PERFORMANCE OF SHELL ELEMENTS

Tunnel lining can be modeled as a circular beam in PLAXIS 2D and as a shell in PLAXIS 3D. By using these elements, three types of deformations are taken into account: shear deformation, compression and bending. This document presents an example that is used to verify that shell elements performance is correctly implemented in PLAXIS.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: The *Tunnel designer* feature is used to model the problem. In both PLAXIS 2D and PLAXIS 3D, a *Plate* is assigned to the generated ring. The selected radius equals R = 1 m. The bottom point of the ring (point C) is fixed with respect to both translations and rotation, while the top point (point A) is allowed to move only in the vertical direction. A prescribed load F = 1 kN/m is assigned to the top of the ring, pointing vertically downwards. A parametric analysis is conducted on the thickness H of the ring cross section. Several values of H are considered so that rings ranging from very thin to very thick are studied: H = 0.01, 0.02, 0.05, 0.10, 0.20, 0.50 m.

In order to generate the tunnel elements a soil cluster is needed. However, because the soil is deactivated for all calculation phases, the selected material properties are arbitrary. Moreover, the model boundaries are designed close to the structure (ring) to avoid redundant elements which would lead to excessive calculation times. For further improvement, an extra soil cluster inside the ring is used to control the number of the generated mesh elements at that region.

In PLAXIS 2D, a 3$\times$3 m soil cluster is used. The extra circular cluster inside the ring has radius R_{in} = 0.75 m. *Point displacements* are used to apply the above mentioned fixities at the top and bottom of the ring. The load F is modeled as *Point load*. Figure 1 illustrates the model geometry in PLAXIS 2D.

In PLAXIS 3D, a 3$\times$3$\times$0.1 m soil volume is considered. The model depth in y-direction is selected to be 0.1 m in order to avoid generation of unnecessary elements. The extra cylindrical cluster inside the ring has radius R_{in} = 0.75 m. *Line displacements* are used to apply the appropriate fixities at points A and C, while the load F is modeled as *Line load*. Figure 2 illustrates the model geometry in PLAXIS 3D. Points A, B and C are selected to be at y = 0.05 m, thus they are not visible.

Materials: As previously mentioned the selected soil material properties are arbitrary. Regarding the *Plates*, six different material sets are used in correspondence with the various selected thicknesses H.

The adopted material parameters are:

Soil:	Linear elastic	γ = 0 kN/m^3	E' = 1 kN/m^2	ν' = 0
Plate:	Linear elastic	γ = 0 kN/m^3	E' = 10^6 kN/m^2	ν' = 0

In PLAXIS 2D, the axial (E'A) and bending (E'I) stiffness are used to define *Plates*. Table 1 presents the corresponding values.

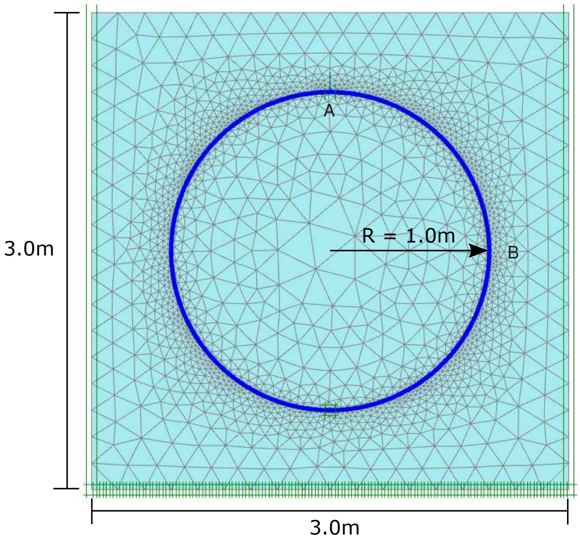

Figure 1 Model geometry (PLAXIS 2D)

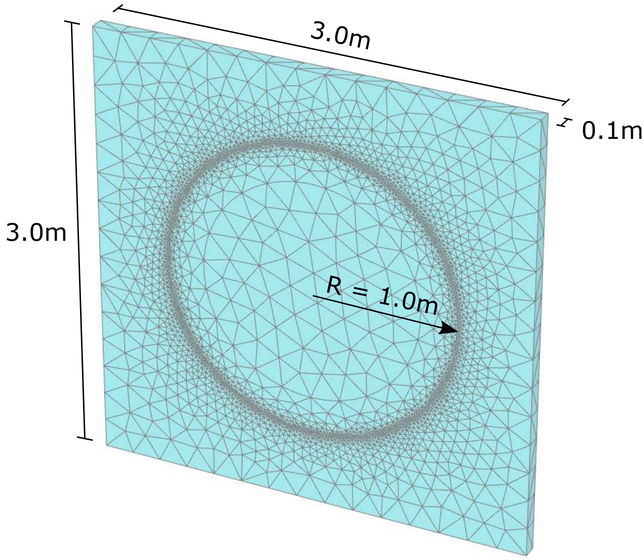

Figure 2 Model geometry (PLAXIS 3D)

Table 1 Material properties for *Plates* (PLAXIS 2D)

H (*m*)	E'A (*kN*/*m*)	I (m^4)	E'I (*kN* m^2/*m*)
0.01	$1 \cdot 10^4$	$0.8 \cdot 10^{-7}$	0.083333
0.02	$2 \cdot 10^4$	$6.7 \cdot 10^{-7}$	0.666667
0.05	$5 \cdot 10^4$	$1.042 \cdot 10^{-5}$	10.416667
0.10	$1 \cdot 10^5$	$8.333 \cdot 10^{-5}$	83.333333
0.20	$2 \cdot 10^5$	$6.6667 \cdot 10^{-4}$	666.666667
0.50	$5 \cdot 10^5$	$1.041667 \cdot 10^{-2}$	10416.6667

Meshing: The *Fine* and *Medium* options are selected for the *Element distribution* in

PLAXIS 2D and PLAXIS 3D respectively. The ring is refined with a *Coarseness factor* of 0.1 in PLAXIS 2D and 0.075 in PLAXIS 3D. The reason for generating such a fine mesh is to obtain more accurate and smooth results, especially in PLAXIS 3D. The generated mesh is illustrated in Figure 1 and Figure 2.

Calculations: Six Phases are used, all starting from the Initial Phase. *Plastic analyses* are performed. In each Phase, a different material set is assigned to the *Plate*. Apart from the soil clusters which are deactivated, every other feature mentioned above is activated. The *Tolerated error* is set equal to 0.001. The default values of the remaining parameters are valid. Geometric non-linearity (*Updated mesh*) is not taken into account. In PLAXIS 3D, both model boundaries in y-direction are set to be *Free*.

Output: As a typical example of the results, total displacements for a ring with H = 0.1 m are illustrated in Figure 3 for both PLAXIS 2D and PLAXIS 3D. For the same thickness H, the resulting normal forces N and bending moments M are presented in Figure 4 and Figure 5 correspondingly. PLAXIS 3D plots are created based on a cross section in the middle of the model in y-direction (y = 0.05 m).

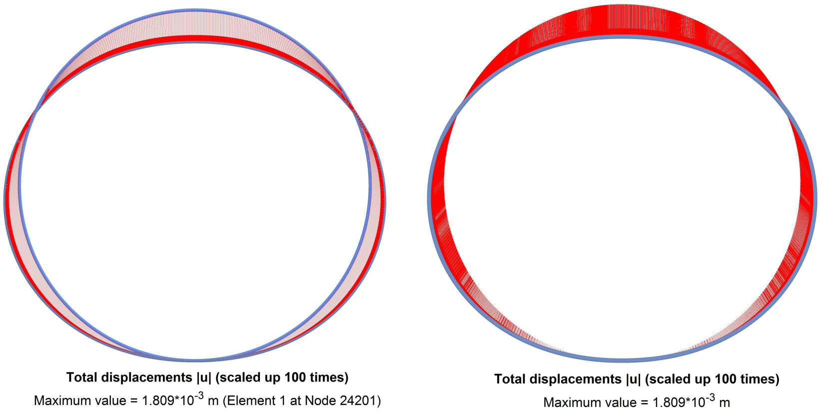

Figure 3 Total displacements for H = 0.1 m in PLAXIS 2D (left) and PLAXIS 3D (right)

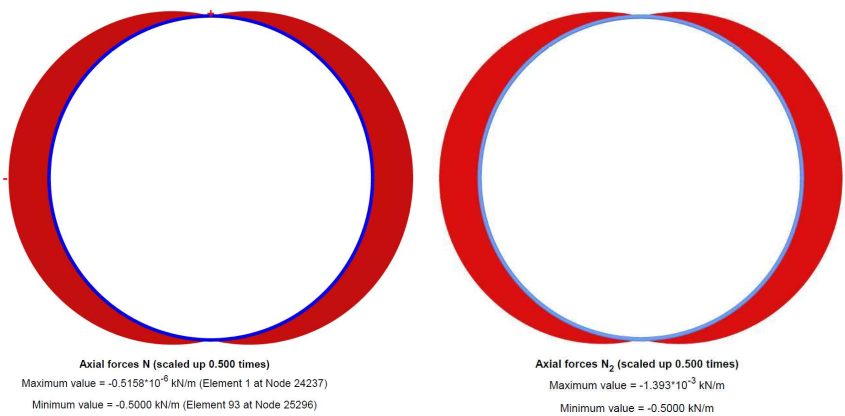

Figure 4 Resulting normal forces N for H = 0.1 m in PLAXIS 2D (left) and PLAXIS 3D (right)

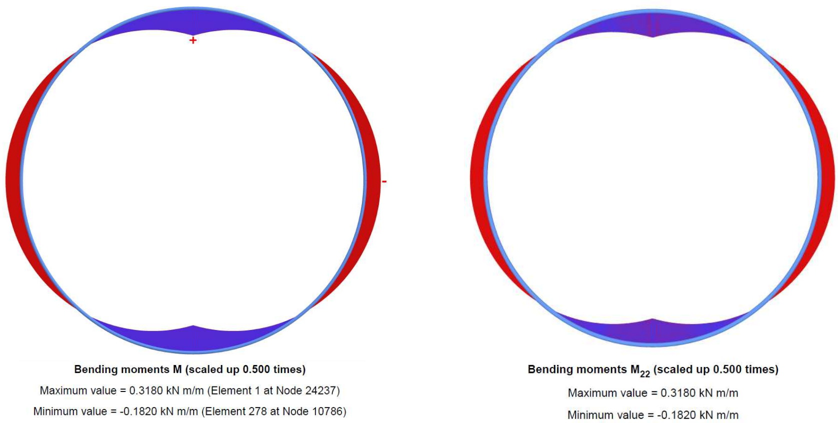

Figure 5 Resulting bending moments M for H = 0.1 m in PLAXIS 2D (left) and PLAXIS 3D (right)

Verification: The analytical solution for the deflection of the ring at point A is given by Blake (1959), and the solution for the bending moment and the normal force at point B can be found based on Roark (1965). The vertical displacement δ at the top of the ring (point A) is given by the following formula:

$$\frac{F\lambda}{E'}\left[1.788\,\lambda^2 + 3.091 - \frac{0.637}{1+12\,\lambda^2}\right] \qquad \text{where} \qquad \lambda = \frac{R}{H}$$

Vertical displacements δ normalized over the ratio F/E' at point A are plotted against the thickness ratio $1/\lambda$ in Figure 6. Results from both PLAXIS 2D and PLAXIS 3D are compared with the analytical solution.

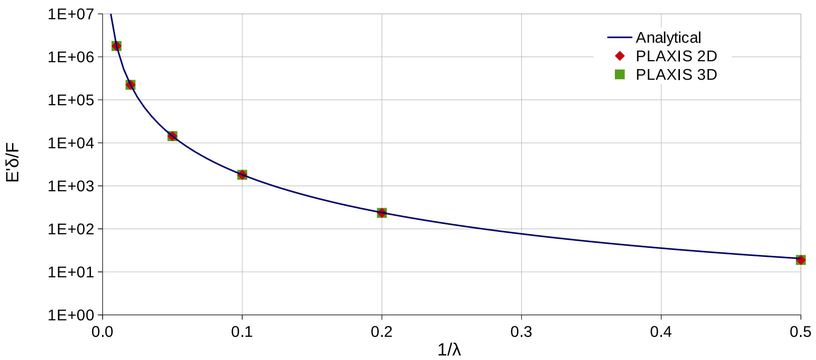

Figure 6 Normalized vertical displacements δ at point A against thickness ratio $1/\lambda$

Table 2, Table 3 and Table 4 present the results obtained from the analytical solution and PLAXIS, with respect to the vertical displacement at point A, the axial force N at point B and the bending moment M at point B. For PLAXIS 3D results a cross section at the middle of the model in y-direction is considered (y = 0.05 m). The resulting relative error increases for a very thick plate with $H = 1/2R$. However, as Figure 6 illustrates, even in this unrealistic case, PLAXIS results are quite accurate. It is concluded that shell elements are correctly implemented in PLAXIS and their performance is in good agreement with the analytical solution.

Table 2 Comparison between analytical solution and PLAXIS results regarding the vertical displacement at point A

Thickness	Vertical displacement at point A (m)			Error	
H (m)	Blake	PLAXIS 2D	PLAXIS 3D	PLAXIS 2D	PLAXIS 3D
0.01	1.7883091	1.78557868	1.78556483	- 0.2 %	- 0.1 %
0.02	0.2236545	0.22328621	0.22328433	- 0.2 %	- 0.2 %
0.05	0.0143658	0.01433017	-0.014330065	- 0.2 %	- 0.2 %
0.10	0.0018189	0.00180905	0.00180903	- 0.5 %	- 0.5 %
0.20	0.0002389	0.00023499	0.00023499	- 1.7 %	- 1.7 %
0.50	0.0000205	0.00001892	1.89232E-5	- 7.5 %	- 7.5 %

Table 3 Comparison between analytical solution and PLAXIS results regarding the axial force N at point B

Thickness	Axial force N at point B (kN/m)			Error	
H (m)	Blake	PLAXIS 2D	PLAXIS 3D	PLAXIS 2D	PLAXIS 3D
0.01	0.5000	0.5000	0.5107	0.0 %	+ 0.1 %
0.02	0.5000	0.5000	-0.5026	0.0 %	0.0 %
0.05	0.5000	0.5000	0.5004	0.0 %	0.0 %
0.10	0.5000	0.5000	0.5001	0.0 %	0.0 %
0.20	0.5000	0.5000	0.5000	0.0 %	0.0 %
0.50	0.5000	0.5000	0.5000	0.0 %	0.0 %

Table 4 Comparison between analytical solution and PLAXIS results regarding the bending moment M at point B

Thickness	Bending moment M at point B (kNm/m)			Error	
H (m)	Blake	PLAXIS 2D	PLAXIS 3D	PLAXIS 2D	PLAXIS 3D
0.01	0.1817	0.1817	0.1817	0.0 %	0.0 %
0.02	0.1817	0.1817	0.1817	0.0 %	0.0 %
0.05	0.1817	0.1818	0.1818	+ 0.1 %	+ 0.1 %
0.10	0.1817	0.1820	0.1820	+ 0.2 %	+ 0.2 %
0.20	0.1817	0.1828	0.1828	+ 0.6 %	+ 0.6 %
0.50	0.1817	0.1883	0.1883	+ 3.6 %	+ 3.6 %

REFERENCES

[1] Blake, A. (1959). Deflection of a thick ring in diametral compression. Am. Soc. Mech. Eng., J. Appl. Mech., 26(2).

[2] Roark, R.J. (1965). Formulas for Stress and Strain. McGraw-Hill Book Company.