ONE-DIMENSIONAL WAVE PROPAGATION

This document describes an example that has been used to verify that one-dimensional (1-D) wave propagation is correctly calculated in PLAXIS.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: A 10 m high soil column with width of 0.25 m is used to simulate the one-dimensional wave propagation. In PLAXIS 2D the problem is modelled as axisymmetric using *15-Noded* elements. A vertically downward *Line displacement* with magnitude equal to $1 \cdot 10^{-3}$ m is applied at the top boundary of the model. A *Displacement multiplier* is assigned to the y-direction in order to apply the displacement abruptly at the beginning of the dynamic calculation. Table 1 presents the selected values:

Table 1 Displacement multiplier

t [s]	Multiplier [-]
0.0	1.0

In PLAXIS 3D, the whole model is built and its dimensions in y-direction and x-direction equal 0.25 m. The prescribed displacement is modelled as *Surface displacement* and the same *Displacement multiplier* is applied in z-direction.

Three points, namely point A (model bottom), B (model middle) and C (model top), are selected for the output results. Figure 1 illustrates the model geometries in PLAXIS 2D and PLAXIS 3D. In PLAXIS 3D the points A and B are not visible as they are internal.

Two different dynamic boundary conditions are considered at the bottom of the model: (A) fully fixed and reflective boundary and (B) absorbent-viscous boundary. Both cases are examined in PLAXIS 2D and PLAXIS 3D. The corresponding models will be named as 'model A' and 'model B'.

Materials: The soil is modelled as *Linear elastic*. The adopted material parameters are:

Soil: Linear elastic γ = 19.62 kN/m^3 E' = 18000 kN/m^2 ν' = 0.2

Meshing: In both PLAXIS 2D and PLAXIS 3D, the *Very fine* option is selected for the *Element distribution*. In PLAXIS 2D the *Coarseness factor* is set equal to 1 for the whole model, while in PLAXIS 3D it is set equal to 0.5 in order to eliminate numerical dispersion. The generated mesh is illustrated in Figure 1.

Calculations: Initial stresses are generated in the Initial Phase by using the *K0 procedure* as *Calculation type*. In Phase 1, *Dynamic* analysis is performed in which the model A is considered and the dynamic bottom boundary is set to *None*. The dynamic component of the prescribed line/surface displacement is activated. A second Phase, starting from the Initial Phase, is introduced and the model B is simulated by setting the dynamic bottom boundary to *Viscous*. In both Phases 1 and 2 the dynamic top and side boundaries are set to *None*.

The *Dynamic time interval* is set equal to 0.25 s. The *Time step determination* is set to *Semi-automatic* and the *Max steps* parameter is selected to be 500. *Newmark alpha* and *Newmark beta* coefficients are set equal to 0.2652 and 0.53 correspondingly. In this way

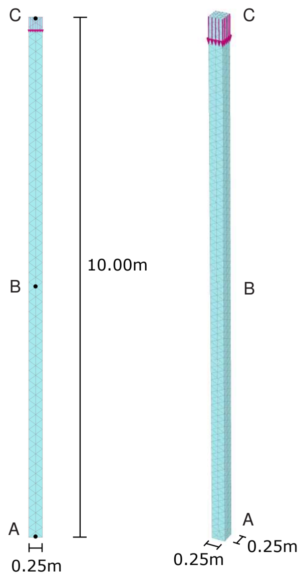

Figure 1 Model geometry in PLAXIS 2D and PLAXIS 3D

numerical damping is introduced and high frequencies due to dispersion are damped out. The *Mass matrix* value is selected equal to 1, considering a consistent mass matrix. The default values of the remaining parameters are valid.

Output: Figures 2 to 3 illustrate the model A results in PLAXIS 2D and PLAXIS 3D respectively, while Figures 4 to 5 present the corresponding results for model B. All Figures depict the vertical displacement of points A, B and C over time.

Verification: The pressure wave velocity V_p, in a confined one-dimensional medium, depends on the stiffness parameter E_{oed} and the density ρ:

$$V_p = \sqrt{\frac{E_{oed}}{\rho}} \tag{1}$$

where $$E_{oed} = \frac{(1 - \nu')E'}{(1 + \nu')(1 - 2\nu')}$$ and $$\rho = \frac{\gamma}{g} \tag{2}$$

in which E' is the Young's modulus, ν' is the Poisson's ratio, γ is the total unit weight and g is the gravity acceleration (9.81 m/s^2).

For the selected material properties, based on Eq. (1), the wave velocity V_p equals 100 m/s. Hence, it takes 0.05 s for a wave generated at the top of the model (point C) to reach the middle of the column (point B), and 0.1 s to reach its bottom (point A).

Model A: Figure 2 shows the vertical displacement over time at the top (prescribed displacement u_y=1 mm at point A), at the middle (point B) and at the bottom (fixed displacement with u_y=0 mm at point C) of the model in PLAXIS 2D. The response at points A and C is as prescribed. Point B starts to move just before 0.05 s and its displacement reaches a value of u_y=1 mm just after 0.05 s. Due to finite element discretisation and integration time step, the response at point B is not as sharp and steady as at point A. The observed perturbation is caused by numerical dispersion.

However, as an average, the displacement at point B agrees with the theory. More precisely, after being reflected at the bottom, the wave reaches point B again at about 0.15 s. As the wave propagates, the displacement diminishes again. The same phenomenon is continuously repeated as no damping has been assigned to the soil material.

The same conclusions are drawn based on Figure 3 in which the PLAXIS 3D results are illustrated.

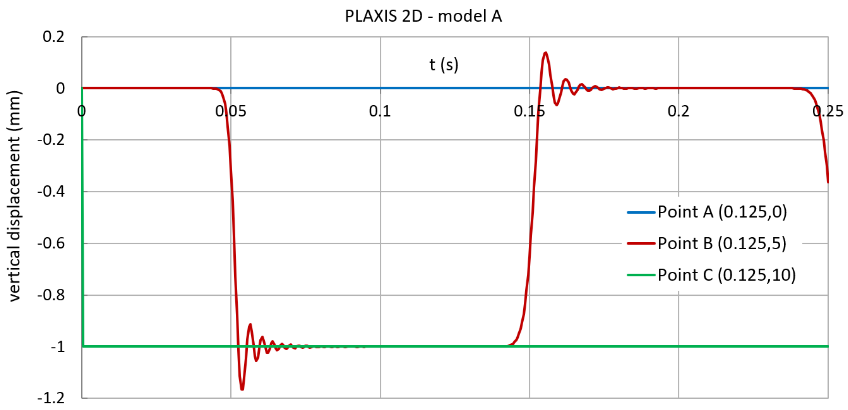

Figure 2 Vertical displacement versus time (PLAXIS 2D - model A)

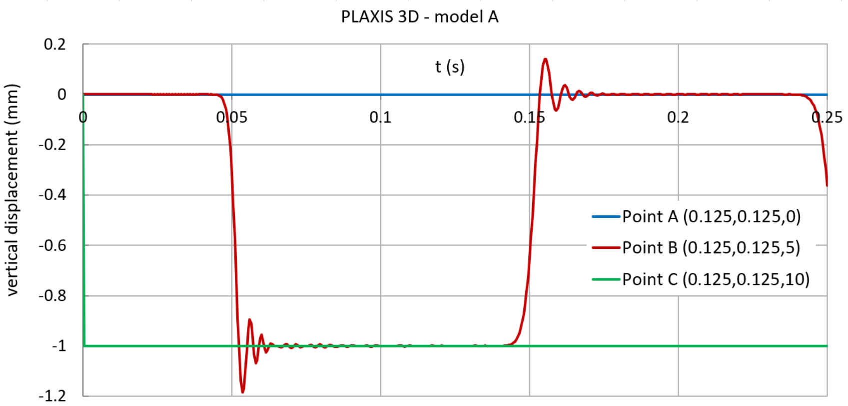

Figure 3 Vertical displacement versus time (PLAXIS 3D - model A)

Model B: Considering model B, Figure 4 shows that the bottom point A also moves downwards by 1 mm around 0.1 s and its displacement becomes steady afterwords. This is due to the absorbent boundary at the model bottom, which prevents wave reflections. This observation is in agreement with the theoretical solution for an infinitely deep soil column. The same results are observed for PLAXIS 3D in Figure 5.

In conclusion, the 1-D wave propagation in PLAXIS is in agreement with the theoretically expected behavior. Any perturbations in the response can be attributed to the numerical discretisation in space and time.

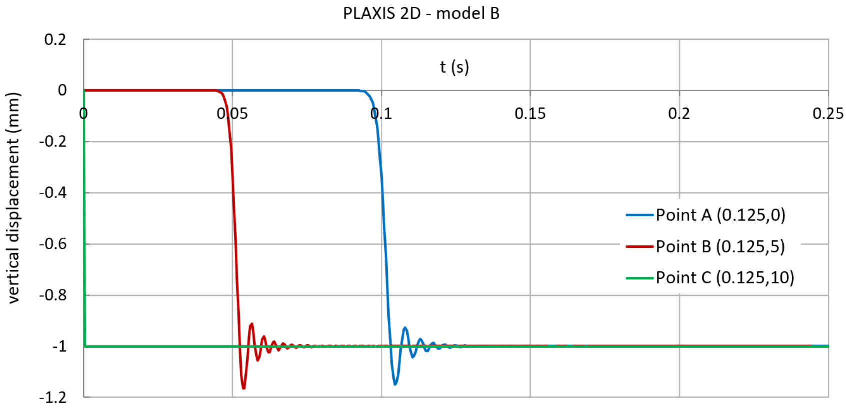

Figure 4 Time-displacement curve; viscous boundary at bottom of column (PLAXIS 2D - model B)

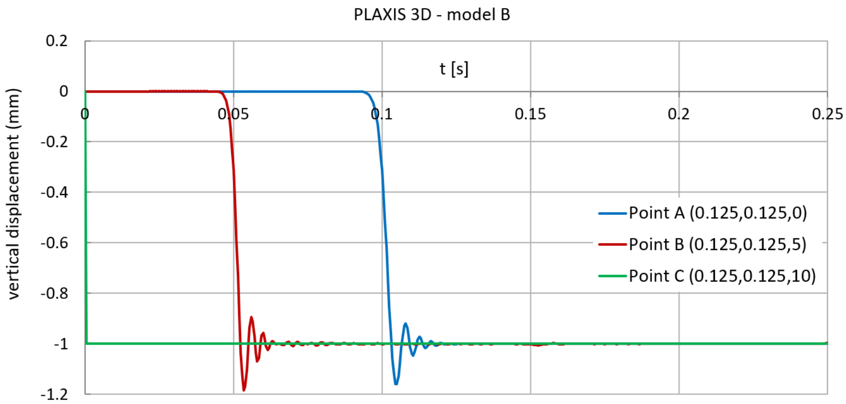

Figure 5 Time-displacement curve; viscous boundary at bottom of column (PLAXIS 3D - model B)