Two-dimensional probabilistic slope stability analysis of Fundão dam

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ABSTRACT
The failure of the Fundão Dam was considered, at its time, the biggest environmental disaster in Brazil and the biggest in the world involving tailings dams. Studies of the failure that followed adopted a deterministic approach, discarding the formal approach to the uncertainties involved in the design. It was identified that the non-designed presence of mud near the downstream zone impaired drainage conditions and simultaneously caused stress redistributions raising the shear stresses in the sandy tailings and leading to contractile rupture and liquefaction. This paper presents two-dimensional probabilistic analyses developed by Monte Carlo methods and following a hybrid point estimation method, considering parameter uncertainties and spatial variability inherent in the materials. With the available data, it was found that the average factor of safety decreases when the section is in the undrained condition. Evaluating the work against traditional limits of threat degrees, it was found that the left section in the undrained condition extrapolated the acceptable limits for ore dam works. However, mainly due to the lack of a borehole or cone penetration test through the total depth of the tailings of the left abutment, where the rupture started, the analyses indicated that the right hand shoulder had a higher probability of rupture.

RÉSUMÉ
La rupture du barrage de Fundão a été considérée, à l'époque, comme la plus grande catastrophe environnementale au Brésil et la plus grande au monde concernant les digues à stériles. Les études de l'échec qui ont suivi ont adopté une approche déterministe, écartant l'approche formelle des incertitudes liées au projet. Il a été identifié que la présence non prévue dans la conception de boue près de la zone en aval a entravé les conditions de drainage et a simultanément provoqué des redistributions de contraintes augmentant les contraintes de cisaillement dans les résidus sableux et conduisant à une rupture contractile et à une liquéfaction. Cet article présente des analyses probabilistes bidimensionnelles développées par des méthodes de Monte Carlo et suivant une méthode hybride d'estimation ponctuelle, en considérant les incertitudes des paramètres et la variabilité spatiale inhérente aux matériaux. Avec les données disponibles, il a été constaté que le facteur de sécurité moyen diminue lorsque la section est dans la condition non drainée. En évaluant l'ouvrage par rapport aux limites traditionnelles des degrés de menace, il a été constaté que la section gauche, à l'état non drainé, dépassait les limites acceptables pour les ouvrages de barrages minéraux. Cependant, principalement en raison de l'absence de sondage ou d'essai de pénétration au cône à travers la profondeur totale des résidus du côté gauche, où la rupture a commencé, les analyses ont indiqué que l'épaule droite avait une plus grande probabilité de rupture.

1 INTRODUCTION

On November 5, 2015, the Fundão Dam, located at the Germano industrial site, Bento Rodrigues subdistrict of the municipality of Mariana-MG, ruptured, causing a rush of mud and mining tailings that caused the destruction of the subdistrict, left 17 dead, more than 600 people homeless, and generated severe environmental and socioeconomic damage to the entire Doce River Basin. The public and private economic losses and material damage to infrastructure assessed reached R$1.2 billion (Government of Minas Gerais, 2016).

The "Committee of Experts for the Analysis of the Fundão Tailings Dam Rupture," herein referred to as the "Investigation Panel," presented a report of studies of the rupture process (Morgenstern et al., 2016). The Panel attributed the rupture to the presence of mud in locations not initially programmed, resulting in redistributions of horizontal stresses in an interlayer of sand. This stress redistribution produced increases in shear stresses in the sand layers, causing localized ruptures, in a contractile process by the high void ratios, which caused the liquefaction process in the left shoulder, quickly propagated in the dam body (Pereira, 2005).

Morgenstern (2016) presents deterministic analyses, despite the various sources of uncertainty in the problem. Figure 1 presents two probability density functions (PDF) that provide an impossible viewpoint for traditional deterministic analysis. The scenarios presented in Figure 1 show that the PDF with the higher mean factor of safety (E[FS]) has a higher probability of failure. the deterministic FS can provide incomplete or even misleading information about slope stability (Gitirana Jr., 2005). Thus, the probabilistic approach provides a systematic and rational way for considering uncertainties in an engineering design and assessing the probability of failure (Lacasse and Nandim, 2007).

Most of the time, failures occur due to a combination of factors. Assessments in the presence of uncertainties are an inherent aspect of engineering, with the goal of projects being to reduce threats and risks to the population, society, and the environment (Lacasse and Nandim, 2007).

Uncertainty is commonly divided into model uncertainty, parameter uncertainty, and human uncertainty (Morgenstern, 1995, cited by El-Ramly and Morgenstern, 2003). Human uncertainty is difficult to account for because it is random errors made in statistical measurements or subjective judgments. Model uncertainty, on the other hand, can be reduced by improving models and analysis theories. Parameter uncertainty is mainly attributed to the spatial variability inherent in each material and to statistical uncertainty due to the number of observations (El-Ramly and Morgenstern, 2003).

Although the rupture process pointed out by Morgenstern et al. (2016) is not a conventional mechanism, analyses by the limit equilibrium method (LEM) remain pertinent, as they are able to reveal the safety state of the dam against one of the possible forms of instability. In this context, this paper presents a two-dimensional probabilistic analysis of dam stability via LEM and an assessment of how the site stood in the face of acceptable threat degree levels as defined by Whitman (1984).

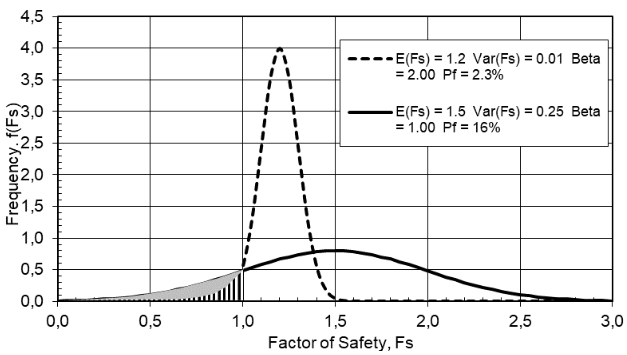

Figure 1. Probability density function of the safety factor, Fs, and the probability of failure density function, Pf

2 HISTORY AND COLLAPSE OF THE FUNDÃO DAM

Iron ore beneficiation was what drove the Germano Complex. As explained by Morgenstern et al. (2016), with the upstream elevation method, the sandy tailings were deposited forming a gently sloping beach, through which the transport water flowed quickly. The clayey tailings, on the other hand, remained in suspension for a longer time and with a slower subsequent densification, producing a less resistant material with low permeability. The upstream method results in conditions more vulnerable to liquefaction. The safety of the dam design was guaranteed as long as there was no mud to impede drainage and the sands remained unsaturated.

In 2005, the Fundão valley was chosen as a new tailings disposal site to meet the needs of the complex. The layout in Figure 2 shows dike 2, which would hold the clayey tailings, and dike 1, which would receive the sandy tailings and should have a higher elevation during the entire raising process, preventing the advancement of the clayey tailings in this region. In 2009, piping or internal erosion was observed on the downstream slope, caused by excessive deposits of fine materials and serious flaws in the construction of the bottom drain and its filters (Hradilek, 2002). Repairs were not possible and therefore the drains, the most important elements of the drained stack concept, were sealed. In parallel, the filling of dike 2 was more accelerated than ideal due to the lower production of sandy tailings, allowing the unwanted advancement of fine tailings downstream of the massif. This required the construction of a third dike, designated Dike 1 A, shown in Figure 3.

As a remediation measure, a drainage mat was built on the surface of the tailings, to replace the inoperative bottom drain positioned below them, as shown in Figure 3. This drainage mat was intended to intercept the flow that could arise on the slope and reduce its stability, following a relatively common drainage design alternative (Assis et al., 2003). However, the sands below this drain would remain saturated, as would most of the tailings upstream.

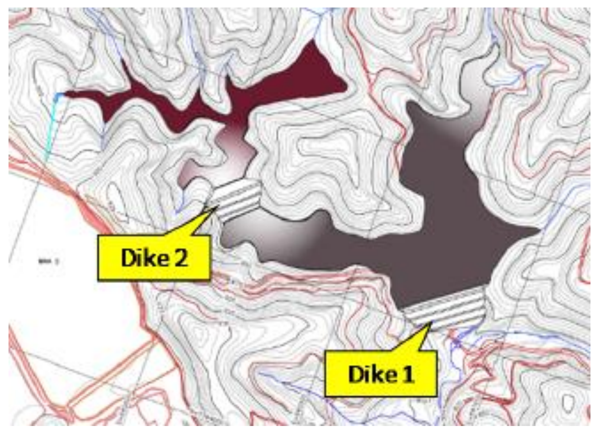

Figure 2. Presentation of the Fundão Valley highlighting the dikes. Source: Morgenstern et al. (2016)

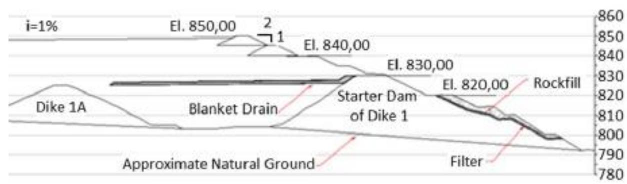

Figure 3. Drainage mat in front of the 1st Dike. Source: Morgenstern et al. (2016)

In late 2012, when the height of the tailings on the left shoulder had already exceeded the carrying capacity of the drains built in that region, it was decided, as a temporary solution, to realign the dam on the left shoulder in order to allow the embankment to continue to rise. This realignment became known as "setback". Due to the retreat, the next year saw episodes of emergence, rutting and slipping at elevation 860 m. Cracks in the dam crest and saturation at the foot of the slope were observed later.

Several small earthquakes occurred on November 5, 2015, the date of rupture. Although seismic tremors of that magnitude are recurrent in the region, this may have been the trigger needed to initiate the rupture. A total of 32.6 million m³ of tailings were released with the rupture. The material flowed over the Santarém dam, which held back a large part of the tailings. Then, the remaining portion of this material reached Bento Rodrigues, a district of the municipality of Mariana (MG), located 8 kilometers away from the Fundão structure, causing destruction of flora and fauna until it reached the ocean (Samarco, 2020).

3 METHODOLOGY

For the analyses performed in this paper, data were mostly taken from the public domain report prepared by the Research Panel (Morgenstern et Al., 2016). The report provides information on geometry and properties of the massif.

With regard to geometry, Morgenstern et Al. (2016), analyzed two representative sections, section AA of the right shoulder and section 01 of the left shoulder, represented by Figure 4. Figure 5 shows the location of the representative sections. The sections were digitized, allowing the sections to be reproduced for the probabilistic analyses, performed using the SVSLOPE program from the SVOFFICE 5 package (SoilVision Systems Ltd., 2018).

Regarding the material parameters, to represent the different drainage conditions, parameters are adopted in the effective stress analysis (ESA), which are the effective friction angle and the effective cohesion, while the undrained strength analysis (USA) uses the undrained strength ratio, for the material below the water level (Morgenstern et al., 2016).

Table 1 presents an overview of the average parameters adopted. The drained strength parameters were obtained by means of CD triaxial type tests. The undrained strength ratio (Su/σ') was determined by CPT tests. CU triaxial tests provided parameters of the drained strength at the reconstituted in-situ stresses. All tests cited were performed by Samarco and presented by Morgenstern et al. (2016).

The Committee did not find any sufficiently detailed liquefaction analysis performed prior to the failure of the Fundão Dam. Nor did it find any borehole or cone penetration test (CPT) traversing the full depth of the tailings, which would have made such an assessment possible.

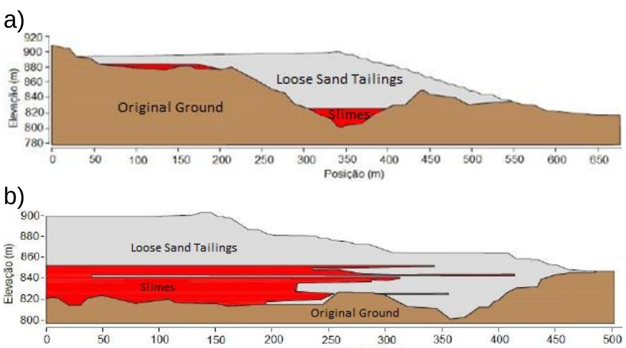

Figure 4. Typical dam sections: a) Section AA, right shoulder; and b) Section 01, left shoulder. Source: Morgenstern et al. (2016), modified by Souza (2008)

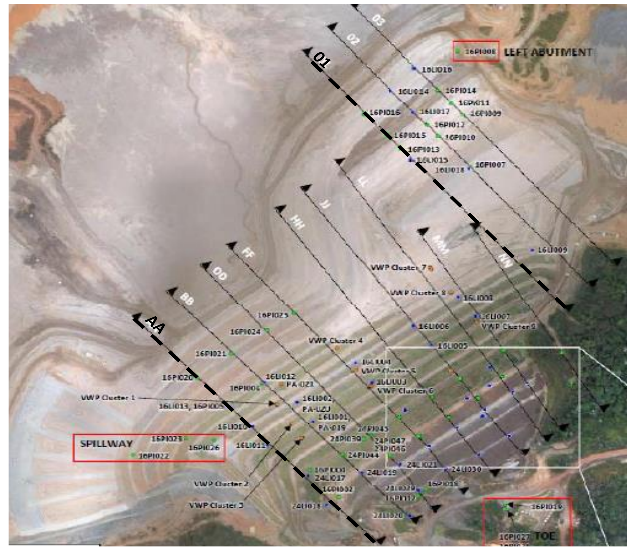

Figure 5. Dike 1 with cross sections. Source: Morgenstern et al. (2016), modified

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To characterize uncertainty in soil properties, one must combine, in addition to actual data, knowledge about the quality of the data, knowledge about geology and its processes, and most importantly, the critical sense of an engineer (Lacasse and Nandim, 2007). A survey of typical values for correlation coefficients (CV) and probability density functions (PDF) found in the literature was conducted during this study. The result of this survey is presented in Table 1. The most commonly adopted PDFs are the Normal (N) and Lognormal (LN).

Considering the frequent unfeasibility of sampling in sufficient quantity for statistical analyses of soil parameters, Phoon & Ching (2014) recommend the employment of typical CV values. In this regard, Table 3 presents the values adopted in the present study, which were based on the typical data presented in Table 2. These data are complemented by the average values presented in Table 1, which were obtained exclusively from samples of the dam materials.

For the strength ratio of the hydraulically thrown clayey tailings, the average of the range of the undrained strength ratio typically adopted for clays was used. The same criterion was used for the hydraulically thrown sandy tailings, using the average of the range of the general property. For the effective cohesion, the upper value typically reported, of 50%, was adopted, because it was admitted that the drainage conditions of the dam, with high variability in the field, should lead to high cohesion CV values.

Table 1. Material parameters

Region	Drained shear strength			Undrained shear strength		
	Unit weight, γ (kN/m³)	Friction angle ϕ (°)	Cohesion c (kPa)'	Unit weight, γ (kN/m³)	Strength ratio Su/σ'ᵥ (kPa)	
	Both sections	Both sections	Both sections	Both sections	Section 01	Section AA
Foundation	22	32	40	22	-	-
Compacted sand tailings	22	35	5	22	-	-
Loose Sand Tailings	22	33	1	22	0.31	0.25
Slimes	22	28	1	22	0.31	0.25

Table 2. Typical material parameter uncertainty

Properties	Type	CV (%)	Reference	PDF
Unit weight	General	3 – 8	Hammitt (1966), Kulhawy (1992); Assumed by Shannon and Wilson (1994)	-
		0 – 10	Scheneider H.R. and Schneider M.A (2013)	N
		10	Lang (2007)	LN
Friction angle	General	2 – 13	Harr (1984), Kulhawy (1992); Shultze (1971), Lacasse & Nadim (1997), Duncan (2000), Phoon & Kulhawy (1999a)	-
		10	Lang (2007)	b
	Sands	3.7 – 15	Shannon and Wilson (1994); Schultze (1972), cited by Harr (1987); Lumb (1974), Hoeg and Murarka 1974, Singh (1971); Schultze (1975)	-
		2 – 5i	Lacasse and Nadim (1996)	N
	Slimes	7.5 – 10.1	Wolff (1985)	-
Undrained strength ratio	Tailings	5 – 20	Baecher et al. (1983)	-
	General	5 – 31	Lacasse and Nadim (1997); Shannon and Wilson (1994); Krahn & Fredlund (1983)	-
	Slimes	5 – 15	Nadim (1996)	N & LN
Effective cohesion	General	10 – 50	Fredlund & Dahlman (1971), Harr (1987), Kulhawy(1992), Tan et al. (1993), Lacasse & Nadim (1997), Phoon & Kulhawy (1999a), Duncan (2000)	-
		30 – 50	Scheneider H.R. and Schneider M.A (2013); Lang (2007)	LN
	Sands	25 – 50	Lumb (1974)	-
	Slimes	20 – 50	Lumb (1974), Singh (1971)	-

With respect to the effective friction angle of the hydraulically released sandy tailings, the adopted value corresponded to the average of the typical range for tailings. For the compacted sand tailings, characterized by the angle of friction of the sands, the adopted value was the average of the range of typical values. For the hydraulically thrown clayey tailings, the average of the friction angle characteristic of clays was used and, for the foundation, the value of 10% was adopted, also due to the intersection of general ranges.

Finally, for the unit weight, the value considered for the foundation and the compacted tailings follow the logic of the average value of the intervals. In the specific case of sandy and clayey tailings, because they present greater variability, the upper limit of the range of typical values was adopted.

Another descriptive parameter of the variability of the materials considered was the spatial variability, quantified by the autocorrelation distance, which determines the distance beyond which adjacent parameters become independent (Lacasse and Nandim, 2007). To characterize this parameter, the theory of random fields is taken into account, which are called stationary (or homogeneous), as described by El-Ramly and Morgenstern (2003), are fields that have an invariant probability distribution when analyzing the parameters of that space.

El Ramly and Morgenstern (2003) applied the autocorrelation distance only to the poropressure and friction angle. However, the analysis of poropressure was simplified in this paper. Thus, the spatial variability was only defined for the angle of friction, adopting mean values from Table 4, provided by them. The values adopted were 30.8 m for autocorrelation distance in the horizontal and 1.98 m in the vertical.

Table 3. Coefficients of variation adopted

Type	Unit weight		Friction angle		Cohesion		Strength ratio	
	CV (%)	PDF	CV (%)	PDF	CV (%)	PDF	CV (%)	PDF
Foundation	5	N	10	N	50	LN		
Compacted sand tailings	5	N	7.5	N	50	LN		
Loose Sand Tailings	10	N	13	N	50	LN	18	N
Slimes	10	N	8,8	N	50	LN	10	N

For the probabilistic analysis, the methods used were Monte Carlo (MC) and the Hybrid Point Estimation Method (APEM). The Monte Carlo method, first developed by Hammersley and Handscomb (1964), uses random number generators for probabilistic analysis. The APEM method, developed by Gitirana Jr. (2005) and applied to the study of tunnels by Franco et al. (2019), is based on a combination of the Taylor series method and the univariate point estimation method proposed by Rosenblueth (1975, 1981). In all probabilistic analysis the brute force search method, searching for entry and exit points and a circular surface. The deterministic analysis is described by the Brazilian Association of Technical Standards (ABNT), through the regulatory standard NBR 13028:2017: Mining - Preparation and presentation of dam design for tailings disposal, sediment containment and water reservoir - Requirements.

Table 4. Typical spatial variability values

Su	Soil Type	Autocorrelation distance (m)		Reference
		Vertical	Horizontal	
VST	Soft organic clay	1.2	-	Asaoka and A-Grivas (1982)
VST	Soft organic clay	3.1	-	Asaoka and A-Grivas (1982)
VST	Sensitive clay	3	30	Soulie et al. (1990)
VST	Very soft clay	1.1	22.1	Bergado et al. (1994)
VST	Sensitive clay	2	-	Chiasson et al. (1995)
Qu	Chicago Clay	0.4	-	Wu (1974)
Qu	Soft clay	2	40	Honjo and Kuroda (1991)
UU	Offshore Soil	3.6	-	Keaveny et al. (1989)
DSS	Offshore Soil	1.4	-	Keaveny et al. (1989)

VST, vane shear test; Qu, unconsolidated compressive strength test; UU, unconsolidated and undrained triaxial test; DSS, direct shear test.

4 MODEL VERIFICATION ANALYSIS

In order to validate the model used, based on data from the Research Panel (Morgenstern et al., 2016), a comparison was made of the deterministic results obtained with the model reproduced for this study and published by the Panel, with the rupture section predetermined in the SVSLOPE program (SoilVision Systems Ltd., 2018). Figure 6 shows the rupture sections.

At the direct shoulder, the Factor of Safety (FS) for the AA section reported by Morgenstern et al. (2016) is 1.90 for drained conditions and 1.00 for undrained conditions. Comparing the results of the Panel with the sections in Figures 6(a) and 6(b), it can be stated that the developed model adequately represents the dam, presenting a percentage difference of less than 5%.

As for the left shoulder, the FS values presented by Morgenstern et al. (2016) are 2.50 and 1.48 in the drained and undrained conditions, respectively. Comparing with the sections shown in Figures 6(c) and 6(d), the maximum percentage difference is 7%, for the drained case. This difference can be attributed to the difficulty in modeling the clay layers and the adopted rupture section. The difference, even so, was considered relatively low, in face of the other variabilities inherent to the analysis model. Another reason that explains this difference in the FS is due to the fact that the Lognormal PDF was adopted in the probabilistic analysis for cohesion, which is an asymmetric distribution. Thus, one has a right-handed shorthand, raising the average FS in a probabilistic analysis (Palin Droubi, 2018).

5 RESULTS AND DISCUSSIONS

The probabilistic analyses were performed on a machine equipped with an Intel(R) CORE ™ i7- 4770 CPD processor @ 3.400GHz 3.40GHz and with 16 Gb of RAM memory. In terms of processing time, the method requiring the least computational effort is APEM. The Monte Carlo method requires on average 6.3 times the time used by the APEM method, adopting 1000 draws.

Tables 5 and 6 present the results obtained in the probabilistic analyses for the right and left shoulders, respectively. For the right shoulder, the E[FS] values obtained with the different probabilistic methods observed in the same drainage condition are close, with a percentage difference of less than 2.2% between them. On the left shoulder the difference between the E[FS] values obtained by the different methods dropped to 0.5%.

When analyzing E[FS] according to drainage conditions, the analyses done with the drained condition have E[FS] values that are 70% higher, in both shoulders, when compared to the values in the undrained condition. The original project design adopted that the dam should promote good drainage conditions. However, due to the design modifications, Morgenstern et al. (2016) state that there was growth in the degree of saturation of the sandy materials of the massif and detriment to the drainage conditions, a factor that caused the reduction of the FS. Spatial variability played a significant role only in the drained condition, since the parameters receiving it are the

drained strength. Looking at the results for the right shoulder (Table 5), according to the spatial variability

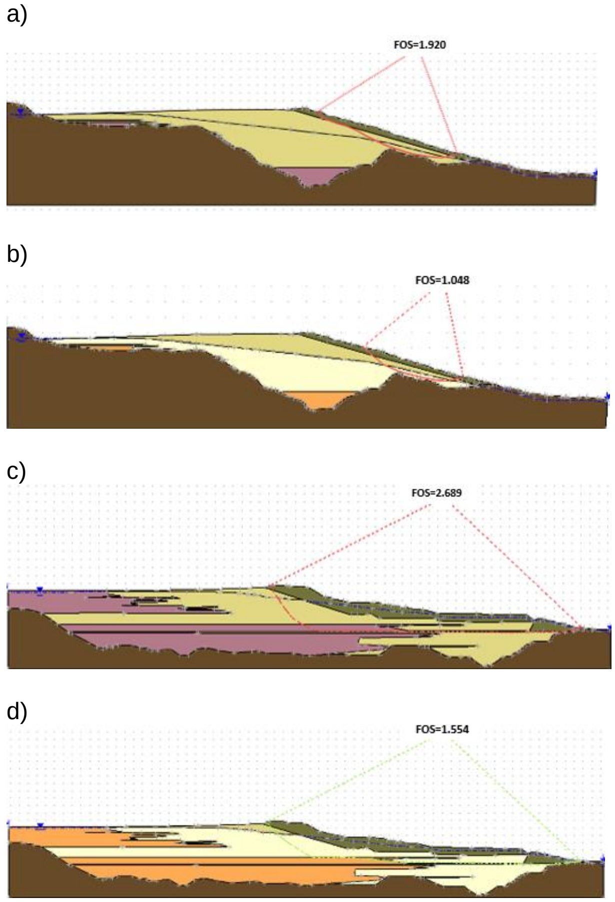

Figure 6- Shear section: a) cross section AA, drained condition; b) cross section AA, undrained condition; c) cross section 01, drained condition; d) cross section 01, undrained condition.

Spatial variability played a significant role only in the drained condition, since the parameters receiving it are the drained strength. Looking at the results for the right shoulder (Table 5), according to the spatial variability scenario, the standard deviation in the sections with drained parameters is on average 2.5 times smaller when spatial variability is applied. This is reflected in the reliability index, which increases on the same scale. As for the left shoulder (Table 6), the standard deviation is on average 10.

The results of the sensitivity analyses show that the parameters that propagated the most uncertainty to the dam performance were those of the materials whose rupture sections pass through them, which is a trivial observation. Only when spatial variability is applied does this cease to be the case, due to the uncertainty reduction explained by Vanmarcke (1977).

The results in Table 5 and 6, from the probabilistic analyses, may be surprising, mainly because E[FS] is higher on the left shoulder where the rupture started. However, Morgenstern et al. (2016) argue that there were a number of factors driving the rupture that are not considered in limit equilibrium analyses. In addition to the lack of borehole or cone penetration test traversing the full depth of the left jamb tailings.

To recover the left shoulder, the initial design was modified, with the construction of a setback. Thus, clayey tailings occupied places not initially programmed, making drainage difficult. With this, Pereira (2005) paraphrases Casagrande, explaining that as saturated sand was subjected to shear under non-drained conditions, the impossibility of volume variation would result, consequently, in a variation of pore pressure. This caused localized ruptures, in a contractile process by the high void ratios. Adding with a trigger mechanism, a tremor, considered normal in Minas Gerais, started the liquefaction process that initiated the fluid slide.

Table 5. Results of the probabilistic analysis of section AA, right abutment

Drainage condition	Method	E[FS]	σ	β	PF (%)	PF-N (%)	PF-LN (%)
			With spatial variability				
Drained	APEM	1.900	0.047	19.25	-	7E-81	4E-148
	MC	1.941	0.080	11.73	1E-40	5E-30	5E-56
Undrained	APEM	1.084	0.112	0.11	-	22.7	23.3
	MC	1.086	0.111	0.78	21.8	21.8	22.3
			Without spatial variability				
Drained	APEM	1.844	0.206	4.1	-	2.07E-3	2.64E-6
	MC	1.851	0.210	4.06	0	2.49E-3	3.46E-6
Undrained	APEM	1.084	0.122	0.69	-	24.6	25.4
	MC	1.061	0.113	0.54	30.3	29.5	30.8

Table 6. Results of the probabilistic analysis of section 01, left abutment

Drainage condition	Method	E[FS]	σ	β	PF (%)	PF-N (%)	PF-LN (%)
			With spatial variability				
Drained	APEM	2.837	0.013	138.7	-	0	0
	MC	2.843	0.091	20.22	0	3E-89	4.2E-231
Undrained	APEM	1.612	0.184	3.32	-	4.49E-4	1.79E-3
	MC	1.604	0.194	3.12	0.60	9.10E-2	5.51E-3
			Without spatial variability				
Drained	APEM	2.838	0.275	6.69	0.60	1.15E-9	3.11E-25
	MC	2.844	0.288	6.44	-	6.20E-9	1.79E-23
Undrained	APEM	1.611	0.184	3.32	0	4.52E-2	1.82E-3
	MC	1.619	0.189	3.28	-	5.31E-2	2.24E-3

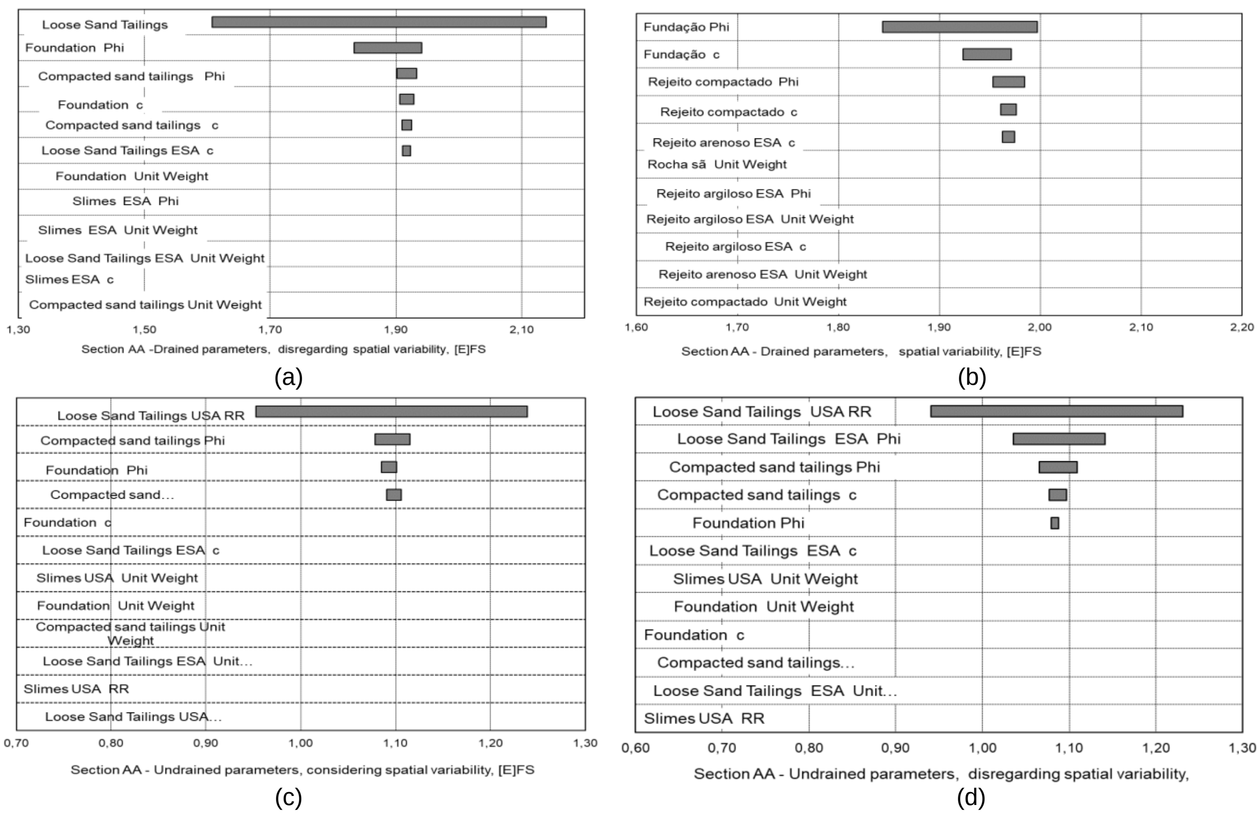

Figure 7. Tornado diagrams of the cross sections: a) Drained cross section AA; b) Undrained cross section AA; c) Drained cross section 01; d) Undrained cross section 01

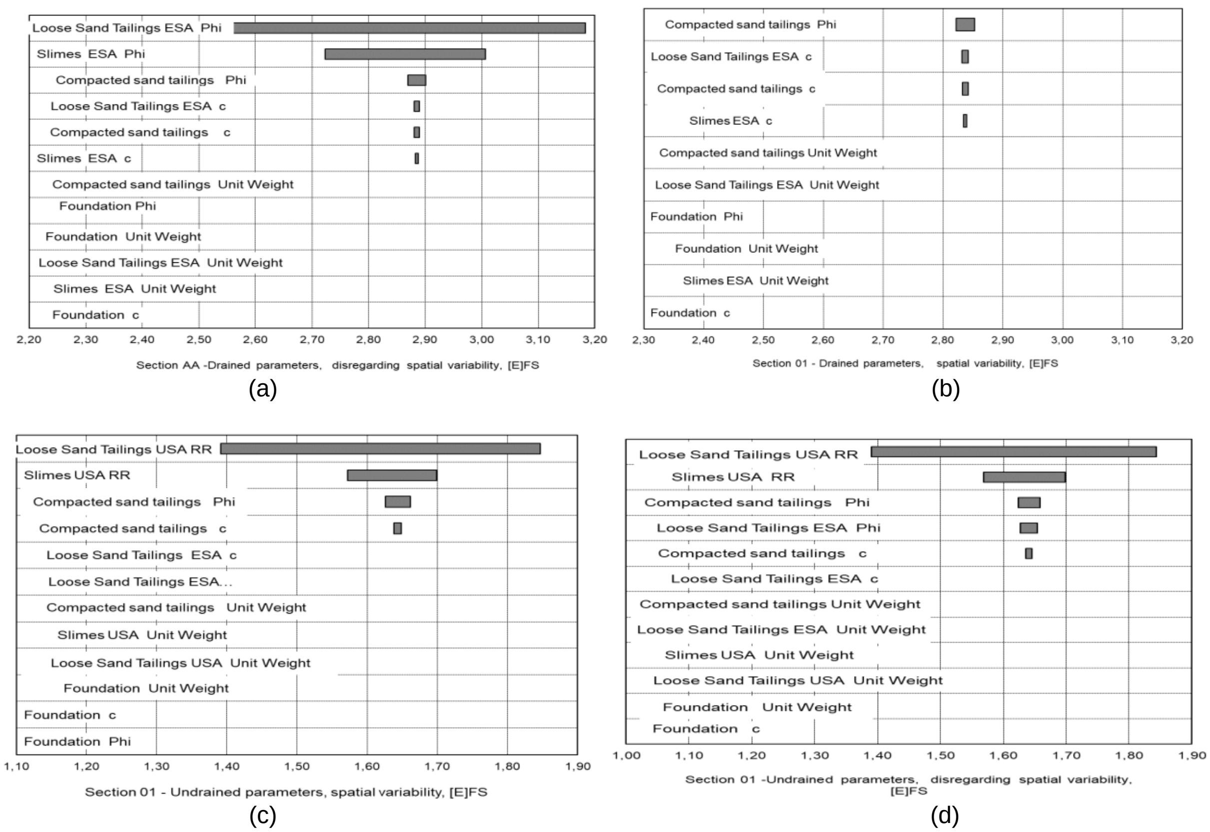

Figure 8. Tornado diagrams of the cross sections: a) Drained cross section AA; b) Undrained cross section AA; c) Drained cross section 01; d) Undrained cross section 01

6 CONCLUDING REMARKS

Lacasse and Nandim (2007) propose the analysis of a framework against risk acceptance criteria for the selection and implementation of measures to manage the identified risks. Whitman (1984) had already proposed a way to make such an assessment, comparing the annual rates of single events and the consequences of poor performance or ruptures, with an analysis of cost and number of fatalities.

Evaluating the work against traditional limits of threat degrees, it was found that the left section (01), in the undrained condition extrapolated the acceptable limits for mineral dam works, especially when evaluated in terms of financial costs.

However, mainly due to the lack of a borehole or cone penetration test through the total depth of the tailings of the left worker, where the rupture started, the analyses pointed out that the right shoulder had a higher probability of rupture. Thus, the range of risk conditions highlights the importance of measures to control drainage in the structure.

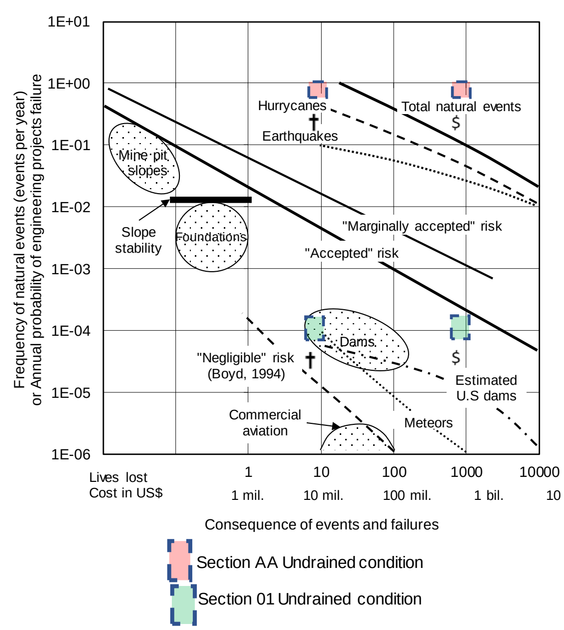

Figure 9 – Risk analysis using the limits established by Whitman (1984)

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