Use of the grain-size distribution for estimation of the soil-water characteristic curve

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Abstract: The implementation of unsaturated soil mechanics into engineering practice is dependent, to a large extent, upon an ability to estimate unsaturated soil property functions. The soil-water characteristic curve (SWCC), along with the saturated soil properties, has proven to provide a satisfactory basis for estimating the permeability function and shear strength functions for an unsaturated soil. The volume change functions have not been totally defined nor applied in geotechnical engineering. The objective of this paper is to present a procedure for estimating the SWCC from information on the grain-size distribution and the volume–mass properties of a soil. SWCCs represent a continuous water content versus soil suction relationship. The proposed method provides an approximate means of estimating the desorption curve corresponding to a soil initially slurried near the liquid limit. The effects of stress history, fabric, confining pressure, and hysteresis are not addressed. A database of published data is used to verify the proposed procedure. The database contains independent measurements of the grain-size distribution and the SWCC. The level of fit between the estimated and measured SWCCs is analyzed statistically. The proposed procedure is compared to previously proposed methods for predicting the SWCC from the grain-size distribution. The results show that the proposed procedure is somewhat superior to previous methods.

Key words: soil-water characteristic curve, grain-size distribution, volume–mass properties, pedo-transfer function, unsaturated soil property functions.

Résumé : L'introduction de la mécanique des sols non saturés dans la pratique de l'ingénieur est dépendante jusqu'à un certain point de l'habileté à estimer les fonctions des propriétés des sols non saturés. La courbe caractéristique sol-eau, avec les propriétés des sols saturés, s'est avérée pouvoir fournir une base satisfaisante pour évaluer la fonction de perméabilité et les fonctions de résistance au cisaillement d'un sol non saturé. Les fonctions de changement de volume n'ont pas été totalement définies ni appliquées à la géotechnique de l'ingénieur. L'objectif de cet article est de présenter une procédure pour évaluer la courbe caractéristique sol-eau en partant de l'information sur la distribution granulométrique et les propriétés masse-volume d'un sol. Les courbes caractéristiques sol-eau représentent une relation continue de teneur en eau en fonction de la succion du sol. La méthode proposée fournit des moyens approximatifs pour évaluer la courbe de désorption correspondant à un sol initialement en boue près de la limite de liquidité. On n'a pas traité des effets de l'histoire des contraintes, de la fabrique, de la pression de confinement et de l'hystérèse. Une base des données publiées est utilisée pour vérifier la méthode proposée. La base de données contient des mesures indépendantes de la distribution granulométrique et de la courbe caractéristique sol-eau. Le niveau de lissage entre les courbes caractéristiques sol-eau mesurées est analysé statistiquement. La procédure proposée est comparée à des méthodes proposées antérieurement pour prédire la courbe caractéristique sol-eau en partant de la distribution granulométrique. Les résultats montrent que la procédure proposée est quelque peu supérieure aux méthode antérieures.

Mots clés : courbe caractéristique sol-eau, distribution granulométrique, propriétés masse-volume, fonction pédotransfert, fonctions de propriétés des sols non saturés.

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Introduction

Unsaturated soil theories have shown significant developments over the past three decades. The soil properties for permeability, shear strength, and volume change are commonly written as nonlinear functions of the negative pore-water pressure (i.e., soil suction). The cost of performing a direct measurement of unsaturated soil property functions in the laboratory is excessive. The costs associated with measuring an entire unsaturated permeability function, or a un-

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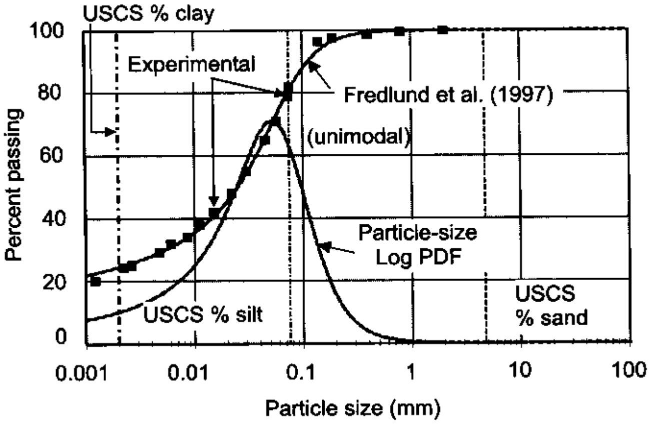
Fig. 1. Fit of a grain-size curve for uniform silt from Ho (1988) with the Fredlund et al. (1997) grain-size equation. PDF, probability density function; USCS, Unified Soil Classification System.



saturated shear strength function, are in the order of 10 times as much as the cost of measuring the saturated soil properties.

The excessive costs associated with direct measurement of unsaturated soil property functions has encouraged the pursuit of a new means of implementing unsaturated soil mechanics into routine geotechnical engineering practice. The newly emerging procedures involve the use of the soil-water characteristic curve (SWCC) and saturated soil properties to estimate the unsaturated soil property functions (Fredlund 1996*a*, 1999). The SWCC can be readily measured in the laboratory, typically for a cost considerably less than that of performing a consolidation test. Costs can be further reduced if it is possible to estimate the SWCC from a grain-size distribution curve (Fredlund 2000).

Estimation techniques are attractive, but the associated assumptions and limitations must be kept in mind. For example, in the method discussed herein, the grain-size distribution curve is first assumed to estimate, and later "trained" to better estimate, an approximate desorption curve for a soil that is initially slurried near the liquid limit. The effects of stress history, fabric, confinement, and hysteresis are not addressed. This assumption must be kept in mind when applying the technique.

A model is proposed in this paper for the estimation of the SWCC based on the grain-size distribution and an assumed "packing arrangement" for each soil particle size. The primary information associated with the grain-size distribution model is shown along with the Fredlund et al. (1997, 2000) fit of the grain-size distribution (Fig. 1). The grain-size distribution curve can be viewed as incremental particle sizes from the smallest to the largest. The results from the various particle sizes are then assembled to build a SWCC. Small increments (on a logarithmic scale) of uniform-sized particles are transposed to form a SWCC representative of a series of average particle sizes. Once the entire grain-size distribution curve has been incrementally analyzed, the individual SWCCs are combined using a superposition technique to give the SWCC for the overall soil.

The SWCC for each uniform particle size range is assumed to be unique when building the overall SWCC. Typical SWCCs for various mixtures of sand, silt, and clay were studied from the research literature. The representative SWCCs were fit using the Fredlund and Xing (1994) equation to provide approximate curve-fitting parameters that could then be classified according to an effective grain-size diameter. A total of 15 soil types ranging from sands to silty clay loams were classified and used in the study.

The shape of an estimated SWCC is predominantly controlled by the grain-size distribution and secondarily influenced by the density of the soil. The Fredlund et al. (1997, 2000) unimodal and bimodal fits of the grain-size distribution curve were used as the basis for the estimation of the SWCC.

Literature review

A number of methods have previously been proposed for the estimation of the SWCC. Three broad categories of estimation techniques are as follows: (*i*) statistical estimates of water contents at various soil suctions (Gupta and Larson 1979), (*ii*) estimation of soil parameters for an algebraic function describing the SWCC (Rawls and Brakensiek 1985, 1989; Vereecken et al. 1989; Scheinost et al. 1997), and (*iii*) physico-empirical models in which the grain-size distribution curve is used in the prediction of the SWCC data (Arya and Paris 1981; Arya et al. 1999; Tyler and Wheatcraft 1989; Fredlund et al. 1997).

This paper presents a new approach to the physico-empirical type model originally presented by Arya and Paris (1981). The original physico-empirical model has been enhanced through information gathered from parametric studies performed on several SWCC data sets. The combination of the physico-empirical model and the parametric study information provides an improvement in the estimation technique for the SWCC.

Experimental SWCCs for similar-sized glass beads were used as one of the reference benchmarks (Nimmo 1997). It was assumed that the shape of the SWCC for glass beads was representative of the shape of the SWCC for uniform coarse particles. The SWCC for very fine material was estimated from the results of soils with increasing clay content. The glass beads and the clay soil results provided limiting values for groups of soils consisting of uniformly sized particles (see Fig. 3).

Definition of terms

A few definitions are useful when identifying conceptual models for the estimation process. (1) A soil property function is a relationship between a physical soil property and either soil suction or the stress state of a soil. (2) A pedotransfer function (PTF) (Bouma 1989) is a function that has as its arguments basic soils data such as the grain-size distribution or porosity and yields a soil property function. (3) A SWCC is either a monotonic, single-valued function that yields water content (expressed in volumetric or gravimetric terms) for a given scale of soil-water potential expressed as soil suction (in kPa); or two functions of the monotonic type to describe the drying and wetting branches (Tietje and Tapkenhinrichs 1993). In other words, the SWCC has hysteresis between the drying and wetting branches, which form limiting conditions.

Representation of the grain-size distribution

The PTF presented herein uses the Fredlund et al. (1997, 2000) unimodal (eq. [1]) and bimodal (eq. [2]) equations to represent the grain-size distribution (Fredlund et al. 2000). The unimodal form can be written as

$$[1]\qquad P_{\mathrm{p}}(d) = \frac{1}{\ln\left[\exp(1) + \left(\dfrac{a_{\mathrm{gr}}}{d}\right)^{n_{\mathrm{gr}}}\right]^{m_{\mathrm{gr}}}} \left\{1 - \left[\frac{\ln\left(1 + \dfrac{d_{\mathrm{r}}}{d}\right)}{\ln\left(1 + \dfrac{d_{\mathrm{r}}}{d_{\mathrm{m}}}\right)}\right]^7\right\}$$

where

a_{gr} is a parameter related to the initial breaking point of the curve;

n_{gr} is a parameter related to the steepest slope of the curve;

m_{gr} is a parameter related to the shape of the fines portion of the curve;

d_{r} is a parameter related to the amount of fines in a soil;

d is the diameter of any particle size under consideration; and

d_{m} is the diameter of the minimum allowable size particle.

The bimodal equation for the grain-size distribution can be written as follows:

$$[2]\qquad P_{\mathrm{p}}(d) = \left[w\left\{\frac{1}{\ln\left[\exp(1) + \left(\dfrac{a_{\mathrm{bi}}}{d}\right)^{n_{\mathrm{bi}}}\right]^{m_{\mathrm{bi}}}}\right\} + (1-w)\right.$$

$$\left.\times\left\{\frac{1}{\ln\left[\exp(1) + \left(\dfrac{j_{\mathrm{bi}}}{d}\right)^{k_{\mathrm{bi}}}\right]^{l_{\mathrm{bi}}}}\right\}\right]\left\{1 - \left[\frac{\ln\left(1 + \dfrac{d_{\mathrm{rbi}}}{d}\right)}{\ln\left(1 + \dfrac{d_{\mathrm{rbi}}}{d_{\mathrm{m}}}\right)}\right]^7\right\}$$

where

a_{bi} is a parameter related to the initial breaking point of the curve;

n_{bi} is a parameter related to the steepest slope of the curve;

m_{bi} is a parameter related to the shape of the curve;

j_{bi} is a parameter related to the second breaking point of the curve;

k_{bi} is a parameter related to the second steep slope of the curve;

l_{bi} is a parameter related to the second shape of the fines portion of the curve;

d_{rbi} is a parameter related to the amount of fines in a soil;

d is the diameter of any particle size under consideration;

d_{m} is the diameter of the minimum allowable size particle; and w is the weighing factor indicating the ratio of the overall sample that constitutes the coarse fraction.

Most PTFs use grain-size distribution information in some form as the basis for the estimation of the SWCC.

Representation of the SWCC

Numerous empirical equations have been proposed to represent the SWCC (Sillers 1996). Three equations have been selected for use in conjunction with the PTFs. Gravimetric water content (w_{w}) can be used to represent the amount of water in the soil.

The Brooks and Corey (1964) equation is one of the earliest equations proposed for the SWCC and has the following form:

$$[3]\qquad w_{\mathrm{w}} = w_{\mathrm{r}} + (w_{\mathrm{s}} - w_{\mathrm{r}})\left(\frac{a_{\mathrm{c}}}{\psi}\right)^{n_{\mathrm{c}}}$$

where

a_{c} is the air-entry value (or bubbling pressure) of the soil expressed as a soil suction (kPa);

n_{c} is the pore-size distribution index;

w_{s} is the saturated gravimetric water content;

w_{r} is the residual gravimetric water content; and

ψ is the soil suction (kPa).

The van Genuchten (1980) equation is an example of a three-parameter equation that has been shown to have flexibility in fitting a wide range of soils and has the following form:

$$[4]\qquad w_{\mathrm{w}} = w_{\mathrm{rvg}} + (w_{\mathrm{s}} - w_{\mathrm{rvg}})\left\{\frac{1}{[1 + (a_{\mathrm{vg}}\psi)^{n_{\mathrm{vg}}}]^{m_{\mathrm{vg}}}}\right\}$$

where

w_{s} is the saturated gravimetric water content;

w_{r} is the residual gravimetric water content of the soil;

a_{vg} is a fitting soil parameter that is the inverse of the soil suction, corresponding to the inflection point on the curve;

n_{vg} is a fitting parameter related to the rate of desaturation of the soil; and

m_{vg} is a fitting soil parameter related to the curvature in the high suction range.

The Fredlund and Xing (1994) equation is a flexible, continuous function extending to a water content of zero at a suction of 1 000 000 kPa, the point at which a soil is considered to be completely dry (Fredlund and Rahardjo 1993), and has the following form:

$$[5]\qquad w_{\mathrm{w}} = w_{\mathrm{s}}\left[1 - \frac{\ln\left(1 + \dfrac{\psi}{h_{\mathrm{r}}}\right)}{\ln\left(1 + \dfrac{10^6}{h_{\mathrm{r}}}\right)}\right]\left[\frac{1}{\left\{\ln\left[\exp(1) + \left(\dfrac{\psi}{a_{\mathrm{f}}}\right)^{n_{\mathrm{f}}}\right]\right\}^{m_{\mathrm{f}}}}\right]$$

where

w_s is the saturated gravimetric water content;

a_f is a fitting parameter corresponding to the soil suction at the inflection point and is somewhat related to the air-entry value of the soil;

n_f is a fitting parameter related to the rate of desaturation of the soil;

m_f is a fitting soil parameter related to the curvature of the function in the high suction range; and

h_r is a constant parameter used to represent soil suction at the residual water content and is selected to be 3000 kPa for this study.

Description of pedo-transfer functions (PTFs)

Two methods that have previously been used to estimate the SWCC are shown in the following two subsections. The new method presented in this paper is based on a physico-empirical model and is compared to previously presented methods to provide a means of verifying the method.

Functional parameter regression method

The functional parameter regression method assumes that parameters of the SWCC equation can be correlated to basic physical properties. An example is the correlation between the air-entry parameter of a SWCC equation and soil properties such as percent sand or porosity. Rawls and Brakensiek (1985) presented regression equations for estimating the parameters for the Brooks and Corey (1964) equation. The regression equations estimated the air-entry (or bubbling) pressure, a_c, the pore-size index, n_c, and the residual water content, w_r, for the Brooks and Corey equation.

The Vereecken et al. (1989) method involved fitting a data set of 40 Belgian soils with the van Genuchten (1980) equation. A one-dimensional sensitivity analysis was performed on the optimized parameters of the SWCC to assess the relative importance and uniqueness of the parameters. A principle factoral analysis was used to examine the relationship between the estimated SWCC and the basic measured soil properties. It was concluded that the SWCC can be estimated to a reasonable level of accuracy using soil properties such as grain-size distribution, dry density, and carbon content. The study focused mainly on the agricultural discipline where organic soils are involved and the emphasis was on water availability for plant growth.

Physico-empirical model method

Arya and Paris (1981) presented the first physico-empirical method to estimate the SWCC. The model made use of basic information from the grain-size distribution curve. Volumetric water contents were calculated based on an estimation of the pore sizes in the soil. The pore radii were converted to equivalent soil suctions through use of the capillary theory (Taylor 1948). The pore radius estimation was based on the assumption of spherical particles and cylindrical pores. The estimation method used empirical factors to account for uncertainties in the estimation.

Arya and Paris (1981) assumed that the pore-size distribution and grain-size distribution of a soil were strongly related, with larger particles producing larger interparticle voids than smaller particles, and vice versa.

Various models have been proposed to estimate the random packing nature of spherical particles in an attempt to improve on the estimation of the pore-size distribution of a heterogeneous system (Iwata et al. 1994). The Arya and Paris (1981) model was later modified by Haverkamp and Parlange (1986), who applied the concept of shape similarity between the SWCC and the cumulative grain-size distribution for sandy soils. Gupta and Ewing (1992) applied the Arya and Paris model to the grain-size distribution to model intra-aggregate pores (i.e., interparticle pores within aggregates) and to the aggregate-size distribution to model the interaggregate pores. Nimmo (1997) presented a method of accounting for the influence of fabric and soil structure through the use of an aggregrate-size distribution.

Description of the proposed model

The physico-empirical method was selected for the new model because of its fundamental and theoretical basis. It was hypothesized that the grain-size distribution provides a physical basis for the estimation technique but is limited in that it does not consider the in situ density (or porosity) of a soil. Furthermore, the fabric of the soil is not considered. The proposed model attempts to represent soil porosity through the use of a packing arrangement factor for the various individual grain sizes. No attempt was made at this stage to represent various soil fabrics. The computed SWCC can be assumed to only approximate a likely SWCC for a soil that is initially prepared from a slurry paste.

Previous models converted the grain-size distribution to equivalent water contents. The pore-size values were used to calculate corresponding volumetric water contents and soil suctions. The method presented in this paper first divides the grain-size distribution into small particle groupings of relatively uniform particle sizes. It is hypothesized that for each uniform group of particles there exists a relatively unique desorption SWCC, and that this curve represents conditions in an initially slurried soil.

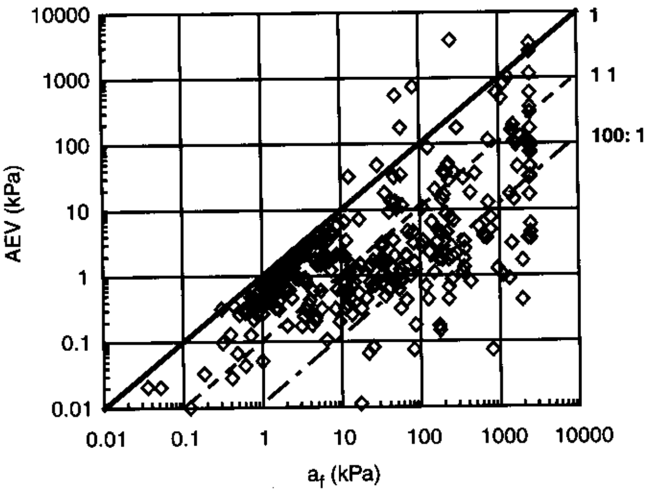
Theory of the proposed model for predicting the SWCC

The Fredlund et al. (1997) unimodal (eq. [1]) and bimodal (eq. [2]) equations can be used to best fit grain-size distribution data and provide a continuous fit of the entire grain-size distribution curve including the coarse and fine extremes (Fredlund et al. 2000). The mathematical fit of the grain-size distribution data provides the basis for a new algorithm for predicting a SWCC. The new model uses a combination of the capillary model and an understanding of the variation of the SWCC with particle sizes, in the estimation of a SWCC. Volume–mass properties and grain-size distribution data form the basic information required for the estimation of the SWCC.

The methodology behind the new approach can be expressed in terms of a series of theorems:

Theorem 1 — A soil composed entirely of uniform, homogeneous particle sizes has a unique drying (or desorption) SWCC.

Fig. 2. Relationship between the air-entry value (AEV) obtained from the Fredlund (1999) construction and the parameter a_f from the Fredlund and Xing (1994) equation for 311 soils from the "training" data set.



Theorem 2 — The capillary model can satisfactorily estimate the air-entry value of each collection of uniform, homogeneous particle sizes.

Theorem 3 — The SWCC for soils composed of more than one particle size can be represented as the summation of the SWCCs for each of the individual particle sizes.

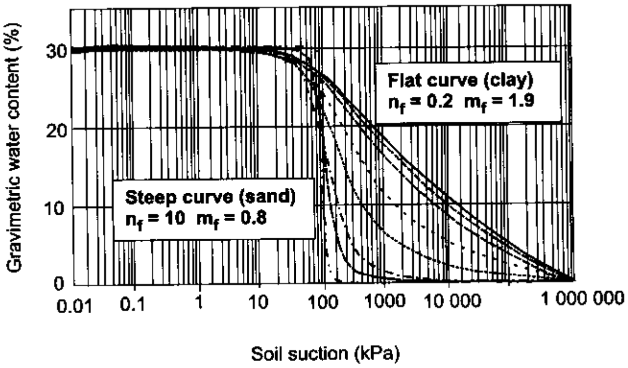
Theorems 1–3 allow the formulation of a method based on the capillary model and SWCCs inferred for each uniform particle-size range. The Fredlund and Xing (1994) equation (i.e., eq. [5]) was selected to model the SWCC because of its ability to fit the entire range of soil suctions.

An estimation of the Fredlund and Xing (1994) equation parameters for each collection of uniform particle sizes is required. The parameter a_f in the Fredlund and Xing model has been shown to be loosely related to the air-entry value of the soil. Figure 2 shows a relationship between the air-entry value for many soils and the fitting parameter a_f used in the Fredlund and Xing equation.

A data set from the SoilVision database (Fredlund 1996*b*) was used to "train" the proposed new PTF. Soils in the database that contained both a grain-size distribution curve and a SWCC measured in the laboratory were used to train the PTF. The database was split into soils with an even index number from those with an odd index number. The soils with an even index number were used to train the PTF. "Training" involved adjusting the packing arrangement factor to obtain the best fit on the laboratory data. The calculated packing arrangement factors were then used as the basis for a neural net algorithm for determining the packing factor for the odd-numbered soils, based on the input of percent sand, percent silt, percent clay, specific gravity, and void ratio of the soil.

Comparisons were then made between the SWCCs predicted using the neural net and the measured SWCCs. Figure 2 shows that the parameter a_f is typically higher than the actual air-entry value, particularly when the parameters n_f and m_f revert to extreme values. The parameter a_f is the primary variable that must be approximated to fix the lateral position of the SWCC.

Fig. 3. Variation of the parameters n_f and m_f according to particle-size diameter while holding a_f constant at 100 kPa.



The variation in the form of the Fredlund and Xing (1994) equation for a range of values of parameters n_f and m_f, with a_f constant at 100 kPa, is shown in Fig. 3. The representative SWCC would shift laterally for other values of a_f. For a given pore radius, the corresponding equivalent air-entry value can be calculated based on the capillary model shown in eq. [6]. The soil suction corresponding to the equivalent air-entry value for a soil with uniform particle sizes can be described as follows:

$$[6] \qquad \psi = 2T_s \frac{\cos\alpha}{\rho_w g r}$$

where

T_s is the surface tension of water (mL/T^2);
α is the contact angle;
ρ_w is the density of water (m/L^3);
g is the acceleration due to gravity (L/T^2);
r is the pore radius (L); and
ψ is the soil suction (m/LT^2).

A SWCC can be computed for each size of particles. Estimating the shape for coarse sand or fine silt can be done with reasonable certainty. It is also necessary to estimate typical SWCCs for uniform grain sizes between coarse, sand-sized particles and clay-sized particles. This was done by incrementally altering the parameters of the Fredlund and Xing (1994) equation for intermediate particle sizes. It was assumed that a smooth transition (on a logarithmic scale) exists for the representation of the SWCC when moving from coarse-sized particles to fine-sized particles (Fig. 4).

A data set combining soils from Rawls and Brakensiek (1985), Sillers (1996), and the CECIL soil survey (Bruce et al. 1983) was used to determine approximate trends in the parameters n_f and m_f for the Fredlund and Xing (1994) equation. An effective grain-size diameter was calculated for each grain-size curve based on the following equation:

$$[7] \qquad \frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_1}{d_1} + \sum_{i=2}^{i=n} \frac{\Delta g_i}{d_i}$$

where

d_e is the effective grain diameter;
d_1 is the largest diameter of the most coarse fraction of the material; and
Δg_1 is the weight of the material of the last fraction in terms of total weight (Vukovic and Soro 1992).

Fig. 4. Assumed limit for the SWCCs for a uniform coarse sand (i.e., sand: $a_f = 1$, $n_f = 20$, $m_f = 2$, $h_r = 3000$) and a clay (i.e., clay: $a_f = 100$, $n_f = 1$, $m_f = 0.5$, $h_r = 3000$).

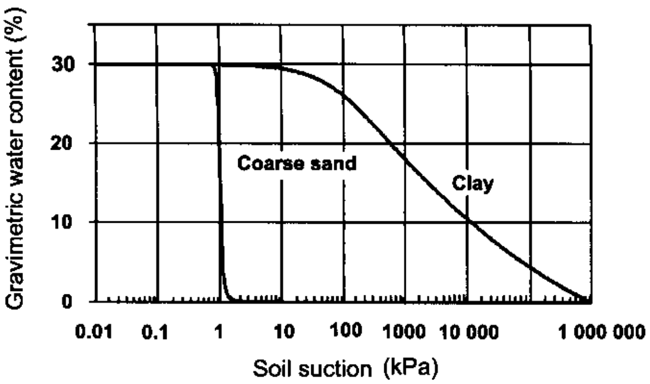


Fig. 5. Variation of the parameter n_f with effective grain-size diameter when the Fredlund and Xing (1994) equation is used to fit the SWCC.

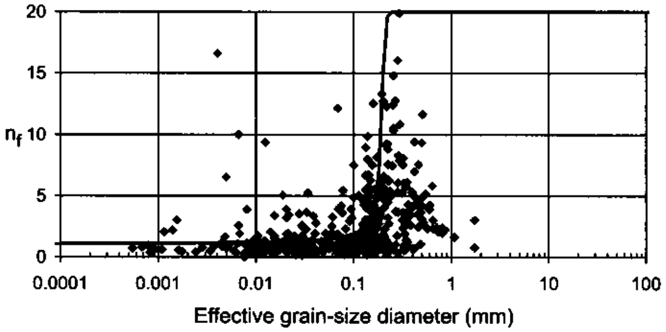


Fig. 6. Variation of the parameter m_f with effective grain-size diameter when the Fredlund and Xing (1994) equation is used to fit the SWCC.

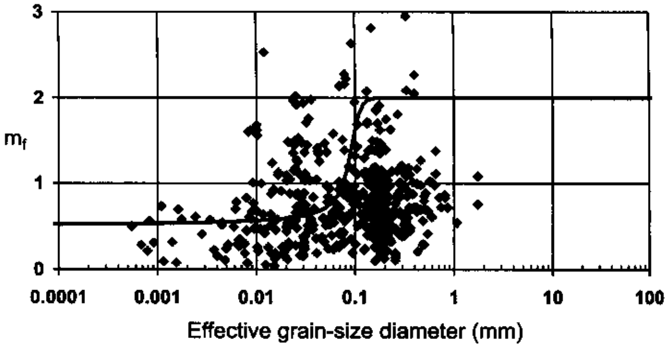
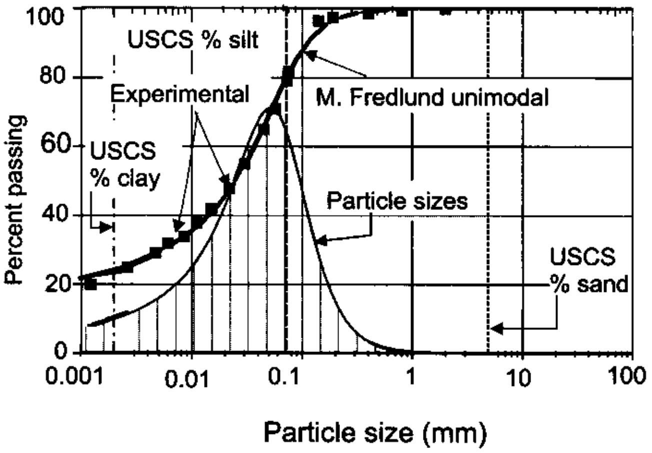


Fig. 7. Small divisions of particle sizes used to build the overall SWCC.



The effective grain-size diameter was then plotted opposite the soil parameters n_f and m_f as shown in Figs. 5 and 6, respectively. The soil parameters n_f and m_f were determined for each soil by fitting experimental data with a least-squares regression algorithm while placing limiting values on n_f and m_f. The data show considerable scatter because there appears to be some interrelationship between n_f and m_f.

An equation with a form similar to that of the van Genuchten (1980) equation was fit through the experimental data to provide an estimate of n_f and m_f based on effective grain-size diameter (i.e., eq [8]):

$$[8] \qquad p(\phi) = p_1 \left[\frac{1}{\ln\left\{ \exp(1) + \left[\frac{10^{-\log(d_e)-1}}{p_2} \right]^{p_3} \right\}} \right]^{p_4} + p_5$$

where

p_1, p_2, p_3, p_4, and p_5 are curve-fitting parameters;
d_e is the effective grain-size diameter; and
$p(\phi)$ is the value for either n_f or m_f.

Equation [8] can be used to represent variation of either n_f or m_f with respect to grain-size diameter. The equation parameters for representing the variation of n_f are as follows: $p_1 = 19$, $p_2 = 50$, $p_3 = 30$, $p_4 = 1$, and $p_5 = 1$ (see Fig. 5). The equation parameters for representing the variation of m_f are as follows: $p_1 = 1.5$, $p_2 = 100$, $p_3 = 10$, $p_4 = 1$, and $p_5 = 0.5$ (see Fig. 6).

The grain-size distribution curve was then divided into small divisions of uniform soil particles. The SWCCs were estimated by starting at the smallest particle size for each division as illustrated in Fig. 7. The divisional SWCCs were then summed starting with the smallest particle size and continuing until the volume of the pore space was equal to that for a combination of all particle sizes. The end result is an estimated SWCC, representative of the desorption curve for an initially slurried soil.

Pore volume

The grain-size distribution curve can be divided into n fractions of uniformly sized particles. Each fraction has an assumed packing arrangement for the particles, and this is referred to as the packing factor. The summation of the pore volumes for the individual particle-size fractions may be greater than the overall porosity of the combined soil fractions. In the assemblage of soil particles, the voids created between larger particles will be filled with smaller particles. This, in essence, reduces the influence that the larger particles have on the SWCC, as has been illustrated through experimental results (Yazdani et al. 2000). Therefore, the pore volumes of the individual fractions are summed until the overall porosity of the soil is reached. After this point, the remaining pore volumes of the particle fractions are ignored.

The assumed packing factor, n_p, for each uniform particle size needs to be approximated. It can be assumed that the variable n_p is the same for each successive particle fraction, but it would also seem reasonable that n_p should be a function of particle diameter. Since the grain-size distribution represents a percentage distribution by weight, the pore volume associated with each fraction is taken as an equivalent proportion of the total pore volume.

There appears to be no fixed relationship between the overall porosity of the soil and the packing factor. The overall porosity of the soil is a macroscopic representation, taking into account the manner in with all the particle sizes are "mixed." The packing factor is assumed for each uniform mass fraction of particles. Ideally, the packing factor should be a function of the particle sizes, but at present it has been assumed to be a constant for all particle sizes. The proposed method is different from other methods in that it attempts to build an overall SWCC from individual SWCCs for each particle-size fraction. The primary limitation lies in the ability to "mix" the individual particle fractions to obtain the overall SWCC.

Description of the data set analyzed in testing the proposed model

A sample data set of 188 soils was selected from the research literature to test the proposed model. The soils included data from Sillers (1996), Rawls and Brakensiek (1985), and Williams et al. (1992). All soils selected had a measured grain-size distribution and a measured SWCC. In addition, the soil parameters associated with the estimation of six PTFs presented in this paper were used.

The selected data set provides a wide distribution of soils from a number of different sources, with no bias towards one particular group of investigators. The experimentally measured SWCCs also allowed for the testing of other estimation techniques.

The reliability of the new PTF was evaluated using cross-validation. With cross-validation (Hjorth 1994), the reliability of a PTF is assessed by (*i*) drawing a random subsample from the data set, (*ii*) developing a PTF for the subsample, and (*iii*) testing the accuracy of the PTF against the data left after subsampling. Data sets were first selected based on the availability of grain-size distribution curves and sufficient volume–mass properties (i.e., void ratio, dry density, and specific gravity) to perform the estimation.

The database was split into two parts, with one part used to train the proposed PTF and the other part used to test the proposed PTF. The data set also needed to have data relevant to the other six PTFs.

Analysis of results of estimation by the proposed model

The comparisons between the experimental and predicted data are shown in Figs. 8–11, and 10 optimal estimations of the SWCC using the proposed method are shown in Fig. 12. The estimation of a SWCC from grain-size distribution has been attempted for all soil types. There appears to be more difficulty in estimating the SWCC for clays, tills, and loams than for silts and sands, although the estimated curves still appear to be quite reasonable. Results tend to be somewhat sensitive to the assumed packing factor, n_p. More research is required to better understand the influence of the packing factor.

The proposed new PTF was found to reasonably estimate the SWCC for a wide range of textural classes. Using a divisional SWCC for each individual grain-size range appears to allow increased accuracy in estimating the composite SWCC for the full grain-size distribution.

There are several groups of soils for which it is particularly difficult to estimate the SWCC. These general categories of soils include (*i*) soils that have a high amount of clay-sized particles, (*ii*) soils that contain large amounts of coarse-size particles mixed with few fines, (*iii*) soils that exhibit bimodal behavior such as sand–bentonite mixtures, and (*iv*) mine tailings and waste rock that have angular particle shapes due to the crushing process.

The proposed PTF appears to allow for a better estimation of the SWCC for the aforementioned soils than that available with previous methods. However, more research is required on how best to take into consideration the influence of fabric, stress history, and initial porosity.

The assumed minimum particle size was found to have an influence on the "closeness of fit" of the SWCC estimation. If the minimum particle-size variable was set too low, there was an over-abundance of clay-sized particles that dominated the estimation. If the minimum particle size was set too high, this caused an absence of smaller particles, with the result that the soil dried out prematurely. It is suggested that the minimum particle size should be set at 0.0001 mm to obtain reasonable results. This value has been arbitrarily set based on a parametric analysis involving various values for the minimum particle size.

Initial packing factor

The initial packing factor, n_p, takes a porosity form and is the only volume–mass variable used in the estimation of the SWCC. The packing factor is important in the estimation of the SWCC, and two techniques were studied in regard to its estimation: (*i*) a statistical method, and (*ii*) the use of a neural net.

The statistical method involved determining the frequency distribution of the packing factor for various textural categories. A mean and variance were computed for the packing factor for different soil types to provide an estimate of the general limits that are reasonable.

The statistical method appeared to be improved when the packing factor was estimated through the use of a neural net. The neural net is an artificial-intelligence technique by which an algorithm is trained to respond to various input conditions. The proposed neural net was trained using soils from a selected data subset. The packing factor of each soil was adjusted to provide an optimal estimation. The adjusted packing factor was then tabulated along with U.S. Department of Agriculture classification variables such as percent clay, percent silt, percent sand, percent coarse, d_{10}, d_{20}, d_{30}, d_{50}, d_{60},

Fig. 8. Comparison of experimental and predicted SWCC and the logarithmic probability density curve (log PDF) for a clay soil from Russam (1958) ($R^2 = 0.982$).

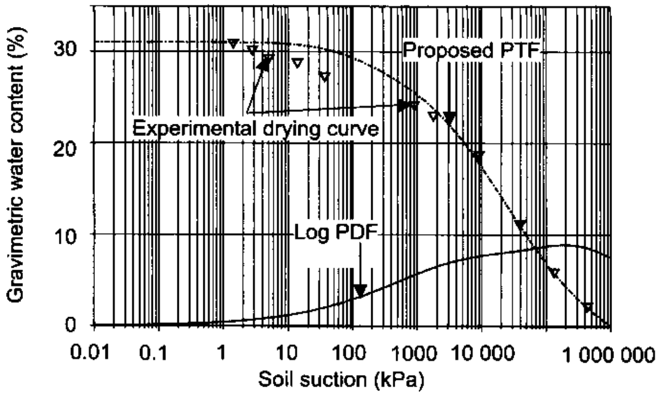


Fig. 9. Comparison of experimental and predicted SWCC and the logarithmic probability density curve for a sand from Dane and Hruska (1983) ($R^2 = 0.969$).

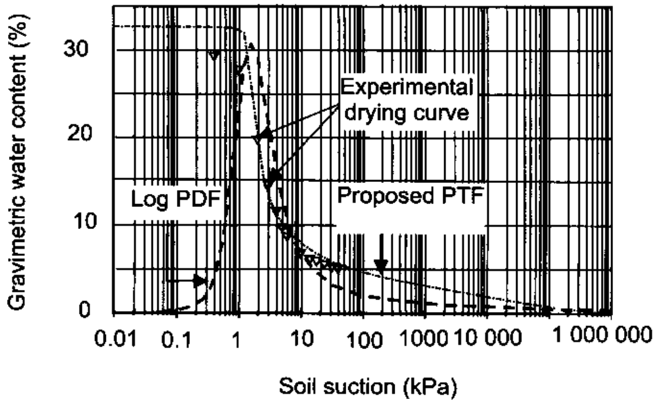


Fig. 10. Comparison of experimental and predicted SWCC and the logarithmic probability density curves for a silt loam originally from Vereecken et al. (1989) ($R^2 = 0.944$).

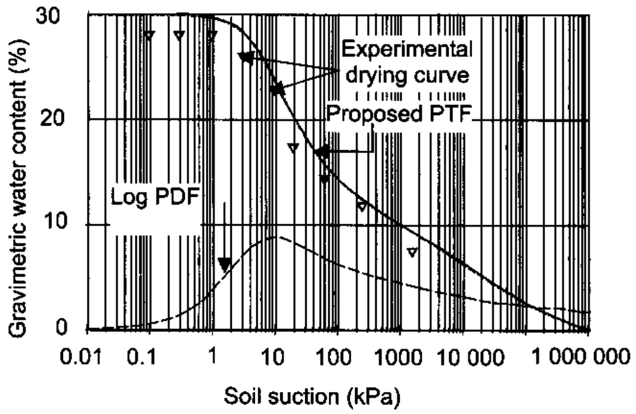
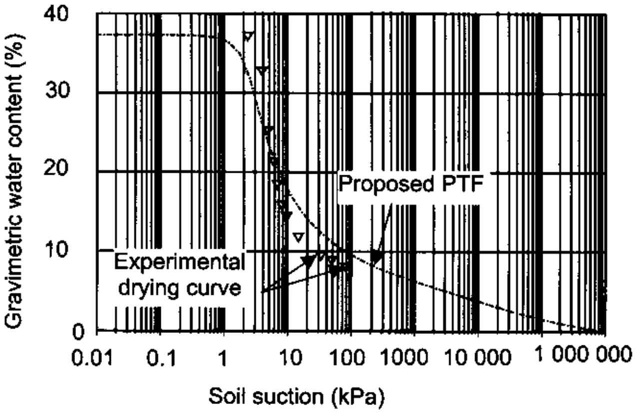


Fig. 11. Comparison of experimental and predicted SWCC data for a sandy loam originally from Schuh et al. (1991) ($R^2 = 0.869$).



and the volume–mass properties. The neural net was trained and yielded sum of squares (R^2) values of 0.830 for the training set. The neural net was then used to estimate the packing factor for the testing data set.

The influence of the packing factor on the estimation of the SWCC was studied and it was found that the packing factor does not always influence the SWCC estimation in the same manner. The effect of the packing factor is illustrated in Fig. 13 for sand and in Fig. 14 for loam. The packing factor has an influence on the entire SWCC. The initial water content of the soil is assumed to be independent of the packing factor. Figure 14 shows that the resulting SWCC estimation does not reach a water content corresponding to 100% saturation with a packing factor n_p equal to 0.36. This condition occurs when the packing factor falls too far below the actual porosity of a soil.

Comparison of results to other PTFs

The proposed estimation technique was compared to various other models, namely, those of Arya and Paris (1981), Scheinost et al. (1997), Rawls and Brakensiek (1985), Vereecken et al. (1989), and Tyler and Wheatcraft (1989). The PTFs were evaluated based on the set of data used to test the proposed method. The set of data contained a wide range of textural classifications and was selected to ensure that sufficient information was available for estimating each of the PTFs.

Arya and Paris (1981)

The Arya and Paris (1981) PTF was originally developed from a small database and then extrapolated to larger databases. A value of 1.38 is generally accepted as a reasonable estimate for the α variable. Later investigations by Arya et al. (1982) showed that the average α value varied among textural classes and ranged from 1.1 for fine-textured soils to 2.5 for coarse-textured soils. The α value for this method was estimated in accordance with the values shown in Table 1. The method requires a reasonably well defined grain-size distribution. Several estimations performed using the Arya and Paris PTF are shown in Fig. 15.

Scheinost et al. (1997)

The Scheinost et al. (1997) PTF uses a linear regression analysis to estimate the parameters for a van Genuchten (1980) type equation. The Scheinost et al. PTF was devel-

Fig. 12. Best estimation for each of five textures of the SWCC using the proposed PTF: silty clay loam $R^2 = 0.80$; loam $R^2 = 0.98$; sand $R^2 = 0.99$; sandy loam $R^2 = 0.97$; silt loam $R^2 = 0.99$.

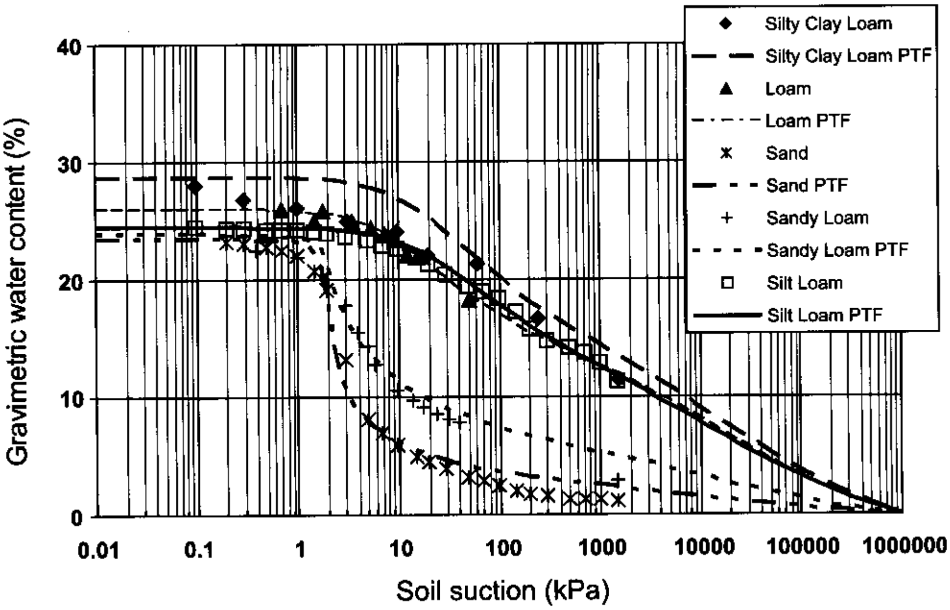
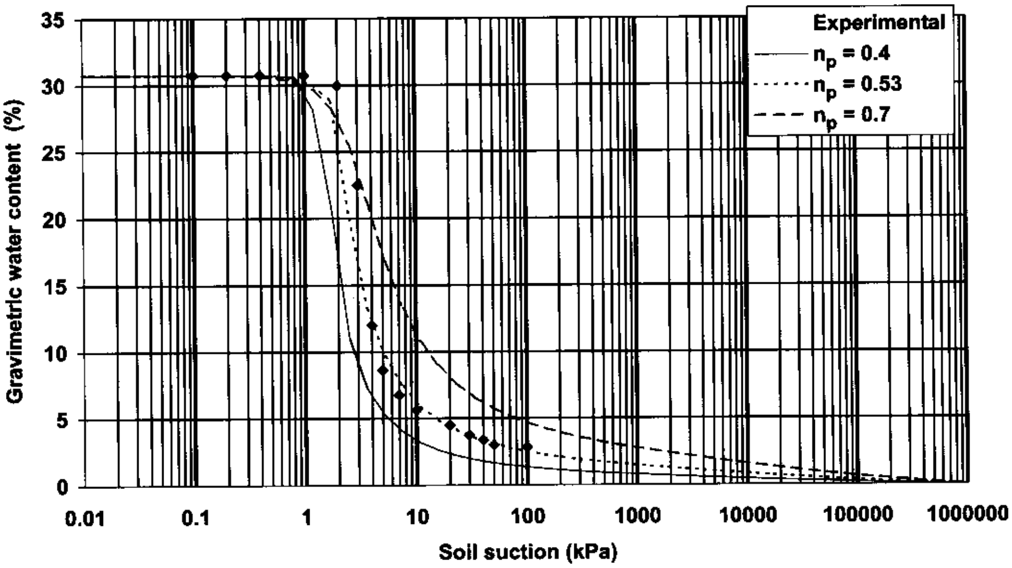


Fig. 13. Effect of varying the “packing” factor, n_p, for a sand originally published by Mualem (1984).



oped to account for extreme variations in the soil parameters, with textures varying from gravels to clays, organic contents varying over a wide range, and bulk densities from 0.80 to 1.85 Mg/m^3. The PTF was trained using a soil data set from near Munich, Germany.

The Scheinost et al. (1997) PTF was able to estimate the desaturation rate of most soils with reasonable accuracy. Five estimations performed using this method are shown in Fig. 16.

Rawls and Brakensiek (1985)

The Rawls and Brakensiek (1985) PTF is based on a regression analysis that estimates parameters for the Brooks and Corey (1964) equation. Although the estimation of the air-entry value for most soils was quite reasonable, the desaturation rate appears to be overestimated for most soils. This is likely due to the sharp initial slope inherent in the Brooks and Corey equation. Results of estimations based on this method are shown in Fig. 17.

Vereecken et al. (1989)

The Vereecken et al. (1989) PTF uses a statistical regression analysis to estimate the parameters for the van Genuchten (1980) equation. The Vereecken et al. PTF has been applied to a wide range of soils and has the ability to account for high organic matter contents. In general, the method performs well for the estimation of desaturation

Fig. 14. Effect of varying the packing factor, n_p, for a loam from Schuh et al. (1991).

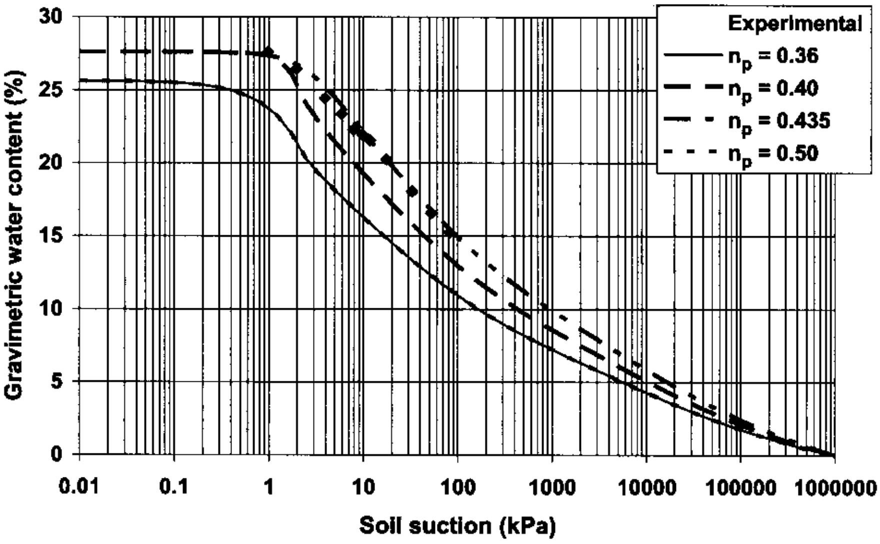


Fig. 15. Best estimation for each of five textures of the SWCC using the Arya and Paris (1981) PTF: silty clay loam $R^2 = 0.85$; loam $R^2 = 0.87$; sand $R^2 = 0.98$; sandy loam $R^2 = 0.96$; silt loam $R^2 = 0.91$.

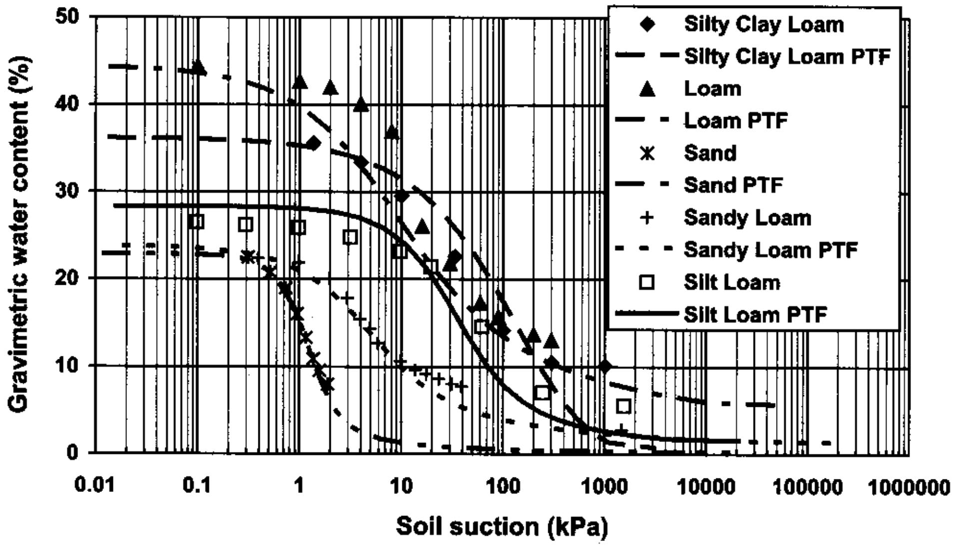


Table 1. Values of the model variable α proposed by Arya et al. (1982).

Texture	α
Sand	1.285
Sandy loam	1.459
Loam	1.375
Silt loam	1.150
Clay	1.160

Note: Texture according to the U.S. Department of Agriculture. The variable α is dimensionless.

rates. Examples of estimations performed using this method are shown in Fig. 18.

Tyler and Wheatcraft (1989)

Tyler and Wheatcraft (1989) use a fractal dimension to estimate the Arya and Paris (1981) α input parameter. The fractal dimension is calculated through the use of a linear regression analysis for particles associated with the grain-size fractions. The method does not appear to improve on the performance of the Arya and Paris estimation. Estimations performed using this method are shown in Fig. 19.

Methods of evaluation of the various PTFs

The six PTFs were evaluated in the following manner: (*i*) the R^2 values produced through the use of each PTF were compared, (*ii*) the differences between the reported and estimated air-entry values were compared, and (*iii*) the differences between the measured and estimated maximum slopes were compared.

Comparison of R^2 results

Common methods of comparison between experimental and estimated results include the use of the mean difference (MD), the root mean squared difference (RMSD), and the

Fig. 16. Best estimation for each of five textures of the SWCC using the Scheinost et al. (1997) PTF: silty clay loam R^2 = 0.96; loam R^2 = 0.76; sand R^2 = 0.64; sandy loam R^2 = 0.95; silt loam R^2 = 0.91.

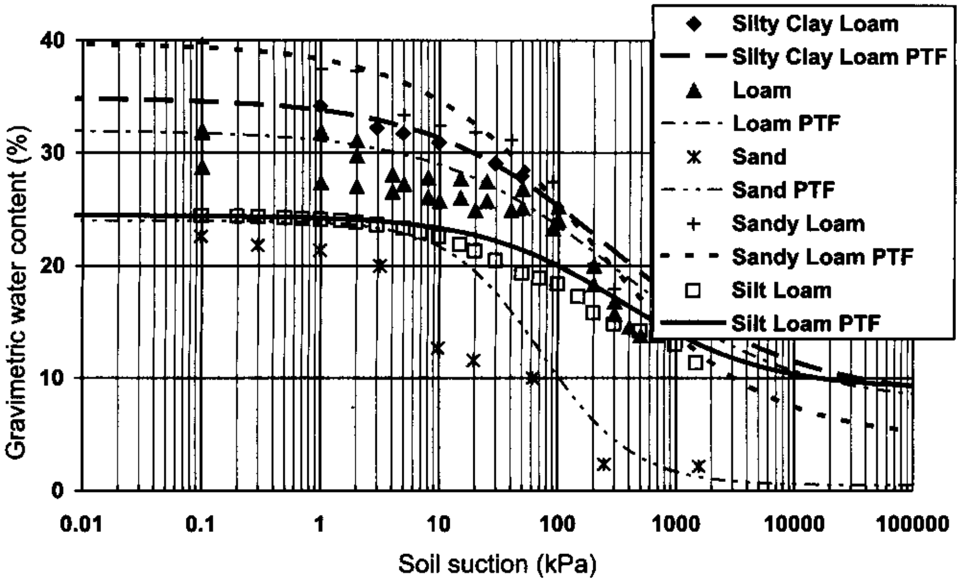
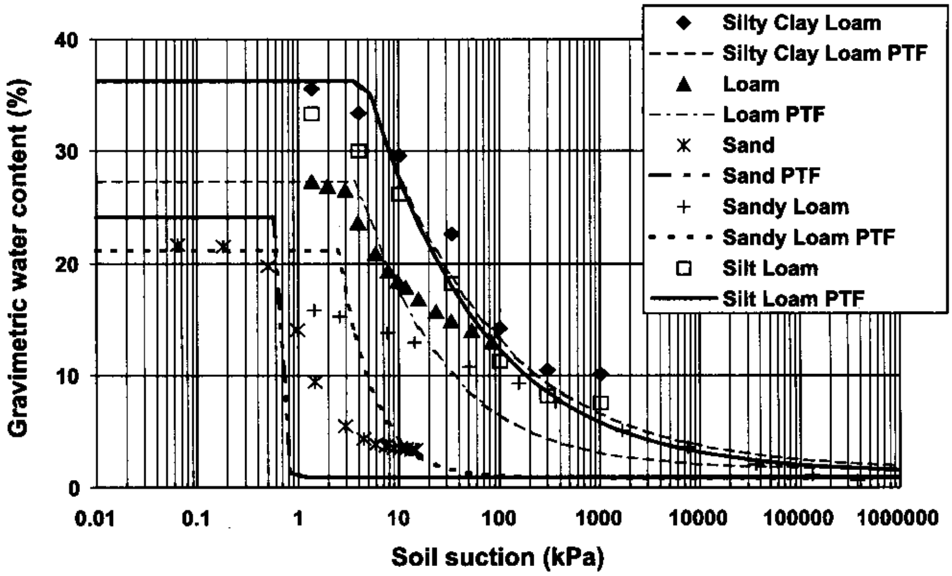


Fig. 17. Best estimation for each of five textures of the SWCC using the Rawls and Brakensiek (1985) PTF: silty clay loam R^2 = 0.94; loam R^2 = 0.58; sand R^2 = 0.50; sandy loam R^2 = –0.29; silt loam R^2 = 0.93.



sum of squares (R^2). The R^2 method was used to compare the results of the six PTFs presented in this paper, and the R^2 values provide an indication of the success of each PTF in estimating the water contents along the SWCC. Mathematically, R^2 is calculated as follows:

$$[9] \qquad R^2 = 1.0 - \frac{SS_{\mathrm{ptf}}}{SS_{\mathrm{tot}}}$$

where

SS_{ptf} is the sum of squares of the experimental points from the PTF; and

SS_{tot} is the sum of squares of the distances of the points from a horizontal line.

Soils from the test data set show R^2 values between 0.0 and 1.0, with a value of 1.0 representing a perfect fit. A value of 0.0 indicates the fit is the same as a horizontal line through the average of the y coordinates. The broken line in Fig. 20*a* indicates a fit where the frequencies increase as the R^2 value approaches 1.0. This trend indicates that the suggested model is performing well. Most of the other models show a somewhat random distribution of R^2 without a significant increase in the frequency as R^2 approaches 1.0.

The frequency distribution was plotted to give an indication of the results of the various PTFs. Comparisons of the R^2 values corresponding to the fit of all models are shown in Fig. 20.

Comparison of air-entry values

The air-entry value of a soil is probably the most relevant parameter associated with the SWCC. It becomes a primary

Fig. 18. Best estimation for each of five textures of the SWCC using the Vereecken et al. (1989) PTF: silty clay loam $R^2 = 0.74$; loam $R^2 = 0.87$; sand $R^2 = 0.73$; sandy loam $R^2 = -0.92$; silt loam $R^2 = 0.91$.

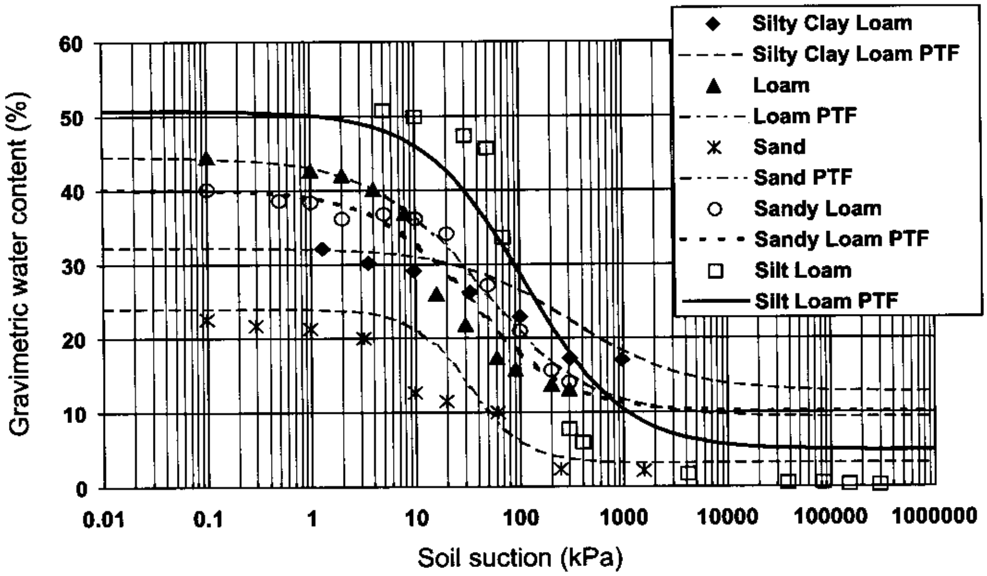
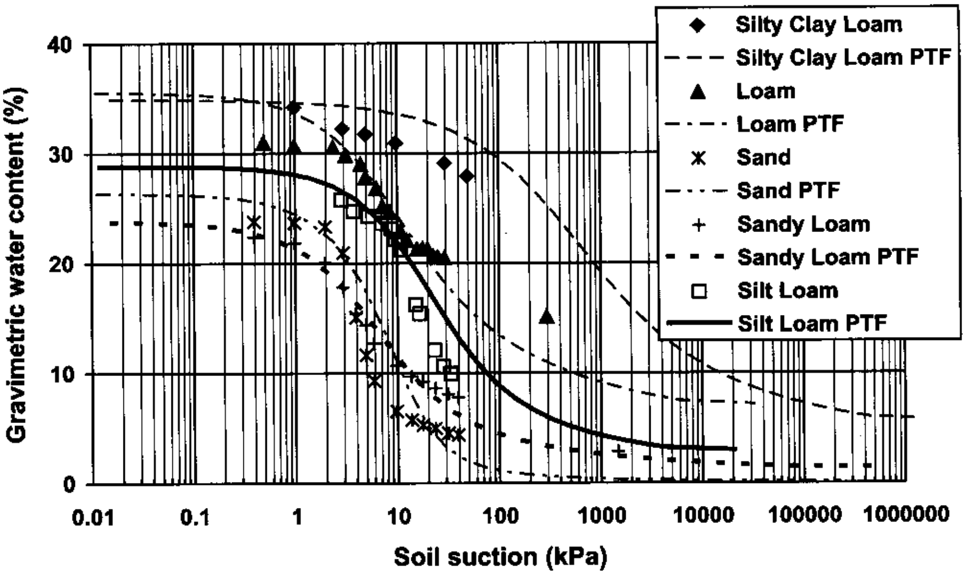


Fig. 19. Best estimation for each of five textures of the SWCC using the Tyler and Wheatcraft (1989) PTF: silty clay loam $R^2 = -0.01$; loam $R^2 = 0.89$; sand $R^2 = 0.86$; sandy loam $R^2 = -0.98$; silt loam $R^2 = 0.77$.



variable in the application of unsaturated soil mechanics in geotechnical engineering. The six PTFs were evaluated based on the accuracy of the estimated air-entry value for a soil. The reference air-entry value for the experimental data was determined by best fitting the data using the Fredlund and Xing (1994) equation. A construction procedure was performed to obtain the air-entry value (Vanapalli et al. 1998). The air-entry value for each PTF was calculated by performing a similar construction on the estimated data points. The air-entry value calculated from the best fit of the experimental data, AEV_e, was then taken to be the reference value and was compared with the air-entry values estimated using each of the PTFs (AEV_{ptf}). The following equation was used for the comparison of any two sets of air-entry values:

$$[10] \quad SD = \frac{1}{N}\sum_{i=1}^{n}[\log(AEV_{ptf}) - \log(AEV_e)]^2$$

where

AEV_{ptf} is the air-entry value of the PTF;
AEV_e is the air-entry value of the experimental data;
N is the number of data points; and
SD is the average squared difference.

A comparison of all of the estimations of the air-entry values is shown in Fig. 21. There is considerable scatter in the data from all of the methods. Figure 21 shows that most of the air-entry values lie between 0.2 and 10.0 kPa. In this range, more of the air-entry values from the PTFs are above the reference line. The Tyler and Wheatcraft (1989) PTF ap-

Fig. 20. Comparison of frequency distribution of R^2 for values between 0.0 and 1.0 for all six PTFs.

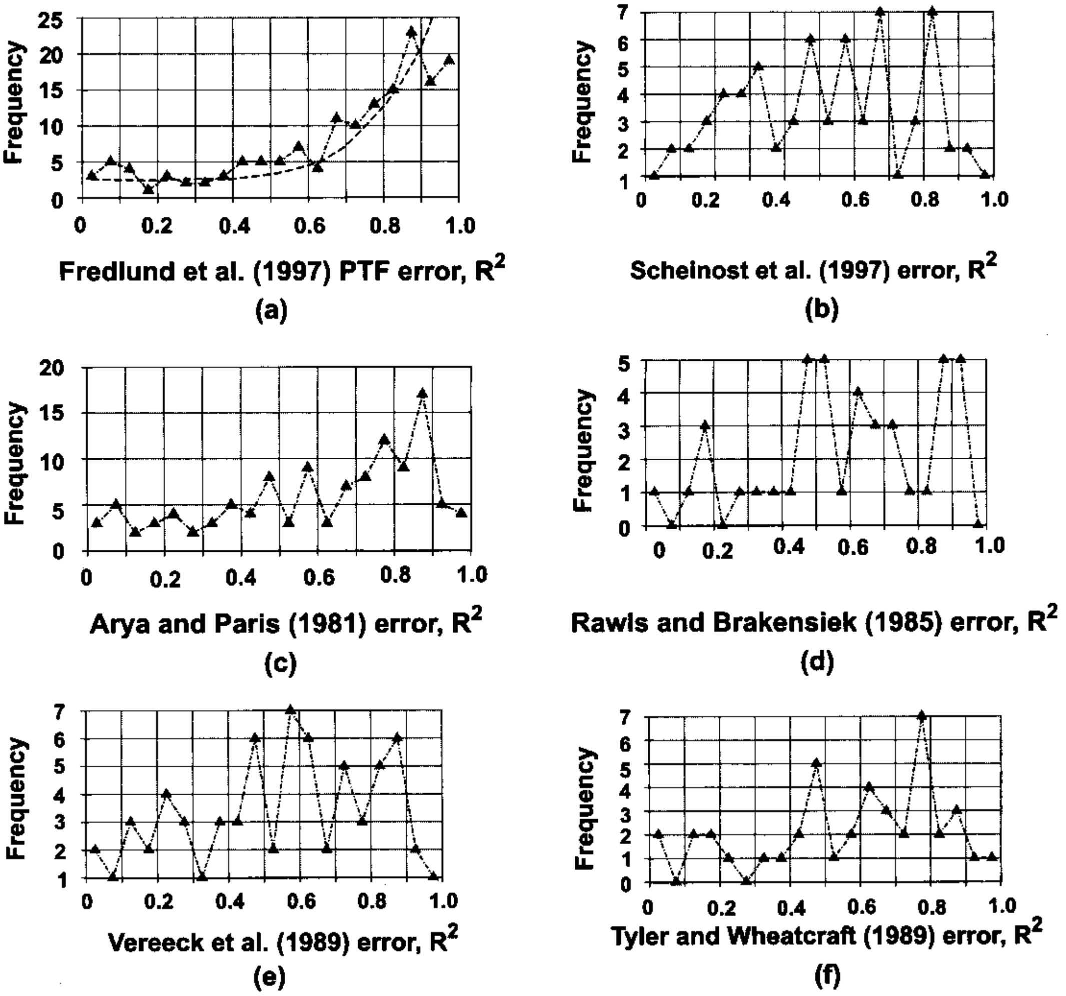


Fig. 21. Difference between the measured and estimated air-entry values (AEV) for all six PTFs.

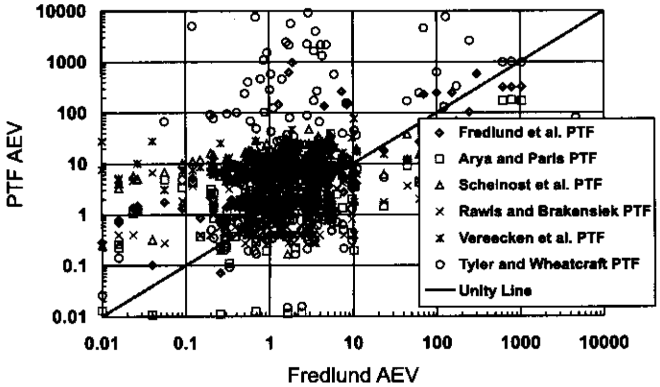
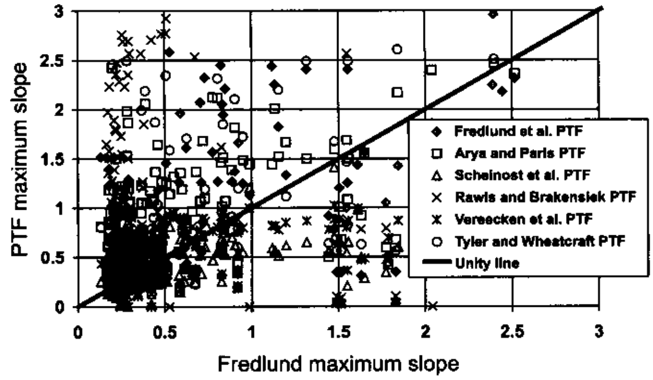


Fig. 22. Difference between the measured and estimated maximum slope for all six PTFs.



pears to give the poorest estimations, giving air-entry values that are far too high.

Comparison of maximum slope

The rate at which a soil desaturates is another soil parameter of importance in geotechnical engineering. The six PTFs were evaluated to assess the estimates of the rate at which a soil desaturates. The representation of the rate of desaturation was taken as the maximum slope of the SWCC. The maximum slope was calculated for the experimental data by fitting the experimental data with the Fredlund and Xing (1994) equation and then determining the point of maximum slope corresponding to the inflection point on the best-fit curve.

The maximum slope is dimensionless and is calculated as a change in the ordinate, Δy, on the normalized SWCC divided by the change in the logarithm of soil suction in kilopascals. Each of the PTFs was evaluated by similarly

Table 2. Squared difference between the logarithm of the estimated and measured air-entry values (AEV) for all six PTFs.

PTF AEV	Squared difference
Fredlund et al. 1997 PTF	0.5850
Arya and Paris 1981	0.8620
Scheinost et al. 1997	1.1911
Rawls and Brakensiek 1985	0.7870
Vereecken et al. 1989	1.3281
Tyler and Wheatcraft 1989	3.4380

Table 3. Squared difference between estimated and measured maximum slopes for all six PTFs.

PTF maximum slope	Squared difference
Fredlund et al. 1997 PTF	0.487
Arya and Paris 1981	0.586
Scheinost et al. 1997	0.476
Rawls and Brakensiek 1985	7.850
Vereecken et al. 1989	0.462
Tyler and Wheatcraft 1989	0.988

calculating the maximum slope from the points along the estimated SWCC. The squared difference between the experimental and estimated results was then calculated according to the following equation:

$$[11] \quad \mathrm{SD} = \frac{1}{N}\sum_{i=1}^{n}[\mathrm{MS_{ptf}} - \mathrm{MS_e}]^2$$

where

$\mathrm{MS_{ptf}}$ is the maximum slope of the PTF;

$\mathrm{MS_e}$ is the maximum slope of the experimental data;

N is the number of data points; and

SD is the average squared difference.

A comparison of the maximum slopes is shown in Fig. 22. Most of the maximum slopes are in the range from 0.2 to 2.5. The predicted maximum slopes from the PTFs are generally higher than the reference line.

Results and discussion of the fit between the predicted and experimental data

The results of the statistical analysis showed that the proposed PTF performed well. Comparisons with previously proposed PTFs indicated a significant improvement in the performance of the PTF. Each PTF was evaluated based on R^2 values, air-entry values, and the maximum slope.

Results of the comparisons between the measured and estimated air-entry values indicated a significant improvement when using the proposed PTF. The average of the logarithm of the squared differences between the experimental and measured results for the test data set is shown in Table 2. The new PTF and the Arya and Paris (1981) methods showed the highest level of confidence in correctly estimating the air-entry value of a soil over the full range of air-entry values in the database (Fig. 21). The Rawls and Brakensiek (1985) method predicted the air-entry value satisfactorily but failed in predicting the maximum slope of the SWCC.

The maximum slope of the SWCC computed using the proposed PTF showed reasonable accuracy when compared with the maximum slope computed using the experimental data. The averages of the logarithm of the squared differences between the experimental and measured results for the test data set are shown in Table 3. Table 3 indicates reasonable performance of the proposed PTF. The best performance was obtained using the Vereecken et al. (1989) method.

Conclusions

Estimation of the soil-water characteristic curve (SWCC) from the grain-size distribution was found to be reasonably reliable for sands and silts. Clays, tills, and loams were more difficult to predict, although the accuracy of the estimation algorithm was still reasonable. More data sets are required to test the algorithms on finer soils, and different algorithms will likely be required to estimate the SWCCs of soils with more complex fabrics.

The analytical results tended to be sensitive to the packing factor, n_p, and more research is needed to fully understand how best to estimate this parameter. The variability in fabric as the clay content of a soil increases is the main limitation associated with using any of the proposed PTF methods.

The proposed PTF was compared to five other PTFs. The success of each PTF was evaluated based on an R^2 value, an air-entry value, and a maximum slope. The proposed PTF showed an improvement in the R^2 error distribution (i.e., a shape approximating a close fit).

The results of the comparisons between the measured and estimated air-entry values indicated significant improvements for the new PTF over existing methods. The new PTF and the Arya and Paris (1981) methods showed the highest level of confidence in estimating the air-entry value and the maximum slope for the SWCC of a soil. The proposed PTF showed reasonable accuracy in estimating the maximum slope of the SWCC. On the selected data set, the best performance was obtained using the Vereecken et al. (1989) method.

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