CYLINDRICAL CAVITY EXPANSION

This document describes an example that has been used to verify that stresses are calculated correctly during the expansion of a cylindrical cavity in an elastic perfectly-plastic cohesive soil (Figure 1). Both small and large displacement calculations are performed.

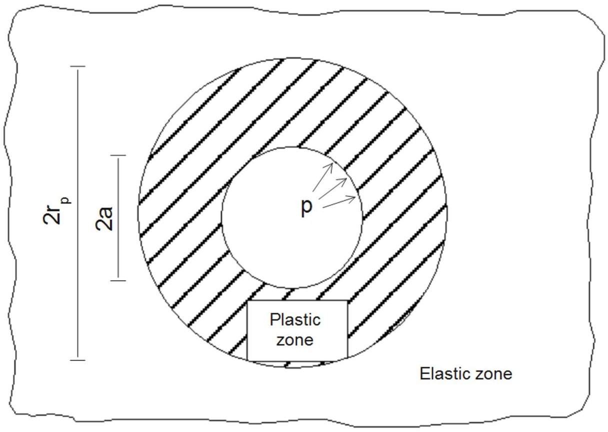

Figure 1 Cylindrical cavity expansion model

Used version:

- PLAXIS 2D - Version 2018.0

Geometry: A cylindrical cavity of initial radius a_0 is expanded to radius a under an internal pressure p which is triggered by a *Prescribed displacement*. The radius of the elastic-plastic boundary is denoted as r_p. An axisymmetric model is considered. The model geometry and the selected boundary conditions are presented in Figure 2. The value of a_0 is 1.0 m. The width of the soil layer is selected to be $64a_0$ and its thickness equals $10a_0$. Since the theoretical solution is based on an infinite continuum, a correcting material cluster is added to the soil perimeter, extended by $64a_0$.

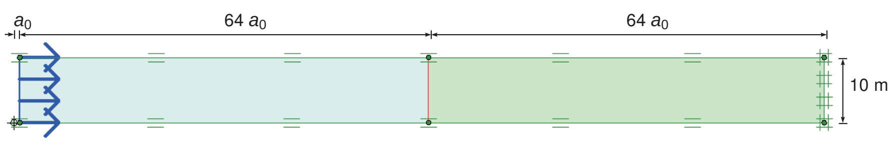

Figure 2 Model geometry and boundary conditions

Materials: The soil is assumed to be nearly incompressible with an angle of friction ϕ' equal to zero. The ratio G/c' equals 100 and the Poisson's ratio is 0.495. The *Tension cut-off* option is deactivated. Regarding the correcting layer, a Poisson's ratio of 0.25 and a Young's modulus of 5*E*'/12 are assigned, where *E*' is the soil Young's modulus. The selected material properties of the correcting layer are presented by Burd & Houlsby (1990). The adopted material parameters are:

Soil:	Mohr-Coulomb	G = 100 kN/m^2	ν' = 0.495	c' = 1.0 kN/m^2
Correcting layer:	Linear elastic	E' = 124.6 kN/m^2	ν' = 0.25	

Meshing: The *Medium* option is selected for the *Element distribution*. The left boundary of the model, where the prescribed displacement is applied, is refined with a *Coarseness factor* of 0.1. Equal refinement is selected for the boundary line between the soil and the correcting layer. The resulting mesh is shown in Figure 3.

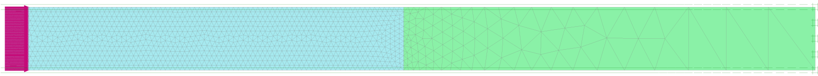

Figure 3 Generated mesh

Calculations: In the Initial phase zero initial stresses are generated by using the *K0 procedure* as *Calculation type* (γ = 0). The small displacement calculation is performed in the first phase with *Plastic analysis*. The *Reset displacements to zero* option is selected. The prescribed displacement is activated and is set equal to 4 m. The *Tolerated error* is selected equal to 0.001. The large displacement calculation is similarly defined in Phase 2, starting from the Initial phase as well. The *Updated mesh* option is selected.

Output: In order to obtain the cavity pressure p (radial stress) from PLAXIS, the computed force per unit radian acting on the cavity surface should be divided by the thickness of the soil layer times the cavity radius. Note that in the small displacement calculation the cavity radius is constant and equal to a_0. In the large displacement calculation, the cavity radius increases from a_0 to a as the calculation evolves. The obtained cavity pressure is normalized over the soil cohesion c'. The variation of p/c' over the normalized radial displacement $(a - a_0)/a_0$ is presented in Figure 4. PLAXIS results for both small and large displacements are plotted.

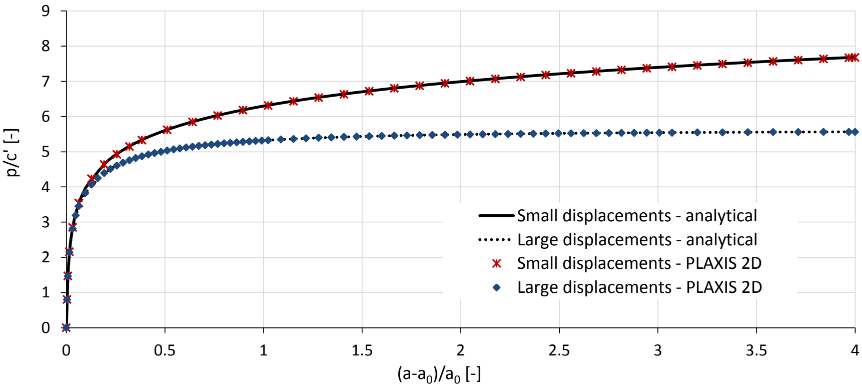

Figure 4 Variation of normalised cavity pressure over normalised radial displacement

Verification: The present problem has been studied by various researchers and a theoretical solution exists for both large and small displacements (Sagaseta, 1984). The analytical solution is obtained as:

Small displacement solution:

$$r_p^2 = 2\left(\frac{G}{c}\right) a_0 (a - a_0)$$ elasto-plastic boundary radius

$$p = \frac{2G(a - a_0)}{a_0}$$ for $r_p < a_0$

$$p = c' - 2c' \ln\left(\frac{a_0}{r_p}\right)$$ for $r_p > a_0$

Large displacement solution:

$$r_p^2 = \frac{1}{\eta_r^2}(a^2 - a_0^2)$$ elasto-plastic boundary radius

$$p = GF(\eta)$$ for $r_p < a$

$$p = GF(\eta_r) + 2c' \ln\left(\frac{\eta}{\eta_r}\right)$$ for $r_p > a$

where:

$$\eta^2 = \frac{a^2 - a_0^2}{a^2} \qquad \eta_r^2 = 1 - \exp\left(\frac{-c'}{G}\right) \qquad F(\eta) = \eta^2 + \frac{\eta^4}{4} + \frac{\eta^6}{9} + ...$$

The analytical results are presented in Figure 4. It is concluded that they are in good agreement with PLAXIS results.

With respect to the elasto-plastic boundary radius r_p when the cavity is expanded to radius a = 5 m, analytical and numerical results are compared in Table 1 and they are found to be in good agreement as well.

Table 1 Comparison between analytical and PLAXIS results regarding r_p when a = 5 m

Approach	Analytical	PLAXIS 2D	Error (%)
Small displacements	28.28 m	28.01 m	1.0
Large displacements	49.11 m	48.85 m	0.5

The plastic region (plastic points distribution) for the small and large displacement approach is illustrated in Figure 5 and Figure 6 correspondingly.

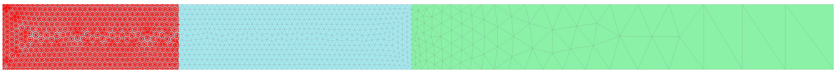

Figure 5 Plastic region: small displacement approach

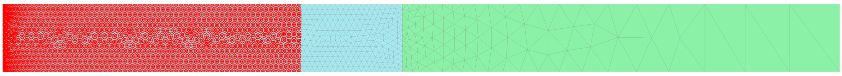

Figure 6 Plastic region: large displacement approach

REFERENCES

[1] Burd, H.J., Houlsby, G.T. (1990). Finite element analysis of two cylindrical expansion problems involving nearly incompressible material behaviour. Int. J. Num. Analyt. Meth. Geomech., 14, 351–366.

[2] Sagaseta, C. (1984). Personal communication.