BIOT THEORY OF POROELASTICITY

This document verifies that groundwater flow principles are correctly implemented in PLAXIS. Poroelasticity theory, first introduced by Biot (1941), counts for coupled hydro-mechanical processes. The present verification example studies the case of a drilled vertical borehole in saturated soil formation (porous rock), subjected to non-hydrostatic in situ stress. PLAXIS results are compared to the analytical solution provided by Detournay & Cheng (1988).

Used version:

- PLAXIS 2D - Version 2018.00

- PLAXIS 3D - Version 2018.00

Geometry: The model geometry in PLAXIS 2D is presented in Figure 1. An axisymmetric model with 15-noded elements is used. To simulate a two-dimensional condition, in which stress variation merely occurs in horizontal direction, the model's width and height equal 5 m and 0.01 m correspondingly. Borehole radius R_{bh} equals 0.1 m. Initial compressive stress equal to 70000 kN/m/m is applied at the outer (right) boundary of the model. Initial pore pressure is *User-defined*, set equal to -40000 kN/m^2. A compressive *Line load* equal to 40000 kN/m/m is applied at the borehole's perimeter to compensate for the remaining pore pressure when the borehole will be drilled. *Deformations* are set to be *Normally fixed* at the top, bottom and left (axis of symmetry) boundaries. All groundwater flow boundaries are set to be *Closed* (impervious).

In PLAXIS 3D, taking advantage of the model's symmetry, only one-quarter of the geometry is modelled. The model is extended 5 m in both x and y directions, while its depth in vertical z-direction equals 0.01 m. Borehole radius R_{bh} equals 0.1 m. *Surface loads* at both outer boundaries (rear and right) are used to generated the same initial stress field as in PLAXIS 2D (compressive stress of 70000 kN/m^2). *User-defined* pore pressure equal to -40000 kN/m^2 is initiated as well. A compressive *Perpendicular Surface load* equal to 40000 kN/m^2 is applied at the borehole's perimeter for the same reason mentioned above. *Deformations* are set to be *Normally fixed* at the top, bottom, left and front boundaries, while they are set to be *Free* for the outer boundaries (rear and right). All groundwater flow boundaries are set to be *Closed* (impervious). Figure 2 illustrates the model geometry in PLAXIS 3D.
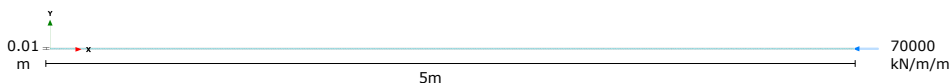

0.01 m ← → 70000 kN/m/m
5m

Figure 1 Problem geometry and generated mesh (PLAXIS 2D)

Materials: The soil (porous rock) is modeled as *Drained*, *Linear elastic*. To specify Biot's α_{Biot} parameter, *Manual - Constant K_w* undrained behaviour is selected. Regarding the groundwater flow properties, *User-defined Saturated* model is used with isotropic permeability k equal to 1 m/s. The adopted material parameters are:

Soil: Linear elastic   Drained γ = 0 kN/m^3 e_{init} = 0.0416

G = 10^6 kN/m^2 ν' = 0.20 K_w = 2.34 · 10^6 kN/m^2 α_{Biot} = 0.893

Meshing: In PLAXIS 2D, the *Very Fine* option is selected for the *Element distribution*,
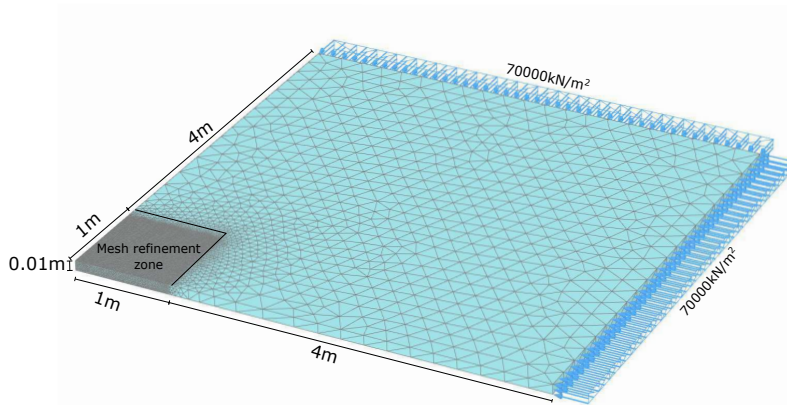
Figure 2 Problem geometry and generated mesh (PLAXIS 3D)

while in PLAXIS 3D, the *Medium* option is used. Additionally in PLAXIS 3D, a refinement zone is used to refine the mesh locally, at the vicinity of the borehole. A *Coarseness factor* of 0.03125 is used for the mesh refinement zone, covering a volume up to 1 m away from the borehole's origin. To reduce the number of mesh elements in PLAXIS 3D, a *Coarseness factor* of 8.0 is used for the rest of the model. The generated mesh is illustrated in Figures 1 and 2 for PLAXIS 2D and PLAXIS 3D respectively.

Calculations: Pore pressure is manually defined in the *Flow conditions* menu. The *Gravity loading* calculation type is used for the Initial phase to ensure stress equilibrium throughout the model. The external loads applied at the model boundaries are activated and the boundary conditions are adjusted as described above. In Phase 1, a *Plastic* analysis is performed. The soil cluster within the borehole is deactivated (drilling) but the water conditions are kept active. Thus, only the effective stress is removed and the pore pressure remains unaffected (water flow is prevented). The line/surface load of 40 MPa acting normal to the borehole perimeter is activated to counteract the remaining pore pressure within the drilled area.

Output: Figure 3 illustrates the active pore pressures after the *Plastic* calculation (Phase 1) in PLAXIS 2D (a) and PLAXIS 3D (b).

Verification: The active pore pressures are defined as:

$$p_{active} = \alpha_{Biot} S_{eff} p_w \tag{1}$$

in which S_{eff} is the effective degree of saturation (equal to 1 for fully saturated condition) and p_w is the pore pressure. Based on the used input parameters, p_{active} equals $35.72 \cdot 10^3$ kPa throughout the entire model. Results presented in Figure 3 are in agreement with the expected value.

Under the above mentioned stress and pore pressure field, the borehole drilling induces the following stress increments at the borehole wall (radius equal to R_{bh}):

$$\Delta\sigma_{rr} = P_0, \ \ \Delta\sigma_{r\theta} = 0, \ \ \Delta p_w = 0 \tag{2}$$

in which $\Delta\sigma_{rr}$ and $\Delta\sigma_{r\theta}$ are the increments of the radial and the tangential total stress, Δp_w is the change in the pore pressure and P_0 is the far-field normal stress, equal to -70
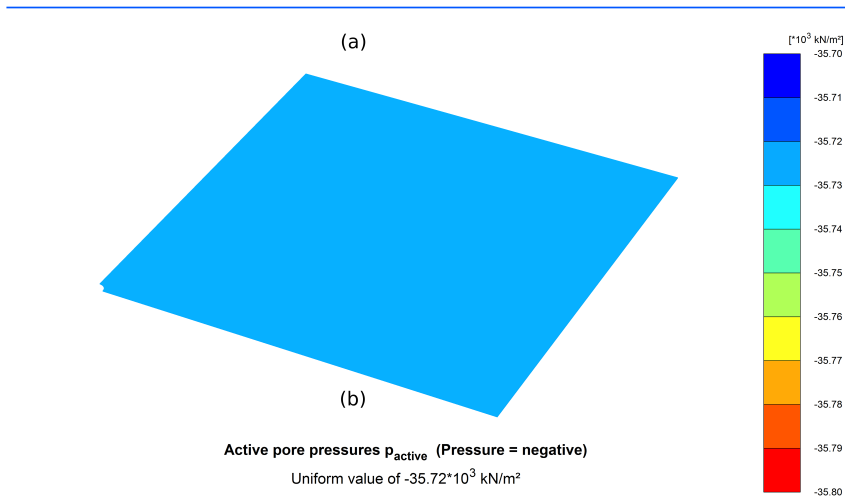
Figure 3 Active pore pressures after the Plastic analysis in PLAXIS 2D (a) and PLAXIS 3D (b)

MPa.

In case of a borehole in an infinite domain, under axisymmetric loading and the above presented boundary conditions, Detournay & Cheng (1988) presents the following analytical solutions for the radial displacement u_r, the total radial stress σ_{rr} and the total tangential stress $\sigma_{\theta\theta}$:

$$\frac{Gu_r}{R_{bh}P_0} = -\frac{1}{2}\frac{R_{bh}}{r} \tag{3}$$

$$\frac{\sigma_{rr}}{P_0} = \frac{R_{bh}^2}{r^2} \tag{4}$$

$$\frac{\sigma_{\theta\theta}}{P_0} = -\frac{R_{bh}^2}{r^2} \tag{5}$$

in which G is the shear modulus and r the distance from the origin (center of the borehole).

Comparison between the analytical solution and PLAXIS results is presented in Figures 4 to 6. For the PLAXIS 3D results, a vertical cross section along the line y = 0 is considered. PLAXIS results are in good match with the results obtained from the analytical formulation. The deviation between PLAXIS and analytical results for the radial displacement u_r close to the outer model boundary (r = 5 m) is caused due to the fact that the PLAXIS model is finite, while the analytical solution is derived based on an infinite domain.
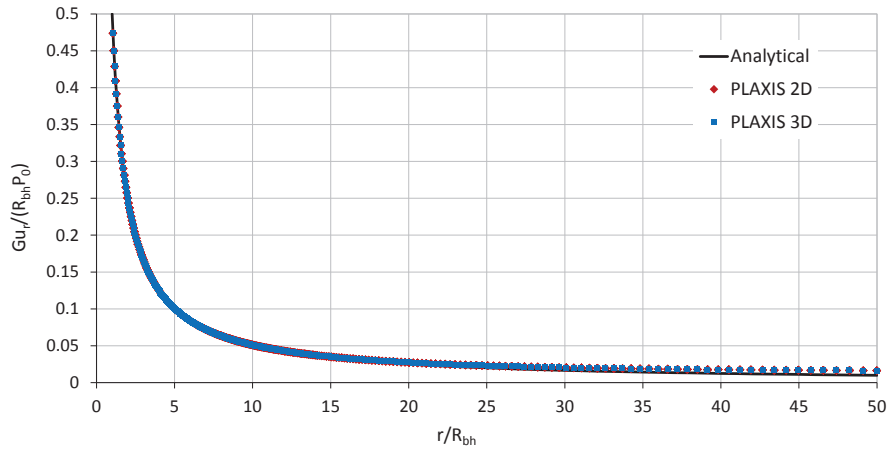
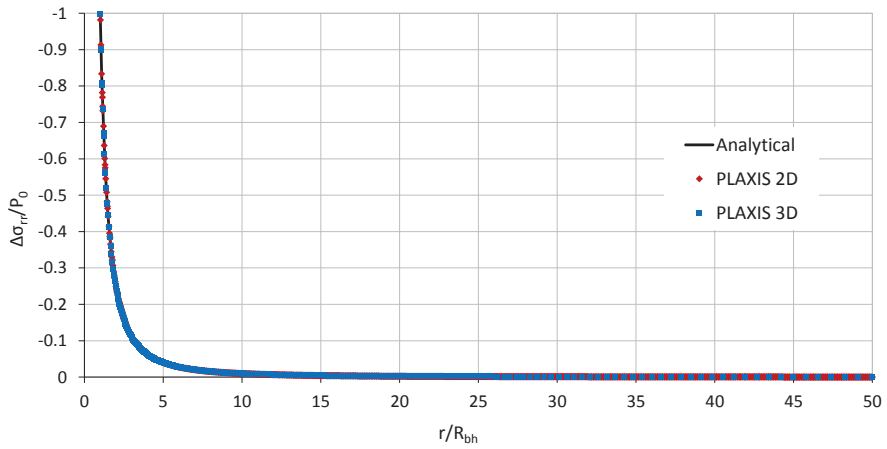Figure 4 Normalized radial displacement over normalized distance from the origin

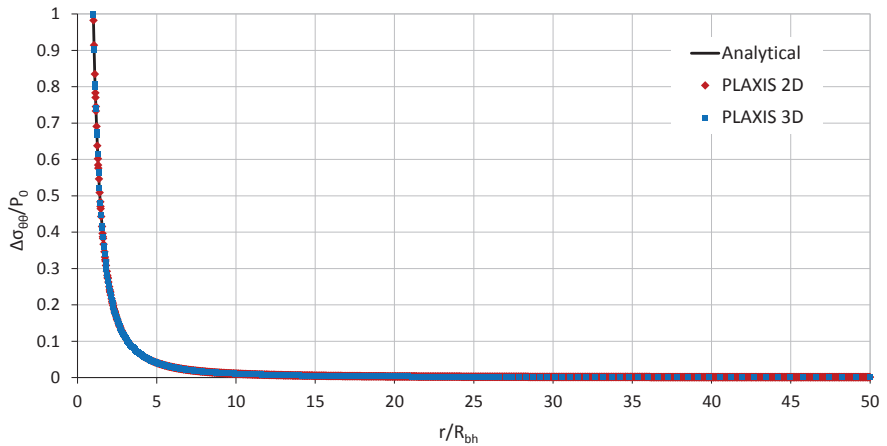Figure 5 Normalized total radial stress over normalized distance from the origin

Figure 6 Normalized total tangential stress over normalized distance from the origin

REFERENCES

[1]   Biot, M.A. (1941). General theory of three-dimensional consolidation. Journal of Applied Physics, 12(2), 155–164.

[2]   Detournay, E., Cheng, A.H.D. (1988). Poroelastic response of a borehole in a non-hydrostatic stress field. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 25(3), 171–182.