BISHOP'S EFFECTIVE STRESS

This document verifies that groundwater flow principles are correctly implemented in PLAXIS. Bishop's effective stress (Bishop & Blight, 1963) is used in PLAXIS to handle unsaturated soil conditions. An example is used to verify the capability of PLAXIS to calculate Bishop's effective stress.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: The model geometry in PLAXIS 2D is presented in Figure 1. A plane-strain model with 15-noded elements is used. Head *h* equal to 1 m is prescribed at the top boundary, while the head at the bottom boundary is set equal to -1 m. The left and right groundwater flow boundaries are set to *Closed* (impervious). The adopted groundwater flow conditions are displayed in Figure 1 as well.

In PLAXIS 3D, the same model as described above is used, extended by 1 m in y-direction. The same groundwater flow boundary conditions are adopted. Additionally, both groundwater flow boundaries in y-direction are set to be *Closed*. Figure 2 illustrates the model geometry in PLAXIS 3D.

The adopted groundwater flow boundaries lead to unsaturated starting flow conditions. Constant suction equal to 10 kPa is generated in the entire soil column.

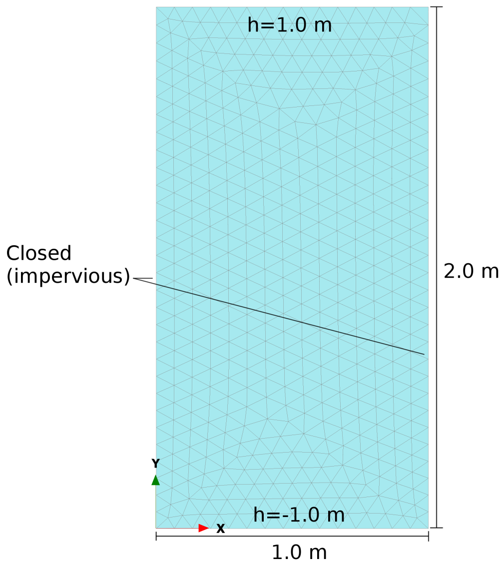

Figure 1 Problem geometry, generated mesh and groundwater flow boundary conditions (PLAXIS 2D)

Materials: The soil is modeled as *Drained*, *Linear elastic*. *User-defined* parameters for the *Van Genuchten* hydraulic model are used to model the unsaturated flow conditions. The adopted material parameters are presented in Table 1.

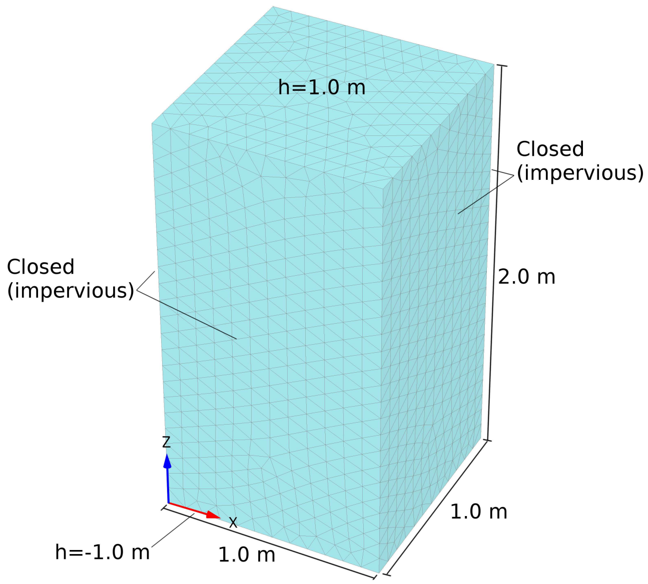

Figure 2 Problem geometry, generated mesh and groundwater flow boundary conditions (PLAXIS 3D)

Table 1 Material parameters

Parameter	Symbol	Unit	Value
Unsaturated unit weight	γ_{unsat}	kN/m^3	20
Saturated unit weight	γ_{sat}	kN/m^3	20
Initial void ratio	e_{init}	-	0.5625
Young's modulus	E'	kN/m^2	10000
Poisson's ratio	ν'	-	0.20
Saturated degree of saturation	S_{sat}	-	1.00
Residual degree of saturation	S_{res}	-	0.02
Van Genuchten fitting parameter	g_n	-	2.286
Van Genuchten fitting parameter	g_a	-	2.240
Van Genuchten fitting parameter	g_l	-	0.000
Permeability of saturated soil	k	m/day	1.0

Based on van Genuchten (1980), the degree of saturation S in unsaturated soil conditions is given as:

$$S(\phi_p) = S_{res} + (S_{sat} - S_{res})\left[1 + \left(g_a|\phi_p|\right)^{g_n}\right]^{g_c} \tag{1}$$

in which ϕ_p is the pressure head, defined as:

$$\phi_p = \frac{-p_w}{\gamma_w} \tag{2}$$

where p_w is the suction and γ_w is the unit weight of the water.

Based on Eq. (1) and the selected input values presented in Table 1, the degree of saturation S is plot against suction in Figure 3.

Meshing: In PLAXIS 2D, the *Very fine* option is selected for the *Element distribution*,

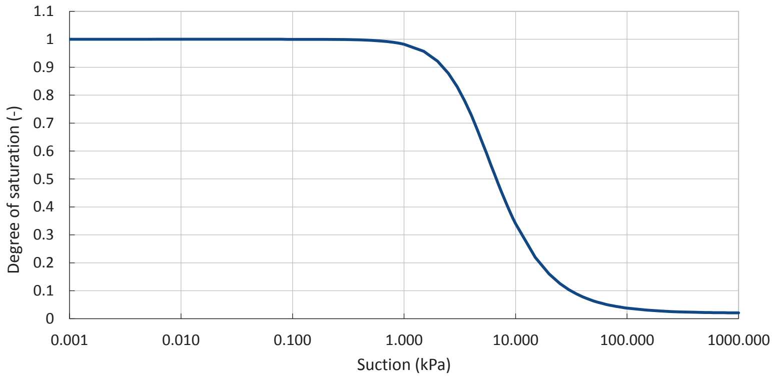

Figure 3 Degree of saturation versus suction

while in PLAXIS 3D, the *Fine* option is selected. The generated mesh is illustrated in Figures 1 and 2 for PLAXIS 2D and PLAXIS 3D respectively.

Calculations: The calculations are performed using the *Gravity loading* calculation type in the Initial phase. The *Steady state groundwater flow* is used as pore pressure calculation type. The groundwater flow boundary conditions are adjusted as described above and the *Ignore suction* option is deactivated.

Output: The degree of saturation under steady-state conditions is presented in Figure 4 for PLAXIS 2D (left) and PLAXIS 3D (right). Figures 5 and 6 illustrate the vertical and horizontal effective stresses correspondingly, for PLAXIS 2D (left) and PLAXIS 3D (right).

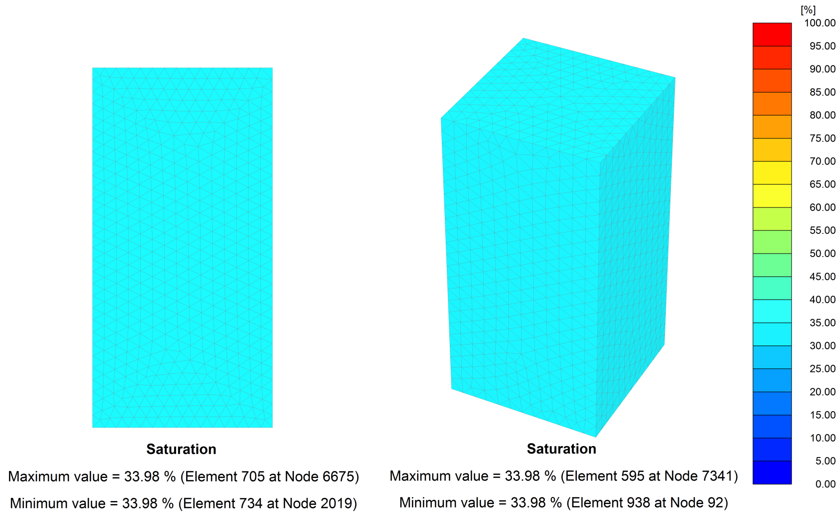

Figure 4 Degree of saturation obtained in PLAXIS 2D (left) and PLAXIS 3D (right)

Verification: Bishop's effective stress (Bishop & Blight, 1963) is given as:

$$\sigma' = \sigma - m \cdot [\chi p_w + (1 - \chi) p_\alpha] \tag{3}$$

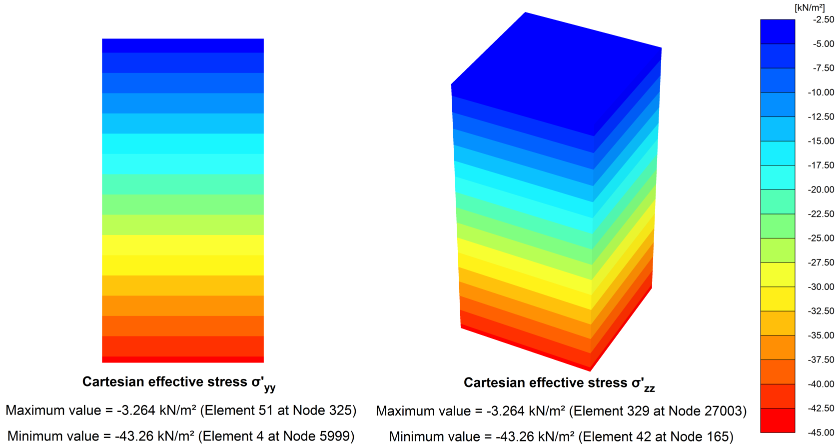

Figure 5 Vertical effective stresses in PLAXIS 2D (left) and PLAXIS 3D (right)

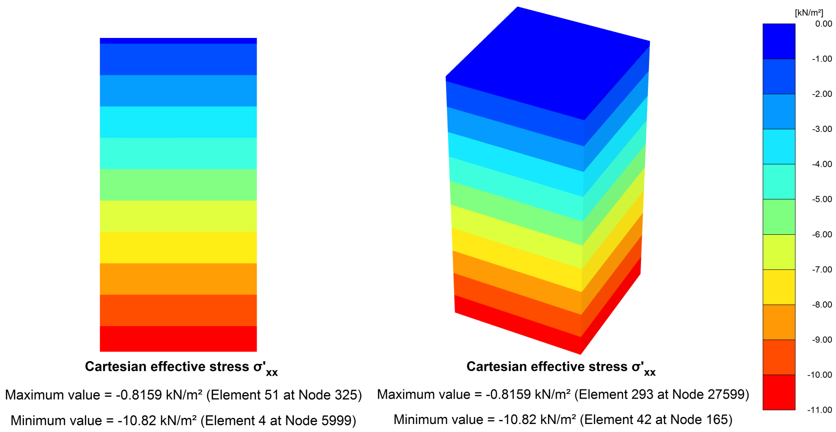

Figure 6 Horizontal effective stresses in PLAXIS 2D (left) and PLAXIS 3D (right)

in which,

$$\begin{aligned} \sigma &= \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \tau_{xy} & \tau_{yz} & \tau_{xz} \end{bmatrix}^T \\ \sigma' &= \begin{bmatrix} \sigma'_{xx} & \sigma'_{yy} & \sigma'_{zz} & \tau_{xy} & \tau_{yz} & \tau_{xz} \end{bmatrix}^T \\ m &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}^T \end{aligned} \quad (4)$$

In Eq. (3), σ is the vector of total stresses, σ' is the vector of effective stresses, p_α is the pore air pressure and χ is the matric suction coefficient. In general, χ is determined experimentally, being a function of the degree of saturation, the porosity and the matric suction $(p_\alpha - p_w)$ (Khalili & Khabbaz, 1998). Due to lack of experimental data, χ is often assumed equal to the effective saturation S_{eff} (Bolzon, Schrefler & Zienkiewicz, 1996), which is given as:

$$S_{eff} = \frac{S - S_{res}}{S_{sat} - S_{res}} \tag{5}$$

Based on Figure 3, for suction equal to 10 kPa, saturation S equals 0.3398. This values is in agreement with PLAXIS results presented in Figure 4. The effective saturation is calculated from Eq. (5) and equals 0.3264.

The vertical effective stresses are calculated under the assumption that p_α is considered as the zero reference level. Thus, Bishop's effective stress in PLAXIS is defined as:

$$\sigma' = \sigma - mS_{eff}p_w \tag{6}$$

The total stress at the top model boundary is $\sigma_{v_{top}}$ = 0 kPa, while the total stress at the bottom model boundary is $\sigma_{v_{bottom}}$ = -40 kPa. Based on Eq. (3), the following results are obtained for the vertical effective stress σ'_v at the top and the bottom model boundaries.

$$\sigma'_{v_{top}} = -3.264 \text{ kN/m}^2 \quad \text{and} \quad \sigma'_{v_{bottom}} = -43.264 \text{ kN/m}^2 \tag{7}$$

The horizontal effective stress σ'_h is obtained as:

$$\sigma'_h = K_0\sigma'_v \tag{8}$$

in which the lateral earth pressure coefficient at rest K_0 is calculated as:

$$K_0 = \frac{\nu'}{1 - \nu'} \tag{9}$$

Based on Eqs. (8) and (9), the horizontal effective stress σ'_h at the top and the bottom model boundaries is:

$$\sigma'_{v_{top}} = -0.816 \text{ kN/m}^2 \quad \text{and} \quad \sigma'_{v_{bottom}} = -10.816 \text{ kN/m}^2 \tag{10}$$

PLAXIS results presented in Figures 5 and 6 are in perfect agreement with the calculated values above. It is concluded that Bishop's effective stresses are calculated correctly in PLAXIS.

REFERENCES

[1] Bishop, A.W., Blight, G.E. (1963). Some aspects of effective stress in saturated and partly saturated soils. Géotechnique, 13(3), 177–197.

[2] Bolzon, G., Schrefler, B., Zienkiewicz, O. (1996). Elastoplastic soil constitutive laws generalized to partially saturated states. Géotechnique, 46(2), 279–289.

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