Soil-Water Characteristic Curve Variability

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Abstract

The most widely-used constitutive relations for unsaturated soils use matric soil suction as a state variable. The soil-water characteristic curve (SWCC), the relationship between soil suction and some measure of the water content, can be measured or predicted based on soil index properties such as the grain-size distribution (GSD) function. Estimation based on index properties is highly desirable due to its simplicity and low cost and would be the path of choice to the SWCC, provided the accuracy of the estimate were adequate. Whether measured or estimated, there is variability and uncertainty associated with the SWCC that in turn will directly impact any model for the unsaturated soil behavior that makes use of this relationship. The variability in the SWCC associated with direct suction measurements as well as the variability associated with the prediction of the SWCC based on GSD was investigated. Three different soils that cover a typical range of soils encountered in practice were used in this study. The investigated sources of variability related to direct suction measurements included the equations used to fit the suction data, the different methods available to measure suction, the operator, the number of data points used to define the SWCC, and the range of suction covered in the measurements. The variability observed in the SWCC when different predictive algorithms are used is also reported. Finally, a new model for predicting the SWCC based on soil index properties is presented, using a database of 190 soils. Results showed that the operator as well as the range of suction covered in the measurements produced significant variability in the measured SWCC. Surprisingly, the variability in the SWCC as predicted by soil index properties and/or GSD-based algorithms was found to be as small or smaller than that associated with the operator (person(s) measuring the SWCC).

Introduction

The soil-water characteristic curve (SWCC) is the relationship between soil water suction and the water content of the soil. The form of water content most often used when soil water suction is initially measured is gravimetric water content. However, when published, the most commonly used form of water content is volumetric water content (Houston et al., 1999). The degree of saturation, S, is also used sometimes as a measure of water content for the SWCC.

The suction used for the SWCC is usually the matric suction, u_a-u_w, but total suction is occasionally used as well. The SWCC directly relates to the shear strength, volume change, and fluid flow characteristics of unsaturated soils. It is now generally

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accepted that the behavior of an unsaturated soil cannot be related to a single effective stress variable. Rather, the total normal stress, σ_n, the pore-air pressure, u_a, and the pore-water pressure, u_w, must be combined in two independent stress state parameters. The two stress variables usually selected are the net normal stress $(\sigma_n - u_a)$, and the matric suction $(u_a - u_w)$ (Fredlund and Morgenstern,1977). The SWCC is commonly used together with the water content to estimate soil suction for use in the calculation of the state variable. Therefore, uncertainty in the SWCC relationship leads to variability in predicting unsaturated soil behavior. Despite the well-recognized importance of suction, it has not become routinely measured in geotechnical laboratories or in the field. An investigation of practice throughout the United States, conducted as a part of the present study, showed that less than 20% of commercial geotechnical laboratories performed suction measurements on a regular basis (Zapata, C., 1999). It also appears that much less than 20% of the laboratories of practicing geotechnical consultants perform suction measurements on a routine basis. This fact suggests that the geotechnical engineering profession would rely on empirically derived methods for estimating the SWCC, if the estimations were of satisfactory accuracy.

To encourage geotechnical engineers to implement unsaturated soil mechanics theory in routine practice, it is necessary to evaluate to what degree of accuracy the SWCC can be ascertained using various approaches. To quantify the variability associated with the determination of the SWCC, it is necessary to also quantify the uncertainty associated with direct suction measurements. This uncertainty will serve as a benchmark to decide whether or not the variability in any given prediction is acceptable. If, for example, the uncertainty associated with the predicted SWCC were comparable to that associated with the directly measured SWCC, then one might be prone to embrace the predictive algorithm.

The objectives of the studies described in this paper were:

1) To evaluate the variability in the SWCC.
2) To develop a new relationship between the SWCC and soil index properties.

Background

The development of a SWCC for a particular soil from suction measurements can require several tests and can take some time to obtain all the necessary data. In order to facilitate the efficient determination of the SWCC, several mathematical models have been developed to describe the SWCC for a particular soil from just a few points. The second important reason why we fit functions to the experimental SWCC raw data is that many applications of the SWCC require that it be differentiated or integrated and be continuous. Therefore, a set of data for a SWCC without a function fitted to it is of limited use. Some of the fitted functions that have been proposed are summarized in Table 1. Generally, the equations in Table 1 have been validated for certain soils and ranges in suction. The equations with three and four parameters seem to be the more suitable to represent the SWCC (Leong and Rahardjo, 1996). However, at this point it is not possible to state that a particular equation is capable of best-fitting every soil encountered. The process of fitting experimental suction data to one of the proposed equations requires a minimum number of experimentally obtained suction measurements, depending upon the number of unknown parameters in the chosen function.

Table 1. Soil-Water Characteristic Curve Equations Used to Fit Experimental Data

Reference	Equation	Parameter Description
Fredlund and Xing(1994) (F&X)	$\theta_w = C(h) \times \left[\frac{\theta_s}{\left[\ln\left[\exp(1) + \left(\frac{h}{a}\right)^b \right] \right]^c} \right]$ ...(1) $C(h) = \left[1 - \frac{\ln\left(1 + \frac{h}{h_r}\right)}{\ln\left(1 + \frac{10^6}{h_r}\right)} \right]$ ....(2)	θ_w volumetric water content a = a soil parameter which is primarily a function of the air entry value of the soil in kPa. b = a soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded. c = a soil parameter which is primarily a function of the residual water content. h_r = a soil parameter which is primarily a function of the suction at which residual water content occurs in kPa.
van Genuchten (1980) (van G.)	$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{h}{a}\right)^b \right]^c}$ ....(3)	θ_r = residual volumetric water content. a = a soil parameter which is primarily a function of the air entry value of the soil in kPa. b = a soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded. c = a soil parameter which is primarily a function of the residual water content.
McKee and Bumb (1987)	$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{1 + \exp(1)^{\left[\frac{(h-a)}{b}\right]}}$ ....(4)	θ_r = residual volumetric water content. a = curve-fitting parameter b = curve-fitting parameter
van Genuchten and Mualem (1980) (van G.&M)	$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{h}{a}\right)^{bm} \right]^{\left(1 - \frac{1}{bm}\right)}}$ ....(5)	θ_r = residual volumetric water content. a = a soil parameter which is primarily a function of the air entry value of the soil in kPa. b_m = a soil parameter which controls the slope at the inflection point in the soil-water characteristic curve.
van Genuchten and Burdine (1980) (van G.&B)	$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{h}{a}\right)^b \right]^{\left(1 - \frac{2}{b}\right)}}$ ....(6)	θ_r = residual volumetric water content. a = a soil parameter which is primarily a function of the air entry value of the soil in kPa. b = a soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded.

Table 1. Continued

Reference	Equation	Parameter Description
Gardner (1958)	$\theta_w = \theta_r + \dfrac{\theta_s - \theta_r}{1 + \left(\dfrac{h}{a}\right)^b}$ ....(7)	θ_r = residual volumetric water content. a = a soil parameter which is primarily a function of the air entry value of the soil in kPa. b = a soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded.
Brooks and Corey (1964) (B&C)	$\theta_w = \theta_r + (\theta_s - \theta_r)\left(\dfrac{a_b}{h}\right)^{b_b}$ ....(8)	θ_r = residual volumetric water content. a_b = bubbling pressure in kPa. b_b = pore size index.
Williams et al. (1983)	$\ln \Theta_e = A + B \ln h$ ....(9)	A = fitting parameter B = fitting parameter
Farrel and Larson (1972)	$h = (u_a - u_w)_b \exp[\alpha(\theta_s - \theta_w)]$ ....(10)	α = empirical constant $(u_a - u_w)_b$ = air-entry value
Assouline et al. (1998)	$\theta_w = \theta_L + (\theta_s - \theta_L)\left[1 - \exp\left[-\xi\left(\dfrac{1}{\psi} - \dfrac{1}{\psi_L}\right)^{\eta}\right]\right]$ ....(11)	Ψ = capillary head Ψ_L = capillary head that corresponds to a very low water content, at which the hydraulic conductivity is negligible. θ_L = volumetric water content at capillary head Ψ_L. η = fitting parameter ξ = fitting parameter

Soil suction measurement methods can be generally classified as either direct or indirect. Direct methods include pressure plates, pressure membranes, and tensiometers. These methods measure the pore pressure in the soil or impose a known pressure to the soil and allow the water content to come to equilibrium with the imposed pressure. Indirect methods include filter paper, porous blocks, and heat dissipation sensors. These methods use measurements or indicators of water content or a physical property that is sensitive to a change in water content (e.g., water content in the filter paper method, electrical resistance, and thermal conductivity). Before suction measurements are taken, a baseline calibration run is made and all suction measurements are based on this calibration. Thorough descriptions of methods to measure soil suction can be found in Ridley and Wray (1995), and Lee and Wray (1995).

Another option for obtaining the SWCC is to predict it, based on the GSD and other soil properties. The vast majority of the proposed predictive algorithms can be classified into three major approaches. The first of these approaches is based upon statistical estimation of water contents at selected matric suction values. These water contents, at each suction value, are correlated to soil properties. This process generally requires a regression analysis followed by a curve fitting procedure. The final product may or may not involve an equation describing the SWCC. This approach has been followed by several researchers including Visser (1969), Gupta and Larson (1979), Rawls t al. (1982), Cassel, Ratliff, and Ritchie (1983), Hutson and Cass (1987), Rawls, Gish, nd Brakensiek (1991), Rajkai, K. and Varallyay, Gy. (1992), Rawls and Brakensiek 1982), Gregson et al. (1987), Williams et al. (1992), Livneh, M., Kinsky, J., and Zaslavsky, D. (1970), Reddi, L. and Poduri, R. (1997), Aubertin, M., Ricard, J., and Chapuis, R. (1998), and Mbagwu, J. and Mbah, C. (1998).

The second approach includes those methods that correlate, by regression analysis, soil properties with the fitting parameters of an analytical equation that represents the SWCC. This statistical approach has been followed by Ghosh, R. (1980), Williams et al. (1983), Ahuja et al. (1985), Rawls, W., Ahuja, L., and Brakensiek, D. (1992), Cresswell, H. and Paydar, Z. (1996), and Tomasella, J. and Hodnett, M. (1998), as well as others.

Although the first two approaches both use a statistical correlation between soil texture and other soil data and water contents, they differ in the parameters which are correlated with soil properties. van Genuchten and Leij (1992) suggest that the first of these two approaches should be preferred over the second for reasons of simplicity.

The third approach includes the methods that estimate the SWCC using a physics-based conceptual model. It involves physical models based upon the conversion of the GSD (textural information) into a pore-size distribution, which in turn is related to a distribution of water contents and associated pore pressures. This approach was first presented by Arya, L. and Paris, J. in 1981, and followed by Haverkamp, R. and Parlange, J. (1986); Mishra, S., Parker, J. C., and Singhal, N. (1989); Arya and Dierolf (1992); Smettem, K. and Gregory, P. (1996); Basile, A. and D'Urso, G. (1997), and Fredlund, M. et al. (1997), among others. This approach has generated considerable interest, although there is still a need for improvement in the representation of the pore space.

For those models that use statistical correlation (approaches one and two above) the quality of the results obtained obviously depends heavily on the database use to fit and test the equations. Those models using a physical conceptual base are generally limited to soils with large pores, namely sandy soils, and to low matric suction ranges. The predictions for fine-grained soils remain rather unreliable. It seems that the representation of the pore-size distribution (PSD) is the most critical factor in formulating relationships between pore and particle radii, which in turn are used to estimate the SWCC.

Comparison of the different models can be found in van Genuchten and Leij (1992); Williams, R. D. and Ahuja, L. R. (1992); Kern, J. S. (1995); and Nandagiri, L. and Prasad, R. (1997), among others.

Variability in the SWCC

The first research objective is subdivided into two parts. Part 1 focuses on factors that are believed to cause variability in the SWCC when experimental measurements of suction are used. Part 2 is directed toward assessing the variability in the SWCC when it is predicted from correlations with soil index properties or when first principles are used to predict its position. The approaches used under Part 2 are based on either empirical relationships or surface tension and calculated radii of menisci.

Part 1: Variability in the SWCC when Estimated from Direct Suction Measurements

Variability should be expected when direct measurement of matric suction is used to define the SWCC because it is very difficult to measure suction with precision. Several factors may contribute to this variability, including:

1) The equation used to fit the experimental data
2) The operators' ability and experience
3) The number of data points used to define the SWCC
4) The range in suction covered by the actual measurements
5) The method used to acquire matric suction data

Soil Characterization

In an effort to quantify the variability in SWCC, a soil characterization program was conducted for three different soils. The soils were selected to cover a typical range of soils frequently encountered in engineering practice. The three test soils are referred to as El Paso sand, Price Club silt, and Fountain Hills clay. Each soil was thoroughly mixed and split in such a way that all sub-samples were representative and very close to identical.

Soil characterization involved sieve analysis, hydrometer analysis, Atterberg limits, and specific gravity of solids. Grain-size distribution curves for the three soils are shown in Figure 1. The soils were classified as a poorly graded sand, SP (El Paso), a sandy-low plasticity silt, ML (Price Club), and a silty-highly plastic clay, CH (Fountain Hills). The results from the soil characterization tests are summarized in Table 2.

Matric Suction Data Acquisition

About 80 commercial geotechnical and agricultural laboratories and educational institutions that performed soil testing on a regular basis were contacted and surveyed. Survey results showed that 14 institutions were willing to perform soil matric suction measurements for a SWCC, using at least one method. The most commonly used methods were the pressure plate, the pressure membrane, and the buried filter paper.

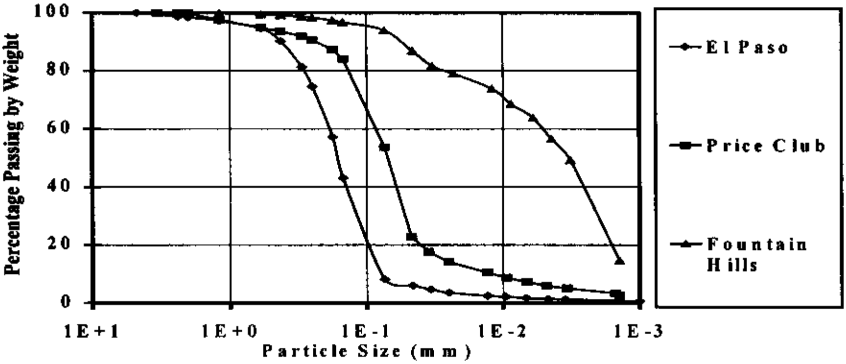

Figure 1. Grain-Size Distribution Curves

Table 2. Soil Characterization

Characteristics	**El Paso**	**Price Club**	**Fountain Hills**
Grain Size Distribution			
Percent sand	91.9	46.4	6.1
Percent silt	6.7	47.6	34.7
Percent clay	1.4	6.0	59.2
Smallest Particle Size (mm)	1e-4	1e-5	1e-5
Atterberg Limits			
Liquid Limit, LL (%)	---	22	70
Plastic Limit, PL (%)	NP	18	29
Plasticity Index, PI (%)	---	4	41
Specific Gravity of Solids, G_s	2.65	2.73	2.77
Dry Density, ρ_d (gm/cm^3)	1.39	1.33	1.14
USCS Classification	SP	ML	CH

Those facilities that used pressure plate extractors exclusively had limitations in the range of suction they could provide. Generally, pressure plate and pressure membrane measurements were limited to 1500 kPa due to the air-entry value of the ceramic plates or membranes used. Thus, the full range of suction was not covered when pressure plates and membranes were used exclusively. On the other hand, laboratories that measured matric suction by means of buried filter paper could give a wider range of measured values.

The three soils were sent to nine different laboratories chosen from those contacted to acquire the matric suction information. In addition, two more data sets were acquired in our laboratory by different operators using different suction determination methods. Table 3 is a summary of the data reported by each laboratory. Relationships between matric suction and volumetric water content reported by the different laboratories are shown in Figures 2, 3, and 4 for El Paso sand, Price Club silt, and Fountain Hills clay, respectively.

A preliminary data analysis was performed in order to identify outlying data. Based on statistical considerations and visual inspection, the data provided by laboratories 2, 10, and 11 for El Paso sand were considered outliers and were eliminated from further consideration. For Price Club silt, data from laboratories 2 and 10 were also eliminated from the analysis, while for the Fountain Hills clay, the statistical analysis did not identify any particular outlier. The reduction of the database resulted in retention of 43, 40, and 43 data points, for the sand, silt, and clay, respectively.

Best-Estimate SWCC and its Confidence Band

To perform variability studies, it was deemed necessary to fit the suction data with a continuous function. Based on previous studies (e.g., Leong and Rahardjo, 1996), seven equation were chosen to fit the data from those listed in Table 1. The equations are:

1) Equations (1) and (2) by Fredlund and Xing (F&X)
2) Equation (1) by Fredlund and Xing with C(h) = 1
3) Equation (3) by van Genuchten (van G.)

4) Equation (5) by van Genuchten and Mualem (van G.&M)
5) Equation (6) by van Genuchten and Burdine (van G.&B)
6) Equation (7) by Gardner, and
7) Equation (8) by Brooks and Corey (B&C)

Table 3. Data Collected from Each Laboratory

Lab #	**Number of Suction Points**			**Dry Density (gm/cm^3)**			**Method**
	El Paso	**Price Club**	**Fountain Hills**	**El Paso**	**Price Club**	**Fountain Hills**	
Lab 1	4	4	4	1.39	1.33	1.11 1.02	pp
Lab 2	2	2	2	1.38	1.39	1.19	pp
Lab 3	3	3	3	1.31	1.43	1.01	pp
Lab 4	1	3	4	1.38	1.28	1.18	fp
Lab 5	12	10	7	1.38	1.39 1.28	1.18	fp
Lab 6	13	7	6	1.38	1.28	1.18	pm
Lab 7	3	3	4	1.36 1.23 1.20	1.37 1.10 1.32	0.92 0.94 1.19 1.21	fp
Lab 8	4	4	5	1.25 1.45 1.57 1.60	1.39 1.43 1.50 1.73	1.25 1.16 1.13 1.26 1.17	fp
Lab 9	3	3	3	1.54	1.48	1.12	pp
Lab 10	5	5	5	1.38	1.28	1.18	fp
Lab 11	3	3	---	1.38	1.28	---	fp

pp = pressure plate; pm = pressure membrane; fp = filter paper

The fit to the experimental data was performed by an iteration process that minimized the sum of the squared residuals (SS_E). Once the best-estimate SWCC was found for each soil, its 95% confidence band (95% c.b.) was determined. The confidence band is associated with the fitting process and represents the uncertainty in the fitting parameters of the equations used to represent the data. Those interested in the details of the procedure for finding the confidence band for the response function at any abscissa are referred to Bates and Watts (1988), Mishra et al. (1989), and Huet et al. (1996).

The best-estimate SWCCs and their confidence bands are depicted in Figures 5, 6, and 7, for the sand, the silt, and the clay, respectively. The equation proposed by Fredlund and Xing (eq. 1) was found to best fit the data for the sand, while the van Genuchten (eq. 3) was found to be the best fit to the experimental data for the silt and the clay, although the Fredlund and Xing equation was a fairly close second for these soils as well.

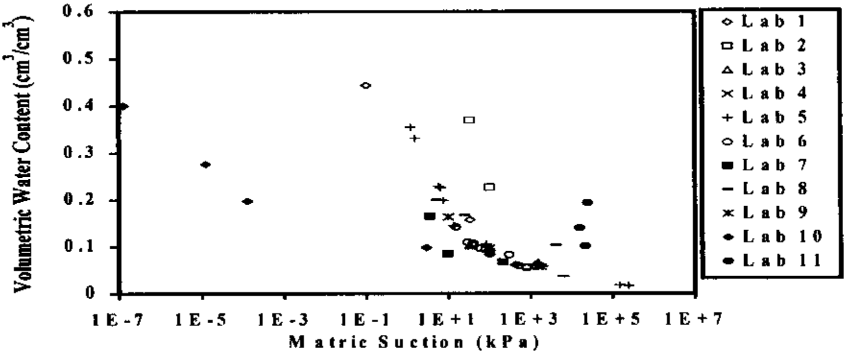

Figure 2. Matric Suction Experimental Results for El Paso Sand

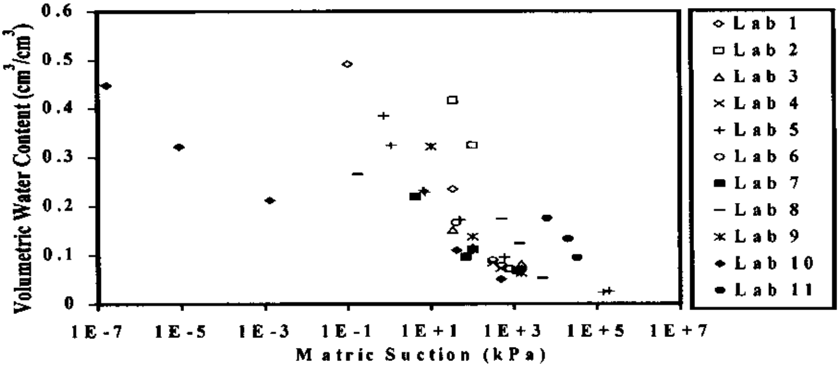

Figure 3. Matric Suction Experimental Results for Price Club Silt

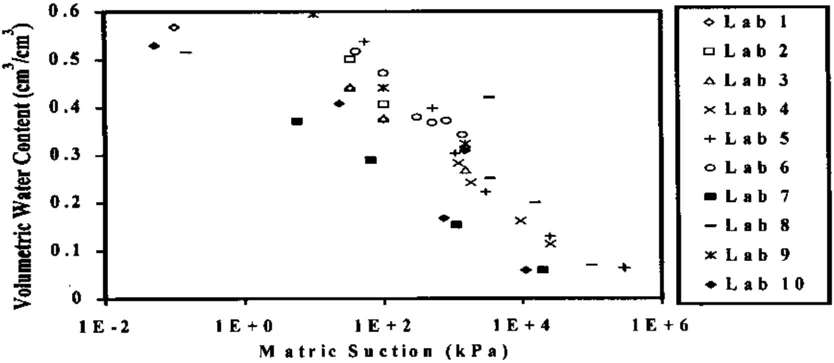

Figure 4. Matric Suction Experimental Results for Fountain Hills Clay

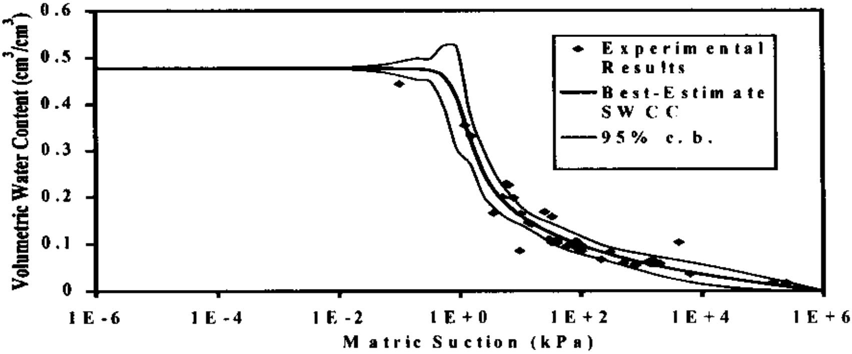

Figure 5. Best-Estimate SWCC and Confidence Band - El Paso Sand

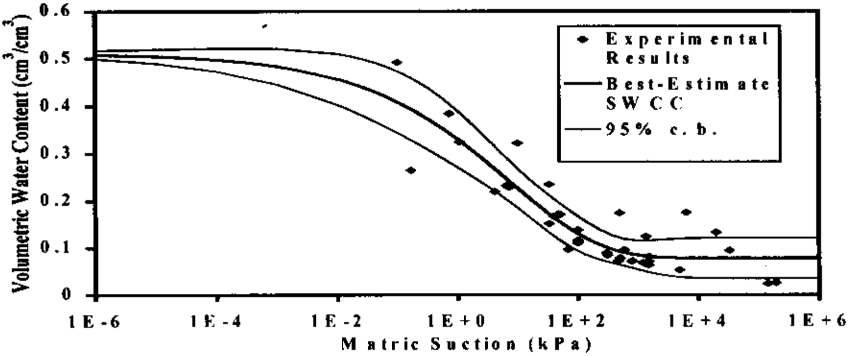

Figure 6. Best-Estimate SWCC and Confidence Band - Price Club Silt

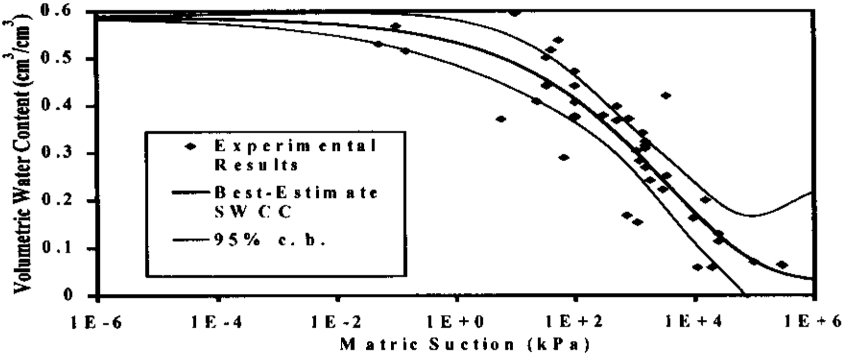

Figure 7. Best-Estimate SWCC and Confidence Band – Fountain Hills Clay

Variability in the SWCC as a Function of the Type of Equation Used to Fit Experimental Data

In practice, an engineer is unlikely to examine seven different equations to fit experimental suction data. Therefore, it may be useful to know what error can be expected from choosing a single equation that may not give the *best* fit to the data. For this purpose, the seven equations previously discussed were fitted to the data obtained for the three soils. The range of variability in water content determination related with the type of equation used to fit the experimental data is depicted in Figures 8, 9, and 10 for El Paso sand, Price Club silt, and Fountain Hills clay, respectively.

To quantify the variability around the best estimate value, the predicted volumetric water contents obtained by each equation were compared with the ordinates from the best-fit equation, at each level of suction. The absolute difference of those two values was plotted as a percentage of the saturated volumetric water content in Figures 11, 12, and 13, corresponding to El Paso sand, Price Club silt, and Fountain Hills clay, respectively. The variability was computed as follows:

$$\%\ \text{Variability} = \frac{\left|\hat{\theta}_i - \overline{\theta}_i\right|}{\theta_{sat}} \times 100\% \qquad (12)$$

Where:

$\hat{\theta}_i$ = predicted volumetric water content at i suction level, for the equation being tested

$\overline{\theta}_i$ = best estimate volumetric water content at i suction level, from the best fit equation

θ_{sat} = saturated volumetric water content

Notice that the variability is measured in terms of degree of saturation of the soil, based on the assumption that the volume change of the soil structure is negligible. This definition of variability allows for comparison of different materials, as well as for extrapolation to materials similar to the ones in this study. To evaluate the magnitude of the variability, the 95% confidence band for the best-estimate SWCC was plotted in each figure, also as a percentage of the saturated volumetric water content.

A summary of the results is presented in Table 4. Among the four-parameter equations, the one proposed by van Genuchten had the best overall performance for the range of soils studied, while the Fredlund and Xing equation, using a correction factor equal to 1, performed the best among the three-parameter equations. The rest of the equations had an acceptable performance except for the Brooks and Corey equation, which deviated substantially near the air-entry region for the three soils (see bold numbers in Table 4).

For El Paso sand, the predicted θ_w given by the different equations deviated from the one given by Fredlund and Xing. The Fredlund and Xing equation provided the best fit for the El Paso sand and it forces the curve to zero volumetric water content at 1,000,000 kPa. Fredlund and Xing (1994) provide rather convincing arguments that all curves should have essentially zero water content at 1,000,000 kPa, perhaps the most convincing of which is the fact that the relative humidity at oven dryness is extremely low and corresponds approximately to

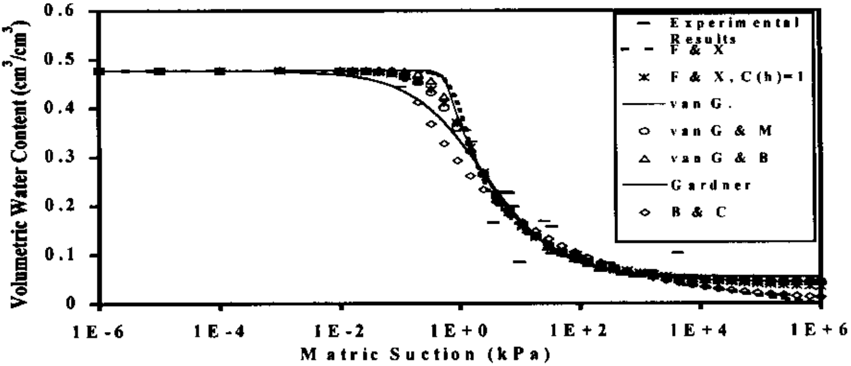

Figure 8. El Paso Sand Data Fitted with Different Equations

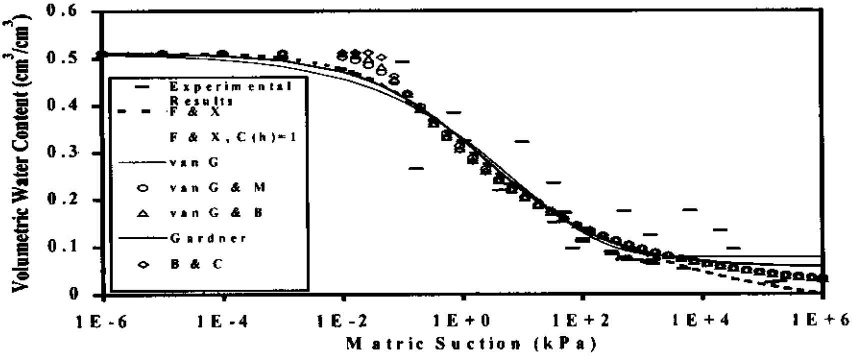

Figure 9. Price Club Silt Data Fitted with Different Equations

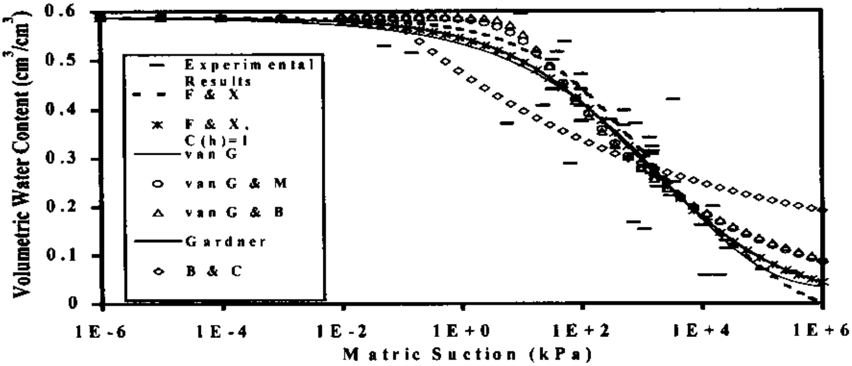

Figure 10. Fountain Hills Clay Data Fitted with Different Equations

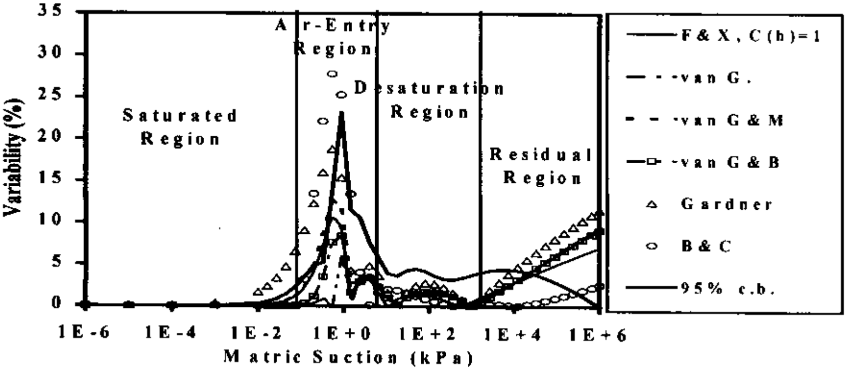

Figure 11. Variability due to Different Equations - El Paso Sand

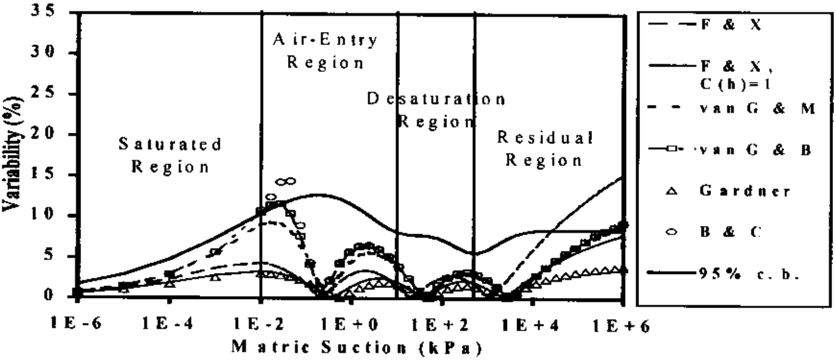

Figure 12. Variability due to Different Equations – Price Club Silt

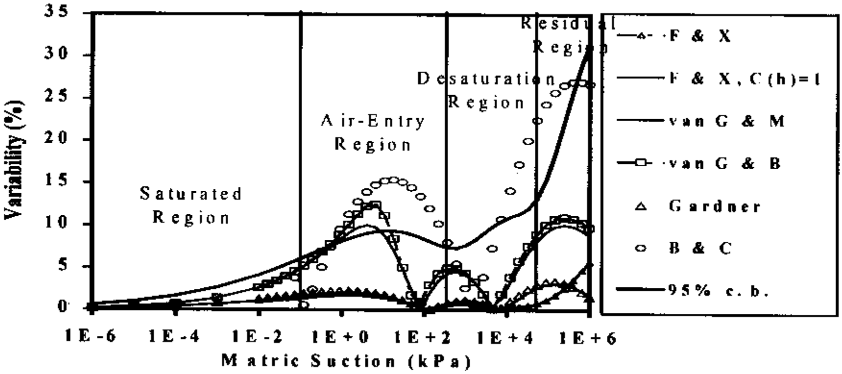

Figure 13. Variability due to Different Equations – Fountain Hills Clay

Table 4. Maximum Percentage of Variability due to Different Equations

Equation	Saturated Region	Air-Entry Region	Desaturation Region	Residual Region
		El Paso Sand		
95% Confidence Band	2	23	6	4
Fredlund & Xing	0	0	0	0
Fredlund &. Xing, C(h)=1	1	11	2	**7**
van Genuchten	0	6	2	**9**
van Genuchten & Mualem	1	9	3	**9**
van Genuchten & Burdine	0	**8**	2	**9**
Gardner	**7**	19	**7**	**12**
Brooks and Corey	0	**28**	2	3
		Price Club Silt		
95% Confidence Band	10	13	8	8
Fredlund & Xing	4	4	3	**15**
Fredlund &. Xing, C(h)=1	3	3	2	8
van Genuchten	0	0	0	0
van Genuchten & Mualem	9	9	4	**9**
van Genuchten & Burdine	10	12	4	**9**
Gardner	3	3	2	4
Brooks and Corey	**11**	**14**	4	**9**
		Fountain Hills Clay		
95% Confidence Band	6	9	12	31
Fredlund & Xing	2	2	1	6
Fredlund &. Xing, C(h)=1	2	2	2	3
van Genuchten	0	0	0	0
van Genuchten & Mualem	5	**10**	7	10
van Genuchten & Burdine	5	**12**	**8**	11
Gardner	2	2	2	3
Brooks and Corey	4	**15**	**20**	27

1,000,000 kPa. Studies described in the latter part of this paper show that SWCCs passing through 1,000,000 kPa at zero water content fit very well for essentially all sands and fairly well for most silts and clays.

Variability in the SWCC due to Different Operators

The variability in the SWCC due to different operators was analyzed by fitting a SWCC to each set of data collected from the different laboratories. Each of these SWCCs was then compared to the best fit equation for each soil (i.e., Fredlund and Xing for the sand, van Genuchten for the silt and the clay). The results are depicted in Figures 14, 15, and 16, after excluding outliers and laboratories reporting less than 3 data points due to the minimum requirement when fitting a nonlinear equation. The variability was accounted the same way as it was for the previous analysis. Results are shown in Figures 17, 18, and 19 for the sand, the silt and the clay, respectively. A summary of the results is presented in Table 5.

El Paso sand results show that the function fitted to data obtained from laboratory 8 gives a misrepresentation of the SWCC with a variability as high as 47% in the air-entry region (see bold numbers in Table 5). Laboratories 3, 5, and 6 provided what appears to be the best representation of the SWCC throughout the entire range of suction measurements. For the Price Club silt, data reported by laboratory 11 showed high deviation, as high as 55%, in the desaturation region. Note that the SWCC produced by Laboratory 11 is skewed to the right of the best-estimate SWCC by at least two orders of magnitude (Figure 15). Data from laboratories 3, 4, and 5 gave a good fit compared to the best-estimate SWCC for this soil. Fountain Hills clay data show laboratories 4, 8, and 10 as the best predictors. The rest of the laboratories failed to produce SWCCs which matched well throughout the entire range of suction (labs 3, 6, 7, 9, and 10) or in a particular region, showing variability in the results as high as 54% (laboratory 6).

Variability in the SWCC as Related to the Number of Data Points Reported

An attempt was made to correlate the number of data points and the range in suction measurements reported, with the performance of each laboratory. The results are shown in Figure 20. For El Paso sand, those laboratories that reported 12 and 13 data points (laboratories 5 and 6, respectively) provided a good estimate of the SWCC. However, data from lab 3 also provided a good estimate of the SWCC with only three data points. Therefore, some correlation was found for the sand, but there is not very clear evidence that the quality of the resulting SWCC increases substantially with number of points.

For the silt and the clay, poor correlation was found between the number of reported data points and the maximum variability. It seems that three data points are enough to make a good representation of the entire SWCC, provided of course that all three are relatively reliable. Comparisons within the overall data set (sand, silt, and clay together) support this conclusion.

Variability in the SWCC as Related to the Range in Suction Measurements Reported and The Method Used for Measurement

The range in suction provided by each lab is determined by the method used to acquire the data. Therefore, the effects of these factors were considered together. The method employed by each laboratory for determining the matric suction of the soil is listed in Table 3. Out of eleven, six laboratories used the buried filter paper method, four used the pressure plate method, and one used the pressure membrane method. It is reasonable to expect the pressure plate and the pressure membrane methods to produce comparable results, because both of them give a desorption curve. It is not clear whether the filter paper method gives a desorption or an adsorption curve, as this depends on the method used to calibrate the filter paper.

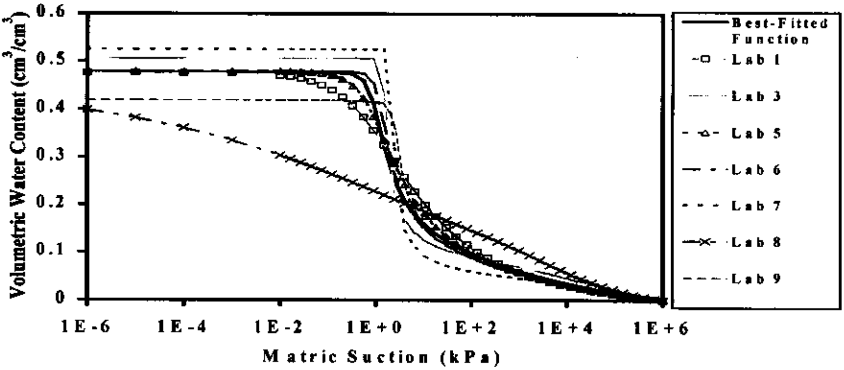

Figure 14. El Paso Sand SWCCs by Different Operators

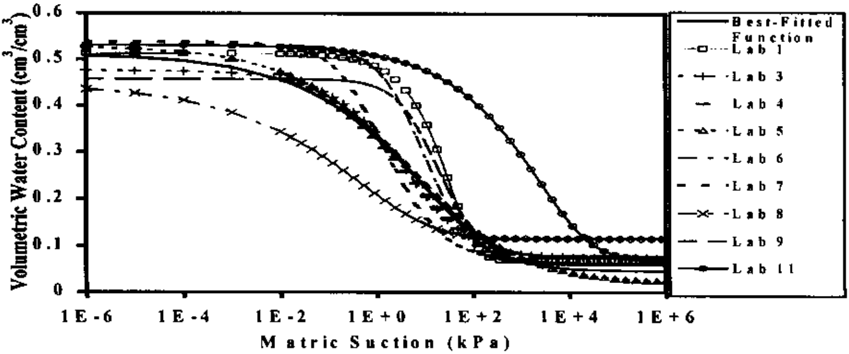

Figure 15. Price Club Silt SWCCs by Different Operators

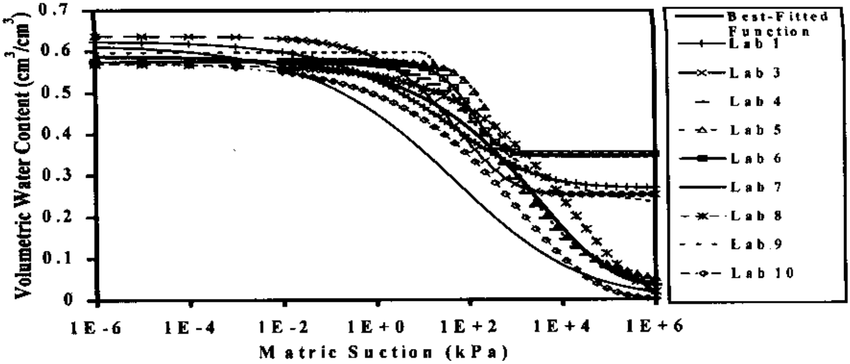

Figure 16. Fountain Hills Clay SWCCs by Different Operators

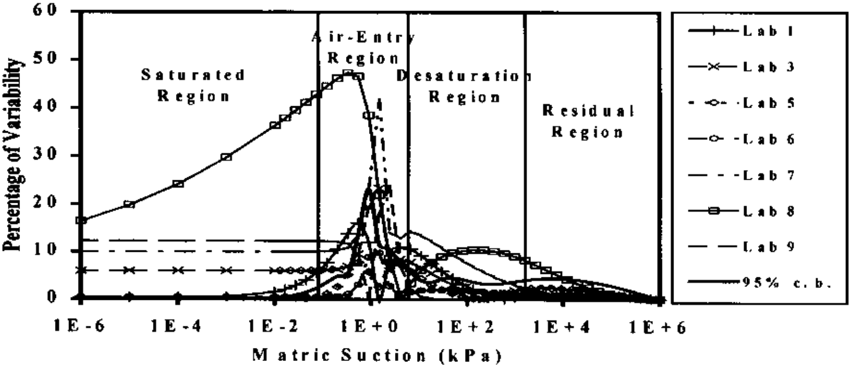

Figure 17. Variability due to Different Operators – El Paso Sand

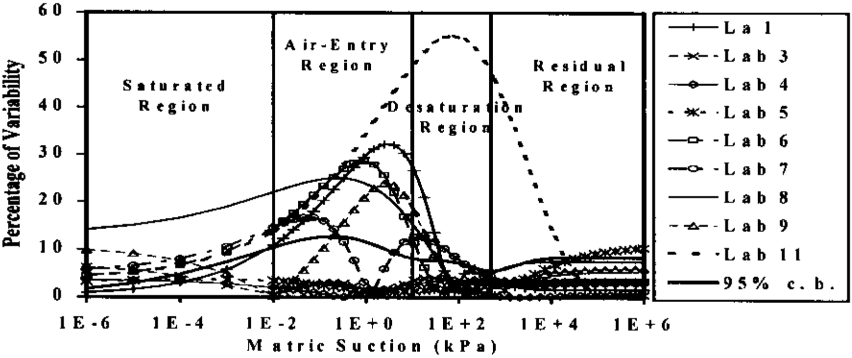

Figure 18. Variability due to Different Operators – Price Club Silt

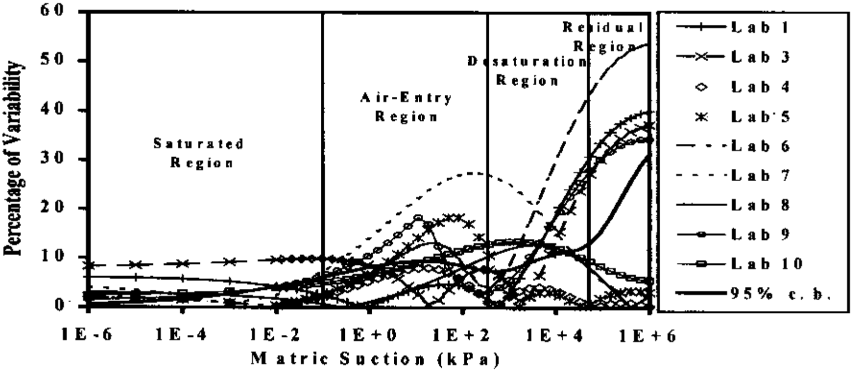

Figure 19. Variability due to Different Operators – Fountain Hills Clay

Table 5. Maximum Percentage of Variability due to Different Operators

Laboratory	Saturated Region	Air-Entry Region	Desaturation Region	Residual Region
		El Paso Sand		
Conf. Band	2	23	6	4
1	**6**	11	**11**	1
3	**6**	19	**8**	2
5	0	**8**	6	1
6	0	10	2	1
7	**10**	**42**	**14**	2
8	**43**	**47**	**10**	7
9	**12**	**24**	2	0
		Price Club Silt		
Conf. Band	10	13	8	8
1	**11**	**32**	**27**	4
3	6	3	4	1
4	4	2	4	4
5	4	3	2	**10**
6	**14**	**29**	**11**	3
7	**15**	**17**	**13**	4
8	**22**	**25**	**15**	8
9	10	**24**	**18**	6
11	**14**	**47**	**55**	**46**
		Fountain Hills Clay		
Conf. Band	6	9	12	31
1	6	5	**28**	**40**
3	**10**	**10**	**24**	**37**
4	2	**8**	4	1
5	2	**18**	11	3
6	2	**13**	**40**	**54**
7	**7**	**27**	**27**	7
8	3	**10**	**13**	8
9	6	**18**	**25**	**34**
10	5	**12**	**13**	9

Numerical comparisons are limited due to the range of suction covered by each method. The range of measurements obtained with the pressure plate and pressure membrane is limited by the air-entry value of the ceramic stone or membrane used, whic is about 1,500 kPa. On the other hand, the filter paper can give a wider range of suction measurements. Thus, filter paper suction values cannot be compared with the other two methods in the extreme ranges of suction. For this reason, the data obtained by the different methods were compared: 1) throughout the entire range in suction, by fitting th data obtained with each method with a SWCC equation, and 2) in a limited range of suction where data were available from all three methods, by comparing the absolute residuals of the experimental data.

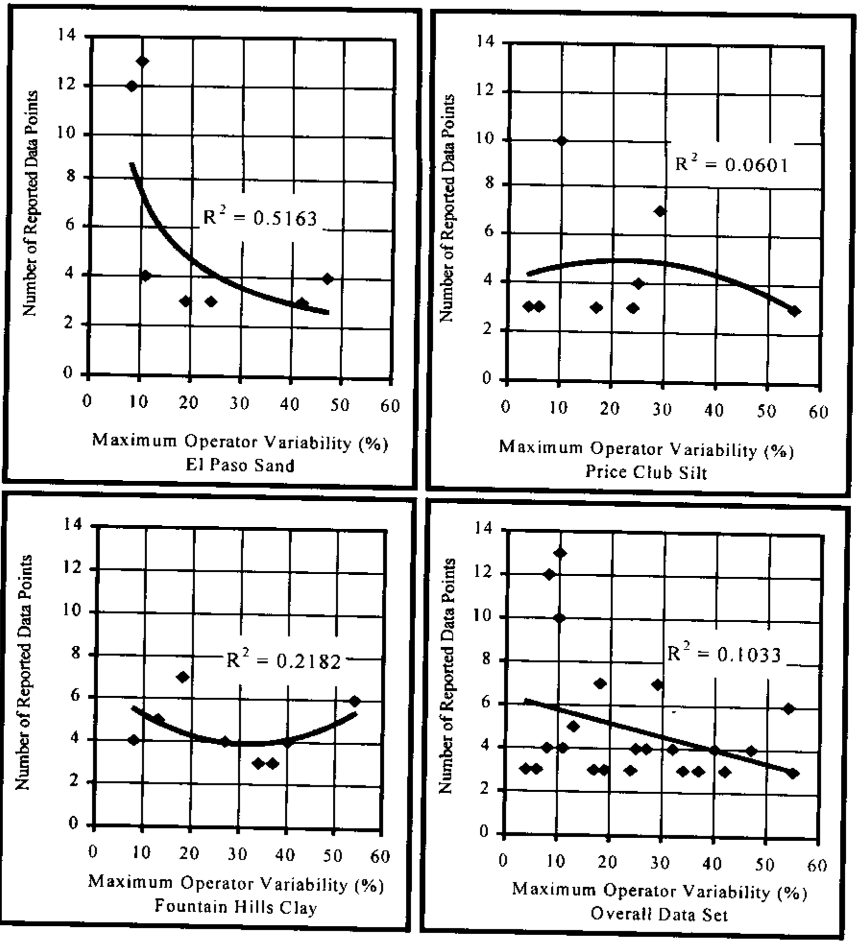

Figure 20. Correlation between Number of Data Points and Maximum Operator Variability

Variability in the SWCC throughout the Entire Range of Suction due to the Method Used to Measure Suction. To find the θ_w variability over the full range of suction, the experimental measurements were divided by method used. Each data set for each method of measurement was fitted to an equation. Then the SWCC for each test method was compared to the best-estimate SWCC. Figures 21 through 23 show the results corresponding to El Paso sand, Price Club silt, and Fountain Hills clay, respectively.

The variability in the predicted θ_w among methods was compared with the best-predicted θ_w at each level of suction. This difference was plotted as a percentage of the saturated volumetric water content in figures 24 through 26. For comparison, the 95% confidence band of the best-estimate SWCC was plotted in the same figure, also as a percentage of the saturated volumetric water content.

Results showed that none of the methods used produced higher variability than that corresponding to the 95% confidence band of the best-estimate SWCC. Although the variability increased generally as the fines content increased, no strong trend was seen. The air-entry region appears to be the most critical region for sand, and the desaturation region the most critical for the clay. The silt data showed a more or less uniform variability throughout the entire range of suction measurements.

Variability in a Limited Suction Range of the SWCC due to Method Used to Measure Suction. For comparison in the range where all three methods produced data, the absolute residuals were plotted. Absolute residuals, e, are obtained from the difference between the measured volumetric water content and its corresponding fitted value from the best-estimate SWCC:

$$e = \left|\theta_i - \hat{\theta}_i\right| \quad \text{.........} \quad (13)$$

Figure 27 shows the absolute residuals, in terms of volumetric water content, for the three soils analyzed. The horizontal lines represent the data mean or average.

Three statistical methods were used to check the scatter of the residuals: Analysis of Variance or ANOVA test, Duncan's multiple range test, and the Barlett test. Information about these tests can be found in Neter et al. (1985) and Hines and Montgomery (1990).

The ANOVA, or Analysis of Variance, determines whether or not the scatter of the measurements obtained by each method affected the overall fit of the data to the chosen function (Fredlund and Xing in the case of the sand, van Genuchten in the cases of the silt and the clay). The ANOVA indicated with 95% confidence, that there was not enough evidence to prove that having different test methods significantly affected the overall fit in any of the soils analyzed. This result is in agreement with the analysis performed in the previous section.

The second test performed on the data was the Barlett test. This test was used to check if the scatter of the residuals within each method statistically differed from each other. The variance of the data was used as the measure of the scatter. The Barlett test

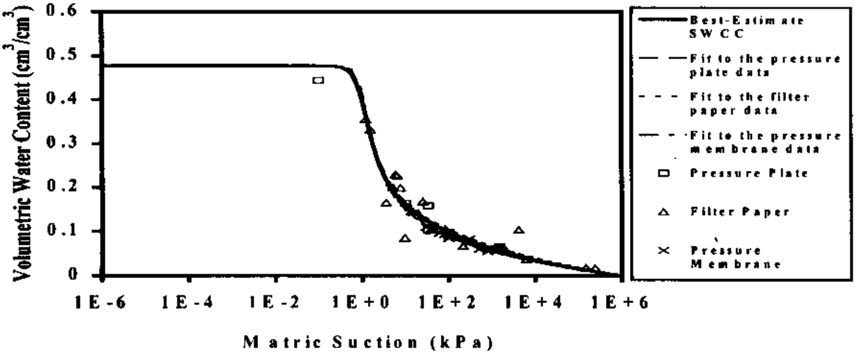

Figure 21. SWCC Obtained by Different Suction Methods - El Paso Sand

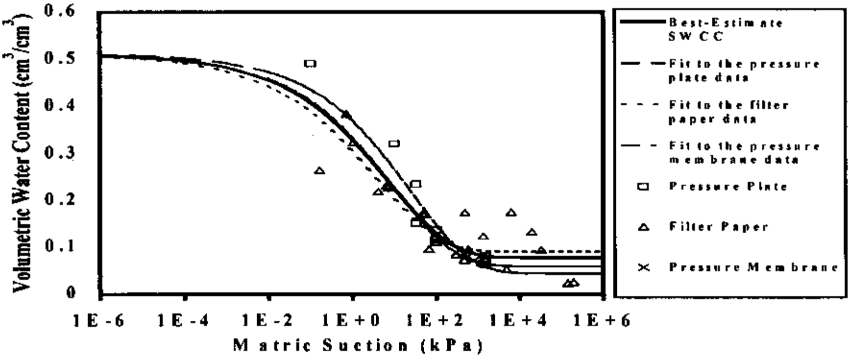

Figure 22. SWCC Obtained by Different Suction Methods – Price Club Silt

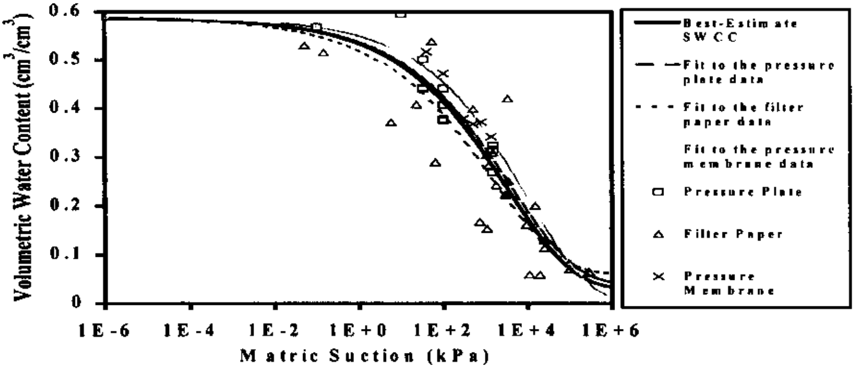

Figure 23. SWCC Obtained by Different Suction Methods – Fountain Hills Clay

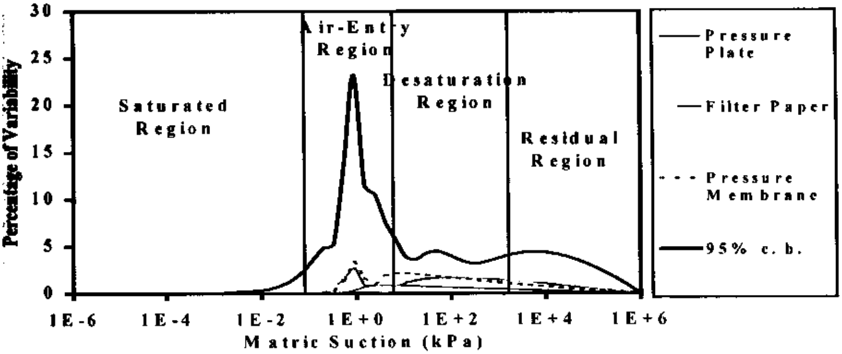

Figure 24. Variability Encountered due to Different Methods - El Paso Sand

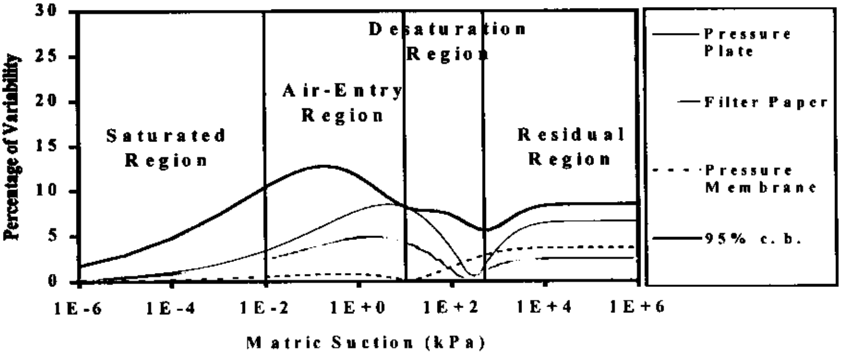

Figure 25. Variability Encountered due to Different Methods - Price Club Silt

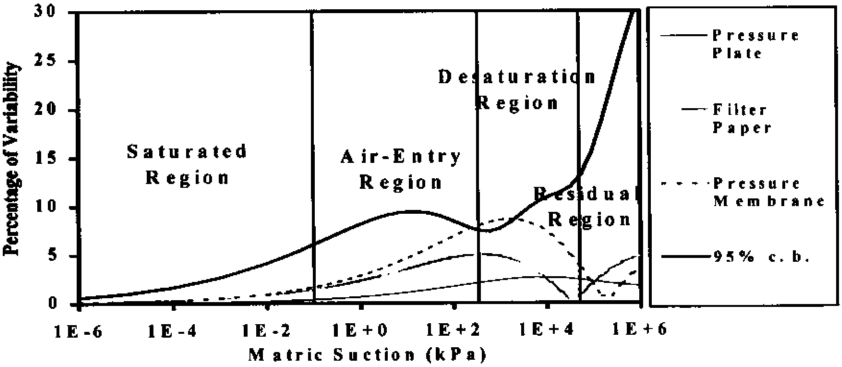

Figure 26. Variability Encountered due to Different Methods - Fountain Hills Clay

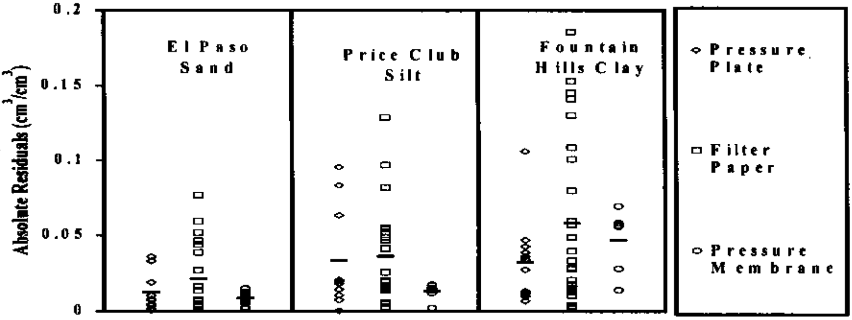

Figure 27. Absolute Residuals Sorted by Method for the Three Types of Soil

showed, with 95% confidence, that the variances were significantly different for the three soils analyzed. For the sand and the clay, the variance of the residuals obtained by the filter paper technique was higher than the variance of the data obtained by the other two methods. For the silt, the highest scatter or variance of the data residuals was for the data obtained by the pressure plate method. Pressure membrane residuals had the lowest variance for the three soils.

The third test performed on the absolute residuals was the Duncan's multiple range test. This test was performed to establish if the different methods' residual means were significantly different from each other. The Duncan test showed with 95% confidence, that the filter paper residual's mean was significantly different from the pressure membrane residual's mean for two of the soils: the sand and the silt. No significant differences were found among the residual means for the clay. Pressure plate and pressure membrane means were not significantly different from each other for the three types of soil.

Part 2: Variability in the SWCC when Predicted from Soil Index Properties

Several reported models used to predict the SWCC from the grain-size distribution (GSD) and soil index properties were reviewed. From those models, 16 were implemented to estimate the SWCC for the three soils previously described. These models were chosen because (i) they have remained popular throughout the years and several researchers have used them, (ii) they use commonly measured soil properties, and (iii) they are relatively easy to implement.

The following models were used to predict the SWCC for the three soils of this study:

1) *Approach 1*: Models using statistical estimation of the water content at a specified matric suction from GSD and/or volume-mass soil properties: Gupta-Larson (1979), Rajkai-Varallyay (1992), Rawls-Brakensiek (1982), Aubertin et al. (1998), and Mbagwu-Mbah (1998).
2) *Approach 2*: Models based on the statistical estimation of water content by correlation between soil properties and fitting parameters of a SWCC

equation: Ghosh (1980), Williams et al. (1983), Rawls et al. (1992), Cresswell-Paydar (1996), and Tomasella-Hodnett (1998).

3) *Approach 3*: Models that based the prediction of the SWCC on a physics based conceptual model using textural information: Arya-Paris (1981), Haverkamp-Parlange (1986), Mishra et al. (1989), Arya-Dierolf (1992). Smettem-Gregory (1996), and Fredlund, M. et al. (1997).

Figures 28 through 30 show the SWCCs predicted using Approach 1, along with the best-estimate SWCC (experimental) for El Paso sand, Price Club silt, and Fountain Hills clay respectively. The Rawls-Brakensiek model provided the best prediction of all the models. Nevertheless, it consistently produced over-predicted values of water content within the air-entry region for all three soils and under-predicted values at higher levels of suction for the clay.

Figures 31 through 33 show the SWCCs predicted using Approach 2, along with the best-fit SWCCs for the three soils. The model by Rawls et al. estimated the Brooks and Corey (BC) and the van Genuchten (vG) parameters. The Williams et al. model yielded the best prediction for the three soils even though the function presents a discontinuity at the air-entry value.

Figures 34 through 36 show the SWCCs predicted using Approach 3, along with the best-fit SWCCs for the three soils. The best predictive equation was that presented by Fredlund, M., even though under-prediction of the θ_w was observed for the three soils at high suction levels.

From the aforementioned 16 models, the best three predictive models for each soil were compared to the experimentally determined SWCC to obtain the variability in the different regions of the SWCC. To statistically determine the best predictive models of the SWCC for the three soils analyzed the experimentally obtained volumetric water content, θ_e, was compared to the predicted volumetric water content, θ_p, by means of the mean squared error. The mean squared error is given by:

$$\mathrm{MSE} = \frac{\mathrm{SSE}}{n} \qquad (14)$$

Where:
MSE = mean squared error
SSE = sum of the squared error
n = number of compared data points

Table 6 shows the MSE for the models in each of the approaches for the three soils analyzed. The MSE for the best three predictive models is shown in bold.

For El Paso sand, the best predictive models were those proposed by Rawls and Brakensiek (1982), Williams et al. (1983), and Fredlund, M. et al. (1997). These models use Approaches 1, 2, and 3 respectively, indicating that any of the three approaches can be used to predict an acceptable SWCC for this type of soil. For the Price Club silt, the best predictors were the models proposed by Rawls and Brakensiek (1982), Williams et al. (1983), and Tomasella and Hodnett (1998). The Rawls-Brakensiek model uses Approach 1, while the remaining two models use Approach 2. Lastly, the best predictive models for the Fountain Hills clay were Rawls and Brakensiek (1982), Ghosh (1980), and

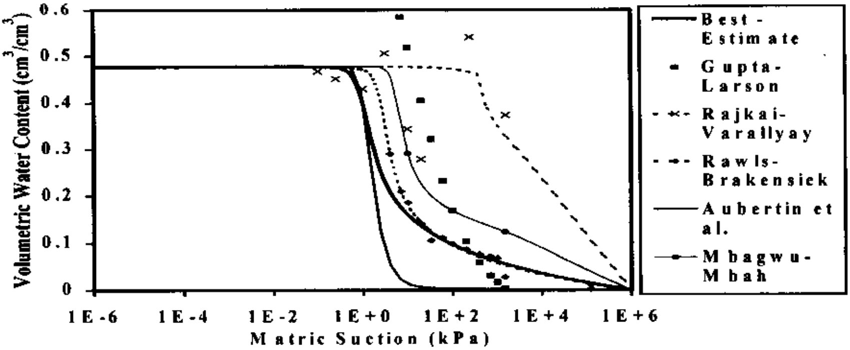

Figure 28. Predicted SWCCs Using Approach 1 - El Paso Sand

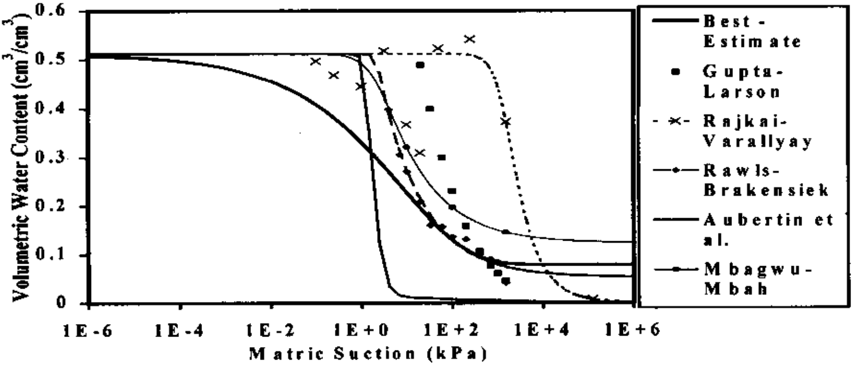

Figure 29. Predicted SWCCs Using Approach 1 – Price Club Silt

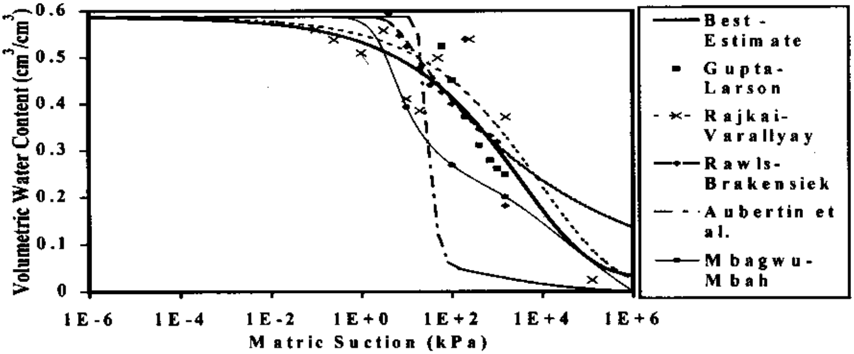

Figure 30. Predicted SWCCs Using Approach 1 – Fountain Hills Clay

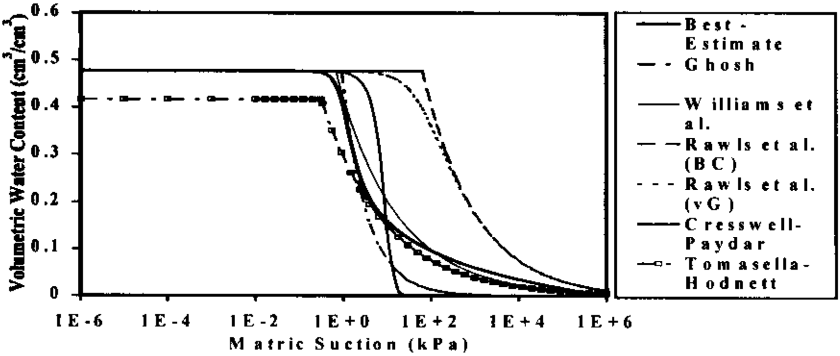

Figure 31. Predicted SWCCs Using Approach 2 – El Paso Sand

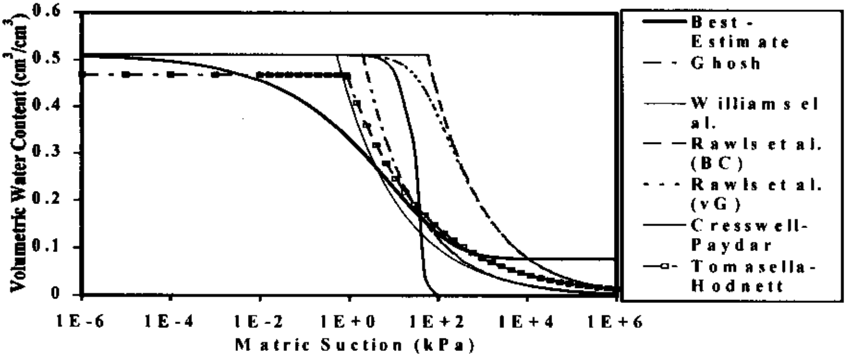

Figure 32. Predicted SWCCs Using Approach 2 – Price Club Silt

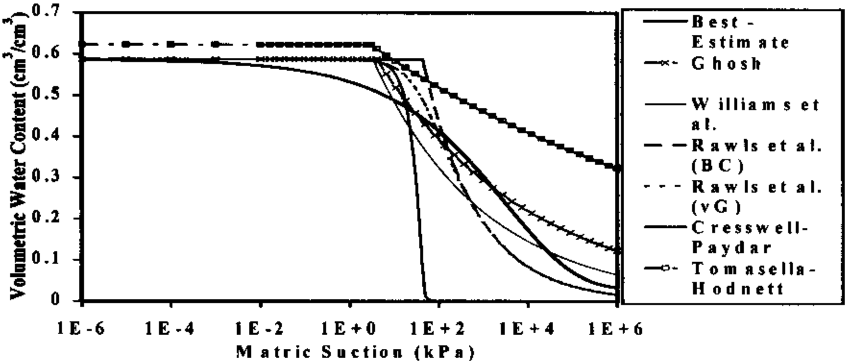

Figure 33. Predicted SWCCs Using Approach 2 – Fountain Hills Clay

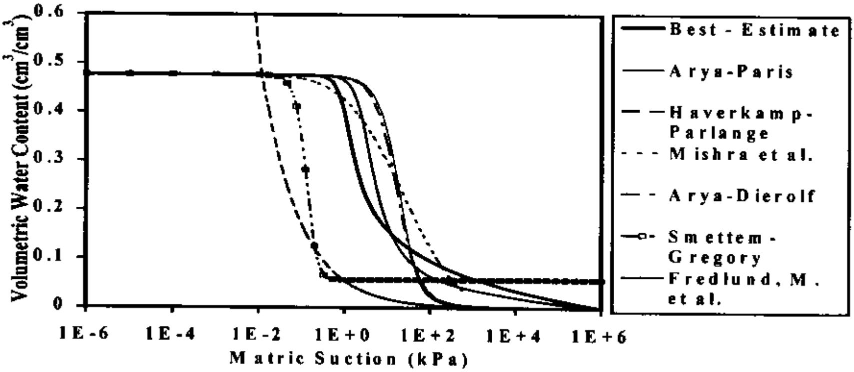

Figure 34. Predicted SWCCs Using Approach 3 – El Paso Sand

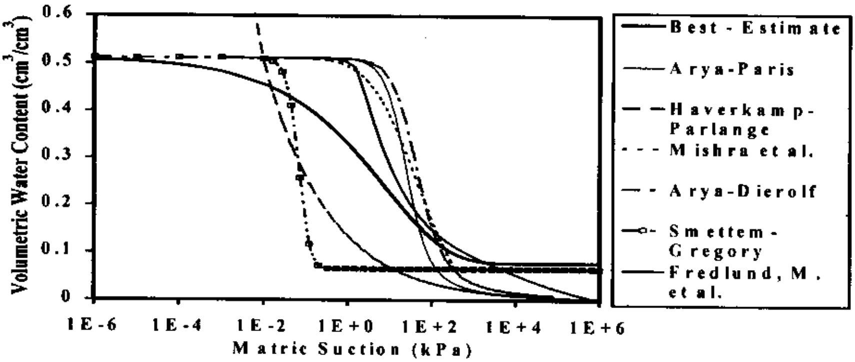

Figure 35. Predicted SWCCs Using Approach 3 – Price Club Silt

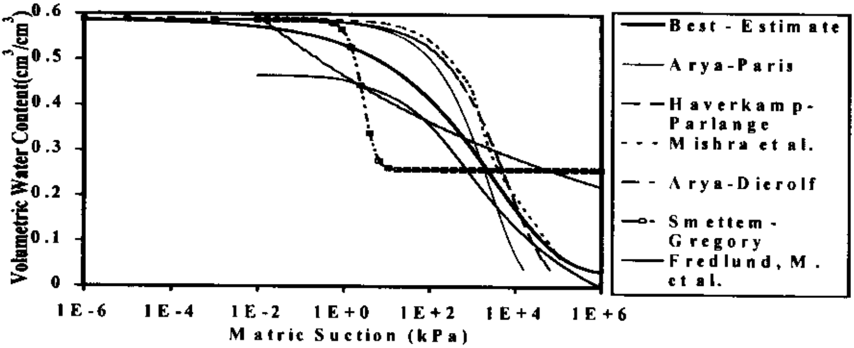

Figure 36. Predicted SWCCs Using Approach 3 – Fountain Hills Clay

Table 6. Mean Squared Error for the Predicted SWCC

Predicted Model	El Paso Sand	Price Club Silt	Fountain Hills Clay
Approach 1			
Gupta-Larson	0.058030	0.065569	0.049108
Rajkai-Varallyay	0.074219	0.053200	0.005673
Rawls-Brakensiek	**0.001302**	**0.005494**	**0.003205**
Aubertin et al.	0.004274	0.012960	0.024997
Mbagwu-Mbah	0.006422	0.006927	0.004629
Approach 2			
Ghosh	0.002123	0.007725	**0.002639**
Williams et al.	**0.000431**	**0.004605**	**0.002581**
Rawls et al. (BC)	0.027459	0.027726	0.004927
Rawls et al. (vG)	0.022665	0.022811	0.003554
Cresswell-Paydar	0.005351	0.014693	0.032291
Tomasella-Hodnett	0.001910	**0.002912**	0.027930
Approach 3			
Arya-Paris	0.007221	0.012544	0.003431
Haverkamp-Parlange	53.28111	1.300468	0.015440
Mishra et al.	0.003489	0.009895	0.004753
Arya-Dierolf	0.006248	0.013939	0.004347
Smetten-Gregory	0.019319	0.017434	0.016242
Fredlund, M. et al.	**0.001440**	0.006290	0.004268

Williams et al. (1983). The first model uses Approach 1, while the other two are based on Approach 2.

The variability, as previously defined, associated with the prediction of the SWCC is depicted in figures 37 through 39 for El Paso sand, Price Club silt, and Fountain Hills clay, respectively. Included on these figures is the 95% confidence band for the best estimate of the experimentally determined SWCC, shown for comparison.

The results showed that for the sand and the silt, the highest variability was found at the air-entry region, whereas for the clay, the variability at the residual region was the highest. Table 7 is a summary of the maximum variability encountered within each region of the SWCC for the three soils. It can be observed that although the magnitude of the variability in the results is considerable, it was lower than the variability found when different operators experimentally determined the SWCC (compare Table 7 with Table 5).

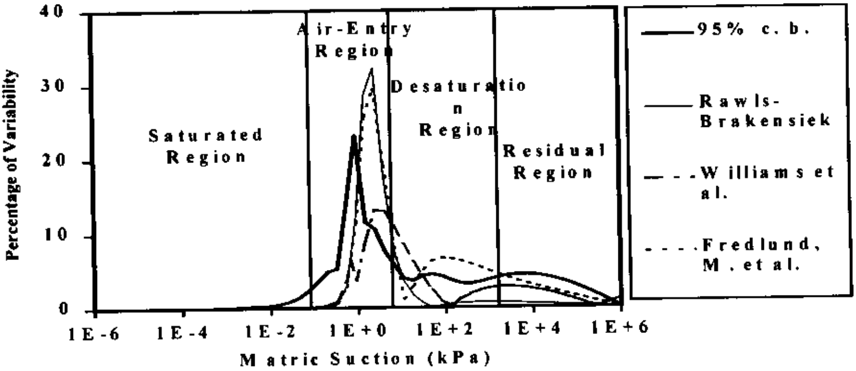

Figure 37. Variability in Predicted SWCC - El Paso Sand

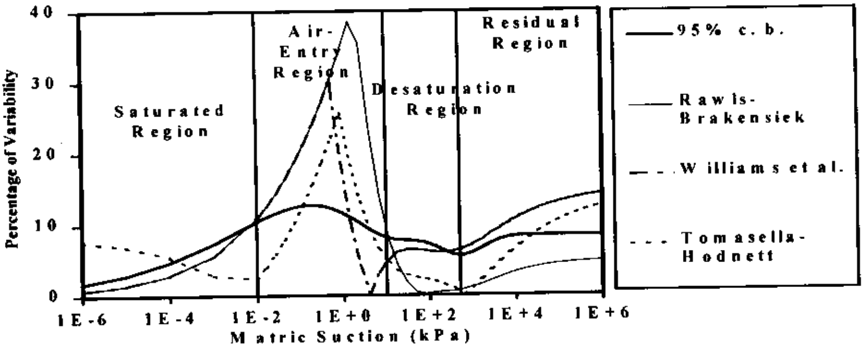

Figure 38. Variability in Predicted SWCC – Price Club Silt

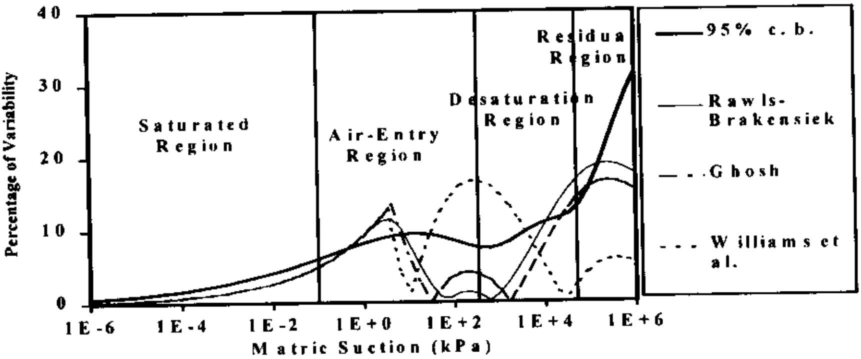

Figure 39. Variability in Predicted SWCC – Fountain Hills Clay

Table 7. Maximum Percentage of Variability when SWCC is Predicted from Soil Index Properties

Predicted Model	Saturated Region	Air-Entry Region	Desaturation Region	Residual Region
			El Paso Sand	
95% Confidence Band	2	23	6	4
Rawls-Brakensiek	0	**32**	**8**	1
Williams et al.	0	13	**11**	3
Fredlund	0	**29**	**8**	4
			Price Club Silt	
95% Confidence Band	10	13	8	8
Rawls-Brakensiek	10	**39**	8	5
Williams et al.	10	**30**	6	**14**
Tomasella-Hodnett	8	**26**	5	**13**
			Fountain Hills Clay	
95% Confidence Band	6	9	12	31
Rawls-Brakensiek	5	**11**	**15**	19
Ghosh	5	**13**	**13**	17
Williams et al.	5	**17**	**17**	6

Given the high variability that is still present in the experimental determination of the SWCC compared with the variability found for the investigated predictive models, a new predictive model will be presented. This new model is based on statistical correlations of very simple and easy to measure soil index properties with the fitting parameters of the SWCC function proposed by Fredlund and Xing (equation 1). This approach was selected because it is clear from Table 6 that Approach 2 was clearly the most succesful.

A New Empirical Model for Predicting the SWCC from Index Properties

A database characterizing approximately 190 soils was assembled from research papers and a knowledge-based program developed by Soilvision Systems Ltd (Dunn and Palmer, 1994; Escario and Juca, 1989; Fredlund, 1995; Fredlund et al., 1995; Gan et al., 1988; Ghosh, 1980; Haverkamp and Parlange, 1986; Houston et al., 1999; Krahn and Fredlund, 1972; Livneh et al., 1970; Marinho and Stuermer, 1998; Oberg and Sallfors, 1997; Rahardjo et al., 1995; Rohm and Vilar, 1995; Sabbagh, 1995; Vanapalli et al., 1998, SoilVision, 1997). The soils were divided into two categories: soils having a Plasticity Index (PI) greater than zero and soils having a PI equal to zero. Data for approximately 70 soils with PI greater than zero and 120 soils with PI equal to zero were collected.

The data assembled for the soils with PI greater than zero included the percentage passing #200 sieve and the Atterberg Limits, particularly the Plasticity Index. For soils with PI equal to zero (non-plastic soils), the diameter D_{60} was gathered. Included in the collected data was a measured and fairly well-defined SWCC.

For the soils with PI greater than zero, the product of the percentage passing the #200 sieve, as a decimal, was multiplied by the PI as a percentage, to form the weighted PI. This value was designated as *wPI,* and used as the main soil property for correlation. The reasoning behind this choice is as follows. The equilibrium soil suction at a given degree of saturation was expected to be proportional to the specific surface area of the soil. The PI is a fair indicator of surface area and the use of PI alone was considered. However, a soil with a small percentage of highly active clay would have a high PI but only a moderate specific surface area. Therefore, the weighted PI, wPI, was considered a better indicator of soil particle surface area available for water adsorption and retention.

For non-plastic soils, several functions of gradation parameters were tried. It was decided finally that correlation with D_{60} worked as well as any of the other correlations. Therefore, the D_{60} was the main soil property for correlation for the soils with PI equal to zero. For simplicity, it was decided to use the Fredlund and Xing equation for all soils, regardless of PI.

For Soils with PI > 0

The Fredlund and Xing equation fitting parameters in equation 1 (parameters a, b, c, and h_r, defined in Table 1), were correlated with the new *wPI* parameter. The equations found are the following:

$$a = 0.00364(\mathrm{wPI})^{3.35} + 4(\mathrm{wPI}) + 11 \qquad (15)$$

$$\frac{b}{c} = -2.313(\mathrm{wPI})^{0.14} + 5 \qquad (16)$$

$$c = 0.0514(\mathrm{wPI})^{0.465} + 0.5 \qquad (17)$$

$$\frac{h_r}{a} = 32.44e^{0.0186(\mathrm{wPI})} \qquad (18)$$

The wPI parameter in equations 15 through 18 is defined as:

$$\mathrm{wPI} = \text{Passing \#200} \times \mathrm{PI} \qquad (19)$$

Where:

Passing #200 = Material passing the #200 U.S. Standard Sieve expressed as a decimal

PI = Plasticity Index (%) = Liquid Limit – Plastic Limit

In those cases where the saturated volumetric water content, θ_{sat}, is unknown, the user can make use of the following correlation:

$$\theta_{sat} = 0.0143(\mathrm{wPI})^{0.75} + 0.36 \qquad (20)$$

However, although equation (20) produces a more or less unbiased estimate of the θ_{sat}, the scatter is very considerable and it is highly desirable to have direct measurements of density, or better yet, density and specific gravity, G_s, so that θ_{sat} can be calculated from direct measurements. Equation (21) for estimating G_s can be used with only small to moderate error when directly measured G_s values are not available:

$$G_s = 0.041(wPI)^{0.29} + 2.65 \qquad (21)$$

For Soils with PI = 0

For granular soils with Plasticity Index equal to zero, the parameter used to relate to the SWCC was the Diameter D_{60} from the grain-size distribution (GSD) curve. The correlations found are the following:

$$a = 0.8627(D_{60})^{-0.751} \qquad (22)$$

$$\bar{b} = 7.5 \qquad (23)$$

$$c = 0.1772\ln(D_{60}) + 0.7734 \qquad (24)$$

$$\frac{h_r}{a} = \frac{1}{D_{60} + 9.7e^{-4}} \qquad (25)$$

Where:

D_{60} = Grain diameter corresponding to 60% passing by weight or mass (mm)

$\bar{b}$ = Average value of fitting parameter b

No correlation between the 'b' parameter and D_{60} was found. Therefore, a constant average b value is suggested. In those cases where the θ_{sat} is unknown, the following average value is recommended for soils with PI equal to zero:

$$\bar{\theta}_{sat} = 0.36 \qquad (26)$$

The individual fitting parameters for the Fredlund and Xing SWCC equation, a, b, c, and h_r, did not correlate very well with the index properties wPI and D_{60}. Perhaps not surprisingly, the R^2 values were quite low. It has been experience of the authors that when a complex function that requires three or four parameters for fitting is used, the fitting parameters themselves rarely correlate well with soil properties. For the case in point, the SWCC, this result is probably related to the fact that numerous sets of fitting parameters, which differed substantially, could be found which would produce SWCCs which were nearly the same. In the final analysis, what the user wants is a predictive model that will predict the SWCC with acceptable accuracy. Therefore, based on this experience, it was decided to delay the judgment of goodness of fit until the final predicted SWCCs could be compared to measured SWCCs. This comparison is made in the following section.

Comparison of Measured SWCCs and SWCCs Predicted by the New Model

Figures 40 through 46 shows the results obtained. Figures 40 through 43 show the SWCCs for the soils with wPI greater than zero. Figures 44 and 45 show the SWCCs for the soils with wPI equal to zero. In figures 40 through 45, the solid curves represent the "predicted" band corresponding to the wPI and D_{60} values indicated. The prediction is derived from the correlations obtained from the database (190 soils), using equations 15 through 25. The data points shown in figures 40 through 45 represent the actual, measured SWCCs, after some smoothing. Each symbol type represents an experimental curve. The goodness of the fit can be judged by observing the extent to which the "predicted" band is centered on and envelops the experimental data. For each figure, the experimental data subset represents the same range in wPI (or D_{60}), as does the predicted band given by the solid curves. Figure 46 summarizes the results obtained for both groups of soils.

Data for the 190 soils used in this study and hundreds of other SWCCs examined by the authors show clearly that the SWCC moves gradually to the right with increasing plasticity. The correlations and algorithms developed for this study provide a smooth transition across the spectrum of soils with no plasticity to those with high plasticity. Although Figures 40 through 45 show that the predicted SWCC curves match the measured SWCC fairly well, and probably about as well as can be done at this time, there is nevertheless considerable scatter. Data presented in the earlier sections of this paper show that if a single soil is sent out to a dozen laboratories across the country for SWCC measurement, the results show a variability greater than that of the experimental data in Figure 42, for example. Likewise, if a single laboratory is asked to reproduce the SWCC for a single soil, i. e., make multiple measurements of the SWCC for a single soil, the variability can be as great as the difference between the wPI = 10 curve and the wPI = 30 curve in Figure 42.

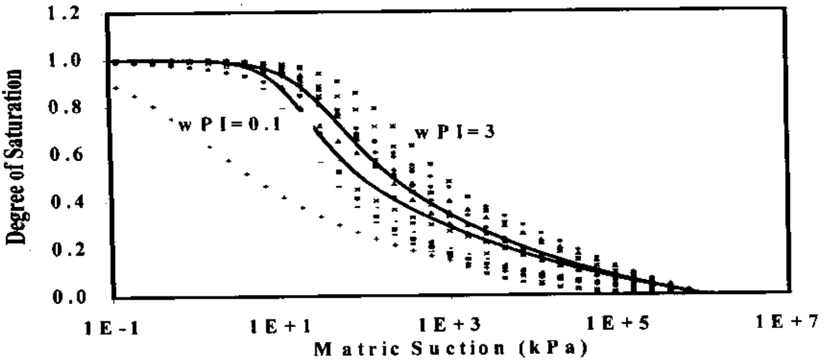

Figure 40. Range of SWCCs for 14 Soils with wPI between 0.1 and 3

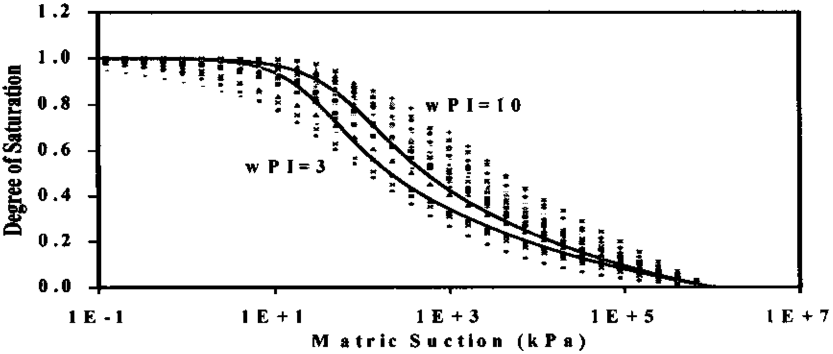

Figure 41. Range of SWCCs for 21 Soils with wPI between 3 and 10

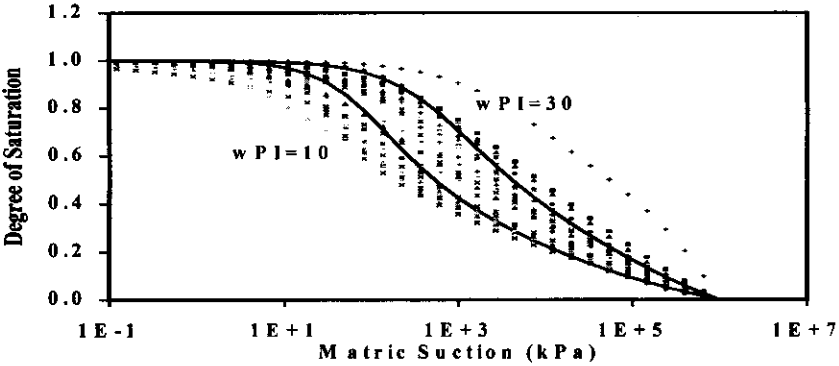

Figure 42. Range of SWCCs for 24 Soils with wPI between 10 and 30

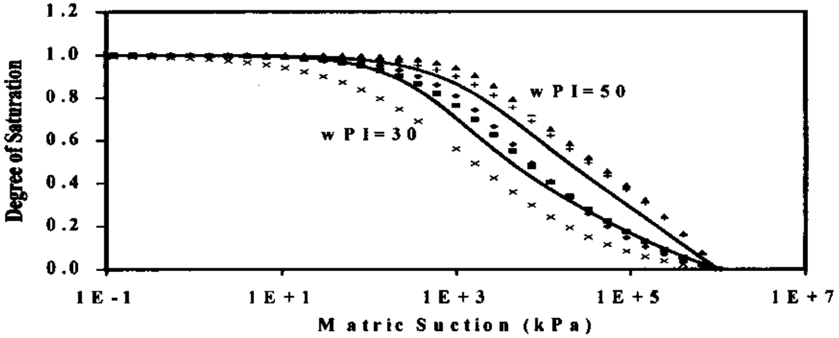

Figure 43. Range of SWCCs for 5 Soils with wPI between 30 and 50

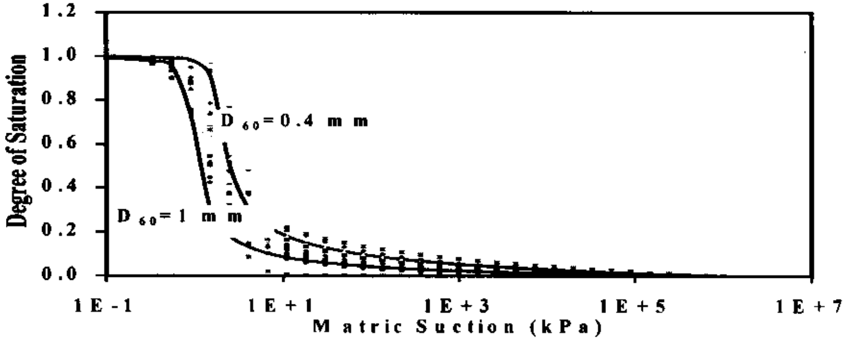

Figure 44. Range of SWCCs for 18 Soils with D_{60} between 1 and 0.4 mm.

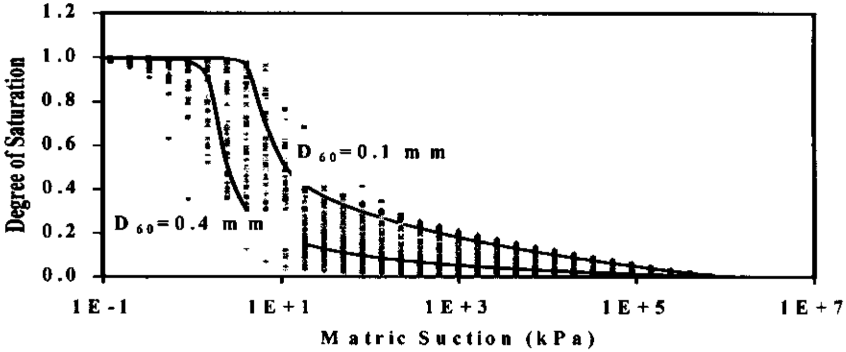

Figure 45. Range of SWCCs for 108 Soils with D_{60} between 0.4 and 0.1 mm.

The Case for Compensating Errors

The use of models to predict engineering behavior of materials dates back to the inception of engineering itself. Even the creator of each model, however, would not claim that his/her model is perfect. The imperfections may lie in the fitting parameters, which characterize the model, or in the functional form of the model itself. Because imperfections are the norm rather than the exception, it has long been an important tenet of engineering practice to check, and if possible calibrate, a predictive model against actual performance under prototype conditions. This calibration normally takes the form of adjustment of the fitting parameters or coefficients in the model. To the extent that the calibration has been done well, any remaining errors in the coefficients or the form of the model tend to compensate so that the model predicts the correct, observed performance. The engineer using the model for subsequent applications checks to see that the new conditions are sufficiently similar to the calibration conditions that good results can be expected.

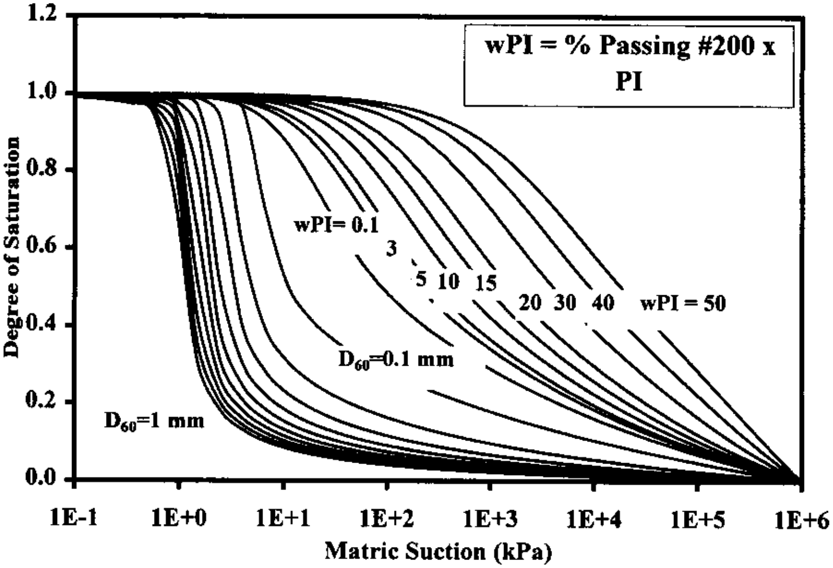

Figure 46. Predicted SWCC based on D_{60} and wPI

This utilization of compensating errors is common in all branches of engineering and science and is the feature that allows us to get satisfactorily good predictions with imperfect models. Obviously, the better the model and the smaller the compensating errors, the more confidently we can extrapolate to field conditions differing somewhat from calibration conditions.

The utilization of compensating errors in connection with SWCCs can be postulated as follows. Suppose the shear strength, τ_f, is to be estimated from Equation (27).

$$\tau_f = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad \text{.......}(27)$$

Normally c' and ϕ' would be determined by a lab test program on saturated soil, but tan ϕ^b must be assessed from the testing of unsaturated soil. It is well known that tan ϕ^b approaches tan ϕ' as S approaches 100%, but it is also well known that tan ϕ^b decreases as S decreases, usually non-linearly. Thus, a lab test program on unsaturated specimens is needed to establish the functions given by (28a) or (28b).

$$\tan\phi^b = f(S) \quad \text{.......}(28a)$$

or

$$\tan\phi^b = f(u_a - u_w) \quad \text{.......}(28b)$$

The values of $u_a - u_w$ for use in Equation (27) and (28b) can be measured directly or estimated from a SWCC, such as those depicted by Figure 46. Suppose the SWCC from Figure 46 is adopted and the wPI=30. As discussed at length in this paper, the SWCC for wPI=30 will have some error associated with it. Assume the actual SWCC for the soil lies to the right of the wPI=30 curve. This means that u_a - u_w will be under-estimated for each S value, both in Equation (27) and (28b). This under-estimation of u_a - u_w produces a corresponding error in the tan ϕ^b function.

Now, when the results of the lab test program are applied to the field, the same wPI=30 curve is used to estimate field values of u_a - u_w, which are again under-estimated. When these values of u_a - u_w are used in Equations (28a) and (27), compensating errors can be expected to produce a fairly good estimate of shear strength, τ_f. Note that compensating errors should not be expected if $u_a - u_w$ were directly measured for the lab test program, but the SWCC from Figure 46 were used to estimate u_a - u_w for the field.

Conclusions

The high variability encountered in the experimentally obtained suction measurements and the numerous sources of error in the measured values leads to the conclusion that even some of the most experienced researchers have difficulties in getting a unique SWCC for a soil. It appears that using a large database in conjunction with a knowledge-based system is perhaps the best way to achieve the best fit and to select the optimal SWCC.

These observations have led the authors to conclude that soil suction and SWCCs simply cannot be measured with great precision at the present time. Researchers and practitioners dealing with unsaturated soils need to recognize and acknowledge this condition. There are two corollaries to this conclusion: The first is that the SWCC can probably be estimated from D_{60} or wPI (see Figure 46) about as accurately as it can be measured, unless the laboratory or person making the measurement is highly experienced. Secondly, it is difficult to develop a predictive model for SWCCs that is consistent with all of the SWCCs reported in the literature because of the fairly high probability that any given measured SWCC has significant experimental error associated with it. It therefore follows that the agreement between predicted and measured results depicted in Figures 40 through 45 could be improved if the experimental error from the measured results could be removed. Of course, this latter conclusion is optimistic and rests on the contention that the experimental errors tend to be both positive and negative.

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