling increasingly complex geological settings in 3D , is the requirement to correctly account for the shear strength of anisotropic rock masses in LE models.

Rock anisotropy is a well-known factor that has attracted considerable interest in the field of rock mechanics and engineering over the past several decades. However, up until 2000 there had been relatively little effort to model the shear strength of anisotropic rock masses using LE methods. To correctly account for anisotropy using LE methods, the angle between the plane of shear at the base of slice or column and the predominant plane of weakness of the rock fabric (or the predominant orientation of major structural weakness) needs to be considered. This angle is known as the angle of anisotropy or AoA (Mercer, 2012). In a sedimentary environment this plane of weakness typically follows the bedding.

The relationship between the shear strength of the rock mass and the AoA suggests a shear strength continuum should exist, which is referred to as an anisotropic transition model (ATM). This shear strength continuum, which was defined by numerical modelling, is a nonlinear transition of the shear strength relationship within a continuum as defined by the AoA that varies between rock mass strength and bedding shear strength (also referred to as the shear strength differential) (Mercer, 2012). The two aspects of importance are (i) the manner in which shear strength transitions from bedding to rock mass strength as the AoA changes, and (ii) the magnitude of both the rock mass and the bedding shear strengths.

The anisotropic linear model (ALM) was developed in about 1999 when the Snowden Geotechncial Group recognized that a fast and holistic approach was needed for analysing the stability of open pits using LE methods suited to the complex structural/geotechnical settings in the Pilbara. The ALM basically approximates the nonlinear ATM as a linear symetrical model. It was found that using an ALM was computationally substantially more efficient than using a nonlinear ATM.

Anistropic linear model (ALM1)

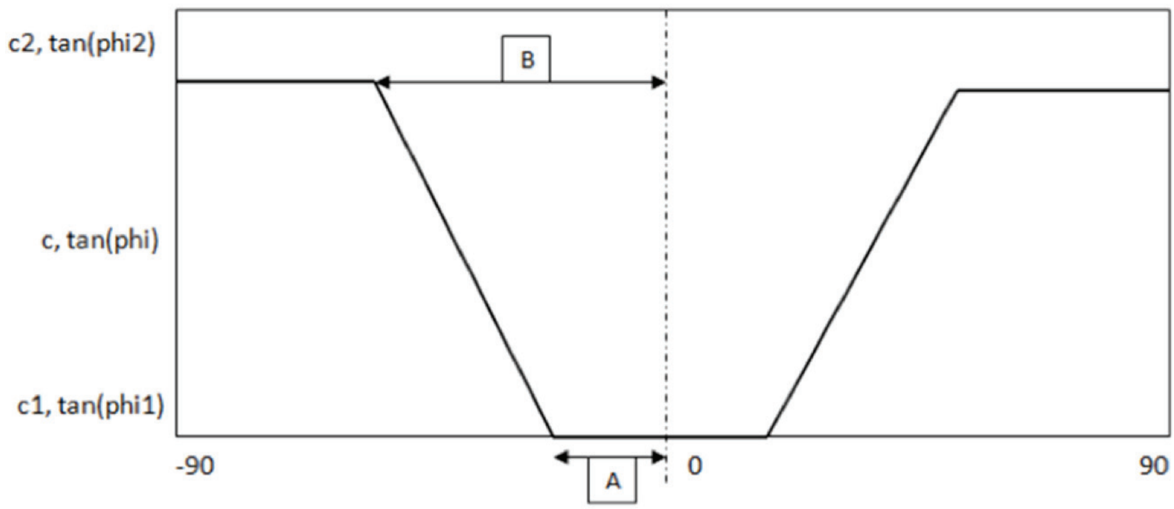

Figure 9 – Anisotropic linear model (ALM1)

The ALM1 was originally developed by the Snowden Geotechnical Group in Perth, Australia in 2005 following an in-house research programme. Two generations of the model are now available. The first generation (ALM1) is based on the Mohr-Coulomb criterion. The bedding and rock mass shear strengths for ALM1 are defined in terms of the Mohr-Coulomb parameters of cohesion and friction angle. In this model, the shear strength continuum is defined using the following parameters:

- The Mohr-Coulomb shear strength along the plane of weakness (usually the bedding plane): cohesion and friction angle (c_1, $\emptyset_1$). This corresponds to the minimum shear strength
- The Mohr-Coulomb shear strength of the rock mass: cohesion and friction angle (c_2, $\emptyset_2$). This corresponds to the maximum shear strength
- Parameters A and B, which define a linear transition from bedding plane strength to rock mass strength, with respect to AoA.

The ALM1 defines the shear strength relationship to the AoA, rock mass strength, and bedding shear strength as shown in Figure 9.

Modified anistropic linear model (ALM2)

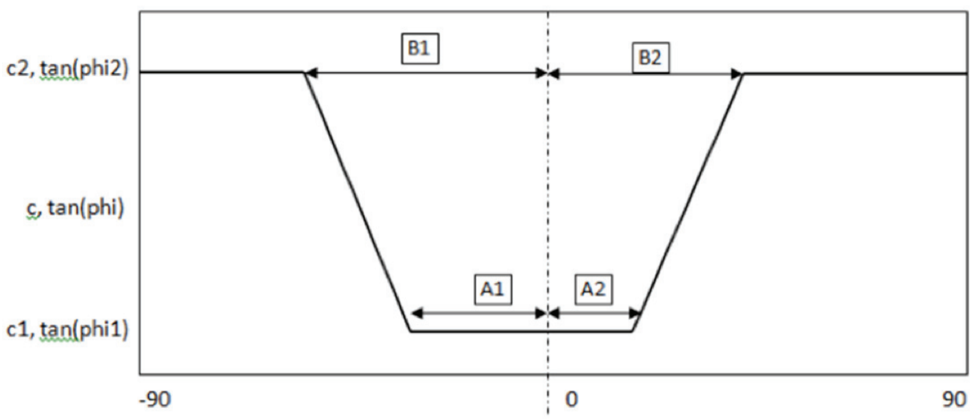

Figure 10 – Modified anisotropic linear model (ALM2)

Since 2009 (Mercer, 2013), Snowden has been undertaking further research and development of the ALM1 method. In the second generation of ALM, referred to as ALM2, Snowden recognized that cohesion, *c* and friction angle, φ for a typical rock mass and bedding plane are a function of the stress state within the rock mass and along the bedding plane. As a result, the model requires either a shear stress *versus* normal stress function or a function relating cohesion and friction angle to normal stress.

In ALM2, as shown in Figure 9, with the parameters A1, A2, B1, and B2, the non-symmetrical shape of the shear strength transition either side of an AoA of 0° is modelled. The rate and shape of the transition depends on the bedding to rock mass strength ratio as well as the normal stress. Both the rock mass and bedding shear strengths are now modelled nonlinearly in terms of the normal stress. The ALM2 constitutive model was implemented in the SVSLOPE 2D software in 2013.

ALM 2-3D

ALM2 was implemented in the new 3D version of the software in 2014. In order to accommodate ALM2 in 3D, a 3D orientation of AoA was required to be defined. At this stage the AoA is calculated by simple resolution of the 3D orientation into two directions, parallel and normal to the direction of slip. Only the ALM shear contribution in the direction of slip is accounted for in the factor of safety.

Combined ellipsoid and wedge searching with faults

Analysis of typical open pit scenarios led to scenarios where the critical slip surface (CSS) may be a combination of an ellipsoid shape and a block-type mechanism in which one or more surfaces may slip along a known single or multi-plane fault. Traditional searching in 3D enables the trial slip surfaces to be specified (i) as ellipsoids where the searching method is either grid and tangent or entry and exit, or (ii) block failure mechanisms where the failure mass is defined through a series of interlocking planes. If we consider a partial ellipsoidal slip surface in which the failure surface would always follow one or more faults, then the traditional method requires modification.

A methodology of combining ellipsoidal and block searching methods was developed and implemented. The resulting method allows the specification of ellipsoidal searching mechanisms but with the consideration that if the ellipsoid intersects a particular fault then the fault geometry takes priority. The fault itself can also be assigned weaker material strength properties. This methodology allows the specification of a combined ellipsoidal slip surface and intersecting faults.

Such a methodology was employed to solve a real-world case study illustrated in Figure 11. In this case study the primary searching mechanism was ellipsoidal-based but the base of the slip surface was designated by a fault plane. The resulting critical slip surface was determined by searching thousands of trial slip surfaces that intersected the fault plane at various locations. The analysis was successfully completed and demonstrated the concept of searching for the CSS using combined methodologies.

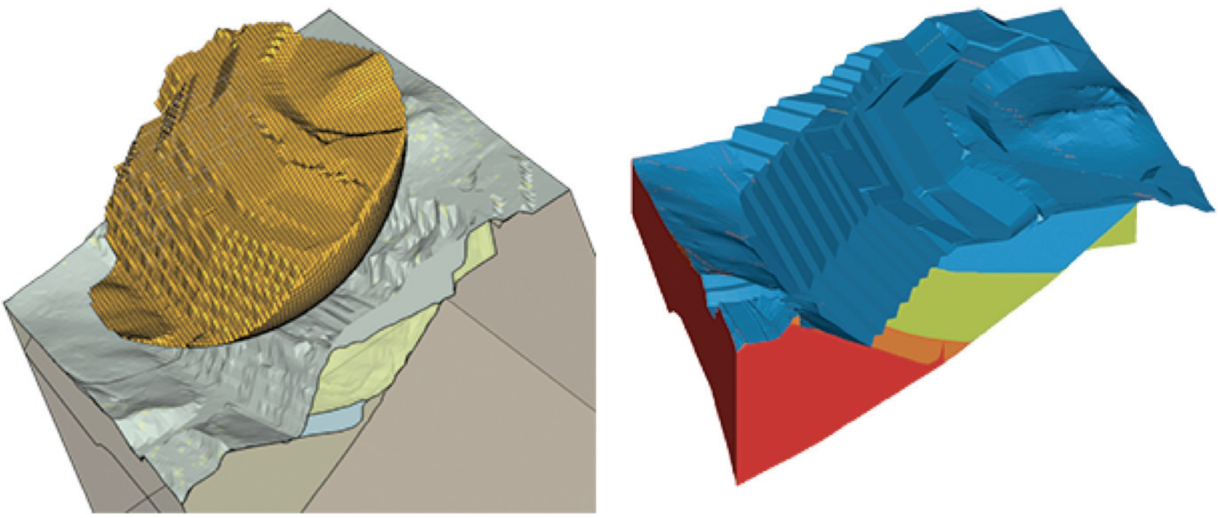

Figure 11 – Example of combined ellipsoidal and block fault search methods

Slicing back to a 2D model

One of the benefits of this methodology is that the 3D conceptual model-builder can easily create 2D slices from any 3D section and export them to a 2D analysis tool to carry out seepage or slope stability analyses. Traditional slope stability analysis has been carried out using 2D LE, and this methodology allows for continuity with current well-established proceedures results to facilitate comparison with more sophisticated 3D LE modelling results.

The process is illustrated with the following 2D cross-section, which is taken along the plan view cross-section line illustrated in Figure 12. The 2D models may be exported with regional material properties automatically included.

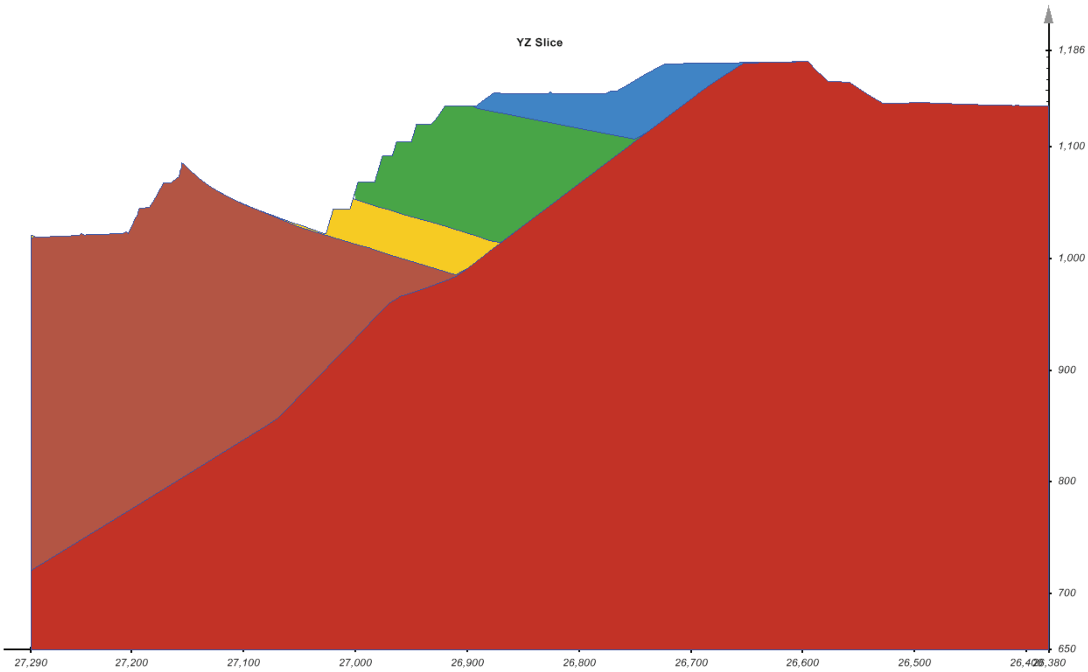

Figure 12 – Example slice of 3D geometry to equivalent 2D geometry

It is worth noting that the the 3D factor of safety (FOS) can be considered a more rigorous calculation of the 2D FOS.

Conclusions

SVS has undertaken extensive research and development to apply conventional 2D LE methods in analysing 3D models containing complex geology and pit shapes. In particular, considerable effort was undertaken to developing algorithims for importing and optimizing TINs and handling intersections. Existing unidirectional 3D LE methods were implemented and modified to undertake stability analyses in multiple directions in a very time-efficient way. The implementation of the anisotropic linear models were particulary challenging in the 3D environment. Of particular note was the development of methodology for combining ellipsoidal and block searching methods in 3D. Finally, all 3D models are capble of being reduced into 2D sections and analysed using conventional 2D LE methods. Ongoing research work is aimed at further validating and refining the implementation of these methods for 3D LE analyses.

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