51st Canadian Geotechnical Conference, Edmonton, Alberta, October 4-7, 1998

THE MEANING AND RELEVANCE OF RESIDUAL STATE TO UNSATURATED SOILS

S.K. Vanapalli, W.S. Sillers, and M.D. Fredlund
Department of Civil Engineering, University of Saskatchewan,
Saskatoon, Saskatchewan, Canada, S7N 5A9

ABSTRACT

All soils irrespective of type, structure, or mineralogy are in a limiting saturated state condition at zero suction and in a dry condition at a suction value of 1,000,000 kPa. As the stress state (i.e., soil suction) of the soil changes from a saturated condition to a dry condition, the distribution of soil-water-air interphase relationships also change. The soil-water-air interphase relationships have a predominant influence on the engineering behavior of unsaturated soils. The residual state is an important point along the soil-water-air interphase in the mechanics for unsaturated soils. This paper summarizes and discusses research on the concept of residual state. The meaning and relevance of residual state to unsaturated soils is detailed. A graphical construction is proposed which uses a computational technique to estimate the residual state and the air-entry value from soil-water characteristic curve data. The significance of residual state with respect to the shear strength behavior of unsaturated soils is discussed. The analysis of these results suggests that the shear strength increases non-linearly up to the residual suction value and may lose strength at higher values of suction.

RÉSUMÉ

Les sols de tout type, structure ou minéralogie sont dans une condition d'état limitant saturé à succion zéro et dans une condition sèche à une valeur de succion de 1,000,000 kPa. Comme le sol passe d'une condition saturée à une condition sèche, la distribution de la relation d'interphase sol-eau-air change avec l'état de stresse (i.e succion du sol). Les relations de l'interphase sol-eau-air ont une influence prédominante sur le comportement en ingénierie des sols insaturés. Le contenu en eau résiduelle est une relation importante d'interphase dans la mécanique pour les sols insaturés. Cet article résume et discute la recherche sur le concept du contenu disponible en eau résiduelle dans la littérature. La signification et la pertinence du contenu en eau résiduelle acordées aux sols insaturés sont décrites. Une construction graphique est proposée laquelle utilise une technique informatique pour estimer le contenu en eau résiduelle et la valeur d'entrée de l'air à partir de données d'une courbe caractéristique eau-sol. L'importance d'une condition d'état résiduel en relation avec le comportement de la force de cisaillement des sols insaturés est discutée. L'analyse de ces résultats suggère que la force de cisaillement augmente de façon non-linéaire jusqu'à la valeur de succion résiduelle et commence à perdre de la force à des valeurs plus élevées de succion.

1. INTRODUCTION

A framework is available to describe the engineering behavior of unsaturated soils in terms of two stress state variables, namely net normal stress, $(\sigma - u_a)$, and suction, $(u_a - u_w)$ (Fredlund and Rahardjo 1993). This approach is however costly and time consuming. Engineering properties of unsaturated soils such as the shear strength, coefficient of permeability and contaminant transport properties can be predicted reasonably well using the saturated soil properties and the soil-water characteristic curve (Fredlund 1997).

The soil-water characteristic curve is the relationship between the water content (i.e., either volumetric or gravimetric) or degree of saturation and soil suction. The soil is in a limiting saturated state condition at zero suction and is in a dry condition at approximately 1,000,000 kPa. These limiting states of saturated and dry conditions are true for all soils irrespective of type, soil structure, mineralogy and the stress state. As the soil moves from a saturated condition to a dry condition, the distribution of soil-water-air interphase relationship changes as the stress state changes. The influence of these phases on the soil-water characteristic curve and hence on the engineering behavior of unsaturated soils, is of importance in the prediction of unsaturated soil properties. The residual state condition (i.e. residual water content or degree of saturation or volumetric water content and soil suction) represents one location along the soil-water characteristic curve and is of interest to geotechnical, agricultural and geo-environmental engineers and soil scientists.

It is often necessary to define the residual state condition from the soil-water characteristic curve in order to obtain the fitting parameters and use in numerical models to predict the coefficient of permeability (Brooks and Corey 1964, van Genuchten 1980) and shear strength (Fredlund et al. 1996 and Vanapalli et al. 1996). Such equations offer computational advantages and are useful in developing mathematical equations for numerical solutions.

The residual state concept is interpreted in several ways in the literature. Some are empirical procedures which use the soil-water characteristic curve data for a limited suction range (i.e., between 0 to 1,500 kPa) to define the residual water content (Brooks and Corey 1964, White et al 1970 and van Genucthen 1980). For fine-grained such as tills and clays the residual state conditions may occur at suction values that are higher than 1,500 kPa. Hence, models based on soil-water characteristic curve data that were developed for a limited range of suctions may not be suitable for the prediction of unsaturated soil properties at low water contents and high suctions.

Recent experimental evidence shows that shear strength increases non-linearly and may even start to decrease at large values of suction for unsaturated soils (Escario and Juca, 1989). Similarly, at suction values beyond residual state conditions the variation of coefficient of permeability may not be due to the liquid phase any more but due to the combined flow of liquid and vapor phase. It is important to use the entire soil-water characteristic curve (i.e., from 0 to 1,000,000 kPa) to model the unsaturated soil behavior and estimate residual state condition. Such an estimate would be useful in understanding the relevance of the term with respect to the engineering behavior of unsaturated soils.

This paper summarizes and discusses research on the concept of residual state. A graphical construction is proposed which uses a computational technique to estimate the residual state using the entire soil-water characteristic data (i.e., from 0 to 1,000,000 kPa). The meaning and relevance of the term residual state with respect to unsaturated soil behavior is discussed. The significance of residual state condition on the shear strength behavior of unsaturated soils is studied in more detail.

2. THE CONCEPT OF RESIDUAL STATE – REVIEW

The definition of residual water content or residual state available in literature are based on empirical, physical or construction procedures. The common element between all these definitions is that at residual state condition, the water phase is discontinuous and isolated with thin films of water surrounding the soil and air. Figure 1 shows the probable soil-water-air interphase relationship at residual state condition.

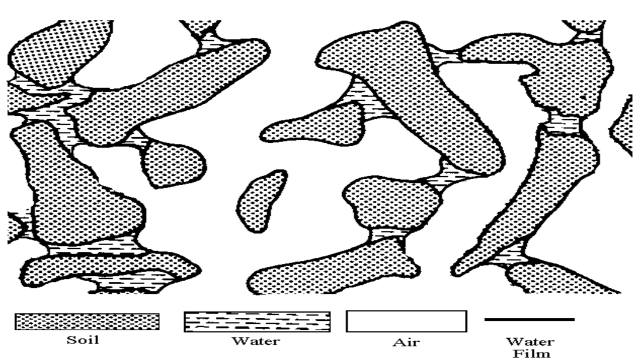

Figure 1 The probable soil-water-air interphase relationship at residual state condition (after Fairbridge and Finkl 1979)

Lebedeff (1927) studied the distribution of water within sand as a function of the height of above a water table. From these findings he noted that below a certain value of water content, an increase in suction had little affect on the soil-water content. The term “maximum molecular moisture-holding capacity” was used to describe residual water content. Residual water content tends to become essentially a constant value for further increments of soil suction according to this definition. The residual water content as defined by Lebedeff (1927) is not a clearly defined value and is a parameter that is extrapolated from the available data. However, it should be noted that the initial desaturation or drainage of soil is in liquid phase at low suction values and at higher values of suction the desaturation is due both to liquid and vapor phases.

Brooks and Corey (1964) defined residual water content as the water content at which suction reaches infinity. The difficulty with this definition is that it is not possible to extend the soil-water characteristic curve to infinity. Both Lebedeff (1927) and Brooks and Corey (1964) definitions have limitations. A soil suction of 1,000,000 kPa is generally accepted to represent dry conditions within a soil (Mitchell 1976). Experimental evidence supporting this value is provided by several investigators (Croney and Coleman, 1961; Russam 1958 and Fredlund, 1964; and Vanapalli, 1994). Thermodynamic principles also support zero water content at 1,000,000 kPa suction (Richards, 1965; Wilson et al. 1994).

van Genuchten (1980) defined the residual water content as the water content at a soil suction of 1500 kPa. This is suction limit of measurement for most soil suction testing equipment. Also, a suction value of 1500 kPa is defined as the wilting point. Models used for agricultural purposes are generally not required to model soil-water characteristics beyond the wilting point.

According to van Genuchten et al (1991), residual water content is defined as the water content at which the slope of the soil-water characteristic curve (i.e., $d\theta/d\psi$ for the data plotted as volumetric water content, θ, versus soil suction, ψ) and coefficient of permeability go to zero when soil suction becomes large. This definition is still open to interpretation, since the coefficient of permeability may be small but there is film flow and vapor movement and as a result the coefficient of permeability is a non-zero, finite number (Nitao and Bear, 1996). Also, the slope of the soil-water characteristic curve does not become zero until a soil suction of 1,000,000 kPa when the water content reaches zero. According to Luckner et al (1989), the residual water content specifies the maximum amount of water in a soil that will not contribute to liquid flow because there is a blockage in flow paths or a strong adsorption onto the solid phase.

Nitao and Bear (1996) noted that calculated residual water contents are more a function of the instrumentation used to measure the soil-water characteristic data (i.e., the maximum suction measurable by the instrument) than an actual physical constant. Nitao and Bear (1996) suggest that residual water content is related to the lowest measured water content.

The soil-water characteristic curve is a continuous function and there is no specific point that can be called the residual water content. Currently, most investigators treat residual water content as a fitting parameter with no real physical significance (van Genuchten, 1991; Kosugi, 1994; van Genuchten, 1988; Siddroupoulos and Yannopoulos, 1988; Luckner et al, 1989; Nimmo, 1991; Nielsen and Luckner, 1992; Kosugi, 1994), or avoid the controversy by using a correction factor (Fredlund and Xing, 1994 and Fayer and Simmons, 1995).

One reason for treating residual water content as a fitting parameter with less emphasis on its physical meaning is related to the fact that different models of the soil-water characteristic curve can produce different results (Kosugi, 1994). Another reason is that the residual water content is not the lowest possible water content within soil. Through evaporation, centrifuging or oven drying it is possible for water content to be less than residual water content. The credibility of residual state or residual water content concept diminishes if film flow and vapor flow are taken into consideration. Water can be transferred through the thin films of water adsorbed onto the surface of the soil particles or by transfer in the vapor phase. Sillers (1997) more recently defined residual water content as the water content where the soil-water goes from being held within the soil primarily by capillary action to soil-water being held in the soil primarily by adsorptive forces.

An alternative and perhaps more accurate method of determining the residual state is to use a construction method. A construction method is less dependent on the soil-water characteristic model used and gives more consistent results across different soil-water characteristic models. A graphical construction procedure which uses a computational construction technique to estimate the residual state based upon soil-water characteristic data ranging from 0 to 1,000,000 kPa. This procedure is extended to estimate the air-entry value of the soil. The practical advantages of this construction method are discussed later in the paper.

3. A COMPUTATIONAL METHOD OF CONSTRUCTION FOR ESTIMATING THE RESIDUAL STATE AND THE AIR-ENTRY VALUE

Fredlund and Xing (1994) provided an analytical basis for mathematically defining the soil-water characteristic curve. The equation applies over the entire range of suctions from 0 to 1,000,000 kPa. This equation is most commonly written in terms of volumetric water content, θ_w, as given below:

$$\theta_w = \theta_s \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right] \left[\frac{1}{\ln\left[e + \left(\frac{\psi}{a}\right)^n \right]^m} \right]$$

[Eq. 1]

where:

θ = volumetric water content;
θ_s = saturated volumetric water content;
a = suction related to the air-entry value of the soil;
n = a soil parameter related to the slope at the inflection point on the soil-water characteristic curve;
ψ = soil suction;
m = a soil parameter related to the residual water content;
e = a natural number, 2.71828…; and
C_r = the residual suction

Best-fit parameters for Eq.1 are found in order to describe the soil-water characteristic curve over the entire range of suction. Figure 2 shows a typical soil-water characteristic curve plotted as volumetric water content, θ_w, vs. soil suction relationship. The air-entry and residual water content values are highlighted on this figure. The fitting parameters (i.e., *a, n,* and *m*), the air-entry and the residual state can be determined using a computational technique with the aid of SoilVision™. The procedural steps involved in this technique for estimating the residual state and the air-entry value are given below:

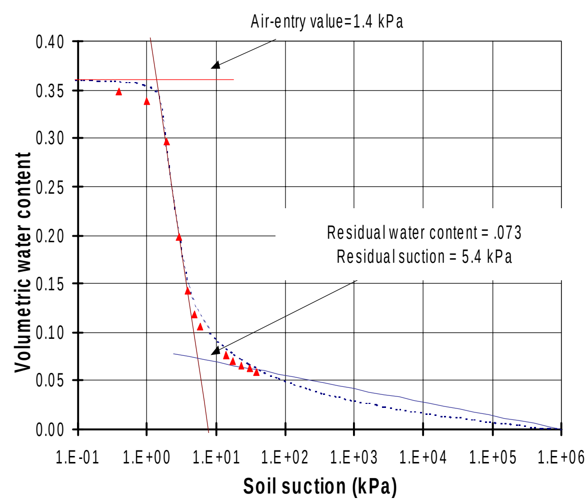

Figure 2 Construction procedure to estimate the residual state and the air-entry value of a Sand. (from Dane et al. 1983) (SoilVision soil #10723)

3.1 Estimation of residual state

Step 1. Using Eq. [1] find the best-fit parameters to describe the experimental data over the entire suction range.

Step 2. Determine the point of maximum slope on the best-fit curve and draw a line tangent to the curve through the point of maximum slope.

Step 3. Determine point of maximum change of slope between the point of maximum slope and 1,000,000 kPa.

Step 4. Move one logarithmic cycle past inflection point and locate point on the best-fit curve.

Step 5. Draw the residual line through the located point and 1,000,000 kPa and zero volumetric water content.

Step 6. The intersection of the two lines indicates the residual state condition (i.e., the residual water content and the residual suction of the soil).

3.2 Air-entry value estimation

Step 1. Step 1 for air-entry value estimation is identical to Step 1 in the estimation of residual state.

Step 2. Step 2 for air-entry value estimation is identical to Step 2 in the estimation of residual state.

Step 3. Draw a line tangent to the curve through the point of maximum slope.

Step 4. Draw a horizontal line through the maximum volumetric water content

Step 5. The intersection of the two lines indicates the air-entry value.

Soil-water characteristic curves are commonly plotted as volumetric or gravimetric water content or degree of saturation versus soil suction relationships. The residual state condition and the air-entry value determined by the computational method will be independent of the way the data is plotted.

Figure 3, Figure 4, and Figure 5 show the results of the residual volumetric water content applied to a variety of different soil types. The construction technique was applied to the 6000 soil-water characteristic curves present in the SoilVision™ database. The performance of the technique was then analyzed by selecting a random sample of soil-water characteristic curves from the database and visually verifying the results.

Typically a value of 3000 kPa has been suggested for the C_r constant in Eq. 1 (Fredlund and Xing 1994). The construction technique presented in this paper provides a method for determining C_r in Eq. 1. The C_r value determined is substituted back into Eq. 1. The fitting parameters *a*, *n*, and *m* are then determined using the C_r value.

The construction technique provided results of the air-entry value and the residual state that matched well with visual inspection for almost all of the soils analyzed. Soils from most of the textural classes such as sands, loams, tills, and clays were inspected. Problem soils encountered were clays and bimodal soils. The problem encountered with dense clays corresponds to the lack of a meaningful inflection point on the fit of the soil-water characteristic curve. The construction technique also does not account for bimodal nature in some soils. This will cause a certain amount of underestimation in the residual suction with bimodal soils. In summary, the construction technique worked well for the majority of the soils analyzed.

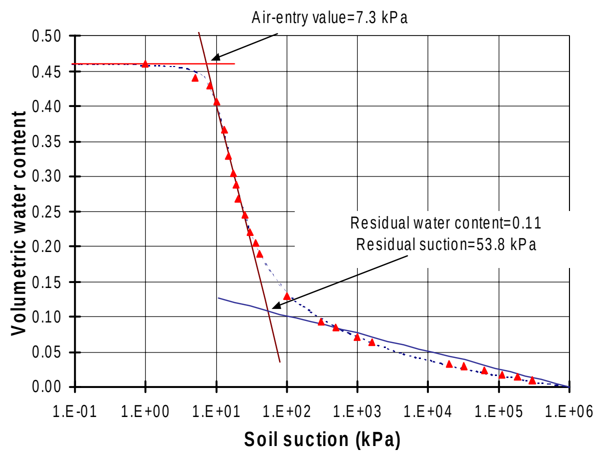

Figure 3 Results of the construction technique for a Loam (from Jackson et al. 1965) (SoilVision soil #91)

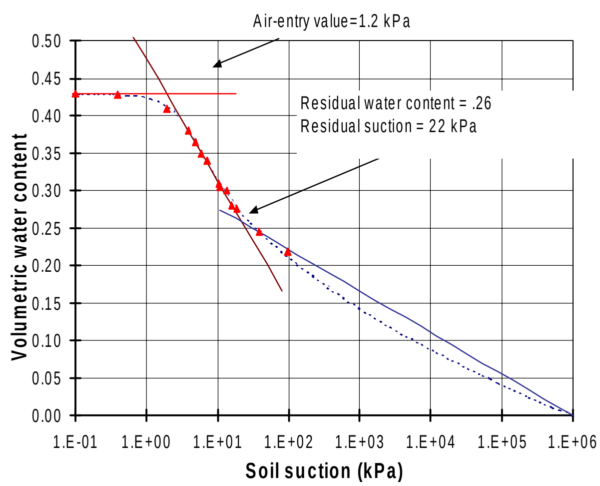

Figure 4 Results of the construction technique for a Loam (from Sisson and van Genuchten, 1991) (SoilVision soil #131)

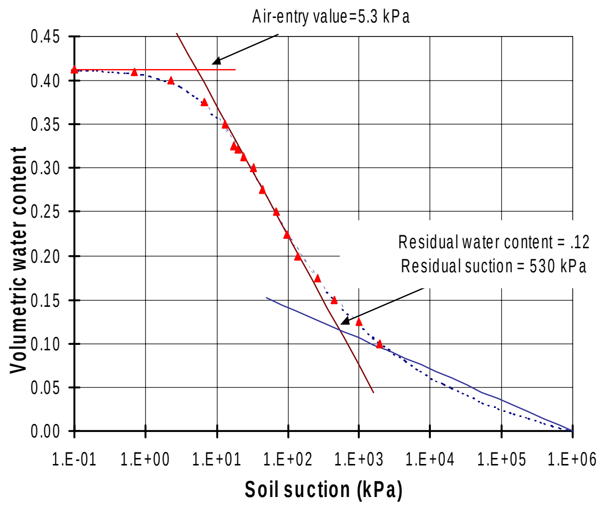

Figure 5 Results of the construction technique for a Silt Loam (from van Dam et al. 1992) (SoilVision soil #133)

4. RESIDUAL STATE CONDITION AND THE SHEAR STRENGTH OF UNSATURATED SOILS

Shear strength equation for an unsaturated soil was proposed by Fredlund et al. (1978) as:

[Eq. 2]

$$\tau_f = [c' + (\sigma_n - u_a)\tan\phi'] + \left[(u_a - u_w)\tan\phi^b\right]$$

where:

τ_f = shear strength of an unsaturated soil,
c' = effective cohesion of the soil,
ϕ' = effective angle of shearing resistance for a saturated soil,
$(\sigma_n - u_a)$ = net normal stress,
$(u_a - u_w)$ = matric suction, and
ϕ^b = angle of shearing resistance relative to an increase in suction

Fredlund et al. (1996) and Vanapalli et al. (1996) suggest several models for predicting the shear strength of an unsaturated soil using the soil-water characteristic curve and the saturated shear strength parameters. Equation [3] given below can be used in predicting the shear strength of an unsaturated soil:

$$\tau_f = [c' + (\sigma_n - u_a)\tan\phi'] + (u_a - u_w)\{\Theta(u_a - u_w)\}^{\kappa}(\tan\phi') \quad \text{[Eq. 3]}$$

where:

κ = fitting parameter used for obtaining better correlation between the measured and predicted values
Θ = normalized volumetric water content

The shear strength contribution due to suction is the second part of Eq. 3, which is:

$$\tau_{us} = (u_a - u_w)\{\Theta(u_a - u_w)\}^{\kappa}(\tan\phi') \quad \text{[Eq. 4]}$$

The incremental shear strength contribution due to suction, $d\tau$, can be obtained by differentiating [Eq. 4] with respect to suction, $(u_a - u_w)$. The result is

$$d\tau = d(u_a - u_w)[\{\Theta(u_a - u_w)\}^{\kappa}(\tan\phi')] + (u_a - u_w)[d\{\Theta(u_a - u_w)\}^{\kappa}](\tan\phi') \quad \text{[Eq. 5]}$$

The value of $(\tan\phi^b)$ at any suction is

$$\tan\phi^b = d\tau / d(u_a - u_w)$$

$$= \left[[\{\Theta(u_a - u_w)\}^{\kappa} + (u_a - u_w)[d\{\Theta(u_a - u_w)\}^{\kappa} / d(u_a - u_w)\}]\right]\tan\phi' \quad \text{[Eq. 6]}$$

Up to the air-entry value of the soil, $\Theta(u_a - u_w)$, is equal to unity and the rate of change of $\Theta(u_a - u_w)$ (i.e., $[d\{\Theta(u_a - u_w)\}^{\kappa}] / [d(u_a - u_w)]$ equals zero. The rate of change of $\Theta(u_a - u_w)$, (i.e., $[d\{\Theta(u_a - u_w)\}^{\kappa}] / [d(u_a - u_w)]$ is always a negative value for increments of suction beyond the air-entry value. However, the net shear strength contribution due to suction, $d\tau$, in [Eq. 6] is positive. At high values of suction (i.e., in the residual state conditions), $\Theta(u_a - u_w)$ is extremely small and the value of $[d\{\Theta(u_a - u_w)\}^{\kappa}] / [d(u_a - u_w)]$ is negative. The net summation of the terms in Eq. 6 start approaching towards negative values. This behavior would result in soil losing strength beyond residual state condition. The shear strength of unsaturated soils at 1,000,000 kPa, which represents the dry state condition of the soil equals the saturated shear strength value. Equation [6] satisfies the conceptual behavior of the shear strength of unsaturated soils and also provides a good understanding of the shear strength behavior at residual state condition and beyond (i.e., at high suction values).

5. EXPERIMENTAL RESULTS

Experimental results of shear strength, coefficient of permeability and contaminant properties for a large range of suction values are not commonly available in the literature. Escario and Juca (1989) have studied the experimental shear strength behavior for Guadalix Red silty clay and Madrid gray clay for a large range of suction (i.e., 0 to 15,000 kPa). The significance of residual state conditions on the shear strength behavior of Guadalix Red silty clay is discussed in this paper. The properties of soil are summarized in Table 1.

Table 1. Properties of Guadalix Red silty clay

Property	Guadalix Red silty clay
Sand (%)	17
Silt (%)	48
Clay (%)	35
Liquid limit (%)	33
Plasticity index (%)	13.6
Specific Gravity, G_s	2.66
Dry density (kN/cu.m)	18
Initial void ratio	0.48
Initial water content (%)	13.6
Coefficient of permeability, k_{sat} (m/sec)	1.0 x 10^{-8}
Cohesion, c' (kPa)	20
Angle of internal friction (ϕ') (degrees)	32.5

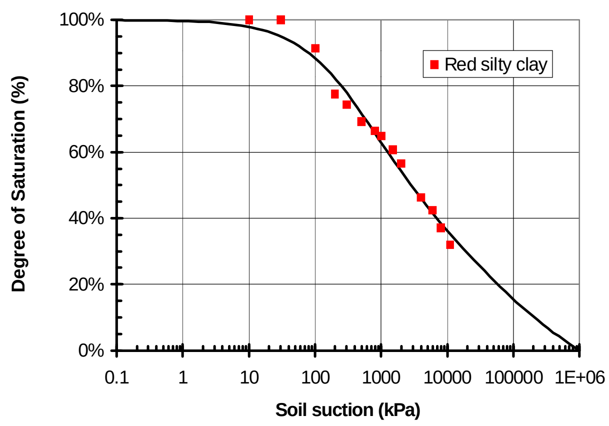

Figure 6 Soil-water characteristic curves for Madrid clayey sand and Guadalix Red silty clay (modified after Escario and Juca 1989).

Several properties such as the applied stress, initial water content, structure and mineralogy influence the soil-water characteristic curve behavior. Shear strength of unsaturated soils can be more accurately predicted using the soil-water characteristic curve that has been determined taking into account the influence of stress state and soil structure. Vanapalli et al. (1998) have shown that shear strength can be predicted with a reasonable degree of accuracy using soil-water characteristic curves that have been determined without considering the influence of the stress state of soil for a net normal stress variation of 25 to 200 kPa. A glacial till was used in this study.

Soil-water characteristic curve for Guadalix Red silty clay measured in the laboratory without the application of any confining or normal pressure is shown in Figure 6. The *a*, *n*, and *m* values required to best-fit the experimental data of soil-water characteristic curves using Fredlund and Xing (1994) equation were determined along with the air-entry value and residual suction values as discussed earlier using SoilVision™. This information is summarized in Table 2.

Table 2. Residual state conditions and *a*, *n*, and *m* values of Guadalix Red silty clay from soil-water characteristic curve data using SoilVision

Parameter/property	Guadalix Red silty clay
a (kPa)	242.1
n	0.81
m	0.79
Air-entry value (kPa)	52.7
Residual suction (kPa)	19,630

Escario and Juca (1989) have tested Guadalix Red silty clay under a net normal stress of 120 to 600 kPa and soil. However, shear strength results under a net normal stress of 120 kPa are only discussed in this paper. The variation of shear strength with respect to suction was measured for a suction range 0 to 15,000 kPa. The saturated shear strength parameters and the soil-water characteristic curve data measured in the laboratory without the application of any applied stress were used in the prediction of the unsaturated shear strength.

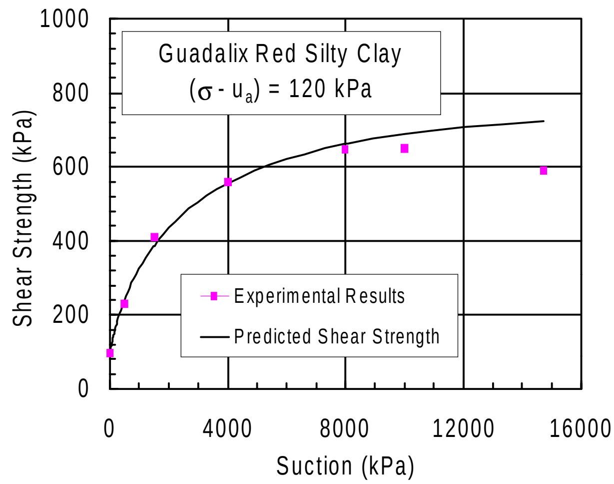

Figure 7. Measured and predicted shear strength values for Guadalix Red silty Clay

The predicted and measured shear strength values for Red silty clay are shown in Fig. 7. A κ value of 2.2 is used for fitting the results using Eq. 3. The saturated shear parameters for the soil used are summarized in Table 1. There is a reasonably good comparison between the measured and predicted values of shear strength for a large range of suction (i.e., 0 to 10,000 kPa). There are certain conceptual limitations with respect to these results. The soil should start losing strength at suction values higher than the estimated residual value of suction in accordance with the discussions in the paper. The residual suction value of this soil estimated using the construction method detailed earlier is 19,630 kPa (see Table 2). The shear strength of this soil, however, shows the trends of losing strength at a suction value of 10,000 kPa, which is lower than the estimated residual suction value of 19, 630 kPa.

A close examination of Escario and Juca (1989) results provide possible reasons for this behavior. These investigators plotted the experimental results of shear strength versus net normal stress for various values of suction for Guadalix Red silty clay and found that the effective angle of shearing resistance (ϕ') to be varying with soil suction. In other words, these results suggest the shear strength envelops were "not congruent". This may be one of the important reasons for the slightly inconsistent behavior. It should also be noted that there are no experimental data for the soil-water characteristic curve for suction values higher than 10,000 kPa. This may have also contributed to be some errors due to a lack of data in the high suction range. Rigorous analysis could not be undertaken at this time due to insufficient experimental data.

More controlled experimental studies for a large suction range for various soils have to be undertaken to better understand the significance of residual state condition on several engineering properties such as the shear strength, coefficient of permeability, volume change and contaminant transport properties.

6. SUMMARY AND CONCLUSIONS

The engineering behavior of unsaturated soils is dependent on the soil-water-air phase relationships. Residual state is one such soil-water-air phase relationship which has considerable influence on the unsaturated soil behavior. There is no accepted definition or procedure for estimating the residual state is presently available in the literature. The present understanding available in the literature on this topic is summarized in this paper. Sillers (1997) defined residual water content as the point on the soil-water characteristic curve where the soil suction phase goes from being held primarily by the capillary action force to being held in the soil primarily by adsorption force (water is present as thin films surrounding the soil particles). Using this principle, a graphical construction is proposed which uses a computational technique to estimate the residual water content from soil-water characteristic curve data from 0 to 1,000,000 kPa. This procedure is also extended to estimate the air-entry value of the soil in this paper.

Shear strength behavior of unsaturated soils with respect to residual state conditions is discussed in this paper. The shear strength increases non-linearly up to the residual state condition. There appears to be a remarkable change in the shear strength behavior of unsaturated soils at residual state conditions. The unsaturated soil may be expected to be lose strength at a decremental rate at values of suction greater than the residual value of suction. Equation [6] satisfies the conceptual behavior of the shear strength for the entire range of suction (i.e., 0 to 1,000,000 kPa).

There are limited experimental studies available in the literature with respect to the engineering behavior of unsaturated soils in the high suction range. Experimental studies on different soils are to be undertaken to better understand the conceptual role of residual state conditions on the engineering behavior of unsaturated soils.

7. REFERENCES

Brooks, R. and Corey, A. 1964. Hydraulic properties of Porous media. Colorado State University, Fort Collins. Hydrology Paper No. 3.

Croney, D. and Coleman, J. 1961. Pore pressure and suction in soils. *In* Proc. of the Conf. on Pore Pressure and Suction in Soils. Butterworths, London. pp. 31-37.

Dane et al. 1983. South. Cooperative Service Bullettin 262, Alabama Agricultural Experimental Station, Auburn University, Alabama.

Escario, V., and Juca, J. 1989. Strength and deformation of partly saturated soils, Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio de Janerio, Vol. 2, pp. 43-46.

Fairbridge, R. and Finkl, C. 1979. The encyclopedia of soil science part 1. Dowden, Hutchinson and Ross, Inc. Stroudsburg Pennsylvania.

Fayer, M. J. and Simmons, C. S. 1995. Modified soil water retention functions for all matric suctions. Water Resour. Res, **31**: 1233-1238.

Fredlund, D.G. 1964. Comparison of soil suction and one-dimensional consolidation characteristics of a highly plastic clay. M.Sc thesis, Department of Civil Engineering, University of Alberta, Edmonton.

Fredlund, D.G., Morgenstern, N.R., and Widger, R.A. (1978). The shear strength of unsaturated soils. Canadian Geotechnical Journal, **15:** 313-321.

Fredlund, D. G. and H. Rahardjo. 1993. Soil mechanics for unsaturated soils. John Wiley & Sons, Inc., New York.

Fredlund, D. G. and Xing, A. 1994. Equations for the soil water characteristic curve. Canadian Geotechnical Journal, **31**: 521-532.

Fredlund, D. G., Xing, A., Fredlund, M.D., and Barbour, S.L. 1996. The relationship of the unsaturated soil shear strength to the soil-water characteristic curve. Canadian Geotechnical Journal, **33**: 440-448.

Fredlund, D.G. 1997. An introduction to unsaturated soil mechanics. Unsaturated Soil Engineering Practice. Geotechnical Speical Publication No. 68. Geo-Logan' 97. Edited S.L. Houston and D.G. Fredlund.

Jackson, R. D.; Reginato, R. J.; and van Bavel, C. H. M. 1965. Comparison of measured and Calculated Hydraulic Conductivities of Unsaturated Soils. Water Resour. Res, **1**: 375-380.

Kosugi, K. 1994. Three parameter lognormal distribution model for soil water retention. Water Resour. Res, **30**: 891-901.

Lebedeff, A.F. 1927. The movement of ground and soil waters. Proc. 1st Int. Cong. Soil Sci. Vol. 1, pp. 459-494.

Luckner, L., van Genuchten M. Th., and Neilsen, D. R. 1989. A consistent set of parameteric models for the flow of water and air as immiscible fluids in the subsurface. Water Resourc. Res. **25**:2187-2189

Mitchell, J.K. 1976. Fundamentals of soil behavior. , John Wiley & Sons Inc., New York.

Nielsen, D. R. and Luckner, L. 1992. Theoretical aspects to estimate reasonable initial parameters and range limits in identification procedures for soil hydraulic properties. In M. Th. Van Genuchten, F. Leij and L. Lund (ed.) Proc Int. Workshop on Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils. University of California, Riverside, Ca.

Nimmo, J. R. 1991. Comment on the treatment of residual water content in " A consistent set of parameteric models for the two-phase flow of immiscible fluids in the subsurface" by L. Luckner et al. Water Resour. Res, **27**: 661-662.

Nitao, J. and Bear, J. 1996. Potentials and their role in transport in porous media. Water Resour. Res, **32**: 225-250.

Russam , K. 1958. An Investigation into the Soil Moisture Conditions Under Roads in Trinidad, B.W.I. Geotechnique, **8**:55-71.

Sillers, W. S. 1997. The mathematical representation of the soil-water characteristic curve. M Sc Thesis University of Saskatchewan.

Richards, B.G. 1965. Measurement of the free energy of soil moisture by the psychrometric technique using thermistors. In Moisture Equilibria and Moisture Changes in Soils Beneath Covered Areas, A Symposium in Print, Sydney, Australia. Edited by G.D. Aitchison. Butterworths & Co. Ltd., pp. 35-46.

Siddroupoulos, E. and Yannopoulos, S. 1988. Sensitivity analysis of closed-form analytical hydraulic conductivity models. J. Hydrol, **101**: 159-172.

Sisson, J.B., and van Genuchten, M.T. 1991. An improved analysis for gravity drainage experiments for estimating the unsaturated soil hydraulic functions. Water Resour. Res, **27**: 569-575

Vanapalli, S.K. 1994. Simple test procedures and their interpretation in evaluating the shear strength of an unsaturated soil. Ph.D. thesis, University of Saskatchewan.

Vanapalli, S.K, Fredlund, D.G. , Pufahl, D.E. and Clifton, A.W. 1996. Model for the prediction of shear strength with respect to soil suction. Canadian Geotechnical Journal, **33**: 379-392.

Vanapalli, S.K, Pufahl, D.E., and Fredlund D. G. 1998. The effect of stress state on the soil-water characteristic of a compacted sandy-clay till. 51st Canadian Geotechncial Conference, Edmonton.

van Dam, J. C.; Stricker, J. N. M. and Droogers, P. 1992. Inverse method for determining soil hydraulic functions from one-step outflow experiments. Soil Science Society of America Journal, **56**: 1042-1050

van Genuchten, M. Th. 1980. A closed form equation predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J, **44**: 892-898.

van Genuchten, M. Th., Leij, F. and Yates, S. 1991. The RETC code for quantifying the hydraulic functions of unsaturated soils. EPA Research Document EPA/600/2-91/065.

Veihmeyer, F.J. and Edlefsen. N.E. 1937. Interpretation of soil-moisture problems by means of energy-changes. Trans Amer. Geophysical Union, 18th Annual Meeting, Hydrology, pp. 302-318.

White, N.F., Duke, H.R., Sunada, D.K., and Corey, A.T. 1970. Physics of desaturation in porous materials, Journal of the Irrigation and Drainage Division, ASCE, **96**: 165-191.

Wilson, G.W., Fredlund, D.G., and Barbour, S.L. 1994. Coupled soil-atmosphere modeling for soil evaporation. Canadian Geotechnical Journal, **31**: 151-161.

8. ACKNOWLEDGEMENTS

The authors appreciate the comments and suggestions of Prof. D.G. Fredlund. The authors would like to acknowledge the use of the SoilVision™ database provided by SoilVision Systems Ltd. SoilVision Systems Ltd. can be found on the world wide web at:

www.quadrant.net/soilvision