SVSOILS

A KNOWLEDGE-BASED DATABASE SYSTEM FOR SATURATED / UNSATURATED SOIL PROPERTIES

Theory Manual

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1 INTRODUCTION

The SVSOILS software is unique among soil database applications in that it provides tools and methodologies to obtain **unsaturated soil properties** that can subsequently be used for numerically simulating saturated/unsaturated physical processes**.** The mathematical representations of soil behavior corresponding to various physical processes are referred to as "constitutive relations". Examples of constitutive relations are: i.) Darcy's law for water flow through a porous media, ii.) Fick's law for the flow of air through a soil, iii.) the Mohr-Coulomb shear strength equation, iv.) Hooke's law describing stress-deformation of soils, v.) and many other empirical laws that have been proposed and verified for describing the physical behavior of saturated and unsaturated soils. Each constitutive relationship contains soil properties.

Constitutive relations contain soil properties that can be either obtained from direct measurements in the laboratory or indirect estimation from other soil property measurements (e.g., from measurements of the soil-water characteristic curve, SWCC, in the case of unsaturated soils). The soil properties contained within constitutive relations most commonly take the form of soil constants when the soil is saturated. However, most soil properties take the form of nonlinear relationships that are functions of soil suction when the soil becomes unsaturated. Consequently, the determination of unsaturated soil properties proves to be increasingly difficult and costly to determine. The SVSOILS software is designed to assist the geotechnical and geo-environmental engineer in determining suitable material parameters for constitutive relations for saturated and unsaturated soils.

In some cases, the equations used to represent soil behavior can be best-fit to measured laboratory data. Various fitting processes are provided within the SVSOILS database system for this purpose. For example, it is possible to best-fit the results from grain-size data and then make use of the fitted mathematical relationship to estimate the SWCC. Then the mathematical relationship for the SWCC can be used to provide the geotechnical engineer with an estimation of the unsaturated soil property functions that describe various unsaturated soil physical processes. It is always possible to obtain unsaturated soil property functions; however, the geotechnical engineer must be aware that there is an accuracy level associated with each of the possible methodologies that might be used.

It is also possible to search and group soil types based on the grain-size parameters. The grain-size distributions can also be designated as lying within a specified band or range. Smooth mathematical representations of unsaturated soil property functions can be determined for usage in the various SoilVision software packages. Convergence problems can be significantly minimized by using soil property functions that are represented as smooth mathematical functions.

The SVSOILS software package is unique in that it provides theoretical algorithms to estimate saturated and unsaturated soil properties (or relationships) for modeling purposes. There are a large number of estimation algorithms have been implemented in the SVSOILS software. The theory behind each of the estimation algorithms is presented in the theory sections of the manual.

2 VOLUME-MASS CALCULATIONS

A soil is a multiphase system comprised of varying amounts of soil solids, water, and air. The volume-mass relations for a soil are a mathematical representation of the relative masses and/or volumes comprising the soil.

SVSOILS makes use of a variety of volume-mass representations. The calculation of the volume-mass variables can be presented in graphical form. A summary of the basic volume-mass relationships for a soil are presented in the following sections. The derivations presented below combine the gravimetric and volumetric properties of a soil. The following presentation of the volume-mass variables is taken from "Soil Mechanics for Unsaturated Soils" by D. G. Fredlund and H. Rahardjo, John Wiley and Sons, Inc., (1993), and "Unsaturated Soil Mechanics in Engineering Practice", by D. G. Fredlund, H. Rahardjo, and M. D. Fredlund, John Wiley and Sons, Inc. (2012).

The following formulations show how the **Volume-Mass** variables are calculated. The calculation of the volume-mass variables are activated by pressing the **State** button. It is also possible to calculate changes in volume-mass properties associated with drying and wetting processes. The volume-mass properties defined are:

Porosity
Void Ratio
Saturation
Gravimetric Water Content
Volumetric Water Content
Soil Density
Basic Relationships
Changes in Volume-Mass Properties

2.1 POROSITY

Figure 1 shows the relative mass and volume proportions in the form of a phase diagram. Porosity, *n*, as a percentage, is defined as the ratio of the volume of voids, V_v, to the total volume, *V* (Figure 1):

$$n = \frac{V_v(100)}{V} \qquad [\ 1\ ]$$

The term "porosity" can also be used to represent the "volume" proportion of any phase relative to the total volume of the soil mixture. Therefore, the "porosity" relative to the solid phase, water phase and air phase can be written as follows:

$$n_s = \frac{V_s(100)}{V} \qquad [\ 2\ ]$$

$$n_w = \frac{V_w(100)}{V} \qquad [\ 3\ ]$$

$$n_a = \frac{V_a(100)}{V} \qquad [\ 4\ ]$$

where:

n_s = *soil particle porosity (%),*
n_w = *water porosity (%), and*
n_a = *air porosity (%).*

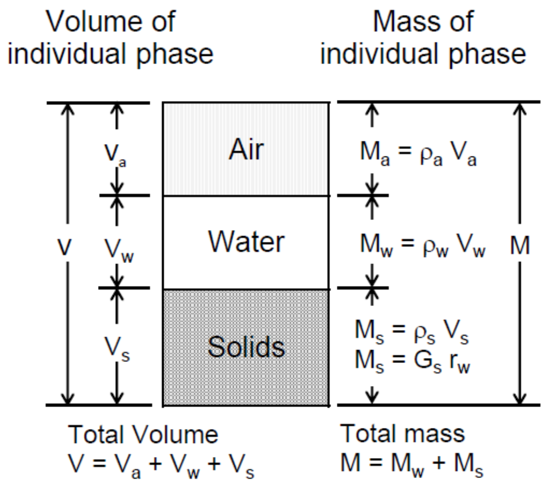


Figure 1 Phase diagram for an unsaturated soil (Fredlund and Rahardjo, 1993)

The water and air porosities represent the overall volumetric percentages corresponding to each phase of the soil. The soil particle porosity can be visualized as the percentage of the overall volume that is comprised of soil particles. The sum of the porosities of all phases must equal 100%. Therefore, the following soil porosity equation can be written:

$$n_s + n = n_s + n_a + n_w = 100(\%) \qquad [5]$$

The water porosity, n_w, expressed in decimal form, is commonly referred to as the volumetric water content, θ_w, of the soil. The term, "volumetric water content", has been extensively used in disciplines such as soil science, soil physics and agronomy whereas the term "gravimetric water content" has been more commonly used in the soil mechanics discipline. The volumetric water content notation is of particular value when formulating partial differential equations describing unsaturated soil behavior. Typical values of porosity for some soils are shown in Table 1.

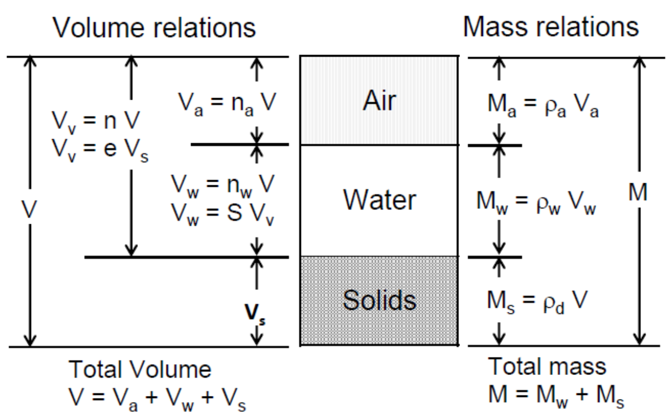


Figure 2 Volume-mass relations (Fredlund and Rahardjo, 1993)

2.2 VOID RATIO

Void ratio, e, is defined as the ratio of the volume of voids, V_v, to the volume of soil solids, V_s (Figure 2):

$$e = \frac{V_v}{V_s} \qquad [6]$$

The relationship between porosity and void ratio is obtained by equating the volume of voids, V_v, from the two equations [i.e., equations [1] and [6]]:

$$n = \frac{e(100)}{1+e} \qquad [7]$$

Typical values for void ratio are shown in Table 1.

2.3 DEGREE OF SATURATION

The percentage of the void space that contains water is expressed as the degree of saturation, *S* (%):

$$S = \frac{V_w(100)}{V_v} \qquad [8]$$

The degree of saturation, *S*, can be used to subdivide soils into three groups.

1. Dry soils (i.e., S = 0%): Dry soil consists of soil particles and air. No water is present.
2. Saturated soils (i.e., S = 100%): All the voids in the soil are filled with water.
3. Unsaturated soils (i.e., 0% < S < 100%): An unsaturated soil can be further subdivided, depending upon whether the air phase is continuous or occluded, and whether the water phase is continuous or discontinuous.

Table 1 Typical Values of Porosity, Void Ratio, and Dry Density (modified from Hough, 1969)

Soil Type	Void Ratio, *e*		Porosity, *n* (%)		Dry Density, *p* (kg/m^3)	
	maximum	minimum	maximum	minimum	maximum	minimum
Granular Materials: 1) Uniform Materials						
a) Equal spheres (theoretical values)	0.92	0.35	47.6	26.0		
b) Standard Ottawa sand	0.80	0.50	44.0	33.0	1762	1474
c) Clean, uniform sand (fine or medium)	1.0	0.40	50.0	29.0	1890	1330
d) Uniform, inorganic silt	1.1	0.40	52.0	29.0	1890	1281
Granular Materials: 2) Well-Graded Materials						
a) Silty sand	0.90	0.30	47.0	23.0	2034	1394
b) Clean, fine to coarse sand	0.95	0.20	49.0	17.0	2210	1362
c) Micaceous sand	1.20	0.40	55.0	29.0	1922	1217
d) Silty sand and gravel	0.85	0.14	46.0	12.0	2239	1426
Mixed Soils						
a) Sandy or silty clay	1.8	0.25	64.0	20.0	2162	961
b) Skip-graded silty clay with stones or rock fragments	1.0	0.20	50.0	17.0	2243	1346
c)Well-graded gravel, sand, silt, and clay mixture	0.70	0.13	41.0	11.0	2371	1602
Clay Soils						
a) Clay (30-50% clay sizes)	2.4	0.50	71.0	33.0	1794	801
b) Colloidal clay (-0.002 mm >= 50%)	12.0	0.60	92.0	37.0	1698	308
Organic Soils						
a) Organic silt	3.0	0.55	75.0	35.0	1762	641
b) Organic clay (30-50% clay sizes)	4.4	0.70	81.0	41.0	1602	481

General Note: Tabulation is based on G_s = 2.65 for granular soils, G_s = 2.70 for clays, and G_s = 2.60 for organic soils.

The subdivisions of soil types shown in Table 1 are based on the relative proportions of various particle-sized material. An unsaturated soil with a continuous air phase generally has a degree of saturation less than approximately 80% (i.e., S < 80%). Occluded air bubbles commonly occur in unsaturated soils having a degree of saturation greater than approximately 90% (i.e., S > 90%). The transition zone between continuous air phase and occluded air bubbles occurs when the degree of saturation is between approximately 80-90% (i.e., 80% < S < 90%). The transition zone between the water being continuous and discontinuous occurs near the residual suction value for the soil.

2.4 GRAVIMETRIC WATER CONTENT

Gravimetric water content, *w*, is defined as the ratio of the mass of water, M_w, to the mass of soil solids, M_s, (Figure 2). It is commonly presented as a percentage [i.e., *w* (%)]:

$$w = \frac{M_w(100)}{M_s} \qquad [9]$$

Gravimetric water content, *w*, is often simply referred to as water content.

2.5 VOLUMETRIC WATER CONTENT

Volumetric water content, θ_w, is defined as the ratio of the volume of water, V_w, to the total volume of the soil, *V:*

$$\theta_w = \frac{V_w}{V} \qquad [\ 10\ ]$$

Historically, volumetric water content has been defined with the "reference" total volume, *V*, being either the original total volume of the soil or else the instantaneous total volume of the soil. Use of the instantaneous total volume of the soil is generally preferred when dealing with unsaturated soil mechanics problems where the overall volume may change as a result of changes in soil suction. It is important that the same reference system for the total volume of the soil, be used for the measurement of the unsaturated soil properties as is used in the theoretical mathematical formulation of the physical process being modeled.

The volumetric water content can also be expressed in terms of porosity, degree of saturation, and void ratio (Figure 2). The volumetric water content can be written as:

$$\theta_w = \frac{S V_v}{V} \qquad [\ 11\ ]$$

Since V_v/V is equal to the porosity of the soil, equation [11] becomes:

$$\theta_w = S n \qquad [\ 12\ ]$$

Substituting equation [7] into equation [12] yields another form for the volumetric water content equation:

$$\theta_w = \frac{S e}{1+e} \qquad [\ 13\ ]$$

2.6 SOIL DENSITY

Two commonly used soil density definitions are total density and dry density. The total density of a soil, ρ, is the ratio of the total mass, *M*, to the total volume of the soil, *V* (Figure 2):

$$\rho = \frac{M}{V} \qquad [\ 14\ ]$$

The total density is sometimes referred to as the bulk density.

The dry density of a soil, ρ_d, is defined as the ratio of the mass of the soil solids, M_s, to the total volume of the soil, *V* (Figure 2):

$$\rho_d = \frac{M_s}{V} \qquad [\ 15\ ]$$

Typical minimum and maximum dry densities for various soils are presented in Table 1.

Other soil density definitions are the saturated density and the buoyant density. The saturated density of a soil is the total density of the soil for the case where the voids are filled with water (i.e., V_a = 0 and *S* = 100%). The buoyant density of a soil is the difference between the saturated density of the soil and the density of water.

2.7 BASIC VOLUME-MASS RELATIONSHIP

The volume and mass for each phase can be related to one another using basic relations from the phase diagram (Figure 1) and the volume-mass relations shown in Figure 2.

The mass of water in a soil, M_w, is the product of the volume and the density of water (Figure 3):

$$M_w = \rho_w V_w \qquad [\ 16\ ]$$

The volume of water, V_w, can also be computed from the volume relations given in Figure 3 (i.e., left-handed side):

$$V_w = SeV_s \qquad [\ 17\ ]$$

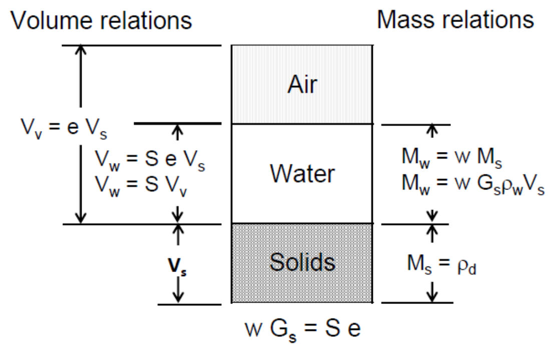


Figure 3 Derivation of the "basic volume-mass relationship" (Fredlund and Rahardjo, 1993)

The relationship given in equation [17] is shown in Figure 3 (i.e., left-hand side). Equation [17] can then be rewritten as:

$$M_w = \rho_w SeV_s \qquad [\ 18\ ]$$

The mass of the water, M_w, can also be related to the mass of soil solids, M_s:

$$M_w = wM_s \qquad [\ 19\ ]$$

The mass of the soil solids, M_s, is obtained from the phase diagram shown in Figure 1.

$$M_w = G_s \rho_w V_s \qquad [\ 20\ ]$$

where:

G_s = *specific gravity of soil solid*

Substituting equation [20] into equation [19] yields:

$$M_w = wG_s \rho_w V_s \qquad [\ 21\ ]$$

Equating equation [21] and equation [21] results in a "basic volume-mass relationship" for soils:

$$Se = wG_s \qquad [\ 22\ ]$$

The total and dry densities of a soil defined in equations [14] and [15], respectively, can also be expressed in terms of the volume-mass properties of the soil (i.e., *S*, *e*, *w*, and G_s). Assuming that the mass of air, M_a, is negligible, the total mass of the soil is the sum of the mass of the water, M_w, and the mass of the soil solids, M_s. The total volume of the soil, *V*, is given as the volume of the soil solids, V_s, and the volume of the voids, V_v. Therefore, the equation for the total density of a soil, ρ, can be rewritten using equations [6], [20] and [21]:

$$\rho = \frac{M_s + M_w}{V_s + V_v} \qquad [\ 23\ ]$$

$$\rho = \frac{G_s \rho_w V_s + wG_s \rho_w V_s}{V_s + eV_s} \qquad [\ 24\ ]$$

$$\rho = \frac{G_w + wG_s}{1+e}\rho_w \qquad [\ 25\ ]$$

Substituting the basic volume-mass relationship (i.e., equation [22]) into equation [25] gives the following equation for the total density:

$$\rho = \frac{G_s + Se}{1+e}\rho_w \qquad [\ 26\ ]$$

The dry density of a soil, ρ_d, is obtained by eliminating the mass of the water, M_w, from equation [23]:

$$\rho_d = \frac{G_s}{1+e}\rho_w \qquad [\ 27\ ]$$

The relationship between total density, ρ, and dry density, ρ_d, for different water contents is presented graphically in Figure 4. If any two of the volume-mass properties of a soil (e.g., *e, w,* or *S*) are known, the total density of the soil, ρ, can be computed in accordance with equations [25] or [26]. The dry density of the soil, ρ_d, is computed using equation [27] provided the void ratio, *e,* or the porosity, *n*, of the soil is known.

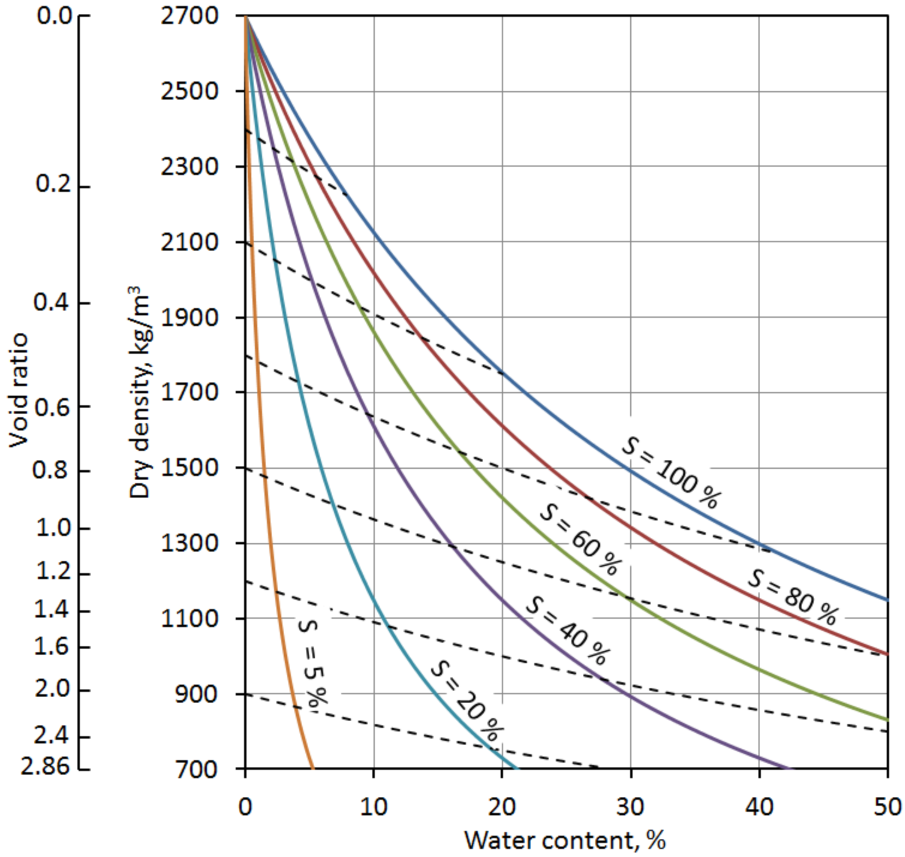


Figure 4 Volume-mass relations for a soil with G_s = 2.70

The dry density curve corresponding to a degree of saturation of 100% is called the "zero air voids" curve. The dry density curves for various degrees of saturation are commonly presented in connection with soil compaction data (Figure 5). Compaction is a mechanical process used to increase the dry density of soils through the expulsion of air (i.e., densification).

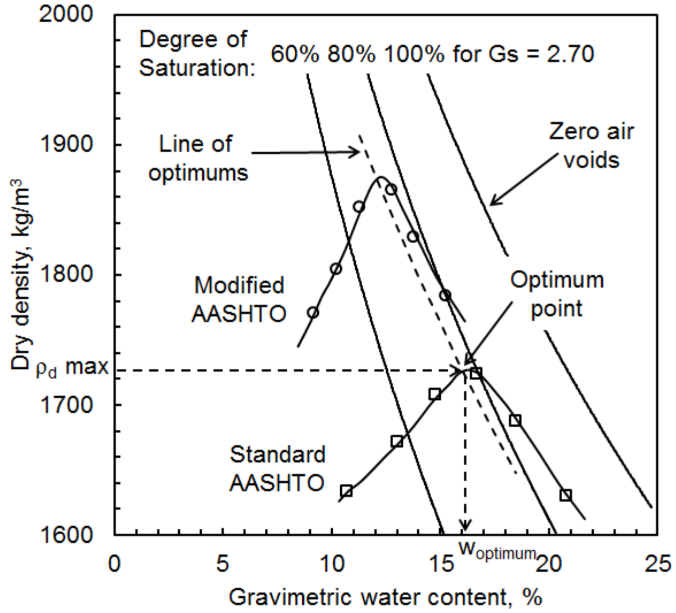


Figure 5 Standard and modified AASHTO compaction curves (Fredlund and Rahardjo, 1993)

The relationship between gravimetric water content, w, and volumetric water content, θ_w, can be established by substituting the basic volume-mass relationship [i.e., equation [22]] into equation [13]:

$$\theta_w = \frac{SwG_s}{S + wG_s} \qquad [\ 28\ ]$$

where:

all the parameters are ratios

The Table 2 shows the relations between various phase variables commonly used in engineering practice. If any three of the basic volume-mass variables are known, the remaining variables can be defined based on these relationships

Table 2 Conversion between volume-mass variables in soils

Parameter		Conversion equations		
S	Degree of Saturation	$S = \frac{w\,G_s}{e}$		
ρ	Bulk Density	$\rho = \frac{(1+w)\,\rho_w}{\left(\frac{1}{G_s} + \frac{w}{S}\right)}$	$\rho = \frac{G_s + (S\,e)}{1+e}\rho_w$	
ρ_d	Dry Density	$\rho_d = \frac{\rho_w}{\left(\frac{1}{G_s} + \frac{w}{S}\right)}$	$\rho_d = \frac{\rho_w G_s}{1+e}$	
n	Porosity	$n = \frac{1}{\frac{S}{w\,G_s} + 1}$	$n = \frac{e}{e+1}$	
G_s	Specific Gravity of Solids (i.e., density of solids)	$G_s = \frac{e\,S}{w}$		
γ	Bulk Unit Weight	$\gamma = \frac{1+w}{\left(\frac{1}{G_s} + \frac{w}{S}\right)}\gamma_w$	$\gamma = \frac{G_s + e\,S}{1+e}\gamma_w$	

γ_d	Dry Unit Weight	$\gamma_d = \dfrac{\gamma_w}{\left(\dfrac{1}{G_s} + \dfrac{w}{S}\right)}$	$\gamma_d = \dfrac{G_s\,\gamma_w}{1+e}$	
e	Void Ratio	$e = \dfrac{n}{1-n}$	$e = \dfrac{w\,G_s}{S}$	$e = \dfrac{\rho_w\,G_s}{\rho_d} - 1$
w	Gravimetric Water Content	$w = \dfrac{e\,S}{G_s}$	$w = \left(\dfrac{\rho_w}{\rho_d} - \dfrac{1}{G_s}\right) S$	$w = \dfrac{\left(1 - \dfrac{\rho}{\rho_w\,G_s}\right)}{\left(\dfrac{\rho}{\rho_w\,S} - 1\right)}$
θ_w	Volumetric Water Content	$\theta_w = \dfrac{SwG_s}{S + wG_s}$		

2.8 CHANGES IN VOLUME-MASS PROPERTIES

The basic volume-mass relationship [equation [22]] applies to any combination of *S, e,* and *w.* Any change in one of these volume-mass properties (i.e., *S, e,* and *w*) may produce changes in the other two properties. Changes in two of the volume-mass quantities must be known in order to compute a change in the third quantity. If changes in the void ratio, *e*, and the water content, *w*, are known, the change in the degree of saturation, *S*, can be computed. Similarly, if the changes in *S* and *e* (or in *S* and *w*) are known, then the change in *w* (or *e*,), can be computed.

The relationship between the changes in the volume-mass properties can be derived from the basic volume-mass relationship expressed in equation [22]. Let us consider a soil that undergoes a physical process such that there are changes in the volume-mass properties of the soil. Prior to the process, the volume-mass properties of the soil have the following relationship:

$$S_i e_i = w_i G_s \qquad [\ 29\ ]$$

where:

S_i = *initial degree of saturation,*
e_i = *initial void ratio, and*
w_i = *initial water content.*

At the end of the process, the soil has final volume-mass properties that are also related by the basic volume-mass relationship:

$$S_f e_f = w_f G_s \qquad [\ 30\ ]$$

where:

S_f = *final degree of saturation,*
e_f = *final void ratio, and*
w_f = *final water content.*

The following relationships between initial and final conditions can be written:

$$S_f = S_i + \Delta S \qquad [\ 31\ ]$$

$$e_f = e_i + \Delta e \qquad [\ 32\ ]$$

$$w_f = w_i + \Delta w \qquad [\ 33\ ]$$

where:

ΔS = *change in the degree of saturation,*
Δe = *change in the void ratio, and*
Δw = *change in the water content.*

Substituting equations [30], [31], [32] and [28] into equation [29] gives:

$$S_i \Delta e + \Delta S e_i + \Delta S \Delta e = \Delta w G_s \qquad [\ 34\ ]$$

The change in the degree of saturation, ΔS, can be written in terms of the change in void ratio, Δe, and the change in water content, Δw:

$$\Delta S = \frac{\left(\Delta w G_w - S_i \Delta e\right)}{e_f} \qquad [\ 35\ ]$$

Similarly, the change in the void ratio, Δe, is obtained by substituting equation [30] into equation [33] and solving for Δe:

$$\Delta e = \frac{\left(\Delta w G_s - \Delta S e_i\right)}{S_f} \qquad [\ 36\ ]$$

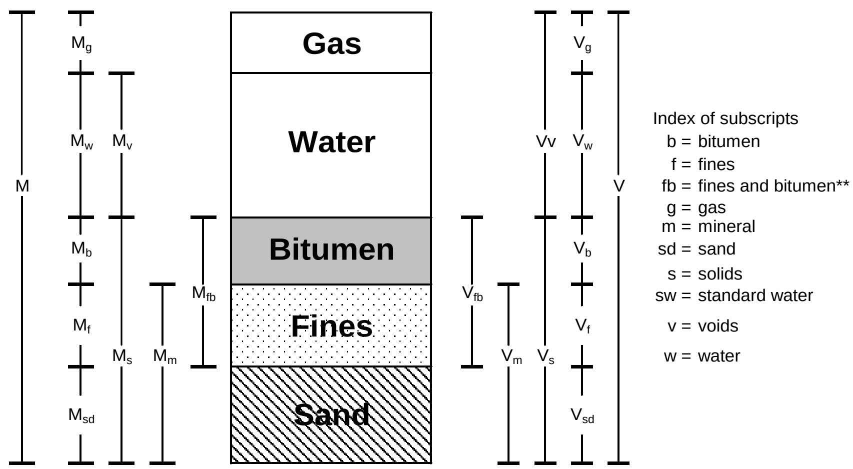
The change in water content, Δw, can be written as follows:

$$\Delta w = \frac{\left(S_f \Delta e + \Delta S e_i\right)}{G_s} \qquad [\ 37\ ]$$

3 VOLUME-MASS CALCULATION: OIL SAND TAILINGS

3.1 PHASE DIAGRAM

Oil sand tailings are an example of a special multiphase material that has more than a solid phase, a gas (air) phase and a liquid (water) phase. Oil sand tailings have an additional phase referred to as the bitumen phase. The bitumen phase mainly affects the geotechnical behavior of the clay (fines) portion. The solid phase can also be further sub-divided into sand and fines portions when dealing with tailings (Figure 6). The volume-mass theory related to oil sands tailings is based on the variable definitions recommended by Scott (2003).



Where:

M = total mass of tailings	V = total volume of tailings
M_g = mass of gas*	V_g = volume of gas
M_w = mass of water	V_w = volume of water
M_b = mass of bitumen	V_b = volume of bitumen
M_f = mass of fines	V_f = volume of fines
M_{sd} = mass of sand	V_{sd} = volume of sand
M_v = mass of voids*	V_v = volume of voids
M_s = mass of solids [bitumen (M_b), fines (M_f) and sand (M_{sd})]	V_s = volume of solids [bitumen (V_b), fines (V_f) and sand (V_{sd})]
M_m = mass of minerals (fines and sand)	V_m = volume of minerals (fines and sand)
M_{fb} = mass of fines and bitumen**	V_{fb} = volume of fines and bitumen**

Notes:
* Mass of gas is taken as zero. Therefore, by definition the mass of voids and mass of water are equivalent.
** Prior to 1991 fines + bitumen were designated as sludge or sludge solids. Since 1991 the term fine tails or mature fine tails has been used.

Figure 6 Phase diagram for saturated and unsaturated oil sands tailings (Scott, 2003)

3.2 TAILINGS PARAMETERS

There are new parameters that need to be defined when dealing with oil sands tailings and other mine tailing materials. The parameters associated with the oil sands mining industry have been defined in terms of both the geotechnical and mining engineering disciplines. Most parameters in this section are based on geotechnical engineering concepts. However, if a particular parameter is defined in accordance with the mining practice, it is labeled as “mining”. Fines are comprised of silt size and clay size particles that are less than 45 μm (U.S No. 325 sieve) or less than 50 μm (Metric No. 50 sieve). Clay size particles are defined as being less than 2.0 μm.

3.2.1 Mass Phase definitions

The following definitions are used in defining the volume and mass quantities associated with oil sands tailings.

Bitumen content, *b*:

$$b = \frac{M_b(100)}{M_s} = \frac{M_b(100)}{M_{sd} + M_f + M_b} \qquad [\ 38\ ]$$

Bitumen content (Mining), b_m:

$$b_m = \frac{M_b(100)}{M} \quad [\ 39\ ]$$

Fine content, f:

$$f = \frac{M_f(100)}{M_s} \quad [\ 40\ ]$$

Fine + bitumen content, fb:

$$fb = \frac{M_{fb}(100)}{M_s} \quad [\ 41\ ]$$

Fine content (Mining), f_m :

$$f_m = \frac{M_f(100)}{M} \quad [\ 42\ ]$$

Fine-Water ratio, FWR:

$$FWR = \frac{M_{fb}(100)}{M_s + M_w} \quad [\ 43\ ]$$

Gas content, A or n_a:

$$A = n_a = \frac{V_a(100)}{V} \quad [\ 44\ ]$$

Sand content, sd:

$$sd = \frac{M_{sd}(100)}{M_s} \quad [\ 45\ ]$$

Sand content (Mining), sd_m:

$$sd_m = \frac{M_{sd}(100)}{M} \quad [\ 46\ ]$$

Sand Fine Ratio, SFR:

$$SFR = \frac{M_{sd}(100)}{M_{fb}} \quad [\ 47\ ]$$

Solids concentration, η:

$$\eta = \frac{M_s}{V} \quad [\ 48\ ]$$

Solids content (Geotechnical), s:

$$s = \frac{M_s(100)}{M} \quad [\ 49\ ]$$

Solids content (Mining), s_m:

$$s_m = \frac{M_m(100)}{M} \quad [\ 50\ ]$$

Water content (Mining), w_m:

$$w_m = \frac{M_w(100)}{M} \quad [\ 51\ ]$$

3.2.2 Void ratio phase definitions

Fine void ratio, e_f:

$$e_f = \frac{V_v}{V_f} \quad [52]$$

Fine-bitumen void ratio, e_{fb}:

$$e_{fb} = \frac{V_v}{V_{fb}} \quad [53]$$

Sand void ratio, e_{sd}:

$$e_{sd} = \frac{V_v}{V_{sd}} \quad [54]$$

Solid volume concentration, ϕ_c:

$$\phi_c = \frac{V_s}{V} \quad [55]$$

As the shown in the above equation, the denominator is not the same in all volume and mass phase equations. Therefore, the sum of various void ratio phase relations are not equal to 1.0 (or 100%).

3.2.3 Density

There are other density definitions for tailings materials in addition to the traditional bulk and dry density relations defined for soils

Bitumen density, ρ_b:

$$\rho_b = \frac{M_b}{V_b} \quad [56]$$

Fine density, ρ_f:

$$\rho_f = \frac{M_f}{V_f} \quad [57]$$

Fine + bitumen density, ρ_{fb}:

$$\rho_{fb} = \frac{M_f + M_b}{V_f + V_b} = \frac{M_{fb}}{V_{fb}} \quad [58]$$

Mineral density, ρ_m:

$$\rho_m = \frac{M_f + M_{sd}}{V_f + V_{sd}} = \frac{M_m}{V_m} \quad [59]$$

Sand density, ρ_{sd}:

$$\rho_{sd} = \frac{M_{sd}}{V_{sd}} \quad [60]$$

Solid density, ρ_s:

$$\rho_s = \frac{M_s}{V_s} \quad [61]$$

3.2.4 Specific Gravity

Besides G_s, there are also other specific gravities defined as the followings:

Specific gravity of bitumen, G_b:

$$G_b = \frac{\rho_b}{\rho_w} \quad [62]$$

Specific gravity of fines, G_f:

$$G_f = \frac{\rho_f}{\rho_w} \quad [63]$$

Specific gravity of fines + bitumen, G_{fb}:

$$G_{fb} = \frac{\rho_{fb}}{\rho_w} \quad [64]$$

Specific gravity of sand, G_{sd}:

$$G_{sd} = \frac{\rho_{sd}}{\rho_w} \quad [65]$$

3.2.5 Summary of phase relations

Following is Table 3 which shows the relationship between the variables used to define volume and mass quantities comprising oil sands tailings.

Table 3 Conversion between volume-mass variables in Tailings

Parameter		Conversion equations			
Note: All mass-volume ratios are expressed as decimals, (as opposed to percentages)					
b	*Bitumen Content (Geotechnical)*	$b = 1 - \frac{s_m}{s}$	$b = \frac{b_m}{s}$	$b = b_m(1+w)$	
b_m	*Bitumen Content (Mining)*	$b_m = 1 - w_m - s_m$	$b_m = \frac{b}{1+w}$	$b_m = b\,s$	
S	*Degree of Saturation*	$S = \frac{w\,G_s}{e}$	$S = \frac{G_s\left(\frac{1}{s}-1\right)}{e}$	$S = 1 - \left(\frac{A}{n}\right)$	
ρ	*Bulk Density*	$\rho = \frac{(1+w)\,\rho_w}{\left(\frac{1}{G_s}+\frac{w}{S}\right)}$	$\rho = \frac{G_s + (S\,e)}{1+e}\rho_w$	$\rho = \frac{\rho_w}{\left(\frac{s}{G_s}+\frac{1-s}{S}\right)}$	
ρ_d	*Dry Density*	$\rho_d = \frac{\rho_w}{\left(\frac{1}{G_s}+\frac{w}{S}\right)}$	$\rho_d = \frac{\rho_s}{1+e}$	$\rho_d = \frac{\rho_w}{\left(\frac{1}{G_s}+\frac{\left(\frac{1}{s}-1\right)}{S}\right)}$	
f	*Fines Content (Geotechnical)*	$f = \frac{w\,G_f}{e_f\,S}$	$f = \frac{\left(\frac{1}{s}-1\right)G_f}{e_f\,S}$	$f = \frac{e\,G_f}{e_f\,G_s}$	$f = \frac{w}{\frac{1}{FWR}-1} - b$
f_m	*Fines Content (Mining)*	$f_m = fb\left(1-\frac{w}{1+w}\right) - b_m$	$f_m = fb(1-w_m) - b_m$		
fb	*Fines bitumen Content (Geotechnical)*	$fb = \frac{e\,G_{fb}}{e_{fb}\,G_s}$	$fb = \frac{1}{SFR+1}$	$fb = \frac{FWR\,w}{1-FWR}$	$fb = \frac{f_m + b_m}{1-w_m}$
FWR	*Fines-Water Ratio*	$FWR = \frac{fb}{fb+w}$	$FWR = \frac{fb}{fb+\frac{S\,e}{G_s}}$	$FWR = \frac{fb}{fb+\left(\frac{1}{s}-1\right)}$	$FWR = \frac{1}{1+\frac{f\,S}{fb}\frac{e_f}{G_f}}$

A	*Gas Content*	$A=\frac{w\,G_s}{S+w\,G_s}(1-S)$	$A=\frac{e}{1+e}(1-S)$	$A=\frac{G_s\left(\frac{1}{s}-1\right)}{S+G_s\left(\frac{1}{s}-1\right)}(1-S)$	$A=n\,(1-S)$
n	*Porosity*	$n=\frac{1}{\frac{S}{w\,G_s}+1}$	$n=\frac{e}{e+1}$	$n=\frac{1}{\frac{S}{\left(\frac{1}{s}-1\right)G_s}+1}$	
sd	*Sand Content (Geotechnical)*	$sd=1-fb$	$sd=1-f-b$		
sd_m	*Sand Content (Mining)*	$sd_m=1-w_m-f_m-b_m$			
SFR	*Sand-Fines Ratio*	$SFR=\frac{1}{fb}-1$			
η	*Solids Concentration*	$\eta=\frac{\rho}{1+w}$	$\eta=\left(1-\frac{e}{1+e}\right)\rho_s$	$\eta=\rho\,s$	$\eta=(1-n)\,\rho_s$
s	*Solids Content (Geotechnical)*	$s=\frac{1}{1+w}$	$s=\frac{G_s}{G_s+e\,S}$	$s=\frac{1}{\left(\frac{\rho_w}{\rho_{dry}}-\frac{1}{G_s}\right)S+1}$	$s=\frac{1}{1+\left(\frac{1-\frac{\rho}{\rho_w\,G_s}}{\frac{\rho}{S\,\rho_w}-1}\right)}$
s_m	*Solids Content (Mining)*	$s_m=\frac{1-b}{1+w}$	$s_m=1-b_m-w_m$		
G_{fb}	*Specific Gravity of Fines + Bitumen*	$G_{fb}=\frac{fb}{\left(\frac{1}{G_s}-\frac{(1-fb)}{G_{sd}}\right)}$	$G_{fb}=\frac{e_{fb}\,G_s\;fb}{e}$		
G_s	*Specific Gravity of Solids*	$G_s=\frac{e\,S}{w}$	$G_s=\frac{e\,S}{\left(\frac{1}{s}-1\right)}$	$G_s=\frac{e\,G_{fb}}{fb\,e_{fb}}$	$G_s=\frac{\frac{b}{1-b}+1}{\left(\frac{b}{G_b\,(1-b)}+\frac{1}{G_m}\right)}$

γ	Bulk Unit Weight	$\gamma = \dfrac{1+w}{\left(\dfrac{1}{G_s} + \dfrac{w}{S}\right)} \gamma_w$	$\gamma = \dfrac{G_s + e\,S}{1+e} \gamma_w$	$\gamma = \dfrac{\gamma_w}{\left(\dfrac{s}{G_s} + \dfrac{1-s}{S}\right)}$	
γ_d	Dry Unit Weight	$\gamma_d = \dfrac{\gamma_w}{\left(\dfrac{1}{G_s} + \dfrac{w}{S}\right)}$	$\gamma_d = \dfrac{G_s\,\gamma_w}{1+e}$	$\gamma_d = \dfrac{\gamma_w}{\left(\dfrac{1}{G_s} + \dfrac{\left(\dfrac{1}{s} - 1\right)}{S}\right)}$	
e	Void Ratio (Geotechnical)	$e = \dfrac{n}{1-n}$	$e = \dfrac{w\,G_s}{S}$	$e = \dfrac{\left(\dfrac{1}{s} - 1\right) G_s}{S}$	$e = \dfrac{e_{fb}\,G_s\;fb}{G_{fb}}$
		$e = \dfrac{\rho_w\,G_s}{\rho_d} - 1$			
e_{fb}	Fines-Bitumen Void Ratio	$e_{fb} = \dfrac{w\,G_{fb}}{fb\;S}$	$e_{fb} = \dfrac{e\,G_{fb}}{fb\,G_s}$	$e_{fb} = \dfrac{\left(\dfrac{1}{s} - 1\right) G_{fb}}{fb\,S}$	
e_f	Fines Void Ratio	$e_f = \dfrac{w\,G_f}{f\;S}$	$e_f = \dfrac{e\,G_f}{f\;G_s}$	$e_f = \dfrac{\left(\dfrac{1}{s} - 1\right) G_f}{f\;S}$	
e_{sd}	Sand Void Ratio	$e_{sd} = \dfrac{e + \dfrac{G_s}{G_{fb}}\,fb}{\dfrac{G_s}{G_{sd}}(1 - fb)}$	$e_{sd} = \dfrac{1 + \dfrac{1}{e_{fb}}}{\dfrac{G_{fb}\,(1 - fb)}{G_{sd}\;e_{fb}\;fb}}$	$e_{sd} = \dfrac{\dfrac{G_{sd}}{S}\left(\dfrac{1}{s} - 1\right) + \dfrac{G_{sd}}{G_{fb}}\,fb}{1 - fb}$	
ϕ_c	Volume Concentration	$\phi_c = \dfrac{\rho}{\rho_s (1+w)}$	$\phi_c = \dfrac{\rho}{\rho_s \left(1 + \dfrac{e\,S}{G_s}\right)}$	$\phi_c = \dfrac{\rho\,s}{\rho_s}$	$\phi_c = \dfrac{\eta}{\rho_s}$

w	*Gravimetric Water Content (Geotechnical)*	$w = \frac{1}{s} - 1$	$w = \frac{e\,S}{G_s}$	$w = \left(\frac{\rho_w}{\rho_d} - \frac{1}{G_s}\right) S$	$w = \frac{\left(1 - \frac{\rho}{\rho_w\,G_s}\right)}{\left(\frac{\rho}{\rho_w\,S} - 1\right)}$
w_m	*Gravimetric Water Content (Mining)*	$w_m = \frac{w}{1+w}$	$w_m = \frac{e\,S}{G_s + e\,S}$	$w_m = \frac{\left(\frac{1}{s} - 1\right)}{1 + \left(\frac{1}{s} - 1\right)}$	$w_m = 1 - b_m - s_m$

4 FITTING THEORY

A database application that simply stores laboratory data points is of limited use in engineering practice. It is possible to transform and store soils information into other formats that are of increased value in engineering practice.

There are two methods that can be used to store equation parameters or physically significant information pertinent to a soil property function. The first method allows the user to identify relevant soil property functions in the form of mathematical equations. The parameters of these equations are first optimized to best-fit laboratory data. The best-fit parameters are then stored in the database. The behavior of the soil can then be quantified by the parameters associated with the best-fit equations.

The second method involves identifying physically significant points on a soil property function. For example, there is a construction technique that can be used to determine residual conditions associated with a soil-water characteristic curve, SWCC. This method uses a construction technique that identifies the residual water content and the air-entry value (or the early break point on the SWCC). It is generally accepted that identifying physically significant points on a soil property function provides a means of identifying likely soil behavior.

Measured points on a soil property function should be interpreted in terms of soil behavior before being used in a numerical solver. The points can typically be interpreted as continuous mathematical functions (e.g., a sigmoidal function). Fitting data points with a spline function that passes through laboratory data points may not be the preferred procedure to use when dealing with unsaturated soil behavior. Spline functions may result in meaningless and unreasonable "humps" and "dips" in the soil property function. These variations can lead to numerical instability in modeling software (Sillers, 1996).

SVSOILS attempts to use closed-form mathematical equations in addition to the data points, to represent (saturated/unsaturated) soil property functions. These equations can be best-fit to laboratory data using a variety of soil parameters. Nonlinear regression algorithms based on the quasi-Newton method are used to adjust equation parameters to obtain the best-fit. Construction techniques are then implemented for the identification of physically significant characteristics for soil property functions. Queries based on the physically significant characteristics of soil property functions provide a reliable means of finding desired soil properties.

A description of the theory of curve fitting implemented in SVSOILS can be found in the following section.

SVSOILS provides fittings of mathematical equations to laboratory data for the following soil properites:

Grain-Size Distribution
Soil-Water Characteristic Curve
Permeability (Hydraulic Conductivity)
Compression and Swelling Curve
Shrinkage Curve
Constitutive Surfaces

4.1 CURVE FITTING THEORY

Nonlinear regression analyses are used to fit equations to data sets within SVSOILS. Conventional fitting procedures are used such as described in Neville and Kennedy (1964) and Spiegel (1961). There are exceptions where non-unique behavior is encountered and two functions must be used. An example is the compression and rebound equations corresponding to loading and unloading a soil

4.1.1 Nonlinear regression analyses

Following is a brief description of the nonlinear regression procedures used for curve-fitting algorithms in SVSOILS. Two, three, and four parameter mathematical equations may be required to accurately define a particular soil property that changes with respect to some other soil variable. The following sections describe the mathematical process implemented in SVSOILS for performing nonlinear regression analyses.

4.1.1.1 Comparison of linear and nonlinear regression

A straight line can be described by a simple equation that calculates *Y* values when *X*, values are input. The straight line is defined by a slope variable and an intercept. The purpose of a linear regression analysis is to find values for the slope and intercept that best define the line that comes closest to the measured data. More precisely, the regression analysis defines the line that minimizes the sum of the squares of the vertical distances of the points from the line. The goal of minimizing the sum-of-squares in a linear regression analysis is straight forward and can be referred to as a "one pass" solution. Consequently, there is no chance for ambiguity in the fitted parameters.

Nonlinear regression analyses are more challenging to perform. A nonlinear regression analysis can be used to fit data to any equation that defines a *Y* variable as a function of an *X* variable and one or more other variables. The regression analysis finds values of those variables that generate a curve that comes closest to the data points. The goal is to minimize the sum-of-squares of the vertical distances of the points from the curve. Except for a few special cases, it is not possible to directly perform the regression analysis (as a one-pass solution) to find the values of all variables that minimize the sum-of-squares. Rather, it is necessary to use an iterative solution approach to finding the fitting variables. In the end, a variety of solutions are possible depending upon the restrictions imposed with respect to convergence of the regression analysis.

4.1.1.2 Iterations used for nonlinear regression analysis

Following are a list of the general steps followed when using a nonlinear regression analysis:

1. Start the regression analysis with an initial estimation for each variable in the equation being fit to the data.
2. Generate the equation defined by using the initial values. Calculate the sum-of-squares (i.e., the sum of the squares of the vertical distances of the points from the curve).
3. Adjust the fitting variables to make the curve come closer to the data points. There are a variety of algorithms that can be used for adjusting the fitting variables. Probably the most common method used for adjusting the fitting variables is called the Levenberg-Marquardt method (Levenberg, 1944 and Marquardt, 1963).
4. Adjust the fitting variables again, such that the curve comes closer to the data points.
5. Continue to adjust the fitting variables until the adjustments make virtually no difference in the sum-of-squares (i.e., changes are less than a designated tolerance value).
6. Report the fitting variables as the "best-fit" results. The final values obtained in the nonlinear regression analysis will depend to some degree, on the initial values selected in step 1, as well as the "stopping criteria" (or tolerance value) associated with step 5. This means that repeat analyses of the same dataset will not always give the exact same results for the fitting variables.

4.1.2 Interpreting Results

The degree to which a nonlinear regression analysis fits the measured dataset is presented in terms of R^2 values (i.e., the sum-of-squares). The sum-of-squares, (*SS*), is the sum of the squares of the vertical distances of the points from the curve. A nonlinear regression analysis varies the fitting variables in an attempt to minimize the sum-of-squares. The fit of the data points is expressed in terms of the square of the units used for the *Y* values. The value R^2 is a measure of goodness of the fit of the equation to the dataset. R^2 is a fraction between 0.0 and 1.0, and it has no units.

When R^2 is equal to 0.0, the best-fit curve fits the data no better than a horizontal line going through the mean of all *Y* values. In this case, knowing *X* does not assist in improving the prediction of *Y*. When R^2 is equal to 1.0, all points lie exactly on the curve and there is no scatter. Therefore, if a *X* value is known, the exact *Y* value can be calculated. R^2 can be viewed as the fraction of the total variance of *Y* as expressed by the equation under consideration. Mathematically, R^2 is defined by the equation; [$R^2 = 1.0 - SS_{reg} / S_{tot}$], where SS_{reg} is the sum-of-squares of the points from the regression curve and SS_{tot} is the sum-of-squares of the distances of the points from a horizontal line where *Y* is equal to the mean of all the data points.

4.1.3 Nonlinear curve-fitting algorithm

The soil-water characteristic curve is commonly used for the estimation of unsaturated soil property functions. The algorithm proposed by Fredlund and Xing (1994) for the soil-water characteristic curve is presented below. The algorithm is both rigorous and flexible, providing a continuous mathematical function over the entire soil suction range. A similar algorithm is also used in the fitting of other soil-property functions.

The proposed equation for the soil-water characteristic curve by Fredlund and Xing, (1994) is:

$$\theta(\psi,a,n,m) = C(\psi)\frac{\theta_s}{\left(\ln\left[\mathrm{e}+(\psi/a)^n\right]\right)^m} \qquad [\ 66\ ]$$

Let $p = (a, n, m)$ denote the unknown vector of three variables, a, n, and m and let us suppose that there are measured data, (θ_i, ψ_i) (i = 1, 2,..., M), where M is the number of measurements. The least squares estimate of p is the vector p^*, which minimizes the following objective function (i.e., the sum of the squared deviations of the measured data from the calculated data).

$$O(p) = O(a,m,n) = \sum_{i=1}^{M}\left[\theta_i - \theta(\psi_i, a, m, n)\right]^2 \qquad [\ 67\ ]$$

In other words, the least squares method determines the three parameters such that the calculated values from equation [66] are as close as possible to the measured values.

A standard requirement of iterative minimization algorithms is that the value of the objective function decreases monotonically from one iteration to another iteration. Let p_i be the estimate of p at the beginning of the i^{th} iteration (i.e., p_0 is the initial guess and, theoretically it is an arbitrary guess). New estimates for p_{i+1} are chosen such that $O(p_{i+1}) < O(p_i)$. The steepest descent method is one of the easiest methods for minimizing a general nonlinear function with several variables. The steepest descent method exploits the fact that from a given starting point, a function decreases the most rapidly in the direction of the negative gradient vector evaluated at the starting point. Let g denote the gradient of $O(p)$ at p_i.

That is:

$$g = \begin{bmatrix} \dfrac{\partial O(p)}{\partial a} \\ \dfrac{\partial O(p)}{\partial n} \\ \dfrac{\partial O(p)}{\partial m} \end{bmatrix}_{p=p_i} \qquad [\ 68\ ]$$

The steepest descent for a subsequent iteration is defined by:

$$p_{i+1} = p_i - \alpha g \qquad [\ 69\ ]$$

where:

α = *equation scalar that determines the length of the step taken in the direction of -g.*

From equation [67] it follows that:

$$\frac{\partial O(p)}{\partial a} = -2\sum_{i=1}^{M}\left[\theta_i - \theta(\psi_i, a, n, m)\right]\frac{\partial \theta(\psi_i, a, n, m)}{\partial a} \qquad [\ 70\ ]$$

Similarly,

$$\frac{\partial O(p)}{\partial n} = -2\sum_{i=1}^{M}\left[\theta_i - \theta(\psi_i, a, n, m)\right]\frac{\partial O(\psi_i, a, n, m)}{\partial n} \qquad [\ 71\ ]$$

$$\frac{\partial O(p)}{\partial m} = -2\sum_{i=1}^{M}\left[\theta_i - \theta(\psi_i, a, n, m)\right]\frac{\partial \theta(\psi_i, a, n, m)}{\partial m} \qquad [\ 72\ ]$$

From equation [66], the partial derivatives in equations [70] to [72] can be obtained as follows:

$$\frac{\partial \theta(\psi_i, a, n, m)}{\partial a} = mC(\psi_i)\theta_s\left\{\ln\left[\mathrm{e} + (\psi_i/a)^n\right]\right\}^{-m-1}\frac{n(\psi_i/a)^{n-1}(\psi_i/a^2)}{\mathrm{e} + (\psi_i/a)^n} \qquad [\ 73\ ]$$

$$\frac{\partial \theta(\psi_i, a, n, m)}{\partial n} = -\,mC(\psi_i)\theta_s\left\{\ln\left[\mathrm{e} + (\psi_i/a)^n\right]\right\}^{-m-1}\frac{(\psi_i/a)^n \ln(\psi_i/a)}{\mathrm{e} + (\psi_i/a)^n} \qquad [\ 74\ ]$$

$$\frac{\partial \theta(\psi_i, a, n, m)}{\partial m} = -\,C(\psi_i)\theta_s\left\{\ln\left[\mathrm{e} + (\psi_i/a)^n\right]\right\}^{-m}\ln\left\{\ln\left[\mathrm{e} + (\psi_i/a)^n\right]\right\} \qquad [\ 75\ ]$$

The steepest descent method is not efficient for practical use, since the rate of convergence is slow, particularly near to the stationary point. The following quasi-Newton method (Sadler, 1975) can be used to make convergence of the curve-fitting program more efficient:

$$p_{i+1} = p_i - A_i g_i \qquad [\ 76\ ]$$

where:

g_i = *gradient of the objective function evaluated at* p_i*, and*
A_i = *operative matrix at the* i^{th} *iteration.*

Equation [76] becomes the steepest descent method if A_i is the identity matrix multiplied by a step length (a scalar). Denote p_{i-1} - p_i by d_i and g_{i+1} - g_i by q_i. Then A_i is updated using the following formula:

$$A_{i+1} = A_i + \frac{(d_i - A_i q_i)(d_i - A_i q_i)^{\mathrm{T}}}{(d_i - A_i q_i)^{\mathrm{T}} q_i} \qquad [\ 77\ ]$$

where:

T = *the transpose of a vector matrix.*

A suitable choice for A_0 is the diagonal matrix defined by:

$$a_{ij} = \begin{cases} \left|\dfrac{\alpha_i}{2\beta_i}\right|, & if\ \ i = j \\ 0, & if\ \ i \neq j \end{cases} \qquad [\ 78\ ]$$

where:

α_i = *i^{th} element of the starting vector p_0, and*

β_i = *i^{th} element of the gradient g_o evaluated at the starting vector.*

The quasi-Newton method does not require a matrix inversion (or an equivalent), since the sequence A_i (i = 0, 1, 2, ...) converges to the inverse Hessian. In practice, the objective function is often approximately quadratic near the minimum, so a second-order convergence can be eventually expected. However, there is no guarantee that A_i remains positive definite, even for a quadratic function. The product $g_i^T d_i$ should be checked and *DI* replaced by its negative value, if $g_i^T d_i > 0$.

Numerical difficulties may also arise when the scalar product $(d_i - A_i q_i)^T q_i$ is very small, resulting in unduly large elements in A_{i+1}. One of several possible strategies can be used to re-initialize A_{i+1} if the cosine of the angle between $(d_i - A_i q_i)$ and q_i is less than 0.0001. For a nonquadratic objective function, it is reasonable to adjust the step length such that the objective function is reduced at each iteration.

4.2 GRAIN-SIZE (PARTICLE-SIZE) DISTRIBUTION

A sieve and hydrometer analysis is commonly performed for soil classification purposes. SVSOILS provides a methodology for best-fitting a mathematical equation to grain-size distribution data. A unimodal equation is used to best-fit a single mode particle-size distribution while the a bimodal equation is used to best-fit grain-size distributions with two modes. The unimodal and bimodal equations appear to cover essentially all types of particle-size distributions encountered in geotechnical engineering practice. The grain-size distribution curve equations are described in the following sections.

4.2.1 Unimodal equation

The Fredlund and Xing (1994) sigmoidal equation was originally proposed for best-fitting soil-water characteristic curve data. The equation was sigmoidal in character and provided a flexible and continuous mathematical equation that could be fitted to laboratory data using a nonlinear regression algorithm. The original Fredlund and Xing (1994) equation was modified by M.D. Fredlund (2000) to also fit grain-size distribution data. The modified grain-size distribution equation provides a continuous fit that defines the coarse and fine extremes of the particle-size distribution curve.

Menu location: Material > Grain-size > Unimodal Fit

Formulation:

$$P_p(d) = \frac{1}{\ln\left(\exp(1) + \left(\dfrac{a_{gr}}{d}\right)^{n_{gr}}\right)^{m_{gr}}} \left[1 - \left(\frac{\ln\left(1 + \dfrac{h_{rgr}}{d}\right)}{\ln\left(1 + \dfrac{h_{rgr}}{d_m}\right)}\right)^7\right] \qquad [\ 79\ ]$$

Definitions:

equation Variable	Dialogue Field Name	Description
P_p		percent passing at any particular grain-size, *d*
a_{gr}	*agr*	fitting parameter corresponding to initial break of equation (i.e., representing the large particle size)
n_{gr}	*ngr*	fitting parameter corresponding to maximum slope of equation
m_{gr}	*mgr*	fitting parameter corresponding to curvature of equation
h_{rgr}	*hrgr*	residual particle diameter (mm)
d_m	*dm*	minimum particle diameter (mm)

d		particle diameter

Fitting method: Least squares nonlinear regression
Required input: Sieve and/or hydrometer data
Applicable soil types: Uniform or well-graded soils

Modified fields:

Dialogue Field Name	Description
agr	fitting parameter
ngr	fitting parameter
mgr	fitting parameter
USDA % Clay	percentage of clay-sized particles as determined by the USDA classification method
USDA % Silt	percentage of silt-sized particles as determined by the USDA classification method
USDA % Sand	percentage of sand-sized particles as determined by the USDA classification method
USDA % Coarse	percentage of coarse-sized particles as determined by the USDA classification method
USCS % Clay	percentage of clay-sized particles as determined by the USCS classification method
USCS % Silt	percentage of silt-sized particles as determined by the USCS classification method
USCS % Sand	percentage of sand-sized particles as determined by the USCS classification method
USCS % Coarse	percentage of coarse-sized particles as determined by the USCS classification method
D_{10}	diameter of the 10% passing cutoff
D_{20}	diameter of the 20% passing cutoff
D_{30}	diameter of the 30% passing cutoff
D_{50}	diameter of the 50% passing cutoff
D_{60}	diameter of the 60% passing cutoff
USDA Texture	USDA textural classification
USCS Texture	USCS textural classification
Unimodal Fit	indicates if the estimation algorithm has been successfully executed on the current data
Unimodal Error	difference between the fit and laboratory values in terms of R^2

The least squares nonlinear regression algorithm optimizes the a_{gr}, n_{gr}, and m_{gr} parameters. The h_{rgr} and d_m equation parameters are considered to be constants.

The **Apply Fit** command performs two additional calculations following the fitting of the grain-size distribution. Please refer to the grain-size section of the SVSOILS User Manual for more information.

4.2.2 Bimodal equation

A gap-graded material means that there is a significant range of particle sizes that are essentially absent from the overall particle-size distribution. Consequently, the overall grain-size distribution curve becomes bimodal in terms of its shape. Applying a fit to the bimodal grain-size equation will initiate fitting algorithm to determine the fitting parameters for two portions of the particle-size distribution curve.

The bimodal equation can be thought of as two superimposed unimodal curves. The fitting algorithm therefore fits the bimodal equation by breaking the curve into smaller and larger particle size portions. Each portion of the particle-size distribution is fit with a nonlinear least squares regression algorithm and the results are then combined into one equation through the use of the principle of superposition. The breaking point between the two portions of the particle-size distribution curve is determined using a *w* parameter.

Menu location: Material > Grain-size > Bimodal Fit

Formulation:

$$P_p(d)=\left\{w\left[\frac{1}{\ln\left(\exp(1)+\left(\frac{a_{bi}}{d}\right)^{n_{bi}}\right)^{m_{bi}}}\right]+(1-w)\left[\frac{1}{\ln\left(\exp(1)+\left(\frac{j_{bi}}{d}\right)^{k_{bi}}\right)^{l_{bi}}}\right]\right\}\left[1-\left(\frac{\ln\left(1+\frac{hr_{bi}}{d}\right)}{\ln\left(1+\frac{hr_{bi}}{d_m}\right)}\right)^7\right] \quad [\ 80\ ]$$

Definitions:

equation Variable	Dialogue Field Name	Description
P_p		percent passing at any particular grain-size, *d*
a_{bi}	abi	fitting parameter related to the initial breaking point of the curve
n_{bi}	nbi	fitting parameter related to the steepest slope of the curve
m_{bi}	mbi	fitting parameter related to the shape of the curve
jbi	jbi	fitting parameter related to the second breaking point
kbi	kbi	fitting parameter related to the maximum slope of the second hump
lbi	lbi	fitting parameter related to the shape of the second hump
h_{rbi}	hrbi	residual particle diameter (mm)
d_m	dm	minimum particle diameter (mm)
w	Bimodal Split	fitting parameter controlling the split between upper and lower portions
d		particle diameter

Fitting method: Least squares nonlinear regression with superposition.
Required input: Sieve and/or hydrometer data.
Applicable soil types: Gap-graded soils.

Modified fields:

Dialogue Field Name	Description
abi	fitting parameter
nbi	fitting parameter
mbi	fitting parameter
jbi	fitting parameter
kbi	fitting parameter
lbi	fitting parameter
Bimodal Split	fitting parameter
USDA % Clay	percentage of clay-sized particles as determined by the USDA classification method
USDA % Silt	percentage of silt-sized particles as determined by the USDA classification method
USDA % Sand	percentage of sand-sized particles as determined by the USDA classification method
USDA % Coarse	percentage of coarse-sized particles as determined by the USDA classification method
USCS % Clay	percentage of clay-sized particles as determined by the USCS classification method
USCS % Silt	percentage of silt-sized particles as determined by the USCS classification method
USCS % Sand	percentage of sand-sized particles as determined by the USCS classification method
USCS % Coarse	percentage of coarse-sized particles as determined by the USCS classification method

D_{10}	diameter of the 10% passing cutoff
D_{20}	diameter of the 20% passing cutoff
D_{30}	diameter of the 30% passing cutoff
D_{50}	diameter of the 50% passing cutoff
D_{60}	diameter of the 60% passing cutoff
USDA Texture	USDA textural classification
USCS Texture	USCS textural classification
Bimodal Fit	indicates if the estimation algorithm has been successfully executed on the current data
Bimodal Error	difference between the fit and laboratory values in terms of R^2

The results of the bimodal fit of the grain-size distribution can be viewed under the Graph or Report menu options.

The **Apply Fit** command also performs two additional calculations following the fitting of the grain-size distribution curve. Please refer to the Grain-size section of the SVSOILS User Manual for more information.

4.2.3 Statistical Distributions for Grain-size

It is possible to undertake a statistical analysis of the particle-size distribution for a single soil. (Note that this is **not** a combined statistical analysis of a number of grain-size distribution curves).

Performing a statistical analysis on the particle-size distributions of a single soil has been found to be of limited value in geotechnical engineering practice. However, the statistical analysis of a particle-size distribution curve has been found to have application in disciplines such as geology, geological engineering, river engineering and hydrology. The ability to undertake a complete particle-size statistical analysis has been included in the SVSOILS software using the definitions suggested by Folk (1980). The statistical distribution variables calculated for the grain-size distribution are described in the following sections. Further details pertaining to the mathematical equations involved can be found in the referred-to references.

4.2.3.1 Mode, *Mo*

Mode (*Mo*) is defined as the most frequently occurring particle size (diameter). Mode is the diameter corresponding to the point of inflection on the cumulative particle-size distribution curve, provided the curve is plotted on an arithmetic frequency scale. The mode can also be viewed as the highest point on the frequency distribution curve. The Mode is independent of the overall grain-size and therefore is not a measure of overall average size.

4.2.3.2 Median, *Md*

Median (*Md*) means that half of the particles by weight are coarser than the median value and half of the particles are finer. The Median is the diameter corresponding to the 50% point on the cumulative curve and can be expressed either in terms of ϕ, (i.e., the negative log base 2 of the diameter in mm), or mm, (i.e., Md_{ϕ} or Md_{mm}). The median is an easy statistical variable to determine. A disadvantage associated with the Median value is the fact that it is not affected by the extremes of the particle-size distribution curve. Stated another way, it does not reflect the overall sizes of the sediments. For bimodal distributions, the Median value is of little value

4.2.3.3 Graphic Mean, *Mz*

The Graphic Mean, (M_Z) (Folk and Ward, 1957), provides a graphic measure of the mean overall particle size. The Graphic Mean is defined by the formula [$M_Z = (\phi\ 16 + \phi\ 50 + \phi\ 84)/3$]. It corresponds closely to the mean as computed by the method of moments.

4.2.3.4 Graphic Standard Deviation, σ_G

The Graphic Standard Deviation, (σ_G), can be computed as [$\phi\ 84 - \phi\ 16)/2$] and is close to the standard deviation defined using the method of moments. The Graphic Standard Deviation is obtained by reading two values on the cumulative particle-size distribution curve. The Graphic Standard Deviation is a sorting value that embraces the central 68% of the distribution. If a material has a Graphic Standard Deviation, σ_G, of 0.5ϕ, it means that two thirds (i.e., 68%) of the particle sizes fall with 1ϕ (or 1 Wentworth grade) centered on the mean (i.e., the mean ± one standard deviation).

4.2.3.5 Inclusive Graphic Standard Deviation, σ_I

The Inclusive Graphiz Standard Deviation, (σ_I), (Folk and Ward, 1957), provides a measure of sorting and is computed as [$\phi\ 84 - \phi\ 16)/2$]. The Inclusive Graphic Standard Deviation takes the central two-thirds of the particle-size distribution into consideration. A better measure of the Inclusive Graphic Standard Deviation, σ_I, is given by the following equation.

$$\sigma_I = \frac{\phi 84 - \phi 16}{4} + \frac{\phi 95 - \phi 5}{6.6} \qquad [\ 81\ ]$$

where:

ϕ *95*	=	*value of ϕ corresponding to cumulative parentage of 95%*
ϕ *84*	=	*value of ϕ corresponding to cumulative parentage of 84%*
ϕ *16*	=	*value of ϕ corresponding to cumulative parentage of 16%*
ϕ *5*	=	*value of ϕ corresponding to cumulative parentage of 5%*

This formula takes 90% of the particle-size distribution into consideration and is a good overall measure of sorting.

The standard deviation is a measure of the spread of the particle-sizes in terms of ϕ, units of the sample. Therefore, the symbol ϕ should always be attached to the value for the Inclusive Graphic Standard Deviation, σ_I. Measurement sorting values for a large number of materials have suggested the following classification scale for sorting purposes:

$\sigma_I < 0.35\phi$,	very well-sorted
$0.35\phi - 0.50\phi$,	well-sorted
$0.50\phi - 0.71\phi$,	moderately well-sorted
$0.71\phi - 1.0\phi$,	moderately sorted
$1.0\phi - 2.0\phi$,	poorly sorted
$2.0\phi - 4.0\phi$,	very poorly sorted
$> 4.0\phi$,	extremely poorly sorted.

The best sorting attained by natural materials is about 0.20ϕ to 0.25ϕ. *For* example, Texas dune and beach sands run about 0.25ϕ to 0.35ϕ. Texas river materials commonly fall within the range between 0.40ϕ and 2.5ϕ. Pipetted floodplain or neritic silts and clays average about 2.0ϕ and 3.5ϕ. Materials such as glacial tills, mudflows, and other materials have σ_I values in the neighborhood of 5ϕ to 8ϕ or even 10ϕ.

ϕ Quartile Skewness, Skq_ϕ. Quartile Skewness can be calculated using the following equation:

$$Skq_\phi = \frac{\phi 25 + \phi 75 - 2Md_\phi}{2} \qquad [\ 82\ ]$$

where:

ϕ *75*	=	*value of ϕ corresponding to cumulative parentage of 75%*
ϕ *25*	=	*value of ϕ corresponding to cumulative parentage of 25%*
Md_ϕ	=	*defined in section 4.2.3.2*

A positive (+) value indicates that the material has an excess amount of fines, (i.e., the frequency curve shows a tail tending towards the fine range), and a negative (-) value indicates a tail in the coarse range). This measure provides a measure of the skewness in the central part of the curve. Consequently, the Quartile Skewness is greatly affected by sorting and is not a "pure" measure of skewness. If two frequency distribution curves have the same amount of asymmetry, the one with poor sorting will have a higher quartile skewness than a well-sorted sample.

Graphic Skewness, Sk_G. Another measure of skewness is Graphic Skewness (Sk_G) which is computed using the following equation proposed by Inman (1952):

$$Sk_G = \frac{\phi 16 + \phi 84 - 2(\phi 50)}{\phi 84 - \phi 16} \qquad [\ 83\ ]$$

where:

ϕ *50*	=	*value of ϕ corresponding to cumulative parentage of 50%*

Other parameters were defined in the previous equations

The Graphic Skewness (Sk_G) measures the displacement of the median from the average of the ϕ 16 and ϕ 84 points (Figure 7), expressed as a fraction of the standard deviation. Consequently, the Graphic Skewness variable is a measure that is geometrically independent of sorting.

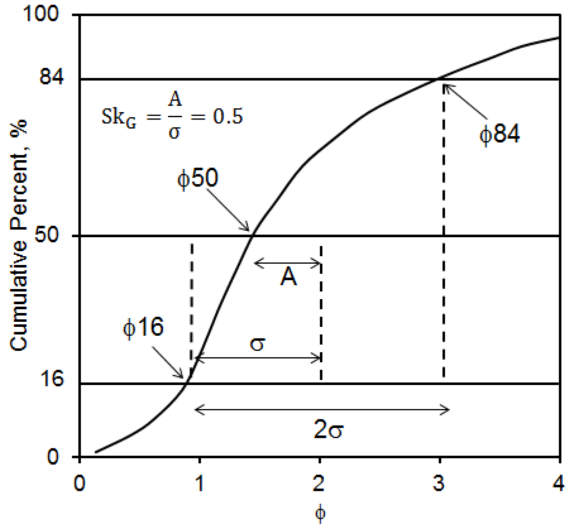


Figure 7 Graphic skewness calculation

The derivation for Graphic Skewness is as follows: Let "*X*" be the midpoint of the ϕ16 and ϕ84 values found by [(ϕ 16 + ϕ 84)/2]. In this case, [(1+3)/2 = 2.0ϕ], and the distance "*A*" (Figure 7) is the displacement of the Median, (ϕ 50), from the midpoint defined as *"X"*. The skewness measure is then *A*/σ. The variables, $A = \frac{\phi 16 + \phi 84}{2} - \phi 50$, and $\sigma = \frac{\phi 84 - \phi 16}{2}$, can be reduced to the following relationship $Sk_G = \frac{\phi 16 + \phi 84 - 2(\phi 50)}{(\phi 84 - \phi 16)}$. In this case, $\frac{1+3-2(1.5)}{(3-1)}$ or Sk_G is equal to +0.50. Note that the median is displaced 0.50 of the way from the "*X*" midpoint to ϕ 16 or the standard deviation mark.

Inclusive Graphic Skewness, Sk_I. (Folk and Ward, 1957). The skewness measure discussed above covers only the central 68% of the particle-size distribution curve. Most skewness occurs in the "tail" portion of the particle-size distribution curve. Therefore the definitions of skewness do not provide a sensitive measure of skewness. A better statistical measure of skewness is the Inclusive Graphic Skewness which includes 90% of the particle-size distribution curve. The Inclusive Graphic Skewness is given by the following equation:

$$Sk_I = \frac{\phi 16 + \phi 84 - 2(\phi 50)}{2(\phi 84 - \phi 16)} + \frac{\phi 5 + \phi 95 - 2(\phi 50)}{2(\phi 95 - \phi 5)} \qquad [\ 84\ ]$$

Equation [84] averages the skewness value obtained when using the ϕ 16 and ϕ 84 points with the skewness obtained when using the ϕ 5 and ϕ 95 points. The Inclusive Graphic Skewness is a good skewness measure to use because it determines the skewness of the "tails" of the distribution curve. The "tails" may contain the most critical differences between samples. Furthermore, the Inclusive Graphic Skewness is geometrically independent of the sorting of the sample. The Inclusive Graphic Skewness equation provides a measure of [phi], ϕ, spread over the numerator and the denominator. The Sk_I value is a pure number and should not be written with ϕ attached. Skewness values should always be recorded with an [a +] or [a –] sign in order to avoid confusion.

Symmetrical curves have an Independent Graphic Skewness, Sk_I equal to 0.0. Materials with excess fines (i.e., a tail to the right) have a positive skewness and those with excess coarse sizes (i.e., a tail to the left) have negative skewness. The greater the skewness value departs from 0.0, the greater is the degree asymmetry. The followings are suggested limits on skewness:

Sk_I: from +1.00 to +0.30 strongly fine-skewed
Sk_I: from +0.30 to +0.10 fine-skewed
Sk_I: from +0.10 to –0.10 near symmetrical
Sk_I: from –0.10 to –0.30 coarse-skewed
Sk_I: from –0.30 to –1.00 strongly coarse-skewed

The absolute mathematical limits for Independent Graphic Skewness are +1.00 to –1.00, and few curves have Sk_I values beyond +0.80 and –0.80.

4.2.3.6 Measures of Kurtosis or Peakedness

The [phi], ϕ, diameter interval should fall between the ϕ 5 and ϕ 95 and be 2.44 times the phi diameter interval between the ϕ 25 and ϕ 75 points if the normal probability curve is defined using the Gaussian equation. If the sample curve forms a straight line when plotted as a probability distribution, (i.e., it follows the normal distribution curve), the distribution is said to have normal kurtosis (i.e., 1.0). Departure from a straight line will alter the kurtosis.

Kurtosis is the quantitative measure used to describe the departure from normality. Kurtosis is a measure of the ratio between the sorting in the "tails" of the curve and the sorting in the central portion of the distribution. If the central portion of the distribution is better sorted than the tails portions, the curve is said to be excessively peaked or leptokurtic. If the tails are better sorted that the central portion of the distribution, the curve is said to be deficient or flat-peaked (i.e., platykurtic). Strongly platykurtic curves are often bimodal with subequal amounts of the two modes. Such a distribution plots as a two-peaked frequency curve with the sag in the middle of the two peaks accounting for its platykurtic character. The following kurtosis measurement is used to represent Graphic Kurtosis, *KG*, (Folk and Ward, 1957).

$$KG = \frac{\phi 95 - \phi 5}{2.44(\phi 75 - \phi 25)} \qquad [\ 85\ ]$$

The Graphic Kurtosis definition responds to the question, "How much is the ϕ 5 to ϕ 95 spread deficient (or in excess) for a given spread of particle sizes between the ϕ 25 and ϕ 75 points?". For a normal distribution curve, *KG* = 1.00. Leptokurtic curves have a value of *KG* in excess of 1.0 (e.g., a curve with *KG* = 2.0 has exactly twice as large a spread in the tails portions as it should have for its [ϕ 25 - ϕ 75] spread). Consequently, the distribution is more poorly sorted in the tails portion than in the central portion. Platykurtic curves have *KG* curves under 1.00 (e.g., a curve with *KG* = 0.70 has tails that have only 0.7 the spread needed to have a given [ϕ 25 - ϕ 75] spread). Kurtosis, like skewness, involves a ratio of spreads and is a pure number. Kurtosis should not be written with the symbol ϕ attached.

$$Sk_G = A/\sigma = +0.50 \qquad [\ 86\ ]$$

4.2.3.7 Representing Grain-size as ϕ

Resultant frequency distributions are highly skewed if the measuring scale is linear or based on equal intervals (Griffiths, 1967). One of the advantages of Wentworth's scale (i.e., using the ϕ variable) is that it is a ratio scale. If each interval of the particle sizes are considered equal, the skewness of many size distributions is materially reduced. Using the ϕ scale is equivalent to drawing a graph of the frequency distribution using a logarithmic scale for the particle size variable. However, statistical computations are difficult and laborious when using the Wentworth's scale (Wentworth, 1929), and there are advantages to converting measurements from the arithmetic scale in say millimeters to a logarithmic scale where the equal ratios of Wentworth's scale become equal arithmetic intervals. Logarithms to any convenient base would suffice, but Krumbein (1938) designed a log transformation scale specially adapted to the Wentworth grade scale. This scale is called the phi (or ϕ) scale and has been adopted for grain-size frequency distributions.

Krumbein (1938) made use of the $\sqrt{2}$ ratio of the Wentworth grade scale by setting:

$$d = 2^{-\phi} \qquad [\ 87\ ]$$

where:

d = *is the diameter in millimeters.*

The logarithm can be taken of both sides of equation [87] to give,

$$\log d = -\phi \log 2 \ \text{ or } \ \log d = -0.30103\phi \qquad [\ 88\ ]$$

Equation [88] can be normalized as follows.

$$\phi = -\frac{\log d}{\log 2} = -\frac{\log d}{0.30103} \qquad [\ 89\ ]$$

Therefore the variable, *d*, can be solved as follows.

$$d = 10^{-0.30103\phi} \qquad [\ 90\ ]$$

4.2.3.8 Effective Grain-size Diameter, d_e

Effective grain-size diameter is defined as the spherical grain diameter of a uniformly sized porous medium that has the same coefficient of permeability as the measured porous medium. Zamarin formulation (Vukovic and Andjelko, 1992) suggested the "effective grain-size diameter" be calculated using equation [91].

$$d_e = \frac{1}{\sum_{i=1}^{n} \frac{f_i}{d_i}} \quad [\ 91\ ]$$

where:

d_i = *grain diameter of the corresponding fraction,*
f_i = *retained weight fraction of soil grain having a diameter, d_i, and*
n = *number of point along the grain size distribution curve to determined d_e*

4.3 SOIL-WATER CHARACTERISTIC CURVE

The soil-water characteristic curve, SWCC, is central to the application of unsaturated soil mechanics in geotechnical engineering practice. There are a number of equations that have been proposed to mathematically represent SWCC data. Each of the proposed equations for the SWCC can be best-fit to a measured data set.

Some of the available fitting methods commonly used in geotechnical engineering practice are as follows:

- Brooks and Corey (1964)
- Gardner (1964)
- van Genuchten (1980)
- van Genuchten (1980) and Mualem (1976)
- van Genuchten (1980) and Burdine (1953)
- Fredlund and Xing (1994)

It is also possible to estimate the SWCC from grain-size distribution data. SVSOILS has implemented a number of the more commonly used algorithms for the estimation of the SWCC from grain-size distribution data. The procedures associated with the estimation of the SWCC from grain-size distribution data are described in Section 5 and are referred to as Pedo-Transfer functions.

Soil-water characteristic curve data is typically recorded in the laboratory as gravimetric water content versus soil suction along the drying (or desorption) branch of an initially saturated soil specimen. It is possible to transform gravimetric water content to other variables such as volumetric water content or degree of saturation versus soil suction using the volume-mass relations. A description of the theory for these transformations can be found in the SWCC Volume-Mass Calculations section. All equations for fitting soil-water characteristic curve data are presented in terms of gravimetric water content in the theory section for the sake of consistency. However, it should be noted that it is necessary to combine the gravimetric water content results with the results of a “shrinkage curve” in order to calculate other volume-mass designations for the SWCC. The geotechnical engineer must be familiar with the role played by each of the designations of the amount of water in a soil (e.g., gravimetric water content, volumetric water content and degree of saturation).

4.3.1 Brooks and Corey (1964) equation

Brooks and Corey (1964) proposed a power-law relationship to represent the soil-water characteristic curve. The SWCC is first divided into two zones; namely, one zone where soil suctions are less than the air-entry value and the other zone where soil suctions are greater than the air-entry value as shown in Figure 8.

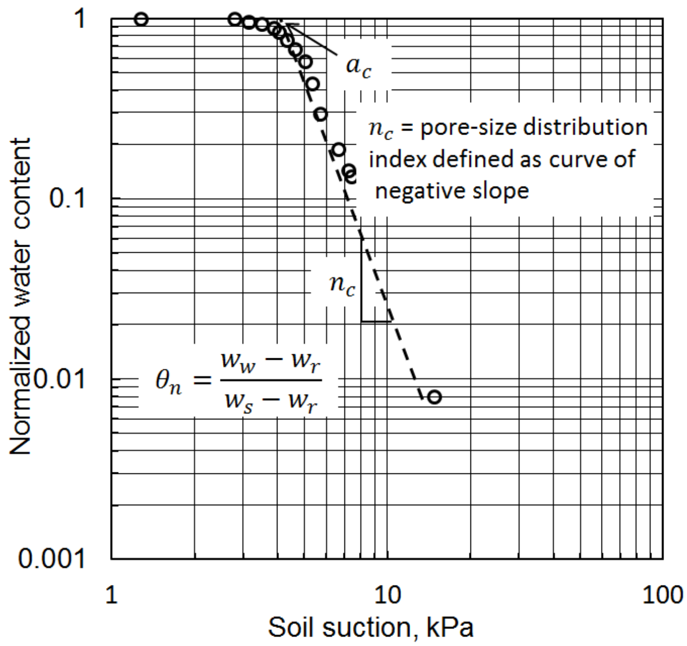


Figure 8 Illustration of the procedure proposed by Brooks and Cory (1964) for the analysis of the SWCC

The amount of water in the soil was described by Brooks and Corey (1964) using the variable, "effective degree of saturation" plotted on a logarithm scale. The same fitting parameters can be obtained using "effective gravimetric water content" provided the soil does not undergo volume change as soil suction is increased. The water content is assumed to remain as a constant value (i.e., saturated coefficient of permeability) prior to reaching the air-entry value. The equation proposed by Brooks and Corey (1964) applies in the suction range between the air-entry value and the residual suction for the soil. The equation for the drying SWCC is written as follows:

Menu location: Material > SWCC > Brooks and Corey Fit

Formulation:

$$w_w = w_r + (w_s - w_r)\left[\frac{a_c}{\psi}\right]^{n_c} \qquad [\ 92\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_r	Residual WC, w_r	residual gravimetric water content
w_s		saturated gravimetric water content
a_c	ac	bubbling pressure (kPa)
n_c	nc	pore size distribution index (dimensionless)
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression

Required input: Drying laboratory data consisting of points on the curve of gravimetric water content versus soil suction. Assumption is made that the soil does not undergo volume change as soil suction is increased.

Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
ac	fitting parameter (kPa)
nc	fitting parameter
Brooks Residual WC	fitting parameter

Brooks SWCC Fit	indicates if the estimation algorithm has been successfully executed on the current data
Brooks Error	difference between the fit and laboratory values in terms of R^2
Brooks AEV	air-entry value as calculated based on the fit of the Brooks and Corey (1964) equation (kPa)
Brooks Max Slope	maximum slope as calculated based on the fit of the Brooks and Corey (1964) equation (kPa)

A nonlinear least-squares regression algorithm is used to determine the fitting parameters for the Brooks and Corey (1964) equation (i.e., a_c and n_c). The regression algorithm can be initiated under the **Apply Fit** button of the **Fitting Method** form.

4.3.2 Gardner (1958) equation

Gardner (1958) presented a continuous equation for the permeability function. The form of the Gardner equation has subsequently been used as the basis for fitting soil-water characteristic curve data. However, it should be noted that the equation was originally proposed as an equation to best-fit measured permeability data.

Menu location: Material > SWCC > Gardner Fit

Formulation:

$$w_w = w_{rg} + \left(w_s - w_{rg}\right)\left[\frac{1}{1 + a_g \psi^{n_g}}\right] \qquad [93]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_{rg}	Residual WC, w_r	residual gravimetric water content
w_s		saturated gravimetric water content
a_g	ag	a soil parameter which is primarily a function of the air-entry value of the soil (kPa)
n_g	ng	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air- entry value has been exceeded
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression
Required input: Drying laboratory data consisting of points on the curve of gravimetric water content versus soil suction.
Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
ag	fitting parameter (kPa)
ng	fitting parameter
Residual WC, w_r	fitting parameter
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
AEV	air-entry value as calculated based on the fit of the Gardner (1958) equation (kPa)
Max Slope	maximum slope as calculated based on the fit of the Gardner (1958) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the Gardner equation (i.e., a_g and n_g). The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.3 Van Genuchten (1980) equation

Van Genuchten (1980) presented a three-parameter equation which provided increased flexibility in best-fitting water content versus soil suction data for a wide range of soils. The parameters of the equation were found using a least-squares fitting algorithm. The proposed equation is a continuous mathematical function; however, the equation is limited to fitting laboratory data in the range up to the residual suction due to the asymptotic nature of the proposed equation.

Menu location: Material > SWCC > van Genuchten Fit

Formulation:

$$w_w = w_{rvg} + \left(w_s - w_{rvg}\right)\left[\frac{1}{\left[1+\left(a_{vg}\psi\right)^{n_{vg}}\right]^{m_{vg}}}\right] \quad [\ 94\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_{rvg}	Residual WC, w_r	residual gravimetric water content
w_s		saturated gravimetric water content
a_{vg}	avg	a soil parameter which is primarily a function of the air-entry value of the soil (kPa)
n_{vg}	nvg	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air-entry value has been exceeded
m_{vg}	mvg	fitting parameter
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression

Required input: Drying laboratory data consisting of points on the curve of gravimetric water content versus soil suction. It should be noted that data points well beyond residual suction conditions may distort the best-fit analysis.

Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
avg	fitting parameter (kPa)
nvg	fitting parameter
mvg	fitting parameter
Residual WC, w_r	fitting parameter
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
AEV	air-entry value as calculated based on the fit of the Genuchten (1980) equation (kPa)
Max Slope	maximum slope as calculated based on the fit of the Genuchten (1980) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the van Genuchten equation (i.e., a_{vg}, n_{vg} and m_{vg}). The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.4 Van Genuchten (1980) and Mualem (1976) equation

Two independent simplifications have been proposed for the van Genuchten's equation. The first simplying assumption suggested that the *m* and *n* parameters be related as follows (i.e., $m = 1 - 1/n_m$). Combining the *n* and *m* variables reduced the reduced the number of fitting parameters from three to two. This simplifying assumption suggested by Mualem (1976) can be seen as a special case of the van Genuchten equation.

Menu location: Material > SWCC > van Genuchten and Mualem Fit

Formulation:

$$w_w = w_{rm} + \left(w_s - w_{rm}\right)\left[\frac{1}{\left[1+\left(a_m\psi\right)^{n_m}\right]^{\left(1-\frac{1}{n_m}\right)}}\right] \quad [\ 95\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_{rm}	Residual WC, w_r	residual gravimetric water content
w_s		saturated gravimetric water content
a_m	am	a soil parameter which is primarily a function of the air-entry value of the soil (kPa)
n_m	nm	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air-entry value has been exceeded
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression

Required input: Drying laboratory data consisting of points on the curve of gravimetric water content versus soil suction. It should be noted that data points well beyond residual suction conditions may distort the best-fit analysis.

Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
am	fitting parameter (kPa)
nm	fitting parameter
Residual WC, w_r	fitting parameter
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
AEV	air-entry value as calculated based on the fit of the Mualem (1976) equation (kPa)
Max Slope	maximum slope as calculated based on the fit of the Mualem (1976) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the van Genuchten (1980) and Mualem (1976) equation. The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.5 Van Genuchten (1980) and Burdine (1953) equation

The second simplying assumption for the van Genuchten (1980) suggested that the m and n parameters be related as follows (i.e., $m = 1 - 2/n_b$). The combination of n and m reduces the number of fitting parameters from three to two. This simplifying assumption suggested by Burdine (1953) can be seen as another special case of the van Genuchten (1980) equation.

Menu location: Material > SWCC > Burdine Fit

Formulation:

$$w_w = w_{rb} + \left(w_s - w_{rb}\right)\left[\frac{1}{\left[1+\left(a_b\psi\right)^{n_b}\right]^{\left(1-\frac{2}{n_b}\right)}}\right] \quad [\ 96\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_{rb}	Residual WC w_r	residual gravimetric water content
w_s		saturated gravimetric water content.
a_b	ab	a soil parameter which is primarily a function of the air-entry value of the soil (kPa)

n_b	nb	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air-entry value has been exceeded
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression

Required input: Drying laboratory data consisting of points on the curve of gravimetric water content versus soil suction. It should be noted that data points well beyond residual suction conditions may distort the best-fit analysis.

Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
ab	fitting parameter (kPa)
nb	fitting parameter
Residual WC, w_r	fitting parameter
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
AEV	air-entry value as calculated based on the fit of the Burdine (1953) equation (kPa)
Max Slope	maximum slope as calculated based on the fit of the Burdine (1953) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the van Genuchten (1980) and Burdine (1953) equation. The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.6 Fredlund and Xing (1994) equation

Fredlund and Xing (1994) presented a three-parameter equation with increased flexibility to fit a wide range of soils. The proposed equation was also modified to provide increased accuracy in the high suction range extending up to 1,000,000 kPa. There may be a limitation in the low suction range (i.e., below the air-entry value) when the soil undergoes volume change in response to an increase in soil suction. The parameters of the equation can be determined using a least-squares algorithm.

Menu location: Material > SWCC > Fredlund and Xing Fit

Formulation:

$$w_w = w_s\left[1-\frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right] \qquad [\ 97\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_w		gravimetric water content at any soil suction
w_s		saturated gravimetric water content.
a_f	af	a soil parameter which is primarily a function of the air entry value of the soil (kPa)
n_f	nf	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air-entry value has been exceeded
m_f	mf	a soil parameter which is primarily a function of the residual water content
h_r	hr	suction at which residual water content occurs (kPa)
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression

Required input: Drying laboratory data consisting of a wide range of gravimetric water content versus soil suction.

Applicable materials: All soils

Modified fields:

Dialogue Field Name	Description
af	fitting parameter (kPa)
nf	fitting parameter
mf	fitting parameter
hr	suction at which residual water content occurs (kPa)
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
Residual WC	gravimetric water content at which residual suction occurs
AEV	air-entry value as calculated based on the fit of the Fredlund (1994) equation (kPa)
Max Slope	maximum slope as calculated based on the fit of the Fredlund (1994) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the Fredlund and Xing (1994) equation. The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.7 Fredlund 2-Point Estimation

The soil-water characteristic curve has two primary defining points: (1) the water content and soil suction at the air-entry value for the soil and (2) the water content and soil suction at residual conditions. Additionally, there are two points that define the extreme limits on the curve: completely saturated conditions under zero suction and completely dry conditions (i.e., zero water content and a soil suction of 1,000,000 kPa) This fit allows the soil-water characteristic curve to be represented by physically meaningful inflection points. The benefit of these physically significant points is that the exact quantification this allows can then lead to an easier statistical analysis.

$$w = w_s + S_1 \log\frac{\psi}{\psi_s} \qquad when\ \psi < \psi_{aev}$$

$$w = w_{aev} + S_2 \log\frac{\psi}{\psi_{aev}} \qquad when\ \psi_{aev} < \psi < \psi_r \qquad [\ 98\ ]$$

$$w = w_r + S_3 \log\frac{\psi}{\psi_r} \qquad when\ \psi_r < \psi$$

Definitions:

Equation Variable	Dialogue Field Name	Description
w_s	Saturated Water Content	gravimetric saturated water content
ψ_s	Saturated Suction	low suction corresponding to saturated conditions (kPa)
w_{aev}		gravimetric water content at air-entry value
ψ_{aev}	Air-Entry Suction	suction at air-entry value (kPa)
w_r		gravimetric residual water content
ψ_r	Residual Suction	residual suction (kPa)
	Air-Entry Saturation	
	Residual Saturation	saturation level at the residual water content expressed as a percent of total saturated volumetric water content.

The slope variables in the above equations are defined as follows:

$$S_1 = \frac{w_{aev} - w_s}{\log(\psi_{aev}) - \log(\psi_s)}$$
$$S_2 = \frac{w_r - w_{aev}}{\log(\psi_r) - \log(\psi_{aev})} \qquad [\ 99\ ]$$
$$S_3 = \frac{-w_r}{\log(10^6) - \log(\psi_r)}$$

The algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.8 Fredlund Bimodal Equation (2000)

The bimodal equation may be thought of as two superimposed unimodal curves. The fitting algorithm therefore fits the bimodal equation by breaking the curve into an upper and lower portion. Each individual portion is fit with a nonlinear least squares regression algorithm and the results are then combined through the use of superposition. The breaking point between the two curves is determined by the s parameter.

$$w = w_s \left\{ s \left[\frac{1}{\ln\left(\exp(1) + \left(\frac{a_{fb}}{\psi} \right)^{n_b} \right)^{m_b}} \right] + (1-s) \left[\frac{1}{\ln\left(\exp(1) + \left(\frac{j_{fb}}{\psi} \right)^{k_{fb}} \right)^{l_{fb}}} \right] \right\} \left[1 - \frac{\ln\left(1 + \frac{\psi}{3000}\right)}{\ln\left(1 + \frac{1000000}{3000}\right)} \right] \qquad [\ 100\ ]$$

where:

Equation Variable	Dialogue Field Name	Description
w_s	w	gravimetric water content at any soil suction
ψ_s	ψ	soil suction (kPa)
a_{fb}	a_{fb}	fitting parameter
n_{fb}	n_{fb}	fitting parameter
m_{fb}	m_{fb}	fitting parameter
j_{fb}	j_{fb}	fitting parameter
k_{fb}	k_{fb}	fitting parameter
l_{fb}	l_{fb}	fitting parameter
K_{fb}	K_{fb}	fitting parameter
s	s	Fredlund bimodal split

Fitting method: Least squares nonlinear regression
Required input: Drying laboratory data consisting of points on the curve of volumetric water content versus soil suction.
Applicable materials: All soils

A nonlinear least-squares regression algorithm is used to determine the parameters for the Fredlund Bimodal Equation (2000) equation. The regression algorithm can be initiated with the **Apply Fit** button of the **Fitting Method** form.

4.3.9 Gitirana and Fredlund (2004)

Gitirana and Fredlund (2004) proposed equations to represent saturated SWCC and it is independent to physical parameters of the SWCC curves. Equations include rotated and translated hyperboles. There are three types of equations to fir various SWCC curves including: 1) Unimodal with one bending point, 2) Unimodal with two bending points, and 3) Biomodal equation. SVSOILS provides curve fitting for Unimodal with one and two bending points.

Menu location: Material > SWCC > Gitirana and Fredlund Fit

Formulation:

- Unimodal with one bending point

$$S = \frac{\tan\theta\left(1+r^2\right)\ln\left(\psi/\psi_b\right)}{1-r^2\tan^2\theta} - \frac{1+\tan^2\theta}{1-r^2\tan^2\theta}\sqrt{r^2\ln^2\left(\psi/\psi_b\right)+\frac{a^2\left(1-r^2\tan^2\theta\right)}{1+\tan^2\theta}}+1 \qquad [\ 101\ ]$$

where:

$\theta = -\lambda/2$ = hyperbole rotation angle,
$r = \tan(\lambda/2)$ = aperture angle tangent, and
$\lambda = \arctan(1/\ln(10^6/\psi_b))$ = desaturation slope

- Unimodal with two bending points

$$S = \frac{S_1 - S_2}{1+\left(\psi/\sqrt{\psi_b\psi_{res}}\right)^d} + S_2 \qquad [\ 102\ ]$$

where:

$$S_i = \frac{\tan\theta_i\left(1+r_i^2\right)\ln\left(\psi/\psi_i^a\right)}{1-r_i^2\tan^2\theta_i} + \left(-1\right)^i \times \frac{1+\tan^2\theta_i}{1-r_i^2\tan^2\theta_i}\sqrt{r_i^2\ln^2\left(\psi/\psi_i^a\right)+\frac{a^2\left(1-r_i^2\tan^2\theta_i\right)}{1+\tan^2\theta_i}}+S_i^a$$

$i = 1, 2$
$\theta_i = -(\lambda_{i-1} + \lambda_i)/2$ = hyperbole rotation angles,
$r_i = \tan((\lambda_{i-1} + \lambda_i)/2)$ = aperture angle tangents,
$\lambda_0 = 0$ and $\lambda_i = \arctan((S_i^a - S_{i+1}^a)/\ln(\psi_{i+1}^a/\psi_i^a))$ = desaturation slopes, and
$S_1^a = 1$; $S_2^a = S_{res}$; $S_3^a = 0$; $\psi_1^a = \psi_b$; $\psi_2^a = \psi_{res}$; ψ_3^a = ***10^6; d = 2exp(1/ln(ψ_{res}/ψ_b)) weight factor***

Definitions:

Equation Variable	Dialogue Field Name	Description
ψ_b	Yb	air-entry value (kPa)
ψ_{res}	Y res	residual soil suction (kPa)
S_{res}	S res	residual degree of saturation
a	agg	a soil parameter which is primarily a function of the rate of water extraction from the soil once the air-entry value has been exceeded
ψ		soil suction (kPa)

Fitting method: Least squares nonlinear regression
Required input: Drying laboratory data consisting of a wide range of gravimetric water content versus soil suction.
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Yb	fitting parameter (kPa)
Y res	fitting parameter (kPa)
S res	fitting parameter
agg	fitting parameter
Fit	indicates if the estimation algorithm has been successfully executed on the current data
Error	difference between the fit and laboratory values in terms of R^2
Residual WC	gravimetric water content at which residual suction occurs
AEV	air-entry value as calculated based on the fit of the Gitirana and Fredlund (2004) equation (kPa)
Max. Slope	maximum slope as calculated based on the fit of the Gitirana and Fredlund (2004) equation

A nonlinear least-squares regression algorithm is used to determine the parameters for the Gitirana and Fredlund (2004) equation. The regression algorithm can be initiated under the **Apply Fit** menu of the **Fitting Method** form.

4.3.10 SWCC Volume-Mass Calculations

SVSOILS provides the ability to calculate other volume-mass versus soil suction relationships. The calculations are either based on an assumption made pertaining to the volume-mass relations or based on the measurement of a "shrinkage curve" for the soil. Calculations can be made converting gravimetric water content to volumetric water content, volumetric air content, degree of saturation, dry density, total density, or normalized curves of the mentioned variables versus soil suction. The calculations used in these conversions are presented in the following sections.

4.3.10.1 Normalized Gravimetric Water Content Relations

Normalization of the soil-water characteristic curve, SWCC, ensures that each curve extends from zero to 1.0 on the vertical axis. A normalized representation is useful when comparing air-entry values from multiple SWCC curves. The equations to present gravimetric water contents in a normalized form is as follows:

$$w_n(\psi) = \frac{w(\psi)}{w_s} \qquad [\ 103\ ]$$

where:

w_n = *normalized water content,*
$w(\psi)$ = *gravimetric water content as a function of soil suction,*
w_s = *saturated gravimetric water content.*

4.3.10.2 Volumetric Water Content (Assuming no volume change)

Gravimetric water content can be converted to volumetric water content assuming no volume change occurs in the soil during the drying process. The calculation is as follows:

$$\theta_w(\psi) = \frac{w(\psi)\, G_s}{1+e} \qquad [\ 104\ ]$$

where:

$w(\psi)$ = *water content as a function of soil suction,*
θ_w = *volumetric water content,*
G_s = *specific gravity of the soil solids, and*
e = *void ratio, which is a constant.*

4.3.10.3 Volumetric Water Content (Including Volume Change)

The volumetric water content of a soil can be calculated as a function of soil suction with the assistance of "shrinkage curve" data. The "shrinkage curve" defines the relationship between gravimetric water content and void ratio (or overall volume) as the soil dries.

Laboratory data for the soil-water characteristic curve is conventionally measured in terms of gravimetric water content. To calculate volumetric water content, it is necessary to define the relationship between gravimetric water content and void ratio, (i.e., $e(w(\psi))$). The relationship can be provided in terms of fitting parameters through shrinkage curve data or through use of an estimated representation of the shrinkage curve. A fit or an estimation of the shrinkage curve must therefore be made available in order to calculate volumetric water content.

$$\theta_w(\psi) = \frac{w(\psi)\, G_s}{1+e\big(w(\psi)\big)} \qquad [\ 105\ ]$$

where:

G_s = *specific gravity of the soil solids,*
θ_w = *volumetric water content,*
$e(w)$ = *void ratio as defined in terms of water content by the shrinkage curve.*
$w(\psi)$ = *gravimetric water content as a function of soil suction.*

4.3.10.4 Volumetric Air Content (Assuming no volume change)

The volumetric air content can be calculated from the gravimetric water content by assuming there is no overall volume change of the soil as soil suction is increased. The calculation is as follows:

$$\theta_a(\psi) = \frac{\big(1-w(\psi)\big)e}{1+e} \qquad [\ 106\ ]$$

where:

$w(\psi)$ = *water content as a function of soil suction,*
θ_a = *volumetric air content,*
G_s = *specific gravity of the soil solids, and*
e = *void ratio.*

4.3.10.5 Degree of Saturation

The degree of saturation of a soil can be calculated as a function of soil suction. Basic laboratory data for a soil-water characteristic curve is assumed to exist as gravimetric water content versus soil suction. To calculate the degree of saturation, it is necessary to assume the volume change behavior as the soil dries or describe how the volume changes as the soil dries through use of the "shrinkage curve". The necessary additional information is provided in the form of a shrinkage curve for the soil.

$$S(\psi) = \frac{w(\psi)\,G_s}{e\big(w(\psi)\big)} \qquad [\ 107\ ]$$

where:

G_s = *specific gravity of the soil solids,*
$w(\psi)$ = *water content as a function of soil suction, and*
$e(w)$ = *void ratio as defined in terms of gravimetric water content by the shrinkage curve.*

4.3.10.6 Dry Density

The dry density of a soil can also be calculated as a function of soil suction. The laboratory data for the soil-water characteristic curve must be available in terms of gravimetric water content. Dry density can be calculated as a function of the volume change as the soil dries. A shrinkage curve must either be estimated or measured in order to calculate the dry density under various suction conditions. The dry density versus soil suction can be calculated as follows.

$$\rho_d(\psi) = \frac{G_s}{1+e\big(w(\psi)\big)}\rho_w \qquad [\ 108\ ]$$

where:

G_s = *specific gravity of the soil solids,*
ρ_w = *density of water (kg/m^3),*
$w(\psi)$ = *water content as a function of soil suction, and*
$e(w)$ = *void ratio as defined in terms of water content by the shrinkage curve.*

4.3.10.7 Total Density

The total density of a soil can be calculated as a function of soil suction. The laboratory data for the soil-water characteristic curve needs to be available in terms of gravimetric water content. Total density can be calculated provided the shrinkage curve of the soil has been measured or is assumed.

$$\rho = \frac{(w(\psi)+1)G_s}{(1+e(w(\psi))}\rho_w \qquad [\ 109\ ]$$

where:

G_s = *specific gravity of the soil solids,*
ρ_w = *density of water (kg/m^3),*
$w(\psi)$ = *water content as a function of soil suction, and*
$e(w)$ = *void ratio as defined in terms of gravimetric water content by the shrinkage curve.*

4.3.11 Determination of Air-Entry Value (AEV)

The air-entry of a soil is defined in terms of the suction at which the soil begins to desaturate. If the soil does not exhibit any significant volume change as soil suction is increased, then the air-entry value can be determined from the gravimetric water content versus soil suction data. However, if the soil undergoes significant volume change as soil suction is increased, the degree of saturation must be calculated and used to determine the "true air-entry" value for the soil.

The air-entry value, AEV, of a soil is the suction at which air begins to enter into the largest pore spaces of the soil. The degree of saturation versus soil suction relationship, and not the gravimetric water content or volumetric water content versus soil suction relationship, must be used to determine the "true AEV" (Zhang and Fredlund, 2014). Zhang and Fredlund (2014) suggested the following procedure to determine the AEV along with the Fredlund and Xing (1994) best-fit of of the SWCC.

Step 1 – Best-fit the degree of saturation SWCC (i.e., S-SWCC), using the Fredlund and Xing (1994) equation.

Step 2 - Substitute soils suction, ψ with a new parameter designated as $\xi = \log_{10}(\psi)$, and write the degree of saturation SWCC as follows:

$$S(\xi) = \frac{1 - \dfrac{\ln\left(1 + \dfrac{10^{\xi}}{\psi_r}\right)}{\ln\left(1 + \dfrac{10^{6}}{\psi_r}\right)}}{\left(\ln\left(\exp(1) + \left(\dfrac{10^{\xi}}{a_f}\right)^{n_f}\right)\right)^{m_f}} \qquad [\ 100\ ]$$

where:

a_f, n_f, m_f and ψ_r = fitting parameters for the Fredlund and Xing (1994) equation

Step 3 - Determine the point of maximum slope (or the inflection point) from the second derivative of equation [100] and set the second derivative to zero:

$$\frac{d^2 S(\xi)}{d\xi^2} = 0 \qquad [\ 101\ ]$$

Solve equation [101] for the variable ξ_i and calculate the point of zero curvature as $(\xi_i, S(\xi_i))$

Step 4 – Draw a line tangent to the curve through the inflection point. The equation for the tangent line at the point of maximum slope is:

$$S(\xi) = \frac{dS(\xi_i)}{d\xi}(\xi - \xi_i) + S(\xi_i) = S'(\xi - \xi_i) + S(\xi_i) \qquad [\ 102\ ]$$

Step 5 – Draw a horizontal line through the maximum degree of saturation (i.e., *S* = 1 or 100%). The intersection of these two lines determines the air-entry value, AEV $\left(\dfrac{1 - S(\xi)}{S'(\xi_i)} + \xi_i, 1\right)$

Step 6 – Back-calculate the AEV (ψ_{AEV}) by setting $\xi = \log_{10}(\psi)$ as $\psi_{AEV} = 10^{\frac{1 - S(\xi_i)}{S'(\xi_i)} + \xi_i}$

The calculated AEV can then be used to calculate the relative permeability function for the soil as proposed by Fredlund et al., (1994) along with the use of the assumption suggested by Childs and Collis George (1950).

$$k_r^s(\psi) = \frac{\displaystyle\int_{\ln(\psi)}^{b} \frac{S(e^y) - S(\psi)}{e^y} S'(e^y)\, dy}{\displaystyle\int_{\ln(\psi_{AEV})}^{b} \frac{S(e^y) - S(\psi_{AEV})}{e^y} S'(e^y)\, dy} \qquad [103]$$

The lower limit of the integration in the denominator of the above equation must be ψ_{AEV} (Fredlund et al., 1994). It should be noted that usage of a lower limit of integration set equal to 0.0 may introduce significant errors into the calculation of the permeability function for a soil. Zhang and Fredlund (2014) showed that starting the limit of integration at 0.0 may under-estimate the relative permeability function for the soil.

4.4 PERMEABILITY (HYDRAULIC CONDUCTIVITY)

The SVSOILS software implements a number of procedures for the computation of the permeability function for an unsaturated soil. The Gardner (1958) equation was proposed as an equation that can be fitted to a laboratory data set of permeability versus soil suction.

4.4.1 Gardner (1958) equation

Gardner (1958) permeability function for unsaturated soils is expressed as a function of soil suction. It should be noted that the Gardner equation was originally proposed as a permeability function. The equation was meant to be fitted to measured laboratory permeability data corresponding to various applied soil suction values. The equation has an *"a"* variable that is related to the air-entry of the soil and a *"n"* variable that is related to the rate at which the coefficient of permeability of the soil decreases as soil suction increases.

The Gardner (1958) equation has also been used to best-fit water content versus soil suction data (i.e., the SWCC). It should be noted that there is no assurance that a *n* variable measured on a soil-water characteristic curve will accurately represent that rate of permeability change as soil suction is increased. The Gardner (1958) permeability function should more correctly be used as a fitting function for measured or independently computed permeability data points.

Menu location: SVFlux > Permeability > Gardner Fit

Formulation:

$$k_w = \frac{k_s}{1 + a\left\{\frac{\psi}{\rho_w g}\right\}^n} \qquad [\ 110\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase (m/s)
k_s		saturated coefficient of permeability of the water phase (m/s)
ρ_w		density of water (kg/m^3)
a	aga	fitting parameter
n	nga	fitting parameter
g		acceleration of gravity (m/s^2)
ψ		soil suction (Pa)

Fitting method: Least squares nonlinear regression
Required input: Laboratory data consisting of at least three points on the curve of permeability versus soil suction.
Applicable soil types: All soils.

Modified fields:

Dialogue Field Name	Description
aga	fitting parameter
nga	fitting parameter
Gardner Fit	indicates if the fit algorithm has been successfully executed on the current data
Gardner Permeability Error	difference between the fit and laboratory values in terms of R^2

This equation provides a flexible permeability function that is defined in terms of two parameters, *a* and *n.* The parameter, *n* defines the slope of the permeability function, and *a* is a parameter related to the breaking point of the function that can be obtained from laboratory data

Figure 9 shows the sensitivity of the permeability function to changes in *a* and *n* parameters. The Gardner permeability function has been used frequently in saturated-unsaturated seepage modeling. A set of data is presented in Figure 10 to demonstrate the application of the Gardner (1958) equation in fitting the laboratory data of coefficient of permeability for various soils. It is also possible to compute data points from the Fredlund et al., (1994) integral permeability equation and then fit the data points (i.e., obtain the *a* and *n* values) using the Gardner permeability function.

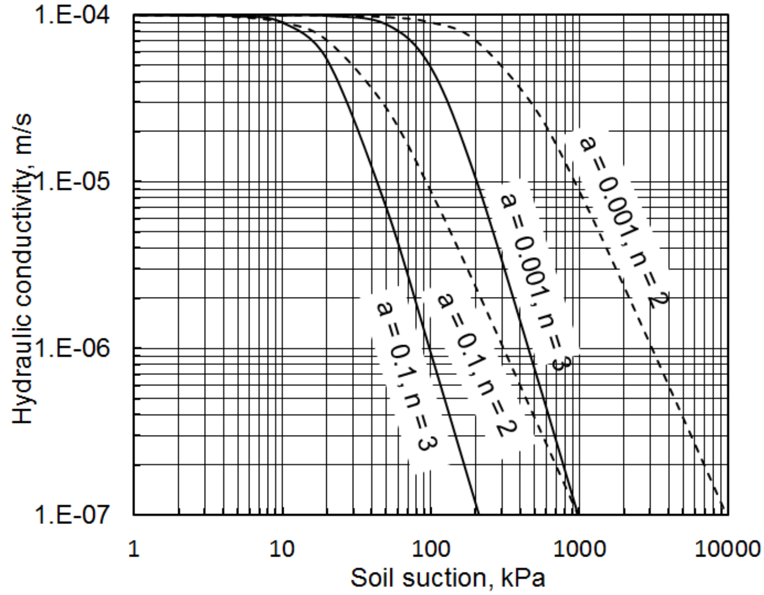


Figure 9 Sensitivity of the Gardner (1958) permeability equation to the *a* and *n* parameters (from Fredlund and Rahardjo, 1993)

4.5 COMPRESSION (AND SWELLING)

Compression curves are defined as the relationship between volume change (e.g., void ratio change) and changes in effective stress. There are numerous stress paths that can be followed when loading a soil. This portion of the theory manual is limited to one-dimensional loading of soils under K_o or one-dimensional conditions. SVSOILS implements the following methods for fitting compression curve data:

Fredlund equation
Two-Slope equation
Weibull Function
Power Function

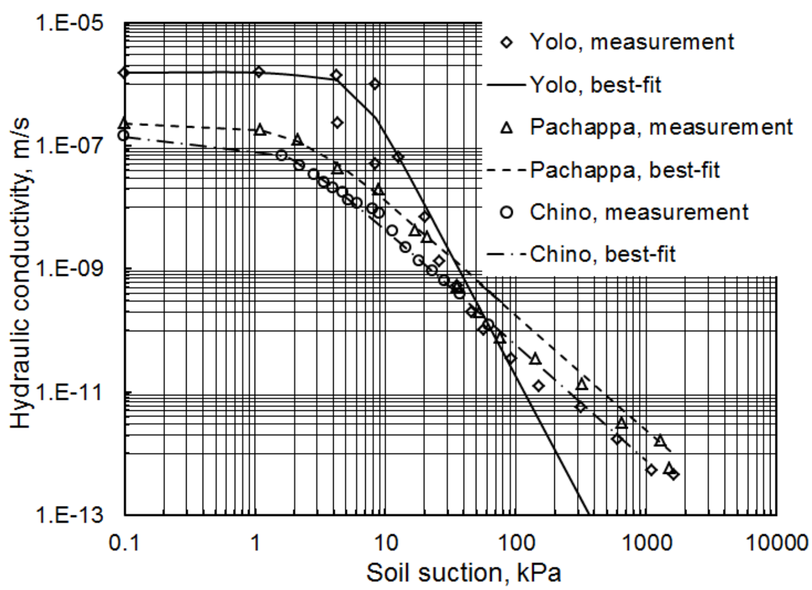


Figure 10 Comparison between the measured and the best-fit coefficient of permeability values for three soils using the Gardner (1958) equation (data from Huang et al., 1995).

4.5.1 Fredlund Fit equation

The shape of the compression curves can be assumed to be similar to the shape of the soil-water characteristic curve. Since the soil-water characteristic curve can be fit with the Fredlund and Xing (1994) equation, it can be assumed that a similar form of the equation can be best-fit to compression data. A modified Fredlund and Xing (1994) equation has been implemented in SVSOILS.

Menu location: SVSolid > Compression > Fredlund Fit

Formulation:

$$e(\sigma_n) = e_o \left[1 - \frac{\ln\left(1 + \frac{\sigma_n}{h_{rco}}\right)}{\ln\left(1 + \frac{3500000}{h_{rco}}\right)} \right] \left[\frac{1}{\ln\left(\exp(1) + \left(\frac{\sigma_n}{a_{co}}\right)^{n_{co}}\right)} \right]^{m_{co}} \qquad [\ 111\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
e		void ratio at any net normal stress
e_o		initial void ratio
σ_n		net normal stress
h_{rco}	hrco	constant curve parameter (kPa)
a_{co}	aco	variable curve parameter related to the breaking point of the curve (kPa)
n_{co}	nco	variable curve parameter related to the maximum slope of the curve
m_{co}	mco	variable curve parameter related to the shape of the curve

Fitting method: Least squares nonlinear regression
Required input: Laboratory data in the form of void ratio versus net normal stress obtained from an oedometer compression test. A minimum of three laboratory points is required on the curve.
Applicable soil types: Normally consolidated soils

Modified fields:

Dialogue Field Name	Description
aco	fitting parameter (kPa)
nco	fitting parameter
mco	fitting parameter
Compression Fit	indicates if the fit algorithm has been successfully executed on the current data
Compression Error	difference between the fit and laboratory values in terms of R^2

The modified form of the Fredlund and Xing (1994) equation contains three soil parameters (a_{co}, n_{co}, and m_{co}) that can be found using a nonlinear regression algorithm. The parameters a_{co}, n_{co}, and m_{co} can be determined in a manner similar to that used for the Fredlund and Xing (1994) SWCC equation.

The 'a_{co}' parameter corresponded to the initial break in the equation while the 'n_{co}' parameter corresponded to the maximum slope of the equation. The 'm_{co}' parameter provides an indication of the curvature of the equation. The number 3,500,000 forces the equation to a void ratio approaching zero at a net normal stress of 3,500,000 kPa. The void ratio of some soils has been shown to approach zero near this loading condition (Ho, 1985)

Fitting the modified Fredlund and Xing (1994) equation to laboratory data produced satisfactory results for many soils. The equation mathematically describes the compression characteristics of the soil. Graphs showing some compression curves for soils are shown below.

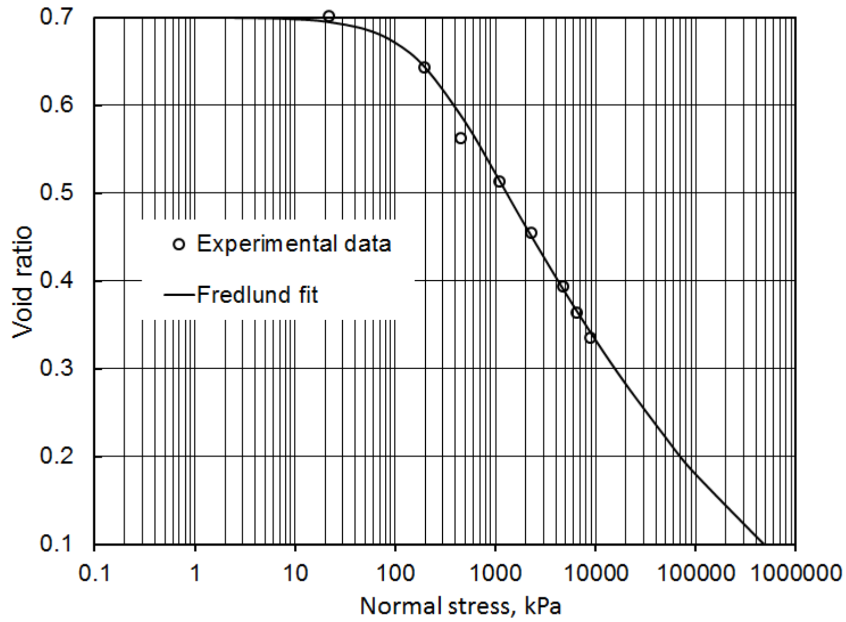


Figure 11 Plot showing the fit of the modified Fredlund and Xing (1994) equation to compression data for an inorganic, low plasticity clay

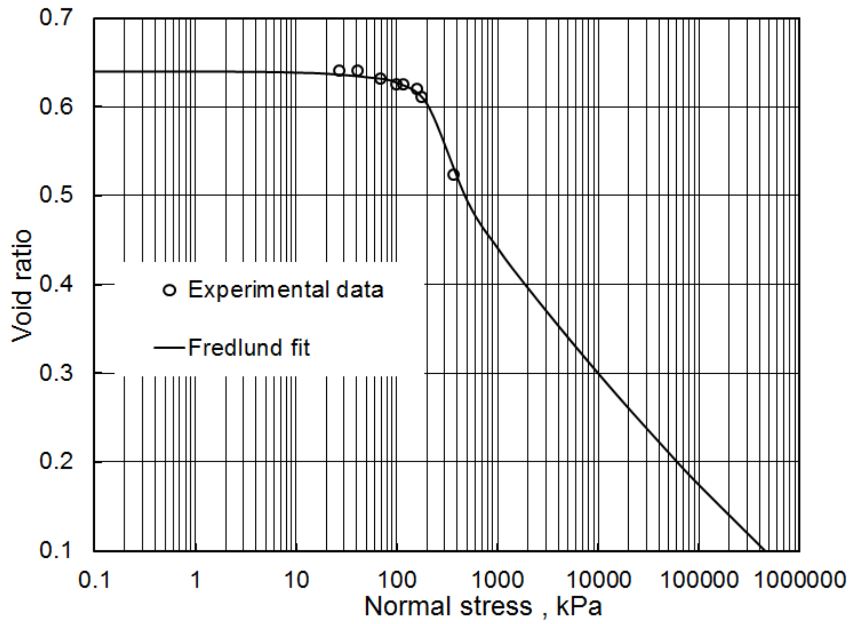


Figure 12 Plot showing the fit of the modified Fredlund and Xing (1994) equation to compression data an for inorganic, low plasticity clay

4.5.2 Two-Slope equation

SVSOILS implements a two-slope equation which allows greater flexibility in terms of the mathematical representation of a compression curve (M.D. Fredlund, 2000). The equation has a shape that is representative of either oedometer data or isotropic triaxial data on a preconsolidated soil. The equation allows for a smooth transition between the recompression and virgin compression branches of the soil. The equation is useful for representing either a normally consolidated compression curve or an over-consolidated compression curve. The equation can also be differentiated to provide a continuous representation of the slope of the compression curve in the form of C_c, m_v, or a_v.

Menu location: SVSolid > Compression > Two Slope Function

Formulation:

$$e(\sigma)=\left[e_o-\frac{C_r}{2}\ln\left[1+\left(\frac{\sigma}{\sigma_s}\right)^2\right]-\frac{C_c-C_r}{2}\ln\left[1+\left(\frac{\sigma}{\sigma_s}\right)^2\right]\right] \qquad [\ 112\ ]$$

Definitions:
Compression

Equation Variable	Dialogue Field Name	Description
e_o		initial void ratio
C_r	Swelling Index	swelling index or recompression index
C_c	Compression Index	compression index
σ_s	Swelling Pressure	swelling pressure (kPa)
σ_p	Preconsolidation Pressure	preconsolidation pressure (kPa).

Rebound

Equation Variable	Dialogue Field Name	Description
e_o	Rebound Void Ratio	initial void ratio obtained from the linear regression of the rebound data
C_r	Rebound Swelling Index	swelling index or recompression index
C_c	Compression Index	compression index
σ_s	Rebound Swelling Pressure	swelling pressure (kPa)
σ_p	Rebound Preconsolidation Pressure	preconsolidation pressure (kPa)

Fitting method: Linear regression
Required input: Laboratory data in the form of void ratio versus net normal stress obtained from an oedometer compression test or an isotropic triaxial test. A minimum of three laboratory points is required on the curve.
Applicable soil types: Normally consolidated or overconsolidated soils

Modified fields:
Compression

Dialogue Field Name	Description
Compression Index	compression index
Preconsolidation Pressure	preconsolidation pressure (kPa)
Swelling Index	swelling index
Swelling Pressure	swelling pressure (kPa)
Maximum Stress	stress at which the compression curve goes to zero (kPa)
Two-Slope Fit	indicates if the fit algorithm has been successfully executed on the current data
Two-Slope Error	difference between the fit and laboratory values in terms of R^2

Rebound

Dialogue Field Name	Description
Rebound Preconsolidation Pressure	preconsolidation pressure
Rebound Swelling Index	swelling index
Rebound Swelling Pressure	swelling pressure (kPa)

Rebound Fit	indicates if the fit algorithm has been successfully executed on the current data
Rebound Error	difference between the fit and laboratory values in terms of R^2

The results of the fit of the two-slope equation to laboratory data can be seen using the options under the Graph or Report menu options. A typical graph representing the two-slope fit of the compression curve can be seen in Figure 13.

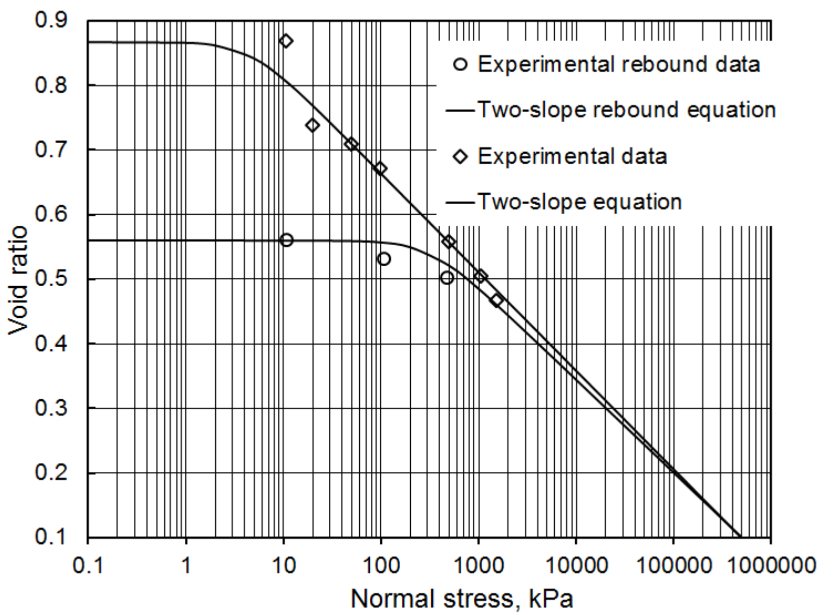


Figure 13 Compression curve showing the relationship between void ratio and net normal stress

4.5.3 Weibull equation

A Weibull function (Priestley, 2012) can be used to represent the compression curve of soft soils and tailings. The Weibull equation also captures the preconsolidation behavior of a soil.

Menu location: SVSolid > Compression > Weibull Function

Formulation:

$$e = a_{wb} - b_{wb} \exp\left(- e_{wb} \sigma'^{f_{wb}}\right) \quad [\ 113\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
e		void ratio at any net normal stress
σ'		effective stress in kPa
a_{wb}	awb	variable curve parameter related to the breaking point of the curve (kPa)
b_{wb}	bwb	variable curve parameter related to the maximum slope of the curve
e_{wb}	ewb	variable curve parameter related to the pre-consolidation of the soil
f_{wb}	fwb	variable curve parameter related to the shape of the curve

Fitting method: Least squares nonlinear regression
Required input: Laboratory data in the form of void ratio versus net normal stress obtained from an oedometer compression test. A minimum of four laboratory points is required on the curve.
Applicable soil types: Normally and under consolidated soils

Modified fields:

Dialogue Field Name	Description
awb	fitting parameter (kPa)

bwb	fitting parameter
ewb	fitting parameter
fwb	fitting parameter
Minimum Stress Limit	is used in plotting compression curve
Compression Fit	indicates if the fit algorithm has been successfully executed on the current data
Compression Error	difference between the fit and laboratory values in terms of R^2

The Weibull function has been used to fit oil sands tailings laboratory data as shown in Figure 14.

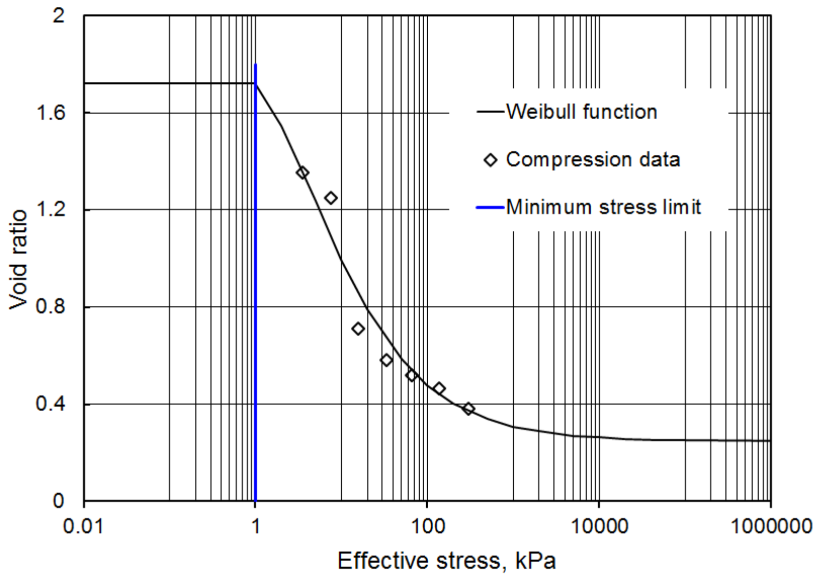


Figure 14 Compression curve showing the relationship between void ratio and effective stress using the Weibull function

4.5.4 Power Function equation

A Power function is another equation form that can be used to describe the compression curve for soft soils and tailings (Priestley, 2012). The Power function is easy to use, but it has the drawback that it cannot capture the preconsolidation pressure of the soil.

Menu location: SVSolid > Compression > Power Function

Formulation:

$$e = a_p + \sigma'^{b_p} \qquad [\ 114\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
e		void ratio at any net normal stress
σ'		effective stress (kPa)
a_p	ap	variable curve parameter related to void ratio at an initial effective stress (kPa)
b_p	bp	variable curve parameter related to the maximum slope of the curve

Fitting method: Least squares nonlinear regression

Required input: Laboratory data in the form of void ratio versus net normal stress obtained from an oedometer compression test. A minimum of two laboratory points is required on the curve.

Applicable soil types: Normally and under consolidated soils

Modified fields:

Dialogue Field Name	Description
ap	fitting parameter (kPa)
bp	fitting parameter
Minimum Stress Limit	is used in plotting compression curve
Compression Fit	indicates if the fit algorithm has been successfully executed on the current data
Compression Error	difference between the fit and laboratory values in terms of R^2

The use of the Power function to describe the volume change characteristics of Oil Sands tailings data is shown in Figure 15. The Power function provides a good fit, but has limitations in describing variations from the shape of the mathematical function.

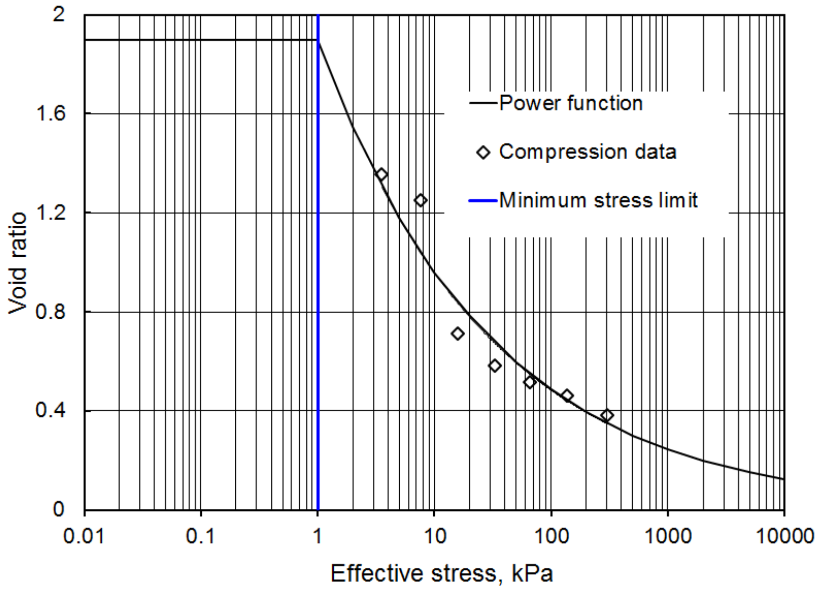


Figure 15 Compression curve showing the relationship between void ratio and effective stress using the Power function

4.5.5 Compression Volume-Mass Calculations

SVSOILS provides a number of calculations that can be used to convert void ratio to sample height, specific volume, porosity, volumetric water content, gravimetric water content, and Young's modulus, *E* as a function of net normal stress. It is also possible to present the compression characteristics of a soil using a variety of variables common to soil mechanics. The calculations used in these conversions are presented in the following sections.

Sample Height
Specific Volume
Porosity
Volumetric Water Content
Gravimetric Water Content
Compression Curve Slope
Young's Modulus

4.5.5.1 Specimen Height

The soil specimen height can be calculated as a function of net normal stress as follows:

$$H_i = H_{i-1} - \frac{\Delta e(\sigma)}{(1+e)} H_o \qquad [\ 115\ ]$$

where:

H_i = *height at depth interval, i,*
H_{i-1} = *height from the last depth interval,*
H_o = *height at the start of the test,*
σ = *net normal stress,*
e = *initial void ratio,*
$\Delta e(\sigma)$ = *void ratio as a function of net normal stress.*

4.5.5.2 Specific Volume

The specific volume of a soil specimen can be calculated as a function of net normal stress as follows:

$$v(\sigma) = e(\sigma) + 1 \qquad [\ 116\ ]$$

where:

σ = *net normal stress,*
e = *void ratio, and*
v = *specific volume.*

4.5.5.3 Porosity

The porosity of a soil specimen can be calculated as a function of net normal stress as follows:

$$n(\sigma) = \frac{e(\sigma)}{(1 + e_o)} \qquad [\ 117\ ]$$

where:

σ = *net normal stress,*
e = *void ratio,*
e_o = *initial in situ void ratio, and*
n = *porosity.*

4.5.5.4 Volumetric Water Content

A soil specimen is allowed to access to water at the start of a compression test in accordance with ASTM standard procedures (ASTM Designation No. D2435). Consequently, the soil tested in a conventional oedometer test is saturated. The volumetric water content of a soil tested in a conventional compression test is therefore equal to the porosity of the soil.

4.5.5.5 Gravimetric Water Content

The gravimetric water content can be calculated as a function of net normal stress as follows:

$$w(\sigma) = \frac{e(\sigma)}{G_s} \qquad [\ 118\ ]$$

where:

σ = *net normal stress,*
e = *void ratio,*
w = *gravimetric water content, and*
G_s = *specific gravity of soil solids.*

4.5.5.6 Compression Curve Slope

One of several terms can be used to express the compression properties of a soil or the slope of the compression curve as a function of net normal stress. SVSOILS calculates the following variables that express the compression characteristics of a soil; namely, the compressibility index, C_c, the coefficient of compressibility, a_v, and the coefficient of volume change, m_v. These compression soil properties are referred to as commonly used soil mechanics terminology. The calculated curves can be performed on either the Modified Fredlund or Two-Slope mathematical representation of a compression curve. The curves can be shown in graphical form or output to the clipboard.

4.5.5.6.1 Coefficient of Compressibility, a_v

The coefficient of compressibility of a soil is defined as follows.

$$a_v(\sigma) = \frac{de(\sigma)}{d\sigma} \qquad [\ 119\ ]$$

where:

a_v = *coefficient of compressibility (kPa^{-1}),*
σ = *net normal stress, and*
e = *void ratio.*

4.5.5.6.2 Compressibility Index, C_c

The compressive index of a soil is defined as follows.

$$C_c(\sigma) = a_v(\sigma)\log(10)\cdot\sigma \qquad [\ 120\ ]$$

where:

a_v = *coefficient of compressibility (kPa^{-1}),*
C_c = *compressibility index,*
σ = *net normal stress, and*
e = *void ratio.*

4.5.5.6.3 Coefficient of Volume Change, m_v

The coefficient of volume change of a soil is defined as follows.

$$m_v(\sigma) = \frac{a_v(\sigma)}{(1+e)} \qquad [\ 121\ ]$$

where:

a_v = *coefficient of compressibility (kPa^{-1}),*
m_v = *coefficient of volume change,*
σ = *net normal stress, and*
e = *void ratio.*

4.5.5.7 Young's Modulus

Many computer software codes have the ability to express deformation properties through the use of a linear elastic model or an incremental elasticity model that makes use of Young's modulus, *E*, and Poisson's ratio, μ. Young's modulus is defined as a change in vertical stress, $\Delta\sigma$, divided by the longitudinal strain, $\Delta\varepsilon$, in a uniaxial compression test (i.e., $E = \Delta\sigma/\Delta\varepsilon$). Young's Modulus is typically assumed to be a constant over a small applied stress change. Under conditions of larger stress changes, (under monotonic stress change conditions), Young's modulus needs to be expressed as a function of net normal stress (Fredlund and Rahardjo, 1993). Therefore, Young's modulus is not a constant value, but can be expressed as a function of the stress state variables.

Poisson's Ratio, μ, and the slope of the compression curve, m_v, can be used to calculate Young's modulus from conventional one-dimensional (or K_o) compression test results. Young's modulus becomes a function of net normal stress since it is dependent on the changing coefficient of volume change of the soil, m_v. The calculation of Young's modulus is also dependent upon the stress path followed for the measurement of the compression curve. Relationships between Young's modulus and stress state for the oedometer and isotropic triaxial loading conditions are shown below.

4.5.5.7.1 Oedometer (or K_o) Compression Test

Soil mechanics compression soil properties can be converted into incremental elasticity soil properties by taking into consideration the differences in the boundary condition between the two tests (Fredlund and Rahardjo, 1993).

$$E(\sigma) = -\frac{[(1+\mu)(1-2\mu)]}{m_v(\sigma)(1-\mu)} \qquad [\ 122\ ]$$

where:

E = *Young's modulus,*
m_v = *coefficient of volume change (kPa^{-1}),*
σ = *net normal stress, and*
μ = *Poisson's ratio.*

4.5.5.7.2 Isotropic Triaxial

The laboratory results of an isotropic compression test can also be converted into incremental elasticity soil properties by taking into consideration the difference in boundary conditions between the two tests (Fredlund and Rahardjo, 1993).

$$E(\sigma) = -3\frac{(1-2\mu)}{m_v(\sigma)} \qquad [\ 123\ ]$$

where:

E = *Young's modulus,*

m_v = *coefficient of volume change (kPa^{-1}),*
σ = *net normal stress, and*
μ = *Poisson's ratio.*

4.6 SHRINKAGE CURVE

SVSOILS provides a hyperbolic equation for mathematical fitting of the shrinkage curve to laboratory data (M.D. Fredlund, 2000). Adjustments can be made to the shrinkage curve in order to represent an initial saturated state. The shrinkage curve can be used to calculate the variation of void ratio with gravimetric water content which can subsequently be related to soil suction through the use of a soil-water characteristic curve. The effect of volume change on the soil-water characteristic curve can be computed through use of the shrinkage curve.

4.6.1 Hyperbolic equation

The shrinkage curve hyperbolic equation provides a continuous mathematical equation to represent the drying process of a soil (M.D. Fredlund, 2000). The fitting algorithm is initiated by selecting **Shrinkage > Hyperbolic Fit > Properties > Apply Fit**. The fitting routine will adjust the parameters of the hyperbolic equation to maximize the R^2 value. The fitting algorithm requires a minimum of three laboratory data points in order to perform the analysis.

Menu location: Material > Shrinkage > Hyperbolic Fit

The fitting algorithm determines three fitting parameters for the shrinkage curve along with an error value and the calculation of the shrinkage limit of the soil.

Formulation:

$$e(w) = a_{sh}\left[\frac{w^{c_{sh}}}{b_{sh}{}^{c_{sh}}} + 1\right]^{\left(\frac{1}{c_{sh}}\right)} \qquad [\ 124\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
a_{sh}	ash	fitting parameter representing minimum void ratio
b_{sh}	bsh	fitting parameter
c_{sh}	csh	fitting parameter
w		gravimetric water content

Fitting Method: Least squares nonlinear regression
Required input: Shrinkage laboratory data in the form of gravimetric water content versus void ratio.
Applicable soil types: All soils, including soils with or without structure.

Modified fields:

Dialogue Field Name	Description
ash	fitting parameter
bsh	fitting parameter
csh	fitting parameter
Shrinkage Limit	shrinkage limit of the soil as calculated by the fit of the shrinkage data
True Air Entry Value	calculated true air-entry value where the shrinkage curve deviates from the saturation line
Shrinkage Fit	indicates if the estimation algorithm has been successfully executed on the current data
Shrinkage Fit Error	difference between the fit and laboratory values in terms of R^2

A typical shrinkage fit for a soil dried from an initial high water content is shown in Figure 16.

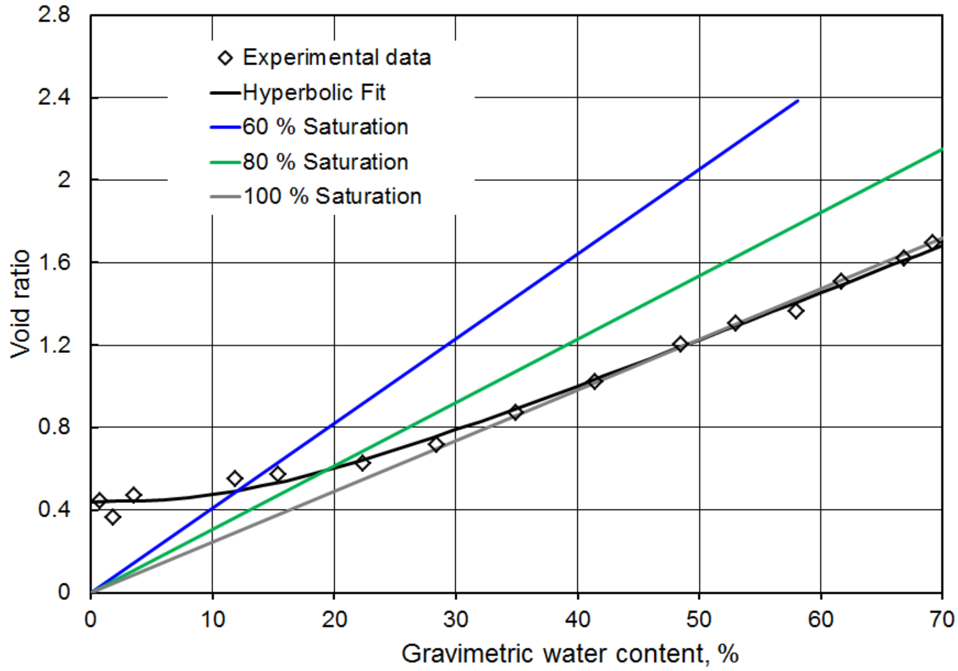


Figure 16 Shrinkage curve showing the relationship between void ratio and gravimetric water content

4.6.2 Adjustment of the Shrinkage Curve for Saturated Conditions

The initial (or starting) conditions for the measurement of the shrinkage curve can be saturated or unsaturated. Most commonly, the measurement of the SWCC begins with the soil sample being saturated or mixed at a slurry water content near to the liquid limit. The soil used for the shrinkage curve and the soil-water characteristic curve are generally saturated at the start of the test; however, the starting water contents may differ in magnitude.

SVSOILS provides a method of estimating the theoretical shrinkage curve that corresponds to the initial saturated conditions for a particular soil. The theoretical shrinkage curve can be matched with the SWCC to determine other volume-mass relations. An example of a laboratory and theoretical shrinkage curve is shown in Figure 17.

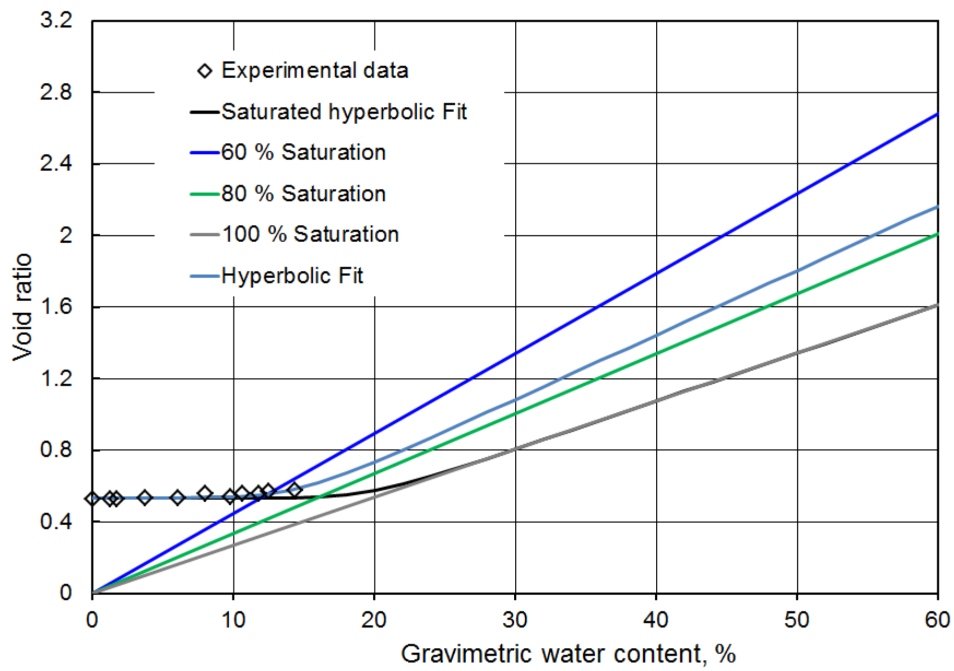


Figure 17 Example of a shrinkage curve modified to conform to initially saturated conditions

Further details pertaining to the calculations related to combining the initial states for the Initial States of Compression, Shrinkage, and Soil-Water Characteristic Laboratory Tests can be found in the section Initial States of Compression, Shrinkage, and Soil-Water Characteristic.

4.6.3 Calculation of Void Ratio versus Soil Suction

The basic soil-water characteristic curve, *w*-SWCC, describes the relationship between the gravimetric water content of a soil and soil suction. The SWCC is extensively used in unsaturated soil mechanics for the estimation of others property functions.

The soil-water characteristic drying curve, *w*-SWCC, is obtained by measuring the mass (or volume) of water that leaves a soil sample while drying the soil under increasing soil suction conditions. It should be noted that volume changes may occur as the soil dries. The volume change under applied suction changes can be characterized by measuring the shrinkage curve for the soil. An example of soil-water characteristic data can be seen in Figure 18.

If a change in volume is measured during the drying process, it is possible to calculate the void ratio and plot the shrinkage curve as shown in Figure 19. It should be noted that the data presented in Figure 19 is for an initially slurry soil specimen where the soil structure has been disturbed due to remolding.

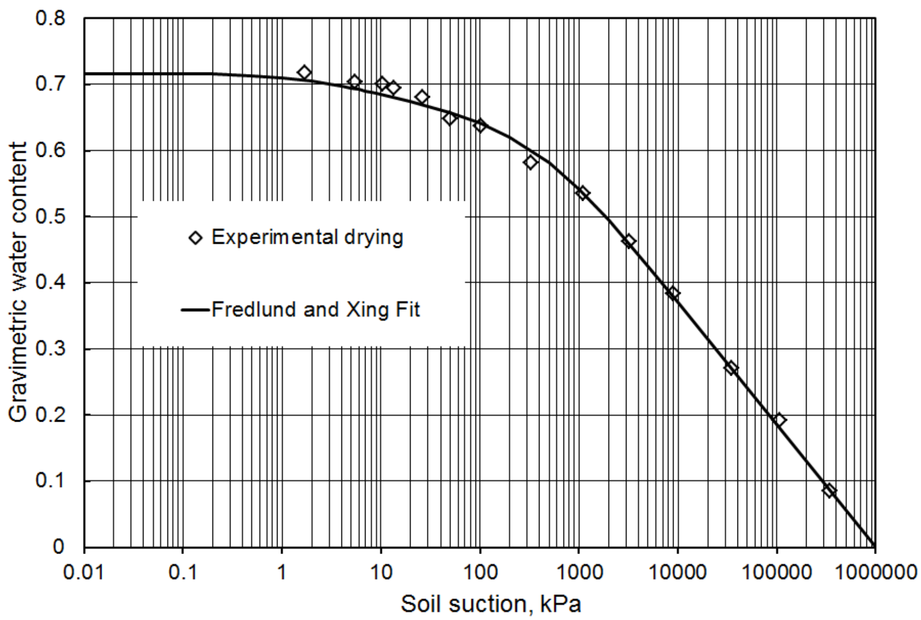


Figure 18 Experimental data for a Black Clay (data from Dagg et al., 1966).

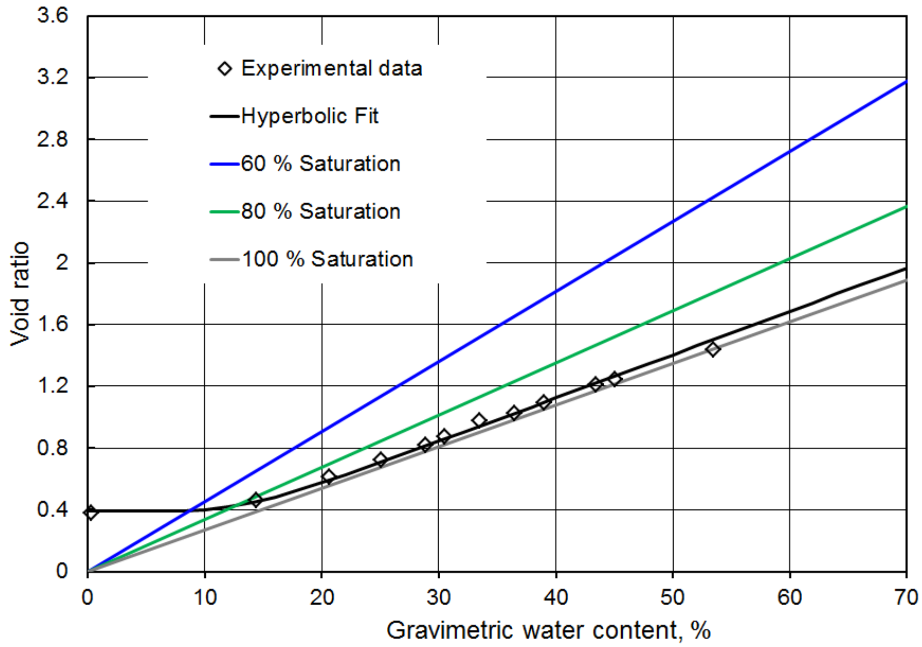


Figure 19 Experimental shrinkage data for a Black Clay originally presented by Dagg et al., (1966), a_{sh} = 0.386, b_{sh} = 0.14, c_{sh} = 5.04, R^2 = 0.993

The shrinkage curve represents the volume change of the soil due to a change in soil suction. Typical measurements for the shrinkage curve of a soil are performed at a net normal stress of zero (ASTM Test Method for Shrinkage Factors of Soils [D 427]).

The shrinkage curve can be visualized as representing the limiting state boundary of the constitutive surface (Figure 20). In reality, there is an independent void ratio constitutive surface and an independent water content constitutive surface. Information is needed regarding the relationship between the two constitutive surfaces when solving unsaturated soil mechanics problems. The *w*-SWCC and the shrinkage curve data provide the minimal required information to analyze unsaturated soil behavior.

The shrinkage curve can be used to calculate the void ratio versus soil suction relationship as shown in Figure 21. The curve shown in Figure 21 was calculated by substituting the gravimetric water content SWCC represented by the Fredlund and Xing (1994) equation (equation [125]) into equation [126]. The relationship between void ratio and soil suction as a limiting boundary condition is shown in Figure 21.

$$w_w(\psi) = w_s\left[1-\frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right] \quad [\ 125\ ]$$

The shrinkage curve equation is shown below (equation 120).

$$e(w) = a_{sh}\left[\frac{w^{c_{sh}}}{b_{sh}{}^{c_{sh}}}+1\right]^{\left(\frac{1}{c_{sh}}\right)} \quad [\ 126\ ]$$

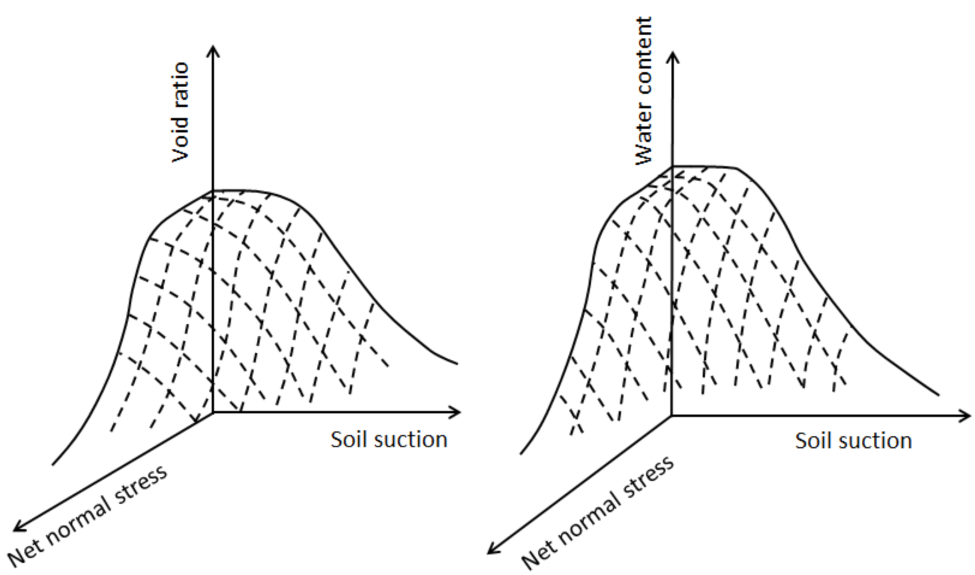


Figure 20 Void ratio and water content constitutive surfaces for an unsaturated soil expressed using soil mechanics terminology (Fredlund and Rahardjo, 1993)

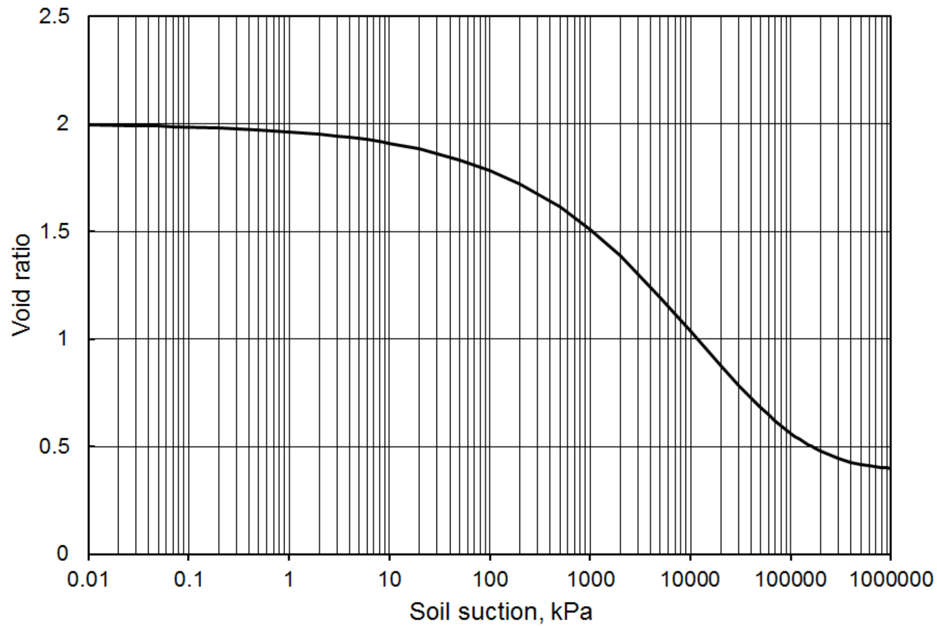


Figure 21 Calculated volume change curve for a Black Clay originally (data from Dagg et al., 1966)

4.7 CONSTITUTIVE SURFACES

The following sections describe the mathematical steps necessary for the generation of a mathematical description of the void ratio and water content constitutive surfaces. Constitutive surfaces are formed in SVSOILS by combining various fitted equations used to describe the state surface boundary conditions. The following topics are covered in this user's manual.

Initial States
Void Ratio Constitutive Surface
Water Content Constitutive Surface

4.7.1 Initial States of Compression, Shrinkage, and Soil-Water Characteristic Laboratory Tests

The selection of initial soil states forms the basis for calculating various constitutive surfaces. Four separate initial soil states are identified based on collected laboratory data. The initial soil states are summarized below.

Laboratory Test	Soil State	
Soil-water characteristic curve	Saturated condition (maximum swell allowed)	initially slurried undisturbed compacted
Shrinkage test	Saturated condition (maximum swell allowed)	initially slurried undisturbed compacted
Consolidation (compression) test	Free-swell	undisturbed compacted
	Constant-volume	initially slurried undisturbed compacted

The generation of constitutive relations requires the selection of specimens which somewhat similar initial soil states were used for all tests. It was determined that the saturated condition obtained at the start of the soil-water characteristic curve is the most common starting condition and, consequently will be used within the SVSOILS software. The soils used within the SVSOILS software vary between being initially slurried, undisturbed, and compacted conditions.

The calculation of the void ratio versus soil suction curve requires the use of a soil-water characteristic curve and a shrinkage curve. It is also necessary that the initial states and initial stress paths be somewhat similar for both tests. Complicating this matter is the fact that shrinkage tests can be either performed on an initially saturated or initially unsaturated (e.g., as-compacted) soil sample. Consequently, the measured shrinkage curve may need to be re-calculated to more closely coincide with initially saturated soil conditions. It is a requirement of the SWCC test that the initial conditions of the soil be close to saturated conditions (i.e., matric suction be released to zero).

The re-calculation of the shrinkage curve is possible after the shrinkage data is best-fit with the shrinkage equation. The equation used to model the shrinkage curve is repeated in equation [114].

$$e(w) = a_{sh}\left[\frac{w^{c_{sh}}}{b_{sh}^{\;c_{sh}}} + 1\right]^{\left(\frac{1}{c_{sh}}\right)} \qquad [\ 127\ ]$$

where:

a_{sh} = *the minimum void ratio, e_{min},*
b_{sh} = *slope of the line of tangency,*
c_{sh} = *curvature of the shrinkage curve,*
and $\frac{a_{sh}}{b_{sh}} = \frac{G_s}{S}$ = *constant for specific soil.*

Re-calculating the shrinkage curve to correspond to initially saturated conditions involves changing the ratio between the a_{sh} and b_{sh} parameters. The c_{sh} parameter controls the curvature of the shrinkage curve and is assumed to remain unchanged. The a_{sh} parameter is equal to the minimum void ratio and is also assumed to remain unchanged. The best-fit curve can, therefore, be adjusted to correspond to complete saturation by re-calculating the b_{sh} according to the relationship presented above. An example of such a calculation can be seen in Figure 22.

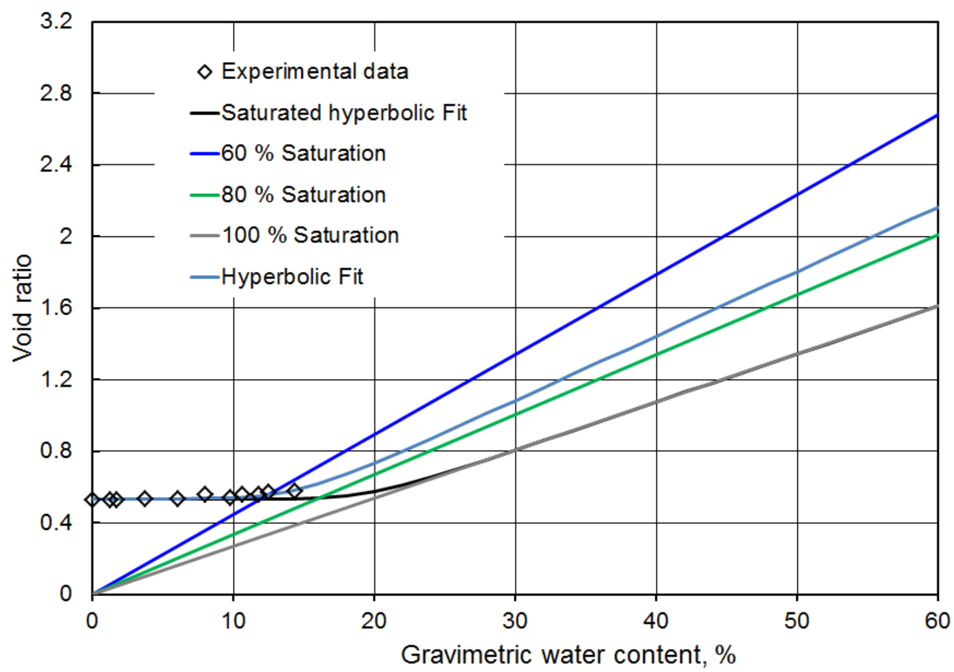


Figure 22 Shrinkage curve for a silty sand starting from unsaturated conditions (data from Russam, 1958)

Re-calculation of the compression equation may also be required when the initial conditions for the experimentally measured compression equation are different from the initial conditions of the soil-water characteristic curve. The compression curve can be best-fit using equation [128]. Therefore, the re-calculation of the compression curve involves the calculation of a new swelling pressure, σ_s, and preconsolidation pressure, σ_p. The rebound index, C_r, and compression index, C_c, for a particular soil remain constant. Modifications to a typical compression curve to account for varying initial conditions are shown in Figure 23.

$$e(\sigma) = \left[e_o - \frac{C_r}{2}\ln\left[1+\left(\frac{\sigma}{\sigma_s}\right)^2\right] - \frac{C_c - C_r}{2}\ln\left[1+\left(\frac{\sigma}{\sigma_p}\right)^2\right]\right] \qquad [\ 128\ ]$$

where:

e_o = *initial or starting void ratio,*
C_r = *recompression index,*
C_c = *compression index,*
σ_s = *swelling pressure (kPa),*
σ_p = *preconsolidation pressure (kPa).*

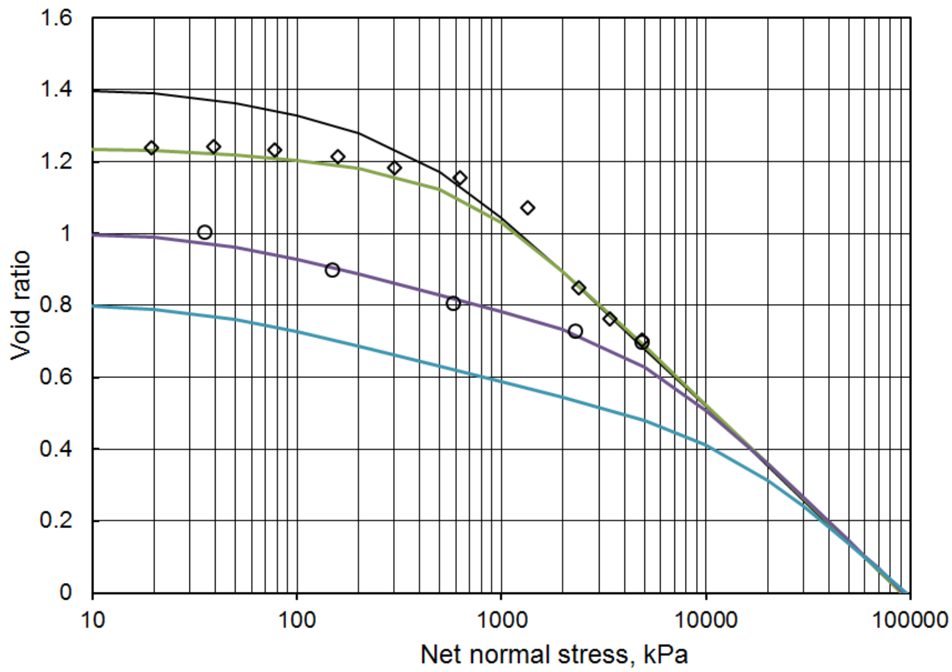


Figure 23 Representation of a family of compression curves generated with the four-parameter equation based on an Albany Clay (data from Schmertmann, 1953)

The calculation of the swelling pressure, σ_s, and the preconsolidation pressure, σ_p, is required for the generation of the series of curves shown in Figure 23. The swelling pressure, σ_s, can be taken as the minimum net normal stress on the rebound curve. If rebound unloading data is not available, σ_s, can be estimated from the compression branch for the soil. The swelling pressure, σ_s, would then be equal to the preconsolidation pressure, σ_p, if the soil is normally consolidated. The preconsolidation pressure can be independently estimated if the soil is over-consolidated.

The preconsolidation pressure, σ_p, can be determined using the conventional procedures described in soil mechanics books (e.g., the Casagrande method). Briefly the procedure involves drawing a straight line on a semilog one-dimensional compression plot to define the virgin compression branch of the curve. Another straight line is then used to represent the rebound portion of the curve. The rebound line is assumed to pass through the initial void ratio of the current soil state and the swelling pressure, σ_s. The intersection of these two straight lines will yield the preconsolidation pressure, σ_p.

4.7.2 Void Ratio Constitutive Surface

The void ratio constitutive surface describes a three-dimensional relationship between void ratio, net normal stress, and soil suction. The void ratio constitutive surface may be formed from the relationships of void ratio versus net normal stress and void ratio versus soil suction. Examples of the final formulation can then be seen in the Calculated Void Ratio Surface section.

4.7.2.1 Void ratio versus net normal stress

The void ratio versus net normal stress boundary constitutive relationship can be formulated either from oedometer test data or an isotropic triaxial compression test data. The formulations presented are assumed to be applicable to both oedometer and triaxial compression test results. Accommodation of the type of the total stress path followed must be done during the calculation of Young's modulus.

Oedometer test data are more common in the literature than isotropic triaxial test data, and will be used to illustrate the void ratio constitutive surfaces. The two-slope compression curve equation (i.e., presented in equation [112]) will be used to mathematically represent the laboratory data.

4.7.2.2 Void ratio versus soil suction

The relationship between void ratio and soil suction can be experimentally determined by combining the soil-water characteristic curve and a shrinkage curve results corresponding to the drying process. The void ratio versus soil suction boundary constitutive relationship can be calculated using a continuous mathematical relationship to represent the soil-water characteristic curve and the shrinkage curve. The Fredlund and Xing (1994) equation can be used to fit laboratory soil-water characteristic curve data using a least-squares algorithm. The shrinkage curve relationship between void ratio and gravimetric water content can be represented using a hyperbolic equation (i.e., equation [124]). Calculations can proceed in the following manner once mathematical expressions are determined for the soil-water characteristic curve and the shrinkage curve.

The soil-water characteristic curve is represented by the Fredlund and Xing (1994) equation (equation [129]).

$$w_w(\psi) = w_s\left[1-\frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right] \qquad [\ 129\ ]$$

where:

w_s	=	*saturated gravimetric water content (in decimal),*
ψ	=	*soil suction,*
a_f	=	*fitting parameter closely related to the air-entry value for the soil (kPa),*
n_f	=	*fitting parameter related to the maximum slope of the curve,*
m_f	=	*fitting parameter related to the curvature of the slope, and*
h_r	=	*constant parameter used to adjust the lower portion of curve (kPa).*

The shrinkage of a soil can be represented using a hyperbolic equation (i.e., equation [124]. The substitution of equation [124] into equation [129] gives the relationship between gravimetric water content and soil suction. The boundary of the constitutive surface describing the relationship between void ratio and soil suction can then be written as follows.

$$e(\psi) = e(w(\psi)) \qquad [\ 130\ ]$$

or

$$e(\psi) = a_{sh}\left[\frac{\left[w_s\left[1-\frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right]\right]^{c_{sh}}}{b_{sh}^{\ c_{sh}}}+1\right]^{\left(\frac{1}{c_{sh}}\right)} \qquad [\ 131\ ]$$

4.7.2.3 Calculated void ratio surface

The goal is to formulate a complete mathematical representation of the void ratio constitutive surface that defines the void ratio over the entire total stress and the soil suction stress range. Due to hysteresis effect it is necessary to restrict equation [112] to monotonic loading conditions (e.g., compression loading only). This restriction allows equation [112] to be simplified to equation [132]. The inverse of equation [132] can then be determined using equation [133].

Equation [133] represents the transition between the void ratio changes produced by suction changes and void ratio changes produced by net normal stress changes. The substitution of equation [131] into equation [133] allows for consideration of both soil suction changes and net normal stress changes. Therefore, the void ratio at any point on the constitutive surface can be represented by adding the net normal stress to an equivalent, suction-induced, net normal stress represented by equation [132]. Equation [134] then represents the equation for the void ratio constitutive surface.

$$e(\sigma) = \left[e_o - \frac{C_c}{2}\ln\left[1+\left(\frac{\sigma}{\sigma_p}\right)^2\right]\right] \qquad [\ 132\ ]$$

$$\sigma(e)=\sqrt{\left[\exp\left(-2\ln(10)\frac{(e(\psi)-e_o)}{C_c}\right)-1\right]}\sigma_p \qquad [\ 133\ ]$$

$$e(\sigma)=\left[e_o-\frac{C_c}{2}\ln\left[1+\left(\frac{\sigma+\sigma(e)}{\sigma_p}\right)^2\right]\right] \qquad [\ 134\ ]$$

The void ratio constitutive surface for a number of different soils are shown in Figure 24 to Figure 25.

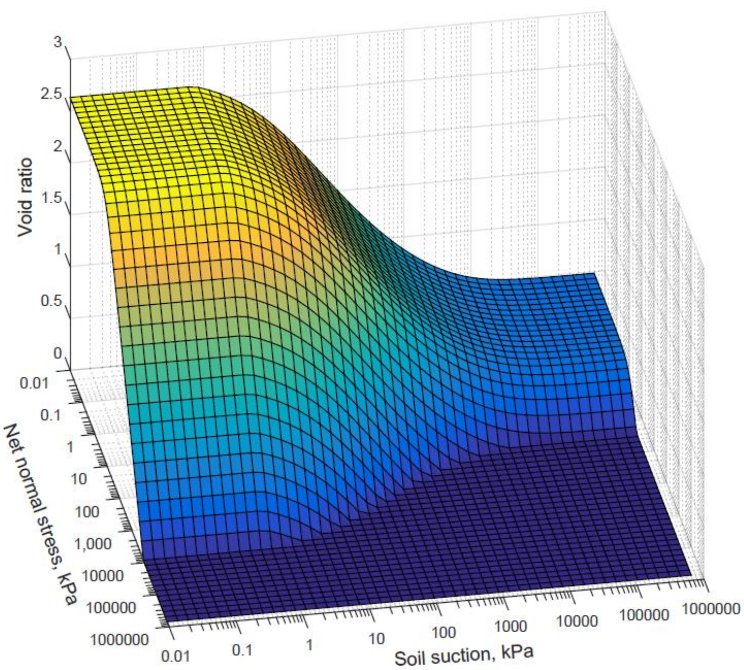


Figure 24 Void ratio constitutive surface for CT (Composite Tailings) oil sands tailings

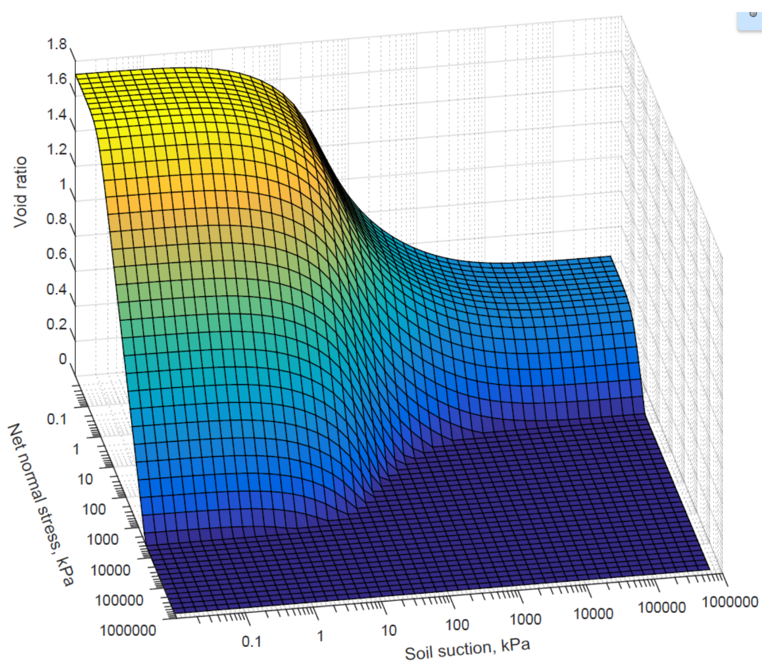


Figure 25 Void ratio constitutive surface for centrifuged oil sands tailings

4.7.3 Water Content Constitutive Surface

The water content constitutive surface describes the relationship between water content, net normal stress, and soil suction. Methods for obtaining the boundaries of the water content constitutive surface and calculations that allow movement across the constitutive surface are presented. The limiting boundaries for the constitutive surface are water content versus soil suction and water content versus net normal stress. The calculations are presented in the Calculated Water Content Surface section.

4.7.3.1 Water content versus soil suction

The water content versus soil suction boundary of the overall constitutive surface can be represented by the water content soil-water characteristic curve. Experimental data can be fit with a nonlinear least squares regression algorithm to mathematically represent laboratory results. The Fredlund and Xing (1994) equation (equation [120]) is used to represent the soil-water characteristic curve due to its ability to model most of the soil suction range. The soil-water characteristic curve is typically measured as the gravimetric water content versus soil suction; therefore, the w-SWCC can simply be applied to the bounding surface.

An adjustment has been introduced to the soil-water characteristic curve which allows the air-entry value determined from of the Fredlund and Xing (1994) equation to increase as compression increases. The effect of various compression levels on the air-entry value has been examined by Vanapalli (1994). Vanapalli (1994) suggested that the air-entry value of a soil followed a line drawn through the steepest slope of the soil-water characteristic curve for soils compacted at optimum water content and dry of optimum water content. A straight line of slightly different slope was followed for soils compacted wet of optimum. A modification to the Fredlund and Xing (1994) equation is therefore presented to account for changes in the air-entry value as the level of compression increases. As a first approximation, the air-entry value is assumed to increase linearly on a semilog plot at a rate equal to the steepest point on the soil-water characteristic curve.

Modification of the air-entry value for the Fredlund and Xing (1994) equation is achieved by making the a_f parameter a function of net normal stress. The a_f parameter has been shown to be related to the air-entry value, therefore, this is considered to be a reasonable assumption. First, a straight line is drawn through the steepest point on the soil-water characteristic curve. The line is then shifted such that the soil-water characteristic curve is unmodified when the net normal stress is at a minimum value. The construction procedure is shown in Figure 26. The final equation for the modified Fredlund and Xing (1994) equation is shown as equation [137].

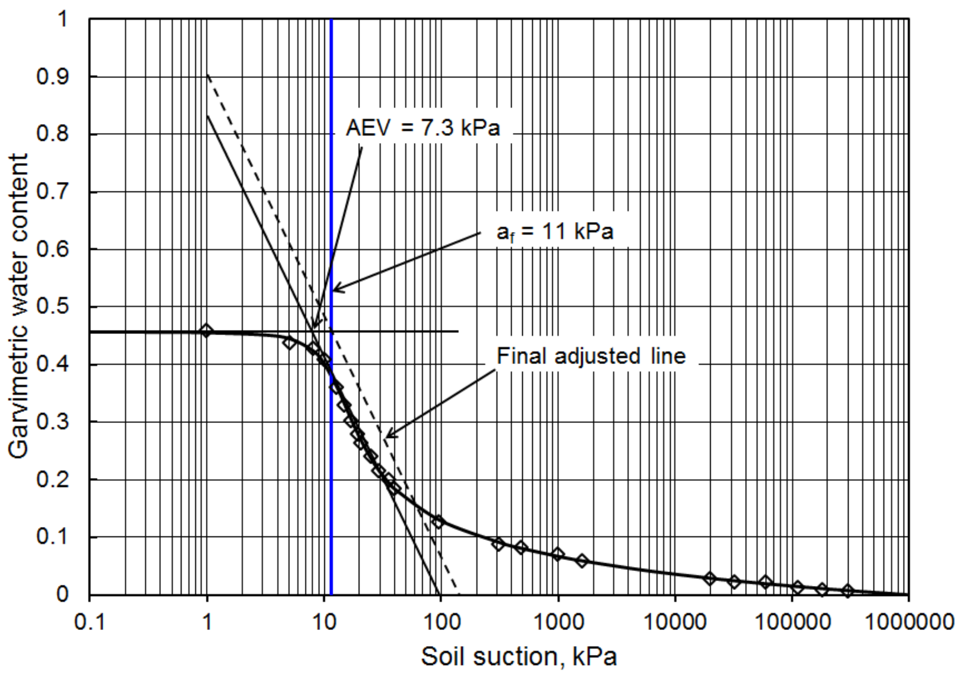


Figure 26 Calculation of modification for air-entry value for a loam using the Fredlund and Xing (1994) equation, a_f = 11.1, n_f = 3.56, m_f = 0.54, h_r = 48.4

The straight line on a semilog plot can be represented by equation [135].

$$w = m \log(a_f) - b \qquad [\ 135\ ]$$

where:

a_f = Fredlund and Xing (1994) parameter related to the air-entry value of the soil,
w = initial water content of the soil sample,

m = slope of the line on a semi-log plot, and
b = y-axis intercept at a value of 1.0.

Equation [135] can be solved for the a_f value.

$$a_f = 10^{\frac{(b-w)}{m}} \qquad [\ 136\ ]$$

The corrected a_f value can be substituted into the Fredlund and Xing (1994) equation. The resulting equation yields the soil-water characteristic curve in terms of soil suction and for a particular initial water content.

$$w_w(\psi) = w_s \left[1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{10^6}{h_r}\right)} \right] \left[\frac{1}{\left[\ln\left[\exp(1) + \left(\frac{\psi}{10^{\frac{(b-w_s)}{m}}} \right)^{n_f} \right] \right]^{m_f}} \right] \qquad [\ 137\ ]$$

4.7.3.2 Water content versus net normal stress

The results of a one-dimensional oedometer test (or an isotropic compression test) can be used to represent the other bounding surface (i.e., the saturated bounding plane) for the overall water content constitutive surface. Experimental data can be best-fit using the four-parameter model (equation [112]). It is not necessary that the initial condition for the compression test be exactly the same as the initial conditions for the measurement of the soil-water characteristic curve. Once the slope of the recompression and the virgin compression branches have been determined and extended to represent the complete compression curve.

The basic volume-mass relationship (i.e., $Se = wG_s$) can be used to convert void ratio to water content on the bounding water content constitutive surface under saturated soil conditions. The soil is saturated (i.e., S = 100%) for all compression tests (i.e., accordance with ASTM D4546). The gravimetric water content under saturated conditions can be expressed as:

$$w = \frac{e}{G_s} \qquad [\ 138\ ]$$

The above relationship can then be expressed as gravimetric water content since void ratio is a function of net normal stress.

$$w(\sigma) = \frac{\left[e_o - \frac{C_r}{2} \ln\left[1 + \left(\frac{\sigma}{\sigma_s} \right)^2 \right] - \frac{C_c - C_r}{2} \ln\left[1 + \left(\frac{\sigma}{\sigma_p} \right)^2 \right] \right]}{G_s} \qquad [\ 139\ ]$$

where:

All parameters were defined in equation [128]

Equation [139] can be used to represent the saturated bounding curve under net total stress loading (Figure 27). The equation parameters are determined using least squares regression.

4.7.3.3 Calculated water content constitutive surfaces

The bounding gravimetric water content surface equations were explained for the water content constitutive surfaces in the preceding sections. The methodology for mathematically representing the interior portion of the water content constitutive surface is as follows.

The compression curve is first used to represent water content conditions along the saturated boundary. The soil-water characteristic curve is then calculated for varying initial conditions and various air-entry values in accordance with equation [137].

Equation [139] is substituted in place of equation [137] to yield the equation for the entire water content constitutive surface. The combination of equation [137] and equation [139] allows for a complete mathematical representation of the water content constitutive surface. The water content constitutive surface for centrifuged oil sands tailings is shown in Figure 27.

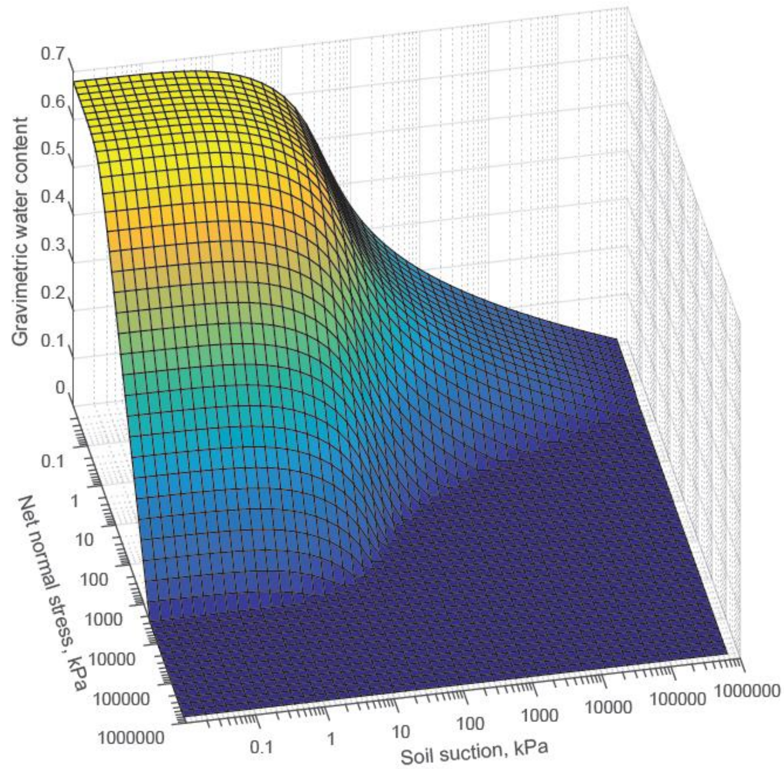


Figure 27 Water content constitutive surface for centrifuged oil sands tailings

4.8 PERMEABILITY VERSUS VOID RATIO

A single power function can be used to fit the relationship between the coefficient of permeability (hydraulic conductivity) and void ratio. The power function is often used for modeling large-strain consolidation of soft soils and mine tailings (Priestley, 2012).

4.8.1 Single power function

Menu location: SVFlux > Ksat vs Void Ratio > Single Power Function

Formulation:

$$k(e) = C\,e^{D} \qquad [\ 140\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k		coefficient of permeability or permeability in [m/s]
C	C	fitting parameter in [m/s]
D	D	fitting parameter related to the curvature of the fitting curve
e		void ratio

Fitting method: Least squares nonlinear regression
Required input: Laboratory data of coefficient of permeability versus void ratio
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Single power Fit	indicates if the fit algorithm has been successfully executed on the current data
Single power Error	difference between the fit and laboratory values in terms of R^2

5 THEORY FOR THE ESTIMATION OF THE SOIL-WATER CHARACTERISTIC CURVE

Unsaturated soil mechanics technologies are expensive and difficult to implement in the consulting practice if the unsaturated soil property functions must be measured in the laboratory. Consequently, there has been a wide range of estimation procedures that have arisen out of worldwide research into unsaturated soil properties. The primary aim of the SVSOILS is to provide engineers with easy-to-implement estimations of unsaturated soil property functions, USPFs. These USPF are mainly based on saturated soil properties and a knowledge of the soil-water characteristic curve for the soil. The primary purpose of the SVSOILS is to provide easy-to-implement procedures to obtain estimations of unsaturated soil property functions.

SoilVision Systems Ltd., does not guarantee the results produced by any particular estimation method. Each estimation method is implemented in a manner consistent with the procedures proposed in the referenced research papers or textbooks. To ensure the quality control with respect to the estimation methods, SoilVision has verified each estimation method with example problems presented in research papers, or textbook. The references to the original documentation used in the implementation of the estimation method are presented in each respective section.

The available estimation algorithms make use of the published forms. Estimation algorithms are also called Pedo-Transfer Functions. Pedo-Transfer Functions are a soil science term that was originally introduced by Bouma (1989).

SVSOILS provides estimations for the following soil property functions:

Soil-Water Characteristic Curve (SWCC)
Permeability (Saturated soil)
Permeability (Unsaturated soil)
k versus Void Ratio (kVoid)

5.1 SOIL-WATER CHARACTERISTIC CURVE

The soil-water characteristic curve, SWCC, has become the means whereby an estimation can be obtained for a variety of unsaturated soil property functions such as the permeability function. The permeability function for a soil, as an example, is time-consuming and costly to measure in the laboratory. Over the past couple of decades, there has been considerable research in geotechnical engineering and the soil science disciplines related to establishing estimation procedures to obtain the soil-water characteristic curve, SWCC, and other unsaturated soil property functions. The desire is to be able to make use of relatively simple tests and basic soil information such as grain-size distribution information for the estimation of the SWCC.

The estimation techniques proposed for the estimation of the soil-water characteristic curve, SWCC, have been labelled as Pedo-Transfer Functions (PTF) within the soil science community. Many of the proposed estimation techniques are quite complex and the intent of the SVSOILS is to alleviate some of the complexity associated with using different algorithms.

SVSOILS has implemented the following methods of estimating the soil-water characteristic curve from simple soils data.

Fredlund and Wilson Estimation Method (1997)
Arya and Paris Estimation Method (1981)
Scheinost Estimation Method (1996)
Rawls Estimation Method (1985)
Vereecken Estimation Method (1989)
Tyler and Wheatcraft Estimation Method (1989)
Gupta and Larson Estimation Method (1979a, 1979b)
Aubertin Estimation Method (2003)

The soil-water characteristic curve section also provides an explanation of the following topics:

Water Storage Function
Estimation of Residual Water Content
Filter Paper Measurement of Soil Suction

The approaches used to estimate the soil-water characteristic curve can be divided into various categories. The categories are referred to as: Point Regression methods, Functional Parameter Estimation methods, and Physio-Empirical methods. A general description of each category can be found in the following sections.

5.1.1.1 Point Regression Method

Some of the point regression methods have been proposed by the following researchers: Husz (1967), Renger (1971), Gupta and Larson (1979a, 1979b), Rawls et al., (1982), and Puckett et al., (1985). The point regression method involves correlating grain-size parameters with the water contents at various suction levels of the soil-water characteristic curve.

5.1.1.2 Functional Parameter Regression Method

The functional parameter regression method assumes that functional parameters of the SWCC equation can be correlated to the basic physical properties of the soil. An example of this method is the correlation between the air-entry parameter of a soil-water characteristic curve equation and basic soil properties such as percent sand or porosity. Research publications by Paclepsky et al., (1982), Cosby et al., (1984), Rawls and Brakensiek (1989), Nicolaeva et al., (1986), and Vereecken et al., (1989) are a few examples of researchers that have used the functional parameter regression method.

The findings of Rawls and Brakensiek (1985) and Vereecken et al., (1989) have been used within SVSOILS as examples of the functional parameter regression method. Rawls and Brakensiek (1985) presented regression equations for estimating the parameters for the Brooks and Corey (1964) equations. The regression equations provide an estimation of the bubbling pressure, a_c, the pore size index, λ, and the residual water content, θ_r, for the Brooks and Corey (1964) equation.

The Vereecken et al., (1984) method involved fitting a dataset of forty Belgian soil series with the van Genuchten (1980) equation. A one-dimensional sensitivity analysis was then performed on the optimized parameters of the soil-water characteristic curve to assess reliability of the methodology. A principle factorial analysis was then used to structure the data and examine the relationship between the SWCC and basic measured soil properties. Regression equations were then proposed. Vereecken et al., (1984) concluded that the SWCC could be estimated with a reasonable level of accuracy using soil properties such as grain-size distribution, dry density, and carbon content.

5.1.1.3 Physical Model Method

Arya and Paris (1981) presented the first physio-empirical method to estimate the soil-water characteristic curve. The model made use of basic soils information such as the grain-size distribution. The volumetric water coefficient was then calculated based on the pore sizes. The pore radii were converted to an equivalent soil suctions through the use of the capillary theory. The estimation method uses an empirical α constant to account for uncertainty in the estimation. The formulations for the pore radius were based on an assumption of spherical particles and cylindrical pores.

Arya and Paris (1981) assumed the pore-size distributions and the grain-size distributions of soils to be approximately congruent. That is, larger particles produce larger inter-particle voids than smaller particles and vice versa. The grain-size distribution was divided into M size fractions. The mass of solid in the i^{th} particle class was equated to the mass of N_i spherical particles with a radius R_i. The volume was given by the following equation:

$$V_{ri} = \frac{4}{3} N_i \, \pi \, R_i^3 \qquad [\ 141\ ]$$

The volume of voids was subsequently represented by a single capillary tube of radius, r_i.

$$V_{ru} = \pi \, r_i^2 h_i \qquad [\ 142\ ]$$

where h_i is the capillary tube length. Arya and Paris (1981) also assumed that the particles were spherical and could be represented by a capillary pore length of an R_i class as:

$$h_i = 2 \, R_i N_i^{\alpha} \qquad [\ 143\ ]$$

where α is an empirical constant between 1 and 2.

Variations of the physical model have been proposed to estimate the random packing nature of spherical particles in an attempt to properly estimate the pore-size distribution of a heterogeneous system (Iwata et al., 1988).

The Arya and Paris (1981) model was later modified by Havercamp and Parlange (1986), who applied the concept of shape similarity between the SWCC and the cumulative grain-size distribution for sandy soils without organic matter. Bupta and Ewing (1992) applied the Arya-Paris model in two ways: i) to the grain-size distribution in order to model intra-aggregate pores, and ii) to the aggregate-size distribution to model the inter-aggregate pores. Nimmo (1997) presented a method to account for the influence of soil structure through the use of aggregate-sized distributions.

Criticisms have been expressed (Havercamp and Parlange 1982 and 1986; Arya and Paris, 1982) regarding the empirical nature of the α parameter presented in the Arya and Paris (1981). Tyler and Wheatcraft (1989) presented an analysis correlating the fitting parameter α to physical properties of the soil using fractal mechanics. It was hypothesized that α was equal to the fractal dimension of the pore trace and expressed a measure of the tortuosity of the pore trace. The fractal dimension of the pore traces ranged from 1.011 to 1.485 for all but one soil tested.

5.1.2 Fredlund and Wilson (1997) Estimation Method

The M.D. Fredlund and Wilson (1997) method for the estimation of the soil-water characteristic curve is based on the physio-empirical method. The methodology is based on the concept of a capillary model along with an understanding of the factors that cause variations in the soil-water characteristic curve. The M.D. Fredlund and Wilson (1997) estimation method has been tested against a subset of the SVSOILS database including 188 soils with varying textures. A comparison was also performed between the M.D. Fredlund and Wilson (1997) method and the other pedo-transfer functions, PTF, (Fredlund, 2000). The theory for the Fredlund and Wilson (1997) algorithm is presented in the following sections.

Menu location:	Material > SWCC > Fredlund and Xing Fit > Fredlund and Wilson Estimation
Formulation:	Algorithm
Fitting method:	N/A
Required input:	*In situ* volume-mass properties and a well-defined grain-size distribution. A hydrometer analysis is recommended for soils with greater than 15% fines.
Applicable soil types:	All soils. The packing porosity used in the estimation will vary based on soil type. The algorithm also estimates the soil-water characteristic curves for waste rock provided the packing porosity is increased appropriately.

Modified fields:

Dialogue Field Name	Description
Fredlund PTF Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Fredlund PTF Error	difference between the fit and laboratory values in terms of R^2
Fredlund PTF AEV	air-entry value as calculated based on the estimated curve
Fredlund PTF Max Slope	maximum slope as calculated based on the estimated curve

Unimodal and bimodal equations can be used to define the grain-size distribution. The grain-size equations allowed for a continuous fit along with a proper description of the extremes of the grain-size distribution curve. The proposed model makes use a combination of the capillary model and a knowledge of the factors that influence the character of the SWCC. The volume-mass properties (including a "packing factor") and the grain-size distribution form the basic information needed for the estimation of the soil-water characteristic curve.

The M.D. Fredlund and Wilson (1997) approach is based on the following theorems.

Theorem 1 – A soil composed entirely of a uniform, homogeneous particle size has a unique drying soil-water characteristic curve.
Theorem 2 – The capillary model is best suited for the estimation of the air-entry value of each collection of uniform, homogeneous particle sizes.
Theorem 3 – The soil-water characteristic curve for more than one particle size is the summation of the SWCCs for each individual particle size.

The Fredlund and Xing (1994) equation was selected to describe the SWCC for each individual particle size. The a_f parameter in the Fredlund and Xing (1994) model was related to the air-entry value of the soil. Figure 28 shows the relationship between the air-entry value of the soil and the a_f fitting parameter of the Fredlund and Xing (1994) equation for the dataset used to train the proposed new pedo-transfer function. The a_f parameter can be seen to be typically higher than the actual air-entry value. The a_f parameters is a relatively close approximation of the air-entry value as calculated using the procedure described by Vanapalli and Fredlund (1998). The variation in the Fredlund and Xing (1994) equation using a range of n_f and m_f parameters and holding the a_f parameter at a constant value of 100 kPa can be seen in Figure 29.

The individual particle-radius in subdivisions along the grain-size distribution can be converted to an equivalent air-entry value using the capillary equation [144]. This soil suction is the air-entry value for a soil with uniform particle sizes.

$$a_f = 2\gamma \frac{\cos\theta}{\rho_w \; g \; r} \qquad [\ 144\ ]$$

where:

γ = *surface tension of water (N),*
θ = *contact angle (degree),*
ρ_w = *density of water (kg/m^3),*
g = *acceleration of gravity (m/s^2),*
r = *pore radius (m), and*
a_f = *parameter related to the air-entry value in the Fredlund and Xing (1994) equation (N/m^2 = Pa).*

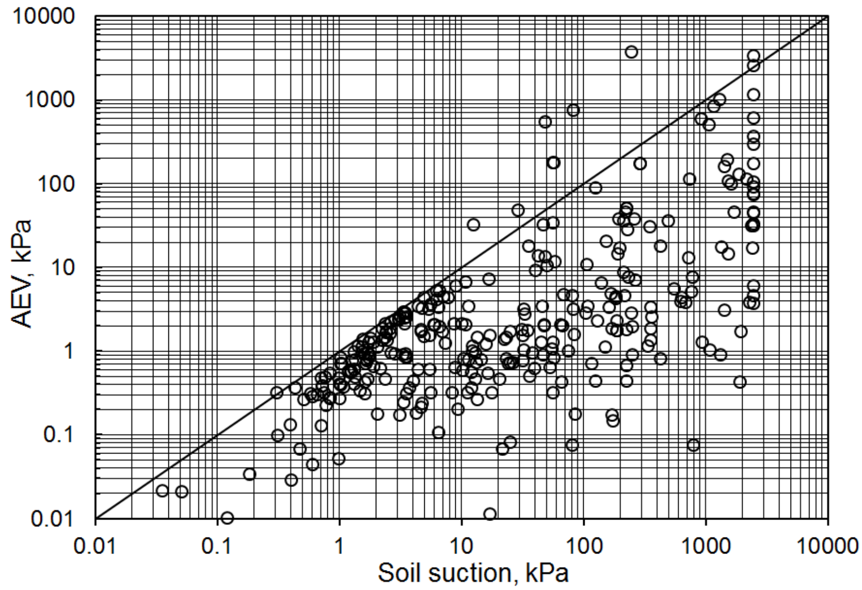


Figure 28 Relationship between the air-entry value from the Vanapalli and Fredlund (1998) construction method and the a_f parameter from the Fredlund and Xing (1994) equation using the training data set

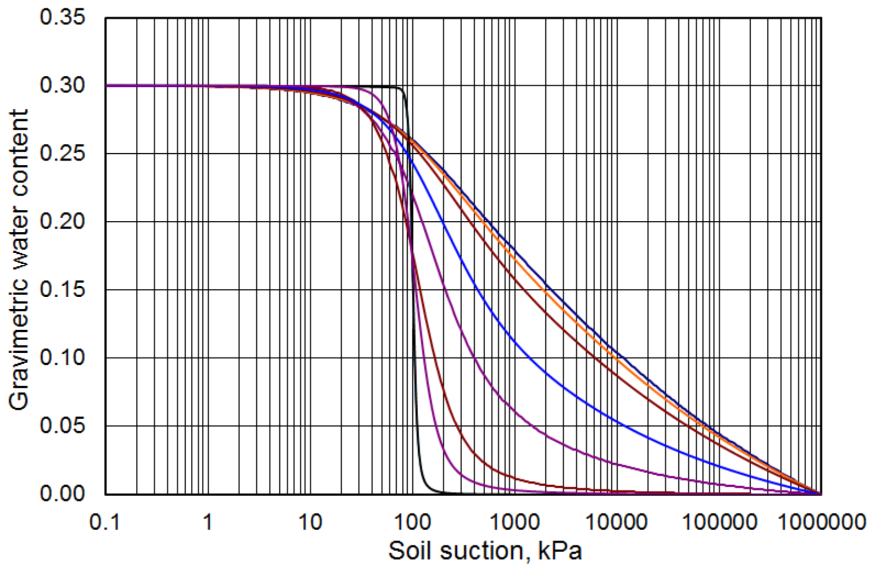


Figure 29 Illustration of variation of n_f and m_f parameters varying with grain-size while holding a_f constant at 100 kPa

An approximate shape for the soil-water characteristic curve can be calculated for each uniform collection of particles. The shape for a uniform coarse sand or a clay material can be estimated using the M.D. Fredlund and Wilson (1997) method.

An experimentally measured SWCC for a collection of glass beads was used as a benchmark. It was assumed that the glass beads were representative of a SWCC for uniform coarse particles. The SWCC for the glass beads is shown in Figure 30. The SWCC for a clay was estimated by plotting the results of a group of soils with a high clay content. The group of clay soils can be seen in Figure 31. The glass beads and the clay soils provide limiting values for groups of soils consisting of uniformly-sized particles. The limiting values were then used as the basis for the estimation of other particle sizes (See Figure 32).

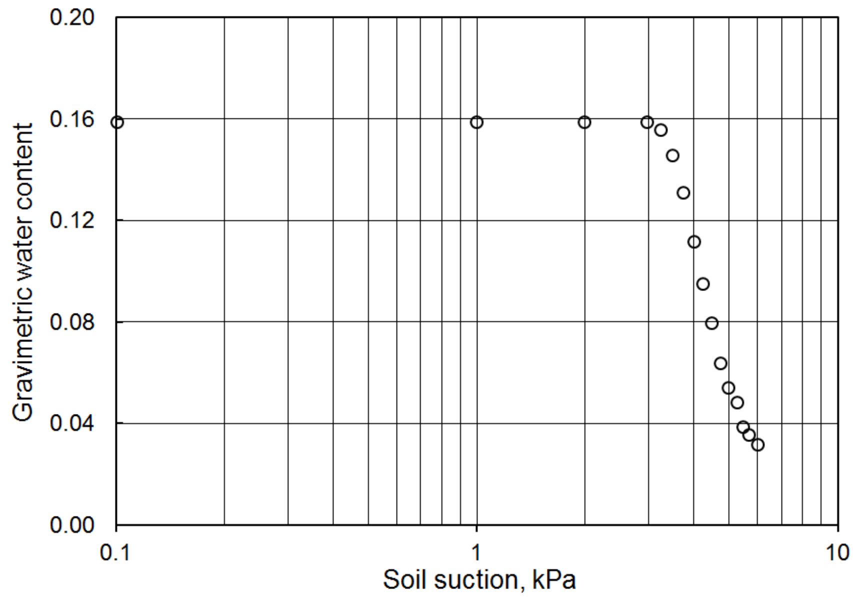


Figure 30 Soil-water characteristic curve for uniform glass beads with a diameter equal to 0.181mm ± 10% (from Nimmo et al., 1996)

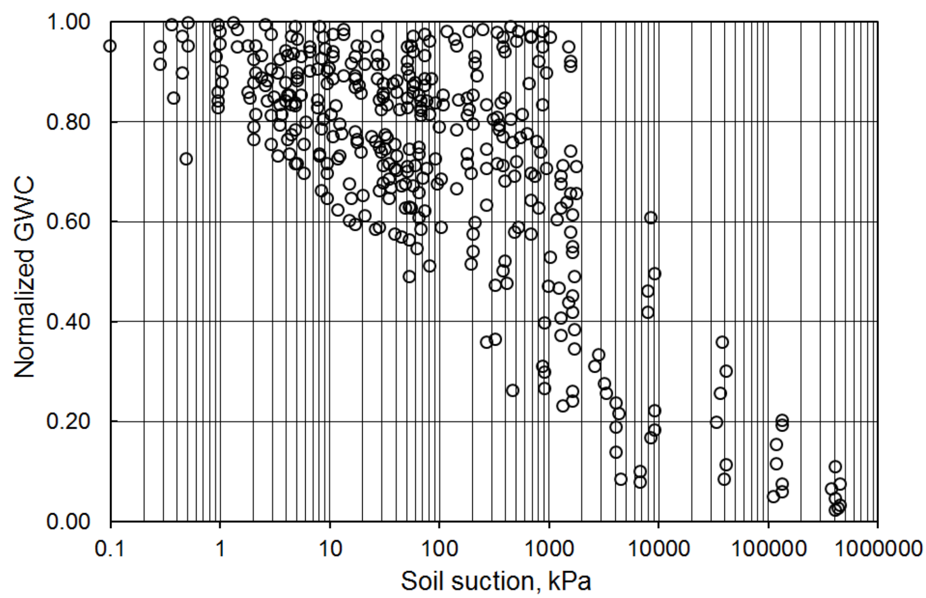


Figure 31 Selection of typical soil-water characteristic curves for clay soils

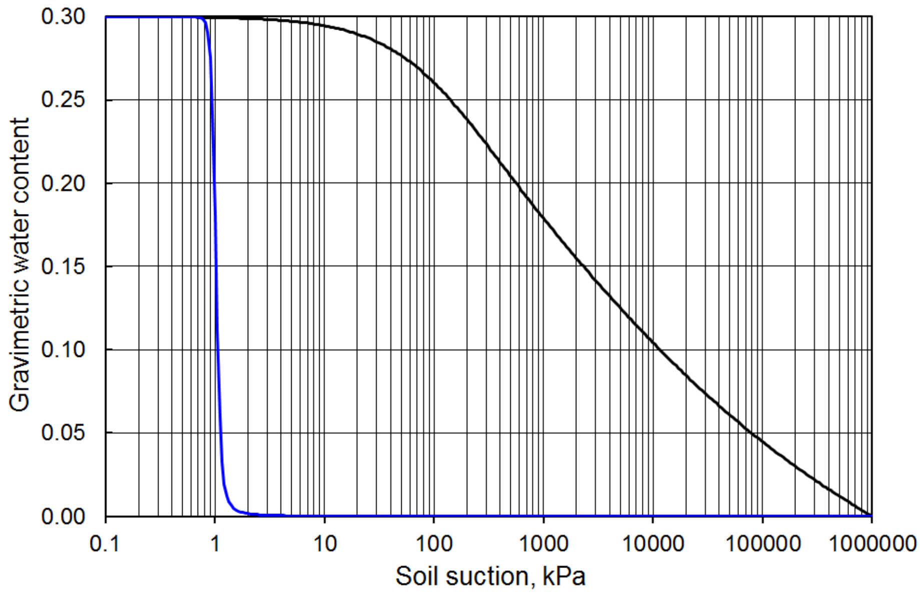


Figure 32 Assumed boundary soil-water characteristic curves for groupings of uniform coarse sand and fine particle sizes; Sand: [a_f = 1, n_f = 20, m_f = 2, h_r = 3000], Clay: [a_f = 100, n_f = 1, m_f = 0.5, h_r = 3000]

Representative SWCC have been established for a sand and a clay. It was assumed that typical SWCCs could be generated for uniform intermediate grain-sizes. Representative SWCCs for intermediate soils were achieved by altering the parameters of the Fredlund and Xing (1994) equation. The boundary SWCCs and the air-entry values for each group of uniform soil particles form the basis for the M.D. Fredlund and Wilson (1997) method to estimate a likely SWCC for any particular particle-size distribution.

The n_f and m_f shape parameters for the Fredlund and Xing (1994) equation are required for each uniform collection of particles. The shape, and resulting n_f and m_f parameters, of the soil-water characteristic curves for uniform sands, silts and clays were estimated as explained above. A dataset containing soils from Rawls et al., (1985) and Sillers, (1997), and the CECIL soil survey was used to determine the approximate trends for the n_f and m_f parameters.

An effective grain-size diameter was calculated for each grain-size curve based on equation [145] (Vukovic et al., 1992). The effective grain-size diameter was then plotted opposite the n_f and m_f parameters. The n_f and m_f parameters were determined for each soil by fitting laboratory data with a least-squares regression algorithm.

$$\frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_l}{d_l} + \sum_{i=2}^{i=n}\frac{\Delta g_i}{d_i} \qquad [\ 145\ ]$$

where:

d_i = *largest diameter of the last fraction of the material,*
Δg_i = *weight of the material of the last, finest fraction, in parts of total weight, and*
d_e = *effective grain diameter.*

The end result of the above analysis is the establishment of representative plots for n_f and m_f. These plots describe reasonable variations in the two parameters with grain-size.

The grain-size distribution curve can then be subdivided into smaller divisions of uniform soil particles. Starting at the smallest particle size, a packing porosity, n_p, can be estimated (Harr, 1977) for each division and a SWCC can be generated as shown in Figure 33. The divisional soil-water characteristic curves can then summed starting with the smallest particle size and continuing until the volume of pore space is equal to that of the entire heterogeneous soil. The end result is an estimated soil-water characteristic curve.

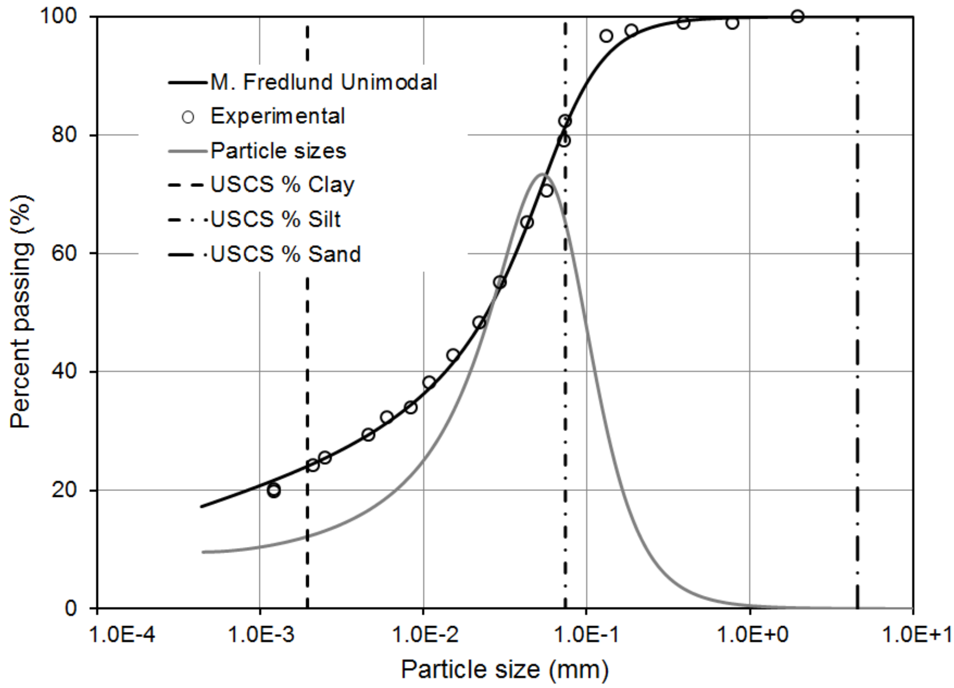


Figure 33 Small divisions of particle size used to build complete soil-water characteristic curve

5.1.2.1 Pore Volume

The grain-size distribution curve can be divided on a logarithm scale into *n* fractions of uniformly-sized particles. Each fraction is assumed to contain its own packing arrangement and porosity. It is assumed that the summation of the individual fraction porosities will be greater than the natural, *in situ* porosity for the soil. The voids created between larger particles will be filled with smaller particles when all soil particles are assembled. The assemblage of particles reduces the influence of the larger particles on the SWCC. Experimental results have shown this to be the case. The porosities of the individual fractions are summed until the measured porosity of the *in situ* soil is reached. The remaining particle fractions are ignored.

The assumed "packing" structure of each uniformly-sized fraction is another important variable . Characteristics of different "packing" structures was calculated by Smith (1929) and has been summarized in Table 4. It was noted that: i) the porosity is independent of particle size, and ii) the porosity varied between 25.95% and 47.64%. These packing porosities are determined for idealized spherical particles. Soil particles are commonly angular and, as such, a greater range of porosity values are possible.

Table 4 Some characteristics of ideal particle packings (Smith, 1929)

Packing	Volume of unit cell	Porosity (%)
Cubic	d^3	47.64
Orthorhombic	$0.87d^3$	39.54
Tetragonal-spheroidal	$0.75d^3$	30.19
Rhombohedral	$0.71d^3$	25.95

The assumed packing porosity, n_p, of each grain-size fraction can either be approximated or else all the packing porosities can be assumed to be equal. Let us assume that the grain-size distribution represents a percent by weight of the total distribution. A unit volume of soil (1 m^3) is analyzed. The weight of soil can be calculated relative to the total unit weight of the soil. Individual weight fractions can then be calculated as follows.

$$W_i = (g_{i+1} - g_i)\rho_t \qquad [\ 146\ ]$$

where:

W_i = *weight of individual fraction (kg),*
g = *function representing percent passing versus particle diameter,*
i = *counter from 1, 2,…, n,*
n = *number of fractions into which grain-size distribution is divided, and*
ρ_t = *total density of the soil sample (kg/m^3).*

The average diameter for each weight fraction can also be calculated by taking the logarithmic average of the i^{th} and $(i+1)^{th}$ particle size divisions or by taking an aritmetic average.

The pore volume associated with each fraction can be computed as being proportional to the total pore volume of the sample. The n_p provides us with an assumed porosity for the i^{th} fraction. The pore volume can be calculated once the porosity is known.

$$V_{v_i} = \frac{W_i}{G_s \rho_w} \frac{n_p}{(1-n_p)} \qquad [\ 147\ ]$$

where:

V_{vi} = *volume of voids (m³),*
G_s = *specific gravity of the soil,*
ρ_w = *density of water (kg/m³), and*
n_p = *assumed packing porosity.*

The contribution of the i^{th} grain-size fraction to the total soil sample can be computed. Since the volume of voids of each fraction, V_{vi}, is known along with the total volume of the sample, (1m^3), the contribution of each fraction to the whole is equal to V_{vi}. The sum of all the voids can be calculated as follows.

$$V_v = \sum_{i=1}^{i=n} V_{v_i} \qquad [\ 148\ ]$$

One of the effects of this technique is that the volume of voids, V_v, can be greater than or less than the actual volume of voids, V_{vt}, in the *in situ* soil. An assumption can then be made when the actual volume of voids is not equal to the analytically computed volume of voids. If the computed volume of voids, V_v, is greater than V_{vt}, the analytical effect of the voids greater than V_{vt} on the SWCC can be ignored. This truncation results in the elimination of the effect of coarse-sized particles on the SWCC since the grain-size curve is evaluated from fine-sized to coarse-sized particles. This is reasonable since the effect of coarse-sized particles can be negligible if the voids are filled with finer-sized particles.

If the sum of voids, V_v, is less than the actual volume of voids, V_{vt}, the resulting SWCC will not reach a saturated condition. A suitable way to analyze this imbalance has not as yet been addressed and further research is needed on this issue.

5.1.2.2 Packing Porosity

Packing porosity, n_p, is one of the variables that has an effect on the estimation of the soil-water characteristic curve. The estimation of a reasonable packing porosity is importantly crucial for the estimation of the SWCC. Following are two methods that were found to be reasonable for estimating the packing porosity: i.) statistical methods, and ii.) the use of a neural net.

Statistical methods involve finding the normal distribution of the packing porosity for the textural category for which a packing porosity is desired. It is then possible to calculate a mean and variance of the packing porosity. This has the advantage that it is possible to obtain confidence limits on possible packing porosity values.

It is also possible to estimate the packing porosity through the use of a neural net. A neural net is an artificial intelligence technique by which an algorithm can be trained to respond to various input stimuli. SVSOILS has developed a neural net that was trained using soils from a training dataset. The packing porosity of each soil was first adjusted to provide an optimal estimation. These adjusted packing porosities were then used in conjunction with the inputs of the USDA classification of % clay, % silt, % sand, % coarse, d_{10}, d_{20}, d_{30}, d_{50}, d_{60}, porosity, water content, dry density, and specific gravity. The neural net was then trained (Goh, 1999) and yielded R^2 equal to 0.830 for the training set. The neural net can be used to estimate packing porosities.

It is often desirable to know the effect of "packing porosity" on an estimated SWCC. The packing porosity does not always influence the estimation of the SWCC in the same manner. The effect of varying the packing porosity is illustrated in Figure 34 and Figure 35 for a sand soil and a silty loam soil, respectively.

It can be seen that the estimated SWCC does not reach 100% saturation as shown in Figure 35 with a packing porosity, n_p, equal to 0.36. This condition occurs when the packing porosity falls below the actual porosity of a soil.

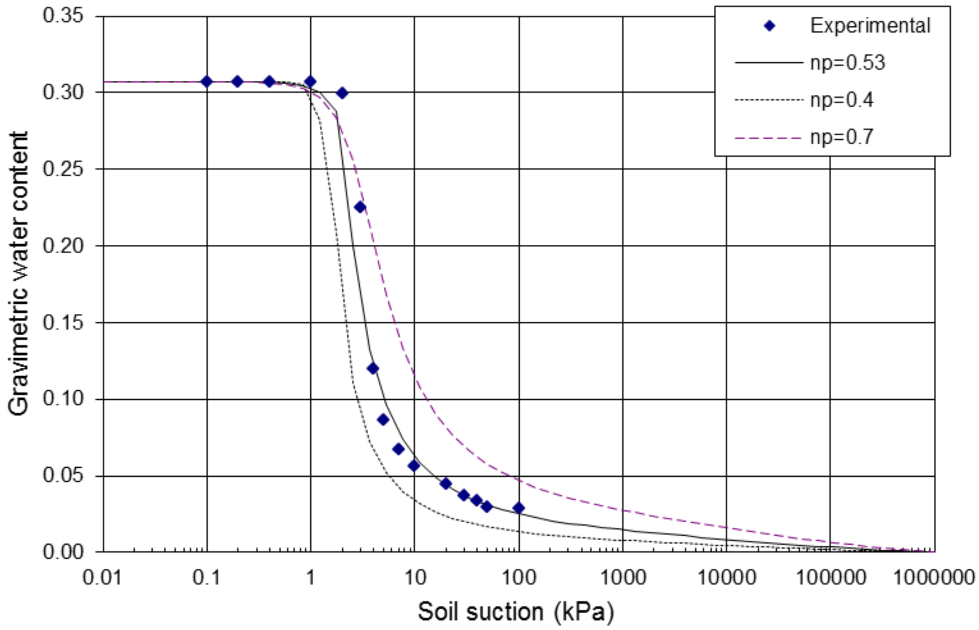


Figure 34 Effect of varying packing porosity, n_p, for sand (data from Mualem, 1984)

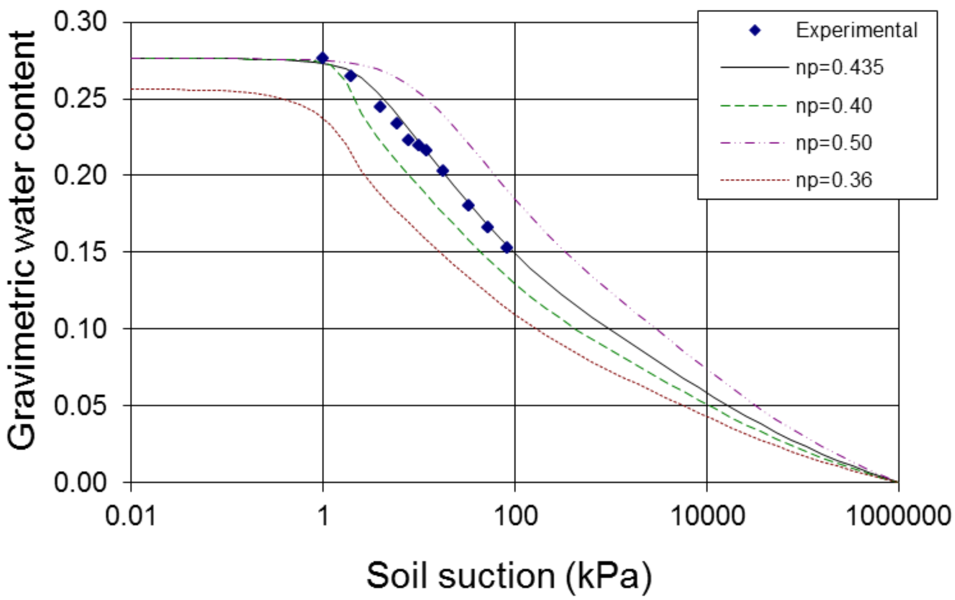


Figure 35 Illustration of the effect of varying packing porosity, n_p, for loam (data from Schuh et al., 1991)

5.1.2.3 Waste Rock

The packing porosity neural net was trained on a data set that did not contain specialty soils such as waste rock and mine tailings. It has been observed that the packing porosity, n_p, can be adjusted to yield reasonable estimates of the SWCC for waste rock (Swanson et al., 2003). It was observed that the packing porosity must be adjusted higher when estimating SWCCs for waste rock. An analysis of waste rock materials has indicated that the packing porosity needs to be increased by an average of 27.9% when estimating SWCCs for waste rock. This analysis was based on five soils obtained from a mine site in Montana, United States. The results of an adjustment of the estimated SWCC are shown in Figure 36.

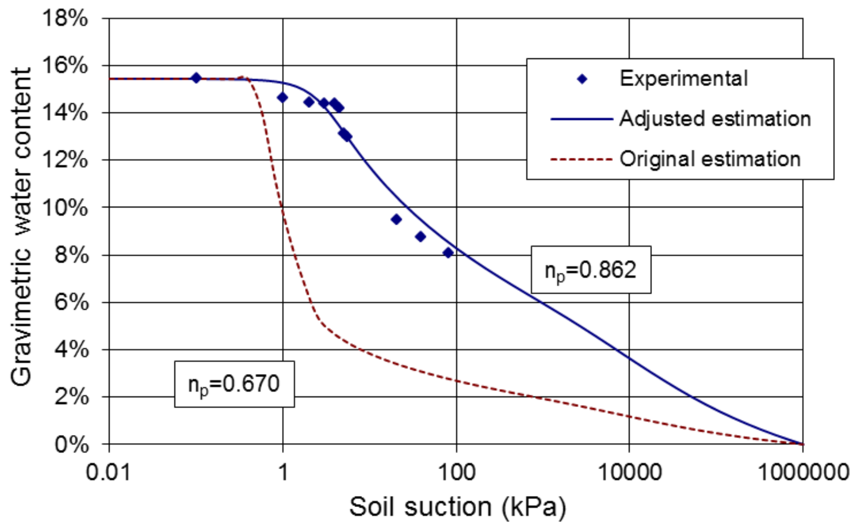


Figure 36 Example of adjustment of the proposed M.D. Fredlund and Wilson (1997) pedo-transfer function for the estimation of the SWCC for a waste rock (data from Herasymuik, 1996)

5.1.3 Arya and Paris (1981) Estimation Method

The Arya and Paris (1981) used a physio-empirical model to estimate the soil-water characteristic curve. The model has been used as the basis for developing other models.

Menu location: Material > SWCC > Fredlund and Xing Fit > Arya and Paris Estimation

Formulation: Algorithm
Fitting method: N/A
Required input: Total density, void ratio, grain-size distribution
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Arya Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Arya Error	difference between the fit and laboratory values in terms of R^2
Arya AEV	air-entry value as calculated based on the estimated curve
Arya Max Slope	maximum slope as calculated based on the estimated curve

5.1.3.1 Assumptions

The solid fraction in each particle-size range can be assembled into a discrete domain having a bulk density equal to that of the natural-structured sample.

The solid volume in any given assemblage can be approximated by uniform-size spheres defined by the mean particle radius for each fraction.

The volume of the resulting pores can be approximated by uniform-size cylindrical capillary tubes whose radii are related to the mean particle radius for the fraction.

5.1.3.2 Theory Associated with the Arya and Paris (1981) Model

The model first transfers a particle-size distribution into a pore-size distribution. Then, the cumulative pore volumes corresponding to progressively increasing pore radii sizes are divided by the bulk volume of the sample to give the volumetric water contents. The pore radii are converted to equivalent soil-water pressures using the capillarity equation.

Pore volumes associated with each fraction size:

$$V_{vi} = (W_i / \rho_p) e\,; \; i = 1, 2, ..., n. \qquad [\ 149\ ]$$

Bulk volume per unit sample mass:

$$V_b = \sum_{i=1}^{n} W_i / \rho_p = 1/\rho_p ; \quad i = 1, 2, ..., n. \qquad [\ 150\ ]$$

Volumetric water content:

$$\theta_{vi} = \sum_{j=1}^{i} V_{vj} / V_b ; \quad i = 1, 2, ..., n. \qquad [\ 151\ ]$$

where:

V_{vi}	=	*pore volume per unit sample mass associated with the solid particles in the* i^{th} particle-*size range,*
V_b	=	*sample bulk volume per unit sample mass,*
W_i	=	*solid mass per unit sample mass in the* i^{th}*particle-size range,*
ρ_p	=	*particle density,*
e	=	*void ratio, and*
θ_{vi}	=	*volumetric water content represented by a pore volume for which the largest size pore corresponds to the upper limit of the* i^{th} *particle-size range.*

5.1.3.3 Particle size and Pore Radius

The pore radius is calculated using the following equation:

$$r_i = R_i \left[\frac{4n_i^{(1-\alpha)}}{6} \right]^{1/2} \qquad [\ 152\ ]$$

where:

r_i	=	*mean pore radius,*
R_i	=	*mean particle radius,*
n_i	=	*the number of spherical particles in the* i^{th} *particle-range, and*
α	=	*an empirical factor and greater than 1.*

The relationship between the pore radius and soil water pressure can be written as follows:

$$\psi_i = \frac{2\gamma \cos\Theta}{\rho_w\, g\, r_i} \qquad [\ 153\ ]$$

where:

ψ_i	=	*soil water pressure (Pa),*
γ	=	*surface tension of water (Pa),*
Θ	=	*contact angle (degree),*
ρ_w	=	*density of water (kg/m*^{3}*),*
g	=	*acceleration due to gravity (m/s*^{2}*), and*
r_i	=	*pore radius (m).*

The required input data for the model includes the particle-size distribution and bulk density.

5.1.3.4 Performance of the model

The present model does not provide close estimations of the SWCC for soils where aggregation, cracking, and root effects are pronounced; otherwise, the model appears to perform quite well.

5.1.4 Scheinost (1996) Estimation Method

The procedure used to estimate the parameters for the van Genuchten (1980) equation involved a multiple regression analysis. The analysis does not appear to produce reliable predictions of the shape parameter α and *n*. A modification of the PTF was proposed by Scheinost to overcome difficulties with the estimations (Scheinost et al., 1996).

Menu location:	Material > SWCC > van Genuchten Fit > Scheinost PTF
Formulation:	Algorithm
Fitting method:	N/A
Required input:	USDA % Coarse, USDA % sand, USDA % silt, USDA % clay, % organic, total density
Applicable soil types:	All soils

Modified fields:

Dialogue Field Name	Description
Scheinost Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Scheinost Error	difference between the fit and laboratory values in terms of R^2
Scheinost AEV	air-entry value as calculated based on the estimated curve
Scheinost Max Slope	maximum slope as calculated based on the estimated curve

5.1.4.1 Function to describe the Soil-Water Characteristic Curve, SWCC (or Moisture Retention Characteristic)

The van Genuchten (1980) function can be is used to provide an estimate of the soil-water characteristic curve.

$$\theta = \theta_r + (\theta_s - \theta_r)(1 + (\alpha h)^n)^m \quad [\ 154\ ]$$

Parameter *m* in this equation was assumed to be (-1)

5.1.4.2 Regression analysis

The particle-size distribution was parameterized using the geometric mean diameter, d_g, and its standard deviation, σ_g. The following relationships between the parameters of the van Genuchten (1980) function and the parameters of particle-size distributions are assumed:

$$\alpha = a_0 + a_1 \cdot d_g \quad [\ 155\ ]$$

$$n = n_0 + n_1 \cdot \sigma_g^{-1} \quad [\ 156\ ]$$

$$\theta_s = s_1 \cdot F + s_2 \cdot clay_x \quad [\ 157\ ]$$

$$\theta_r = r_1 \cdot clay_x + r_2 \cdot C_{org} \quad [\ 158\ ]$$

where:

F = *porosity,*
C_{org} = *organic content,*
d_g = *geometric mean diameter of particle-size distribution,*
σ_g = *standard deviation of particle-size distribution, and*
$clay_x$ = *clay content.*

The above equations were inserted into the van Genuchten (1980) equation replacing the values for θ_s, θ_r, α and *n*, with fitted values from 696 samples. The results provided values of coefficients for the above equations as shown in Table 2. The coefficients in Table 5 were based on 87 datasets. Another 45 datasets were used for the validation of the PTF and a third data set of 37 soils from northern Germany were used to evaluate the PTF.

Table 5 Coefficients for an estimation of the van Genuchten (1980) equation

Coefficients	Using d_{g18}, σ_{g18}		Using d_{g4}, σ_{g4}	
	Estimate	SE [a]	Estimate	SE [a]
s_1	0.85	0.01	0.85	0.01
s_2	0.13	0.02	0.13	0.02
r_1	0.51	0.03	0.52	0.03
r_2	1.7×10^{-3}	0.3×10^{-3}	1.6×10^{-3}	0.3×10^{-3}
a_0	0.23×10^{3}	0.03×10^{-3}	0.25×10^{-3}	0.04×10^{-3}
a_1	7.0×10^{-3}	1.0×10^{-3}	4.3×10^{-3}	0.6×10^{-3}
n_0	0.33	0.04	0.39	0.04
n_1	2.6	0.6	2.2	0.6
SS Model [b]		69.03		69.02
SS Total [c]		69.59		69.59
N		696		696

[a] *Asymptotic standard error*
[b] *Sum of squares of the model*
[c] *Sum of squares of the uncorrected total.*

5.1.4.3 Validation of the Scheinost (1996) pedo-transfer function

To assess the deviation between predicted and measured SWCCs, the roots of the mean squared differences (RMSDs) between measured and predicted water contents (θ_m, θ_p) were calculated:

$$RMSD = \left[\frac{1}{b-a}\int_a^b (\theta_p - \theta_m)^2 \, d\psi\right]^{1/2} \qquad [\ 159\ ]$$

The RMSD equals zero if there is no difference between the predicted and the measured values.

The RMSDs between the measured and the predicted SWCCs for the new PTF (i.e., Scheinost; 1996) and the PTF proposed by Vereecken et al., (1989) are shown in Table 6.

Table 6 Root mean squared differences, (m^3/m^3), between measured and predicted SWCCs

PTF:	New			Vereecken et al. (1989)	
Data set:	PTF	Valid. 1	Valid. 2	PTF	Valid. 2
Mean	0.019	0.017	0.035	0.048	0.037
Min.	0.003	0.001	0.006	0.009	0.004
Max.	0.054	0.054	0.097	0.128	0.104

5.1.5 Rawls and Brakensiek (1985) Estimation Method

The Rawls and Brakensiek (1985) PTF uses multiple linear regression to estimate the parameters for the Brooks and Corey (1964) equation.

Menu location: Material > SWCC > van Genuchten Fit > Rawls PTF

Formulation: Algorithm
Fitting method: N/A
Required input: USDA % sand, USDA % clay, porosity
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Rawls Bubbling Pressure	Rawls estimation of the Brooks and Corey a_c parameter
Rawls Lambda	Rawls estimation of the Brooks and Corey λ parameter.
Rawls Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Rawls Error	difference between the fit and laboratory values in terms of R^2
Rawls AEV	air-entry value as calculated based on the estimated curve
Rawls Max Slope	maximum slope as calculated based on the estimated curve

The Brooks and Corey (1964) equation as well as the regression equations are as follows.

$$\frac{\theta - \theta_r}{\varphi - \theta_r} = \left(\frac{h_b}{h}\right)^{\lambda} \qquad [\ 160\ ]$$

where:

h = *capillary pressure,*
h_b = *bubling pressure,*
λ = *pore size distribution index,*
θ = *water content, and*
θ_r = *residual water content.*

5.1.5.1 Regression equations for the parameters of Brooks-Corey (1964) equation

A number of regressions were performed on a dataset of soils gathered by Rawls and Brakensiek (1985). The following regression equations were presented.

$$\begin{aligned}\ln(h_b) &= 5.3396738 + 0.1845038\,C - 2.4839454\,\phi + 0.00213853\,C^2 \\ &- 0.04356349\,S\phi - 0.61745089\,C\phi + 0.00143598\,S^2\varphi^2 + 0.00855375\,C^2\phi^2 \\ &- 0.0001282\,S^2C + 0.00895359\,C^2\phi - 0.00072472\,S^2\phi + 0.0000054\,C^2S \\ &+ 0.50002860\,\phi^2C\end{aligned} \quad [\ 161\ ]$$

$$\begin{aligned}\ln(h_b) &= 5.3396738 + 0.1845038C - 2.4839454\phi + 0.00213853\,C^2 \\ &- 0.04356349S\phi - 0.61745089C\phi + 0.00143598S^2\varphi^2 + 0.00855375C^2\phi^2 \\ &- 0.0001282S^2C + 0.00895359C^2\phi - 0.00072472S^2\phi + 0.0000054C^2S \\ &+ 0.50002860\phi^2C\end{aligned} \quad [\ 162\ ]$$

$$\begin{aligned}\ln(\lambda) &= 0.7842831 + 0.0177544\,S - 1.062498\,\phi - 0.00005304\,S^2 - 0.00273493\,C^2 \\ &+ 1.11134946\,\phi^2 - 0.03088295\,S\phi + 0.00026587\,S^2\phi^2 - 0.00610522\,C^2\phi^2 \\ &- 0.00000235\,S^2C + 0.00798746\,C^2\phi - 0.00674491\,\phi^2C\end{aligned} \quad [\ 163\ ]$$

where:

C = *percent clay (5 < PC < 60)*
S = *percent sand (5 < PC < 70), and*
ϕ = *porosity (or volume fraction is the ratio of void volume to total volume)*

5.1.6 Vereecken et al., (1989) Estimation Method

Vereecken et al., (1989) PTF uses multiple linear regression to estimate the parameters of the van Genuchten (1980) equation.

Menu location: Material > SWCC > van Genuchten Fit > Vereecken PTF

Formulation: Algorithm
Fitting method: N/A
Required input: USDA % sand, USDA % clay, % Organic Carbon, total density
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Vereecken avg	Vereecken estimation of the van Genuchten a_{vg} parameter
Vereecken nvg	Vereecken estimation of the van Genuchten n_{vg} parameter
Vereecken mvg	Vereecken estimation of the van Genuchten m_{vg} parameter
Vereecken Residual wc	Vereecken estimation of the van Genuchten w_r parameter
Vereecken Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Vereecken Error	difference between the fit and laboratory values in terms of R^2
Vereecken AEV	air-entry value as calculated based on the estimated curve
Vereecken Max Slope	maximum slope as calculated based on the estimated curve

The van Genuchten (1980) equation is written as follows.

$$s_e = (1 + (\alpha\, h)^n)^{-m} \quad [\ 164\ ]$$

where:

$s_e = (\theta - \theta_r)/(\theta_s - \theta_r)$,
θ = *volumetric water content,*
θ_r = *residual water content,*
θ_s = *saturated water content,*
h = *pressure head, and*
α, *n and m* = *parameters defining the SWCC's shape.*

5.1.6.1 The different model structures for the van Genuchten (1980) equation

A number of different forms of the van Genuchten (1980) equation was analyzed by Vereecken et al., (1989). The models considered are shown below.

$$\text{Model 1: } s_e = (1+(\alpha h)^n)^{-m} \qquad [165]$$

$$\text{Model 2: } s_e = (1+(\alpha h)^n)^{-1+1/n} \qquad [166]$$

$$\text{Model 3: } s_e = (1+(\alpha h)^n)^{-1+2/n} \qquad [167]$$

$$\text{Model 4: } s_e = (1+(\alpha h)^n)^{-1} \qquad [168]$$

$$\text{Model 5: } \theta = \theta_s(1+(\alpha h)^n)^{-1} \qquad [169]$$

Statistical analysis results show that model 4 performs considerably better than model 2 and 3. Model 5 has the poorest performance.

5.1.6.2 Sensitivity analysis (model 4)

The saturated volumetric water content, θ_s, is the most sensitive parameter for all types of soils using Model 4. The α and n parameters exhibit a non-symmetric sensitivity with an insensitivity for the positive perturbation of the parameter values. Model 4 shows a strong nonlinear sensitivity for negative perturbations. The residual volumetric water content was the least sensitive parameter

5.1.6.3 Regression analysis (model 4)

The parameter estimation was performed through multiple regression using two sets of soil properties as predictor variables. A first set is composed of the sand, silt, and clay fraction; the carbon content; and the bulk density. The second set contains more detailed information on the particle-size distribution (i.e., the nine textural fractions, the GMPS, and the grain-size distribution). The following regression equations were based on 182 measured soil-water characteristic curves.

$$\begin{aligned}\theta_s &= 0.81 - 0.283\rho + 0.001\,Cl \\ &= 0.84 - 0.010\,f_1 + 0.004\,f_3 - 0.004\,f_4 - 0.288\,\rho\end{aligned} \qquad [170]$$

$$\begin{aligned}\theta_r &= 0.015 + 0.005\,Cl + 0.014\,C \\ &= 0.068 + 0.0333\,f_1 + 0.017\,f_3 - 0.015\,f_4 - 0.009\,f_8 + 0.015\,C\end{aligned} \qquad [171]$$

$$\begin{aligned}log(\alpha) &= -2.486 + 0.025\,sd - 0.351\,C - 2.617\,\rho - 0.023\,Cl \\ &= -1.538 - 0.994\,f_1 - 0.130\,f_6 - 0.147\,f_9 - 0.092\,f_{10}\end{aligned} \qquad [172]$$

$$\begin{aligned}log(n) &= 0.053 - 0.009\,sd - 0.013\,Cl + 0.00015\,sd^2 \\ &= 0.010 - 0.323\,f_1 - 0.062\,f_6 + 0.066\,f_9\end{aligned} \qquad [173]$$

where:

ρ = *bulk density (or total density) (g/cm^3),*
C = *carbon content (%),*
Cl = *clay content (%),*
sd = *sand content (%), and*
f_1, f_3, f_4, f_6, f_9, *and* f_{10} = *the respective principal factors.*

5.1.7 Tyler and Wheatcraft (1989) Estimation Method

Tyler and Wheatcraft (1989) presented an analysis that correlated the fitting parameter, α, in the Arya and Paris (1981) SWCC model to the physical properties of the soil. The model is the same as the Arya and Paris (1981) model with the exception of the α fitting parameter. A summary of the theory involved in the Tyler and Wheatcraft (1989) theory is shown below.

Menu location: Material > SWCC > Fredlund and Xing Fit > Tyler and Wheatcraft Estimation

Formulation: Algorithm

Fitting method: N/A
Required input: Total density, void ratio, grain-size distribution
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Tyler Alpha	Tyler estimation of the Arya and Paris (1981) α parameter (m/s)
Tyler Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Tyler Error	difference between the fit and laboratory values in terms of R^2
Tyler AEV	air-entry value as calculated based on the estimated curve
Tyler Max Slope	maximum slope as calculated based on the estimated curve

Fractal mathematics was used to show that α was equal to the fractal dimension of the pore trace and was an expression of the tortuosity of the pore trace. Arya and Paris (1981) based their estimates of α on a mean squared difference between measured and predicted capillary pressures and found that the predicted results compared well with laboratory data on 15 soils. The fitting coefficient, α, was found to vary between 0.9 and 1.5.

Tyler and Wheatcraft (1989) based their estimation of α on the fractal dimension. The equation used to model the particle sizes, is shown below.

$$N R_i^{\alpha} = \text{constant} \qquad [\ 174\ ]$$

where, N is the total number of particles of radius greater than R_i and α is the fractal dimension of the particle-size distribution. The fractal dimension defines the distribution of particles by size. For α = 0, the distribution is composed solely of particles of equal diameter. A fractal dimension of 3.0 indicates that the number of particles greater than a given radius doubles for each corresponding decrease in particle mass by one-half. In a study by Turcotte (1986), data were presented on the fractal dimension of 21 particle-size distributions. The majority of the soils had fractal dimensions approaching 3.0. Tyler and Wheatcraft (1989) estimated the fractal dimension of the particle-size distribution by plotting the cumulative number of particles larger than a given sieve-size. If equation [174] is rearranged and plotted on a log-log scale, the fractal dimension, α, becomes equivalent to the negative slope value of the plotted line.

5.1.8 Gupta and Larson (1979a, 1979b) Estimation Method

The Gupta and Larson (1979a, 1979b) method used the statistical regression of estimated water contents at various suction levels in order to estimate the SWCC. Statistical relationships were presented for the prediction of the SWCC over a wide range of soil suction (i.e., 4 to 1500 kPa). The proposed relationships are based on percent sand, silt, clay, organic matter, and bulk density.

Menu location: Material > SWCC > van Genuchten Fit > Gupta and Larson PTF

Formulation: Algorithm
Fitting method: N/A
Required input: USDA % sand, USDA % silt, USDA % clay, % organic, total density
Applicable soil types: Coarse-grained soils

Modified fields:

Dialogue Field Name	Description
Gupta Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Gupta Error	difference between the fit and laboratory values in terms of R^2

Models were developed from measured SWCCs of artificially packed cores (7.6 cm × 7.6 cm) for 43 soils.

5.1.8.1 Regression equation

$$\begin{aligned} w_p = {} & a \times sand(\%) + b \times silt(\%) + c \times clay(\%) \\ & + d \times organic\ matter(\%) + e \times bulk\ density(g/cm^3) \end{aligned} \qquad [\ 175\ ]$$

where:

w_p = predicted gravimetric water content, and
a, b, c, d and e = regression coefficients.

5.1.8.2 Results of regression analysis

The results of the regression analysis can be seen in the following table.

Table 7 Regression and correlation coefficients for prediction of soil water content at various soil suctions

Soil suction (kPa)	Regression coefficients					Correlation coefficient R^2
	$a\times10^3$	$b\times10^3$	$c\times10^3$	$d\times10^3$	$e\times10^3$	
4	7.503	10.242	10.070	6.333	-32.120	0.950
7	5.678	9.228	9.135	6.103	-26.960	0.959
10	5.018	8.548	8.833	4.966	-24.230	0.961
20	3.890	7.066	8.408	2.817	-18.780	0.962
33	3.075	5.886	8.039	2.208	-14.340	0.962
60	2.181	4.557	7.557	2.191	-9.276	0.964
100	1.563	3.620	7.154	2.388	-5.759	0.966
200	0.932	2.643	6.636	2.717	-2.214	0.967
400	0.483	1.943	6.128	2.925	-0.204	0.962
700	0.214	1.538	5.908	2.855	1.530	0.954
1000	0.076	1.334	5.802	2.653	2.145	0.951
1500	-0.059	1.142	5.766	2.228	2.671	0.947

5.1.8.3 Performance of the model

The regression models in Table 7 were tested on data from 61 Missouri soils (Janison and Kroth, 1958 and Kroth et al., 1960). The regression analysis was performed using the equation, [($y = \alpha_I + \beta_I x$)] where (y) was the predicted value and (x) was the measured value. The intercept (α_I) values were significantly different than zero and the slope of the line (β_I) was close to 1.0 for all soil suctions, as shown in Table 8. The presence of non-zero intercept values and a slope of 1.0 suggests a constant bias between the predicted and measured water contents. The authors suggested that the bias was due to the differences in the experimental procedures used by Kroth et al., (1960) and the authors.

Table 8 Regression analysis ($y = \alpha_I + \beta_I x$) of predicted (y) and measured (x) water contents at four soil suctions for 61 Missouri soils

Soil suction (kPa)	$\alpha_I \pm se$	$\alpha_I \pm se$	s
10	0.0494 ± 0.0244	0.9934 ± 0.0677	0.0604
33	0.0456 ± 0.0151	0.9489 ± 0.0456	0.0434
100	0.0478 ± 0.0116	0.9173 ± 0.0386	0.0364
1500	0.0555 ± 0.0090	0.9336 ± 0.0412	0.0346

se = standard error
s = standard error of the regression.

5.1.9 Aubertin et al. (2003) Estimation Method

The modified Kovács (Kovács, 1981) model, (or the MK model), was developed to predict the SWCC (or WRC) from easy to obtain basic soil properties (Aubertin et al., 2003). This model was applied to relatively stiff (i.e., incompressible under applied suctions), granular or plastic/cohesive soils, and to other geo-materials such as mine tailings.

Menu location: Material > SWCC > Fredlund and Xing Fit > Aubertin PTF

Formulation: Algorithm
Fitting method: N/A
Required input: Dry Density, porosity, void ratio, *D10*, *D60*
Applicable soil types: Stiff granular and plastic/cohesive materials

Modified fields:

Dailogue Field Name	Description
Aubertin Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Aubertin Error	difference between the fit and laboratory values in terms of R^2
Aubertin AEV	air-entry value as calculated based on the estimated curve
Aubertin Max Slope	maximum slope as calculated based on the estimated curve

Aubertin et al. (2003) estimation method assumes that water is held in the soil by two mechanisms; namely, i) capillary forces responsible for capillary saturation, S_c, and ii) adhesive forces resulting in saturation by adhesion, S_a. The S_c contribution to the SWCC is more important at relatively low suction values while the S_a component becomes dominant at higher suctions. The following set of equations defines the main components of the model:

$$S_r = \frac{\theta}{n} = 1 - \langle 1 - S_a \rangle (1 - S_c) \qquad [\ 176\ ]$$

$$S_c = 1 - \left[(h_{co}/\psi)^2 + 1 \right]^m \exp\left[-m (h_{co}/\psi)^2 \right] \qquad [\ 177\ ]$$

$$S_a = a_c \left(1 - \frac{\ln(1 + \psi/\psi_r)}{\ln(1 + \psi_0/\psi_r)} \right) \frac{(h_{co}/\psi_n)^{2/3}}{e^{1/3} (\psi/\psi_n)^{1/6}} \qquad [\ 178\ ]$$

Equation [176] expresses the total degree of saturation S_r (i.e., equal to θ/n, where n is the porosity and θ the volumetric water content).

The Macauley brackets $\langle\ \rangle$ are defined as $\langle x \rangle = 0.5(x+|x|)$. In equations [177] and [178], h_{co} [L] is the equivalent capillary height which is related to an equivalent pore diameter that depends on the solid surface area; ψ [L] is the matric suction head; m [-] is a pore size coefficient; a_c [-] is the adhesion coefficient; e is the void ratio; ψ_n is a normalization parameter introduced for unit consistencies (i.e., ψ_n = 1 cm when ψ is given in cm, corresponding to a negative pressure of 10^{-3} atmosphere). The equations result in a water content of zero at ψ_0, (i.e., $\theta = 0$ at $\psi = \psi_0 = 10^7$ cm of water).

The MK model parameters; h_{co}, ψ_r, m, and a_c can be obtained using the relations shown in Table 9:

Table 9 Aubertin et al., (2003) parameters for different soil types

Granular soils	Incompressible plastic soils
$h_{co}(cm) = \frac{0.75}{[1.17\log(C_U)+1]\, e\, D_{10}}$	$h_{co}(cm) = \frac{0.15\rho_s}{e}\ w_L^{1.45}$
$\psi_r(cm) = 0.86\ h_{co}^{1.2}$	$\psi_r(cm) = 0.86\ h_{co}^{1.2}$
$m = 1/C_U$	$m = 3\times10^{-5}$
$a_c = 0.01$	$a_c = 7\times10^{-4}$

D_{10} is the diameter corresponding to 10 % passing on the cumulative grain-size distribution curve (in cm),
C_U is the uniformity coefficient (D_{60}/D_{10}),
w_L is the liquid limit (%),
ρ_s is the solid grain density (kg/m^3).

Further information related to the MK model can be found in Aubertin et al., (2003). Users of the model are encouraged to refer to the original and other published papers.

5.1.10 Theory of the Water Storage Function

The water storage curve is the derivative or slope of the soil-water characteristic curve equation written in terms of volumetric water content. Mathematically, water storage, m^2_w, can be written as, $d\theta/d\psi$. The water storage function is required for transient modeling of water seepage through soils. SVSOILS provides the derivative of the Fredlund and Xing (1994) soil-water characteristic curve for the calculation of the water storage function. The derivative of the soil-water characteristic curve is primarily used for modeling transient seepage through unsaturated soils.

5.1.11 Estimation of the Residual Water Content of a Soil

Residual water content of a soil is defined qualitatively as the volumetric water content at which the water phase of a soil becomes largely discontinuous. When a soil is saturated, all voids between soil particles are filled with water. As a soil dries out, soil suction increases and the amount of water trapped in the voids of a soil begins to decrease. Small decreases in water content result in air pockets forming in the larger voids of the soil. There are increasing discontinuities in the water phase as the air volume of the soil increases. The point at which these discontinuities are predominant, and water becomes difficult to remove from the voids is termed the residual gravimetric water content.

Certain engineering properties and physical behavior of an unsaturated soil have been observed to change near residual water content. Residual conditions are also described as the point where moisture flow through soil begins to be dominated by water vapor diffusion rather than liquid flow. The change in behavior associated with suctions in excess of residual conditions requires that residual water content be defined. The computational method used to determine the residual water content of a soil is presented in the following section.

5.1.11.1 A method of construction for estimating the residual state and the air-entry value for soils without volume change

Soil-water characteristic curves are commonly plotted as volumetric or gravimetric water content or degree of saturation versus soil suction relationships. The residual state condition and the air-entry value determined by the computational method will be independent of the manner in which the data is plotted in the case where there is no volume change as soil suction is increased.

Fredlund and Xing (1994) provided an analytical basis for mathematically defining the soil-water characteristic curve. The equation applies over the entire range of suctions from 0 to 1,000,000 kPa. This equation is most commonly written in terms of degree of saturation or volumetric water content, θ_w, as shown below:

$$\theta_w = \theta_s \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right] \left[\frac{1}{\ln\left[e + \left(\frac{\psi}{a}\right)^n \right]^m} \right] \qquad [\ 179\ ]$$

where:

θ_w = *volumetric water content,*
θ_s = *saturated volumetric water content,*
a = *suction related to the air-entry value of the soil,*
n = *a soil parameter related to the slope at the inflection point on the soil-water characteristic curve,*
ψ = *soil suction,*
m = *a soil parameter related to the residual water content,*
e = *a natural number, 2.71828…, and*
C_r = *the residual suction.*

Best-fit parameters for equation [179] are required in order to define the soil-water characteristic curve over the entire range of soil suctions. Figure 37 shows a typical soil-water characteristic curve plotted as volumetric water content, θ_w. The air-entry and residual water content values are highlighted in this figure. The fitting parameters (i.e., *a, n,* and *m*), the air-entry and the residual state can be determined using a computational technique with the aid of SVSOILS. The procedural steps involved in estimating the residual state and the air-entry value are given below (Figure 37):

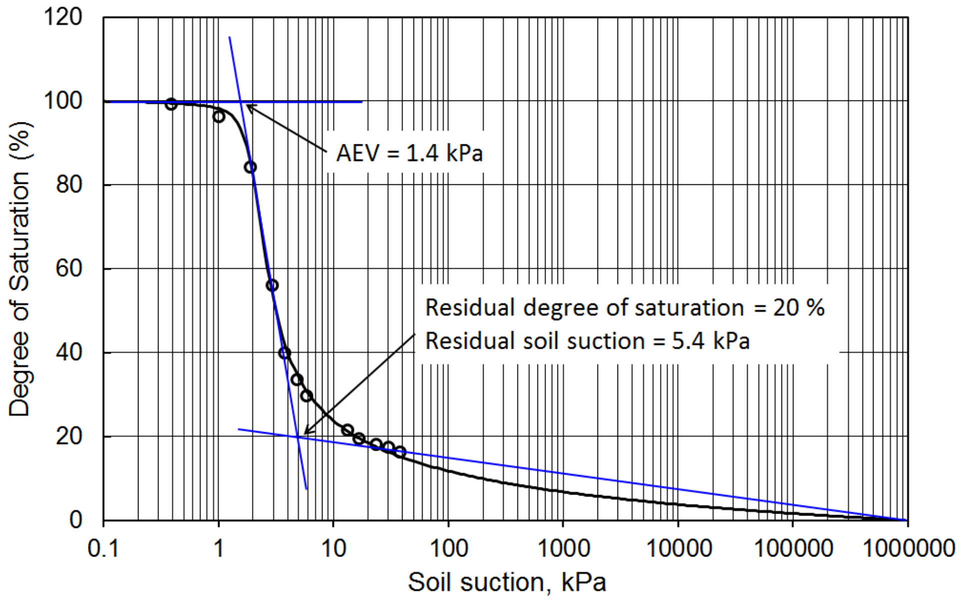


Figure 37 Construction procedure to estimate the residual state and the air-entry value of a Sand (data from Dane and Hruska, 1983)

5.1.11.1.1 Estimation of residual state

The following steps can be taken to estimate the residual state of the soil.

Step 1. Use equation [179] to find the best-fit parameters to describe the laboratory data over the entire suction range.

Step 2. Determine the point of maximum slope on the best-fit curve and draw a line tangent to the curve through the point of maximum slope.

Step 3. Determine point of maximum change of slope between the point of maximum slope and 1,000,000 kPa.

Step 4. Move one logarithmic cycle past inflection point and locate a point on the best-fit curve.

Step 5. Draw the residual line through the located point and 1,000,000 kPa and zero volumetric water content or zero degree of saturation.

Step 6. The intersection of the two lines indicates the residual state condition (i.e., the residual water content and the residual suction of the soil).

5.1.11.1.2 Air-entry value estimation

Step 1. Step 1 for air-entry value estimation is the same as Step 1 in the estimation of residual state.

Step 2. Step 2 for air-entry value estimation is the same as Step 2 in the estimation of residual state.

Step 3. Draw a line tangent to the curve through the point of maximum slope.

Step 4. Draw a horizontal line through the maximum volumetric water content, gravimetric water content or degree of saturation

Step 5. The intersection of the two lines indicates the air-entry value.

Figure 38, Figure 39, and Figure 40 show the results of volumetric water content applied to a variety of different soil types. The construction technique has been applied to the 6000 soil-water characteristic curves present in the SVSOILS database. The performance of the technique was then analyzed by selecting a random sample of soil-water characteristic curves from the database and visually verifying the results.

A value of 3000 kPa has been suggested in the research literature as an estimate for the C_r constant in equation [179] (Fredlund and Xing, 1994). The construction technique presented in this paper provides a method for determining C_r in equation [179]. The C_r value determined is substituted back into equation [179]. The fitting parameters *a*, *n*, and *m* are then determined using the C_r value.

The construction technique provided the results of the air-entry value and the residual state that matched well with visual inspection for most of the soils analyzed. Soils from most of the textural classes such as sands, loams, tills, and clays were studied. Problem soils encountered included clay soils and bimodal soils. The problem encountered with dense clays corresponds to the lack of a clearly defined inflection point on the soil-water characteristic curve. The proposed construction technique does not account for bimodal nature in some soils. Bimodal behavior will result in an under-estimation of the residual suction for bimodal soils. The construction technique appears to work well for the most soils analyzed.

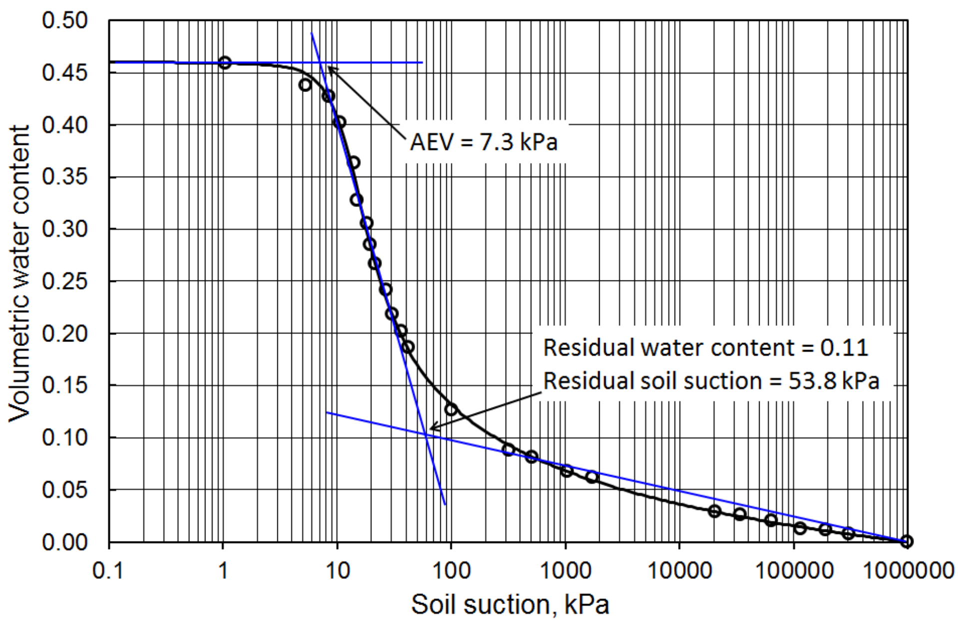


Figure 38 Results of the construction technique for Loam (data from Jackson et al., 1965)

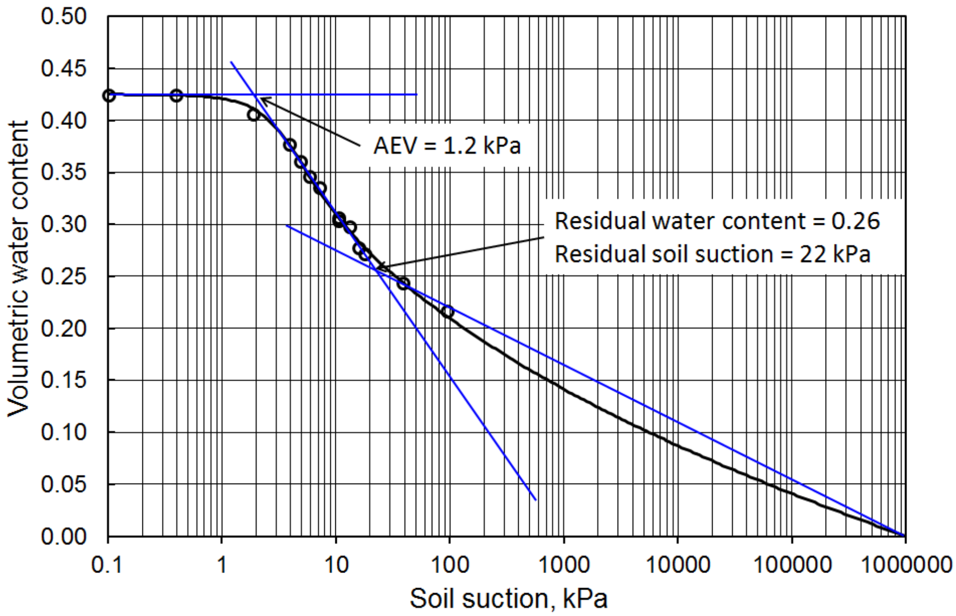


Figure 39 Results of the construction technique for Loam (data from Sisson and van Genuchten, 1991)

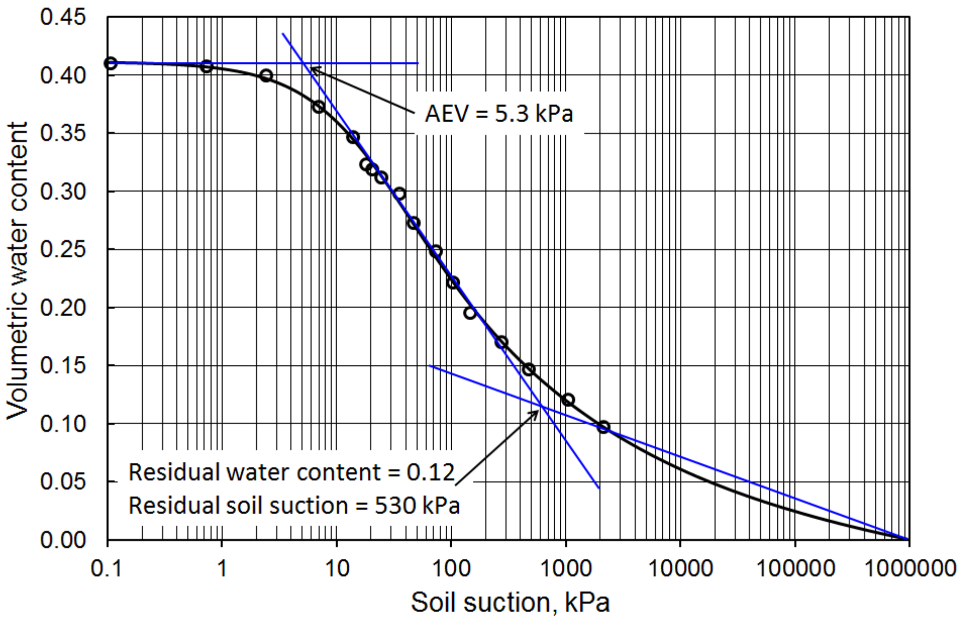


Figure 40 Results of the construction technique for Silt Loam (data from van Dam et al., 1992)

5.1.12 Filter Paper Measurement of Soil Suction

The filter paper method can be used to obtain several points along the soil-water characteristic curve. Soil specimens are prepared at several water content values and allowed to come to equilibrium with a piece of filter paper. Equilibrium times may be in the order of one week.

The filter paper method allows points on the soil-water characteristic curve to be determined without the use of a pressure plate apparatus. The filter paper testing procedure has been used in the soil science discipline, but only to a limited extent in geotechnical engineering.

It is possible to use the filter paper method to measure either total suction or matric suction of a soil. The filter paper is used as a sensor. The filter paper method is classified as an "indirect method" of measuring soil suction. Several soil specimens are prepared by varying the water content and then allow equilibrium to be established between the filter paper and the soil.

The filter paper method is based on the assumption that a filter paper will come to equilibrium (i.e., with respect to either liquid or vapor moisture flow) with the specific suction in the soil. Equilibrium can be reached by either liquid or vapor moisture exchange between the soil and the filter paper. The measured soil suction is assumed to be the matric suction when liquid water equilibrium has been established. The measured soil suction is assumed to be the total suction of the soil when vapor pressure equilibrium is established between the soil and the filter paper. .

The water content of the filter paper corresponds to a calibration soil suction value as illustrated by the filter paper calibration curve shown in Figure 41 for two selected types of filter paper. The same calibration curve is used for both the matric and total suction measurements.

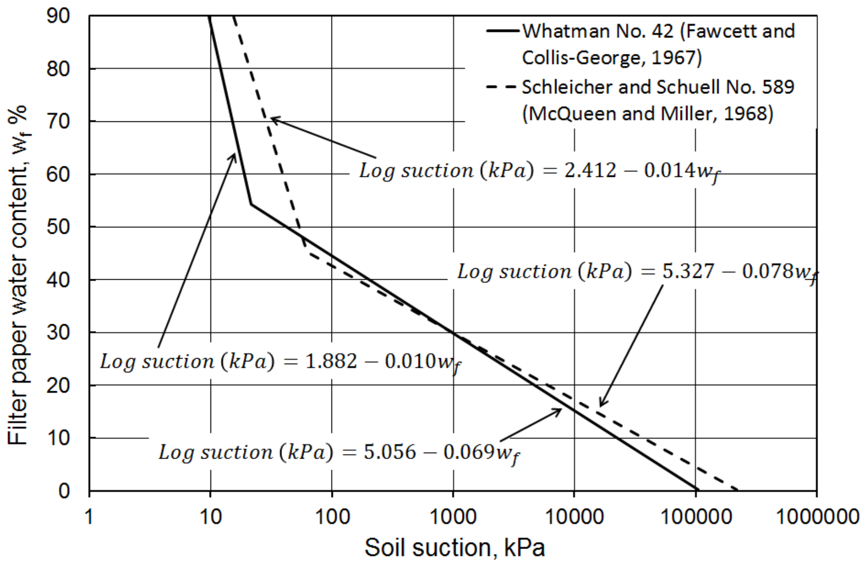


Figure 41 Calibration curves for filter paper measurement of soil suction (data from Fredlund and Rahardjo, 1993)

Gravimetric water content of a soil is measured once the suction of a soil is calculated from the calibration curve presented in Figure 41. It should be noted that the methodology is quite approximate and does not take factors such as hysteresis into account.

5.2 ESTIMATION OF PERMEABILITY (SATURATED)

Estimation models for the prediction of the coefficient of permeability (or hydraulic conductivity) of soils can be divided into three categories. The model either predicts: (1) the saturated coefficient of permeability of a soil (Ahuja, 1989; Russo, 1980; Brakensiek, 1992; Rawls, 1993 and Sperry, 1994), (2) the variation of the coefficient of permeability as a soil desaturates (Fredlund, 1994), or (3) both the saturated coefficient of permeability and the variation in permeability as a soil desaturates (Durner, 1994). The estimation procedures available in the SVSOILS Knowledge-Based system have been divided into two parts: the estimation of the saturated coefficient of permeability (hydraulic conductivity) and the estimation of the permeability function as the soil desaturates.

SVSOILS provides the following methods of estimation for the saturated coefficient of permeability of a soil:

Hazen's Estimation (1911)
Kozeny – Carman Estimation (1989)
Beyer Estimation (1964)
Kruger Estimation (1992)
Zamarin Estimation (1992)
Slichter Estimation (1962)
Terzaghi Estimation (1981)
Kozeny Estimation (1962)
USBR Estimation (1992)
Rawls and Brakensiek Estimation (1983)
Rawls, Brakensiek, and Logsdon Estimation (1993)
Fair-Hatch Estimation (1959)

The following summary is largely from:

> *Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.*

Numerous empirical formulas have been used in engineering practice for the determination of the coefficient of permeability of porous media. These methods were developed at various times, for various materials and different characteristics of the porous media. The empirical formulas are presented in the forms most frequently encountered in the literature.

Frequently used empirical formulas for the saturated coefficient of permeability of a porous medium are presented in a general form that permits estimation comparisons to be made.

$$k_{sat} = \frac{g}{v} \cdot C \cdot \varphi(n) \cdot d_e^2 \qquad [\ 180\ ]$$

where:

k_{sat} = *coefficient of permeability (m/s),*
g = *gravity acceleration (m/s²),*
υ = *kinematic viscosity (m²/s),*
C = *sorting coefficient,*
$\varphi(n)$ = *porosity function, and*
d_e = *effective grain diameter (mm)*

Coefficients of permeability expressed in the form of equation [180] are dimensionally homogeneous, thereby providing the basic prerequisite for comparison and analysis.

5.2.1 Hazen's (1911) Estimation

In 1911 Hazen proposed an empirical equation for the estimation of the saturated coefficient of permeability (Holtz and Kovacs, 1981). The equation was developed for use with clean sands (i.e., with less than 5% passing the No. 200 sieve) and with D_{10} sizes between 0.1 and 3.0 mm.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Hazen's ksat

Formulation:

$$k_{sat} = C \cdot D_{10}^2 \qquad [\ 181\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability or permeability of the water phase (m/s)
C		constant used to vary the estimation
D_{10}		diameter of the 10% passing particle- size (mm)

Required input: D_{10} and Hazen's constant
Applicable soil types: Hazen's equation is only valid for conductivities greater than or equal to 0.00001 m/s or granular soils

Modified fields:

Dialogue Field Name	Description
Hazen's ksat	estimated saturated coefficient of permeability (m/s)

Hazen's (1911) equation (shown in equation [182]), has a C constant that can vary between 0.004 and 0.012 with a commonly used value of 0.01. Hazen's equation gives hydraulic conductivities in m/s when the particle diameter, D_{10} is in mm. The equation is valid for $k_{sat} \geq 10^{-5}$ m/s:

$$k_{sat} = C \cdot D_{10}^2 \qquad [\ 182\ ]$$

In 1892, Hazen developed an equation for the determination of the coefficient of permeability of a porous medium (Vukovic and Andjelko, 1992). The proposed equation still remains the most commonly used estimation equations for permeability and can be presented in the following dimensionally non-homogeneous form:

$$k_{sat} = A \cdot C \cdot \tau \cdot d_e^2 \qquad [\ 183\ ]$$

where:

k_{sat} = coefficient of permeability *(with dimensions depending on the variable A),*
A = *coefficient, which defines the dimension of coefficient of permeability (e.g. for k_{sat} expressed in m/day coefficient which, according to Hazen, depends on the clay fraction content of the porous media. For*

pure and uniform sands it ranges between 800 and 1200, and for clayey soils and non-uniform sands it ranges between 400 and 800[4].

d_e = *the "effective" grain diameter of the porous medium expressed in mm (d_e is most often taken as d_{10})*

τ = *correction temperature determined from $\tau = 0.70 + 0.03\ T$, and*

T = *water temperature in ℃ (If $T = 10$ ℃; $\tau = 0.70 + 0.03 \times 10 = 0.7 + 0.3 = 1.0$)*

The Hazen coefficient, *C,* is sometimes determined using Lange's formula with the coefficient expressed solely as a function of the material porosity:

$$C = 400 + 40(p - 26) \qquad [\ 184\ ]$$

where:

p = *porosity in terms of percentage,*

If the coefficient of permeability is expressed in cm/s and the temperature, *T* is 10 °C, and if the coefficient *C* is set to 860, then the Hazen formula [183] simplifies to:

$$k_{sat}[cm/s] = d_{10}^2[mm]. \qquad [\ 185\ ]$$

The Hazen (1911) equation should not be used for conditions outside the range of specified restrictions; however, it appears that this restriction is often ignored in engineering practice. The specified limitations for applying Hazen's equation are:

- The effective grain diameter, d_e, should be within the range of 0.1 mm < d_e < 3 mm, and
- The coefficient of uniformity, η, should be:

$$\eta = \frac{d_{60}}{d_{10}} < 5. \qquad [\ 186\ ]$$

However, the actual domain of applicability of Hazen's equation is significantly larger.

The Hazen equation [183] can also be written in the form of a dimensionally homogeneous equation:

$$k_{sat}(m/s) = \frac{g}{\nu} \cdot C_H \cdot \varphi(n) \cdot d_{10}^2 \qquad [\ 187\ ]$$

where:

C_H = *6×10^{-4}, and this results of k_{sat} in m/s, and*

$\varphi(n)$ = *is the function that depends on porosity.*

Using a suitable transformation from the Lange formula (Vukovic and Soro, 1992), the $\varphi(n)$ variable can be expressed as $\varphi(n) = [1 + 10(n - 0.26)]$, in which *n* is decimal.

5.2.2 Kozeny – Carman (1989) Estimation

The Kozeny-Carman (1989) equation was presented by Ahuja (1989) and can be used to estimate the saturated coefficient of permeability based on effective pore size (See equation [190]).

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Kozeny-Carman ksat

Formulation:

$$k_{sat} = B \cdot n_e^4 \qquad [\ 188\ ]$$

$$n_e = n - \theta_w \qquad [\ 189\ ]$$

Definitions:

[4] On the basis of experiments involving relatively uniform sands (η < 5) and the effective grain diameters within 0.1mm < d_e < 3 mm, Hazen determined the value of coefficient *C* = 1000. Later [1901], Hazen showed that coefficient *C* is not a constant and that it depends on the coefficient of uniformity of the material (η), shape and mineralogical composition of grains, degree of compactness, clayey fraction content, etc. According to these investigations, Hazen concluded that the value for coefficient *C* might range from 1200 (uniform and ideally pure sand) to 400 (very compact sand including a large amount of clayey fractions or iron hydroxide). As a rule, the empirical coefficient *C* decreases with an increase of non-uniformity of the material (i.e., coefficient of uniformity η) (Vukovic and Soro, 1992).

Equation Variable	Dialogue Field Name	Description
k_{sat}		saturated permeability (m/s)
B		constant equal to 0.002939
n		porosity of the soil
θ_w		volumetric water content when a suction of 33 kPa is applied to the soil

Required input: Porosity, the *B constant*, and a one-point fit of the soil-water characteristic curve
Applicable soil types: Only granular soils.

Modified fields:

Dialogue Field Name	Description
Kozeny-Carman ksat	estimated saturated coefficient of permeability (m/s)

The *B* constant can be taken to be 1058 for most soils. Ahuja (1989) studied the use of the *B* constant for soils in the Cecil, Lakeland, Norfolk, and Wagram series (Williams, 1993). The Cecil, Lakeland, and Norfolk series have been entered into SVSOILS and can be isolated by querying for the respective series names. Estimations using equation [190] are based on an effective porosity, n_e (i.e., equation [191]), which is defined as the total porosity minus the volumetric water content corresponding to a suction of 33 kPa.

$$k_{sat} = B \cdot n_e^4 \quad [\ 190]$$

$$n_e = n - \theta_w \quad [\ 191]$$

where:

k_{sat} = *saturated permeability (m/s),*
B = *constant equal to 0.002939,*
n = *porosity of the soil (decimal), and*
θ_w = *volumetric water content when a suction of 33 kPa is applied to the soil.*

5.2.3 Beyer (1964) Estimation

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Beyer ksat

Formulation:

$$k_{sat} = C \cdot d_e^2 \quad [\ 192\ ]$$

$$C = 4.5 \times 10^{-3} \log \frac{500}{\eta} \quad [\ 193\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (m/s)
d_e		effective grain diameter with 10% coverage on the grain-size distribution curve (mm)
C		empirical coefficient which depends on the coefficient of uniformity η, $C = f(\eta)$
η		the coefficient of uniformity within $1 < \eta < 20$

Required input: *D10*, *D60*, and grain-size distribution (i.e., effective grain diameter, d_e)
Applicable soil types: Only granular soils.

Modified fields:

Dialogue Field Name	Description

Beyer ksat	estimated saturated coefficient of permeability (m/s)

Beyer (1964) suggested the following empirical equation for the determination of the coefficient of permeability:

$$k_{sat} = C \cdot d_e^2 \qquad [\ 194\ ]$$

where:

k_{sat} = *coefficient of permeability (m/s),*
d_e = *effective grain diameter with 10% coverage on the grain-size distribution curve (mm), and*
C = *empirical coefficient, which depends on the coefficient of uniformity η ($C = f(\eta)$).*

The factor of proportionality, *C,* is determined from the relationship presented in Figure 42.

The empirical coefficient, *C,* (Figure 42) can be expressed in the form of following relationship[6] (Vukovic and Andjelko, 1992):

$$C = 4.5 \times 10^{-3} \log \frac{500}{\eta} \qquad [\ 195\ ]$$

The Beyer (1964) equation can be presented in the dimensionally homogeneous form as:

$$k_{sat}(m/s) = \frac{g}{v} \cdot C_B \cdot d_{10}^2 \qquad [\ 196\ ]$$

where:

C_B = *6×10^{-3} log 500/η.*

Equation [196] shows that the Beyer (1964) permeability equation is not a function of the porosity of the porous medium. The Beyer (1964) empirical equation has been recommended for materials with grain-size diameters ranging between [0.06 mm < d_{10} < 0.6 mm], and a coefficient of uniformity between [1 < η < 20].

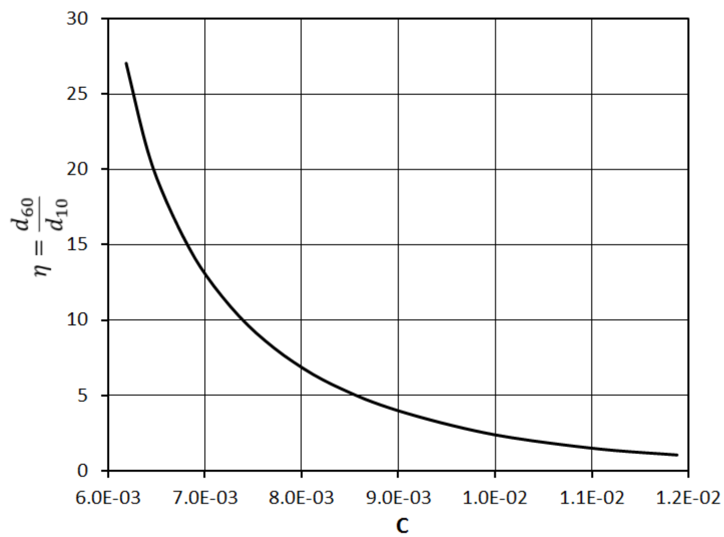


Figure 42 Dependence of empirical coefficient *C* on the coefficient of uniformity η in the Beyer (1964) equation (Vukovic and Andjelko, 1992)

5.2.4 Kruger (1992) Estimation

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Kruger ksat

Formulation:

[6] This approximation introduces a maximum error of ±5% in the calculation.

$$k_{sat} = 240 \frac{n}{(1-n)^2} d_e^2 \quad [197]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability at temperature T = 10 °C (m/day)
d_e		the effective grain diameter (mm) determined on the basis of equation [199]

Required input: Porosity, and grain-size distribution (effective grain diameter, d_e)
Applicable soil types: Medium grain size sands with the coefficient of uniformity $\eta > 5$.

Modified fields:

Dialogue Field Name	Description
Kruger ksat	estimated saturated coefficient of permeability [m/s]

The Kruger (1992) equation for the estimation of the coefficient of permeability at a water temperature T = 10 °C is presented in the following form:

$$k_{sat} = 240 \frac{n}{(1-n)^2} d_e^2 \quad [198]$$

where:

k_{sat} = *coefficient of permeability at 10°C (m/day), and*
d_e = *the effective grain diameter (mm) determined on the basis of formula:*

$$\frac{1}{d_e} = \sum_{i=1}^{i=n} \frac{\Delta g_i}{d_i} = \sum_{i=1}^{i=n} \Delta g_i \frac{2}{d_i^g + d_i^d} \quad [199]$$

where:

Δg_i = *weight content of certain fractions of the material composition, in parts of the total weight,*
d_i = *arithmetic mean grain diameter of the corresponding fraction,*
d_i^g = *maximum grain diameter of the corresponding fraction, and*
d_i^d = *minimum grain diameter of the fraction.*

The Kruger (1992) equation appears to yield the best results for medium grain-size sands with the coefficient of uniformity, $\eta > 5$.

The Kruger (1992) equation [180], can be written in a dimensionally homogeneous form:

$$k_{sat}(m/s) = C_K \frac{g}{\nu} \frac{n}{(1-n)^2} d_e^2 \quad [200]$$

where:

C_K = *numerical constant 4.35×10^{-5} and k_{sat} in m/s.*

5.2.5 Zamarin (1992) Estimation

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Zamarin ksat

Formulation:

$$k_{sat} = 8.07 \frac{n^3}{(1-n)^2} C_n \cdot \tau \cdot d_e^2 \quad [201]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (cm/s)
τ		coefficient of (correction) temperature determined in accordance with Table 10
C_n		empirical coefficient, which depends on the porosity (n)
d_e		effective grain diameter (mm) which depends, in the general case, on all fractions of the analyzed porous medium and is determined from the equation [200].

Required input: Porosity, and grain-size distribution (effective grain diameter, d_e)
Applicable soil types: Large grain sands.

Modified fields:

Dialogue Field Name	Description
Zamarin ksat	estimated saturated coefficient of permeability [m/s]

The most frequently used expression for determination for coefficient of permeability in accordance with the Zamarin (Vukovic and Soro, 1992) equation is:

$$k_{sat} = 8.07 \frac{n^3}{(1-n)^2} C_n \cdot \tau \cdot d_e^2 \qquad [\ 202\ ]$$

where:

k_{sat} = *coefficient of permeability (cm/s),*
τ = *coefficient of (correction) temperature determined in accordance with Table 10,*
C_n = *empirical coefficient, which depends on the porosity (n),*
d_e = *effective grain diameter [mm] which depends, in the general case, on all fractions of the analyzed porous medium and is determined from the expression.*
d_i = *the largest diameter of the finest fraction (d < 0.0025 mm),*
Δg_i = *weight of the material of the finest fraction in parts of the total weight, and*
a_i = *"angular coefficient" in the given interval of the mechanical composition curve, if is viewed as a broken curve, with d_i^g and d_i^d as the extreme diameters of the fraction grains.*

$$\frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_1}{d_1} + \sum_{i=2}^{i=n} a_i \ln \frac{d_i^g}{d_i^d} \qquad [\ 200\ ]$$

Table 10 Coefficient of τ corrected to temperature

T [°C]	τ	*T* [°C]	τ	*T* [°C]	τ	*T* [°C]	τ	*T* [°C]	τ
0	0.588	6	0.712	12	0.854	18	1.000	24	1.155
1	0.612	7	0.744	13	0.874	19	1.025	25	1.180
2	0.635	8	0.766	14	0.902	20	1.052	26	1.313
3	0.656	9	0.786	15	0.926	21	1.080	27	1.620
4	0.676	10	0.807	16	0.950	22	1.107	28	1.926
5	0.698	11	0.837	17	0.975	23	1.131	29	2.231

According to Zamarin (1992), the angular coefficient is determined from the following expression (Figure 43):

$$a_i = \frac{\Delta g_i}{(d_i^g - d_i^d)} \qquad [\ 203\ ]$$

where:

Δg_i = *fraction weight in parts of the total weight.*

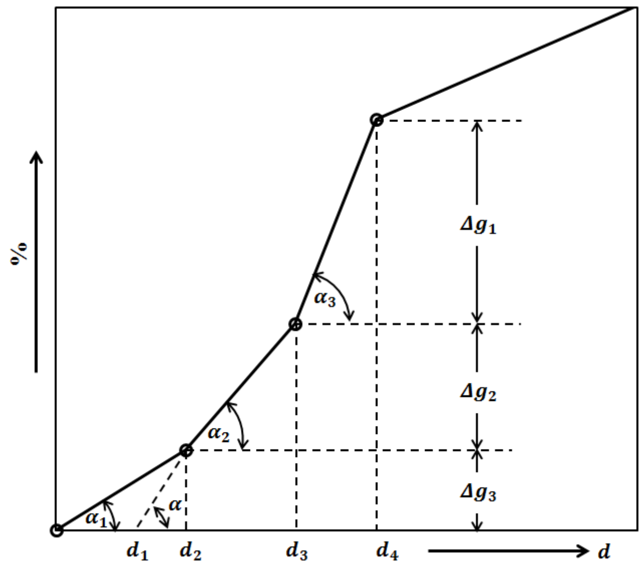


Figure 43 Calculation model for the effective grain diameter according to Zamarin (1992) (reported by Vukovic and Soro, 1992)

A function can be derived for the calculation of the effective grain diameter after Zamarin (1992) by substituting the appropriate expressions into equation [200].

$$\frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_1}{d_1} + \sum_{i=2}^{i=n} \Delta g_i \frac{\ln \frac{d_i^g}{d_i^d}}{d_i^g - d_i^d} \qquad [\ 204\ ]$$

Zamarin (1992) recommended that the effective grain diameter be calculated using the following expression when the mechanical composition of the material does not contain material finer than d = 0.0025 mm:

$$\frac{1}{d_e} = \sum_{i=1}^{i=n} \Delta g_i \frac{\ln \frac{d_i^g}{d_i^d}}{d_i^g - d_i^d} \qquad [\ 205\ ]$$

The values of empirical coefficient C_n, expressed as a function of porosity was determined by Zarmarin (1992) using Table 11.

Table 11 empirical coefficient C_n

n	C_n	n	C_n	n	C_n	n	C_n
0.27	0.757	0.32	0.632	0.37	0.518	0.42	0.416
0.28	0.731	0.33	0.608	0.38	0.497	0.43	0.397
0.29	0.706	0.34	0.585	0.39	0.476	0.44	0.378
0.30	0.680	0.35	0.562	0.40	0.456	0.45	0.360
0.31	0.656	0.36	0.540	0.41	0.435	0.46	0.342

The function of porosity C_n can be expressed analytically in the form:

$$C_n = (1.275 - 1.5n)^2 \qquad [\ 206\ ]$$

It appears that the Zamarin (1992) equation yields the best results for the case of large grain-sized sands.

The Zamarin equation takes on the form of equation [180] when written in the dimensionally homogeneous form.

$$k_{sat}(m/s) = C_z \frac{g}{\nu} \frac{n^3}{(1-n)^2} d_e^2 \qquad [\ 207\ ]$$

where:

C_z = *8.2×10^{-3} and k_{sat} in m/s.*

5.2.6 Slichter (1962) Estimation (Vukovic and Soro, 1992)

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Slichter ksat

Formulation:

$$k_{sat} = 10.0219\frac{\gamma}{\mu}\cdot\varphi(n)\cdot d_e^2 \qquad [\ 208\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (cm/s)
γ		unit weight of water
μ		function of porosity presented in tabular form in the literature
$\varphi(n)$		effective grain diameter (mm) which depends, in the general case, on all fractions of the analyzed porous medium and is determined from expression.
d_e		effective grain diameter (cm), taken as d_{10}

Required input: Porosity, and *D10*

Applicable soil types: Limits defined by the effective grain diameter (*de*) from 0.01 mm to 5 mm. The Slichter (1962) equation does not include nonuniformity of the mechanical composition of the porous medium.

Modified fields:

Dialogue Field Name	Description
Slichter ksat	estimated saturated coefficient of permeability (m/s)

The basic form of Slichter's (1962) empirical equation can be written as follows:

$$k_{sat} = 10.0219\frac{\gamma}{\mu}\varphi(n)\cdot d_e^2 \qquad [\ 209\ ]$$

where:

k_{sat} = *coefficient of permeability (cm/s),*
γ = *unit weight of water,*
μ = *coefficient of absolute viscosity [poise],*
$\varphi(n)$ = *function of porosity presented in tabular form in the literature, and*
d_e = *effective grain diameter (cm), taken as d_{10}.*

The value of porosity function $\varphi(n)$ is determined according to Table 12.

The porosity function may be approximated by:

$$\varphi(n) = n^{3.287} \qquad [\ 210\ ]$$

with an error of the order of ±5%.

Table 12 Porosity function φ(n)

n	φ(n)	n	φ(n)	n	φ(n)
0.26	0.01187	0.34	0.02878	0.42	0.05789
0.27	0.01350	0.35	0.03163	0.43	0.06267
0.28	0.01517	0.36	0.03473	0.44	0.06776
0.29	0.01694	0.37	0.03808	0.45	0.07295
0.30	0.01915	0.38	0.04154	0.46	0.07838
0.31	0.01212	0.39	0.04525	0.47	0.08455
0.32	0.02356	0.40	0.04922		
0.33	0.02601	0.41	0.05339		

The other form of the Slichter (1962) equation that is encountered in the literature is[5]:

$$k_{sat} = 4960 \cdot M \cdot d_e^2 \qquad [\ 211\ ]$$

where:

k_{sat} = coefficient of permeability (m/day),
M = coefficient dependent on porosity (Table 12), and
d_e = effective grain diameter ($d_e = d_{10}$) expressed in mm.

The available literature does not provide instructions for the determination of the effective grain-size diameter d_e. In practice, the effective grain-size is usually taken as the d_{10} value ($d_e = d_{10}$)

The limits of applicability of the Slichter (1962) equation are defined by the effective grain diameter (d_e) which must be between 0.01 mm and 5 mm. The Slichter (1962) equation does not include nonuniformity of the mechanical composition of the porous medium (η).

The Slichter (1962) (Vukovic and Soro, 1992) equation can be expressed in a dimensionally correct form as follows.

$$k_{sat} = \frac{g}{v} \cdot C_s \cdot n^{3.287} \cdot d_{10}^2 \qquad [\ 212\ ]$$

where:

C_s = 1×10^{-2} and other parameters defined in equation [180].

5.2.7 Terzaghi (1925) Estimation

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Terzaghi ksat

Formulation:

$$k_{sat} = C_o \frac{\mu_{10}}{\mu_t} \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 \cdot d_{10}^2 \qquad [\ 213\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (cm/s)
d_{10}		effective grain diameter (cm)
μ_{10}		coefficient of absolute liquid viscosity at temperature 10°C
μ_t		coefficient of absolute liquid viscosity at temperature 10°C
C_o		empirical coefficient which depends on the nature of the grain surface which, in the case of sandy soil,

[5] According to the authors' analysis, this form of the Slichter (1962) (Vukovic and Soro, 1992) equation corresponds to the temperature at 0°C.

		varies from 160 (for coarse grains of irregular shape) to 800 (for rounded, smooth grains).

Required input: Porosity, and *D10*
Applicable soil types: Applied in the case of large grain sands.

Modified fields:

Dialogue Field Name	Description
Terzaghi ksat	estimated saturated coefficient of permeability (m/s)

The Terzaghi [1925] equation is not dimensionally homogeneous, but can be expressed as:

$$k_{sat} = C_o \frac{\mu_{10}}{\mu_t} \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 \cdot d_{10}^2 \quad [\ 214\ ]$$

where:

k_{sat} = *coefficient of permeability (cm/s),*
d_{10} = *effective grain diameter (cm),*
μ_{10} = *coefficient of absolute liquid viscosity at temperature 10°C,*
μ_t = *coefficient of absolute liquid viscosity at temperature T°C, and*
C_o = *empirical coefficient, which depends on the nature of the grain surface which, in the case of sandy soil, varies from 160 (coarse grains of irregular shape) to 800 (rounded, smooth grains).*

The literature offers the following values of the Terzaghi empirical coefficient, C_o:

- Sea sand 750 to 663;
- Dune sand 800;
- Pure river sand 696 to 460;
- Muddy river sand 203.

Another form of the Terzaghi (1925) equation found in the literature is:

$$k_{sat} = \frac{C}{\mu_t} \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 \cdot d_{10}^2 \quad [\ 215\ ]$$

where:

k_{sat} = *coefficient of permeability (cm/s),*
μ_t = *coefficient of absolute water viscosity in pose for liquid temperature T°C,*
d_e = *effective grain diameter, usually d_{10} in cm, and*
C = *empirical coefficient, which depends on the nature of the grain surface (C = 10.48 for smooth grains and C = 6.02 for coarse grains).*

The Terzaghi (1925) equation appears to apply well to large-grained sand.

The Terzaghi (1925) equation can be expressed in a dimensionally homogeneous form as follows:

$$k_{sat} = C_T \frac{g}{v} \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 \cdot d_{10}^2 \quad [\ 216\ ]$$

where:

C_T = *empirical coefficient that depends on grain shape ($C_T = 10.7 \times 10^{-3}$ for coarse grains) and other parameters defined in equation [180].*

5.2.8 Kozeny (1962) Estimation (Vukovic and Soro, 1992)

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Kozeny ksat

Formulation:

$$k_{sat} = 7.94 \frac{n^3}{(1-n)^2} \tau \cdot d_e^2 \qquad [\ 217\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (cm/s)
τ		correction temperature; values of the function τ are given in Table 10
d_e		effective grain diameter in mm, determined from an expression

Required input: Porosity, and grain-size distribution (effective grain-size diameter, d_e)
Applicable soil types: Used in the case of large grain size sands.

Modified fields:

Form Field Name	Table Field Name	Description
Kozeny ksat	Kozeny_ksat	estimated saturated coefficient of permeability (m/s)

Kozeny (1962) (Vukovic and Soro, 1992) recommended the following empirical equation for the calculation of the saturated coefficient of permeability:

$$k_{sat} = 7.94 \frac{n^3}{(1-n)^2} \tau \cdot d_e^2 \qquad [\ 218\ ]$$

where:

k_{sat} = *coefficient of permeability (cm/s),*
τ = *correction temperature – the values of function τ are given in Table 10, and*
d_e = *effective grain diameter in mm, determined in this case from expression:*

$$\frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_1}{d_1} + \sum_{i=2}^{i=n} \frac{\Delta g_i}{d_i} \qquad [\ 219\ ]$$

where:

d_1 = *the largest diameter of the last fraction of the material (d < 0.0025 mm),*
Δg_1 = *weight of the material of the last, finest fraction, in parts of total weight*
Δg_i = *weight of the i^{th} fraction, in parts of total weight, and*
d_i = *the "mean" grain diameter of the observed fraction (I) determined from expression:*

$$\frac{1}{d_i} = \frac{1}{2}\left(\frac{1}{d_i^g} + \frac{1}{d_i^d}\right) \qquad [\ 220\ ]$$

where:

d_i^g *and* d_i^d = *extreme (maximum and minimum) diameters of the observed fraction i.*

The effective grain diameter (d_e), according to Kozeny (1962), is determined using the following equation.

$$\frac{1}{d_e} = \frac{3}{2}\frac{\Delta g_1}{d_1} + \sum_{i=2}^{i=n} \Delta g_i \frac{d_i^g + d_i^d}{2 \cdot d_i^g \cdot d_i^d} \qquad [\ 221\ ]$$

Another form of the Kozeny (1962) equation found in the literature has the d_e *set to* d_{10} and can be written as follows:

$$k_{sat} = 5400 \frac{n^3}{(1-n)^2} d_{10}^2 \qquad [\ 222\ ]$$

where:

C_K = *8.3×10^{-3} and other parameters defined in equation [180].*

5.2.9 USBR (1992) Estimation (Vukovic and Soro, 1992)

The following summary is taken from:

Vukovic, M. and Soro, A. (1992). "Determination of Hydraulic Conductivity of Porous Media from Grain-Size Composition", Water Resources Publications, Littleton, Colorado.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > USBR ksat

Formulation:

$$k_{sat} = 0.36 \cdot d_{20}^{2.3} \quad [\ 223\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		coefficient of permeability (cm/s)
d_{20}		Grain-size diameter in mm, with 20% coverage on the grain-size distribution curve.

Required input: D_{20}
Applicable soil types: Medium-grain sands with a coefficient of uniformity < 5

Modified fields:

Dialogue Field Name	Description
USBR ksat	estimated saturated coefficient of permeability (m/s)

The USBR (1992) equation has been recommended for medium-grain sands with a coefficient of uniformity, $\eta < 5$. The widely used USBR (1992) equation is written in the form:

$$k_{sat} = 0.36 \cdot d_{20}^{2.3} \quad [\ 224\ ]$$

where:

k_{sat} = *coefficient of permeability [cm/s], and*
d_{20} = *grain diameter in mm, with 20 % coverage on the grain-size distribution curve.*

The USBR (1992) equation can be written in the same form as the other proposed equations (i.e., the dimensionally homogeneous form of equation [180]):

$$k_{sat} = \frac{g}{v} C_u \cdot d_{20}^{2} \quad [\ 225\ ]$$

where:

C_u = *coefficient expressed as a function of the effective grain diameter, assuming that the USBR formula was derived for a water temperature of 15°C:*

$$C_u = 4.8 \times 10^{-4} \cdot d_{20}^{0.3} \quad [\ 226\ ]$$

(for C_u in this case, the diameter representing 20% coverage on the grain-size curve (d_{20}) is taken in mm).

5.2.10 Rawls and Brakensiek (1983) Estimation

The following summary is taken from:

Rawls, W.J., Brakensiek, D.L., and Soni, B. (1983). Agricultural Management Effects on Soil Water Processes Part I: Soil Water Retention and Green and Ampt Infiltraion Parameters, Soil and Water Division of ASAE, 26, pp. 1747-1752.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Rawls and Brakensiek 1983 ksat

Formulation:

$$\begin{aligned} k_{sat} = Exp(&19.52348n - 8.96847 - 0.028212C + 0.00018107S^2 \\ &- 0.0094125C^2 - 8.395215n^2 + 0.077718Sn - 0.00298S^2n^2 \\ &- 0.019492C^2n^2 + 0.0000173S^2C + 0.02733C^2n + 0.001434S^2n \\ &- 0.0000035C^2S)\ 2.77 \times 10^{-6} \end{aligned} \quad [\ 227\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
C		Percent clay (5 < *PC* < 60)
S		Percent sand (5 < *PS* < 70)
n		Porosity (vol. fraction)

Required input: % clay, % sand, and porosity
Applicable soil types: 5% < % sand < 70% and 5% < % clay < 60%

Modified fields:

Dialogue Field Name	Description
Rawls 1983 *ksat*	estimated saturated coefficient of permeability (m/s)

Rawls and Brakensiek (1983) reported the above regression equation ([227]) for the Brooks-Corey (1964) permeability equation as a function of soil properties. The soil properties incorporated into the equation include percent sand and clay of the soil fraction (< 2mm) and the soil porosity (volume fraction). The equation is valid for a sand content greater than 5 percent but less than 70% and for a clay content greater than 5 percent and less than 60%.

5.2.11 Rawls, Brakensiek, and Logsdon (1993) Estimation

The following summary is taken from:

Rawls, W.J., Brakensiek, D.L., and Logsdon, S.D. (1993). Predicting saturated hydraulic conductivity utilizing fractal principles, Soil Science Society of America Journal, Madison, WI, Vol. 57, No. 5, pp. 1193-1197.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Rawls and Brakensiek 1993 ksat

Formulation:

$$k_{sat} = 4.41 \times 10^7 \left(\frac{\phi}{n^2} \right) R_1^{\ 2} \quad [\ 228\ ]$$

Definitions:

Equation Variable	Form Field Name	Description
k_{sat}		Saturated coefficient of permeability (cm/hr)
ϕ		total porosity
n		total pore size classes
R_1		average pore radius (cm)

Required input: % clay, % sand, and porosity
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Rawls 1993 ksat	estimated saturated coefficient of permeability (m/s)

The fractal analysis is calculated using the following equation.

$$k_{sat} = 4.41 \times 10^7 \left(\frac{\phi}{n^2} \right) R_1^{\ 2} \qquad [\ 229\ ]$$

where:

k_{sat}	=	*saturated coefficient of permeability (cm/hr)*
ϕ	=	*total porosity*
n	=	*total pore size classes and*
R_1	=	*average pore radius (cm).*

The largest equivalent pore radius, R_1, is determined from the capillary rise equation using the methodology of Tyler and Wheatcraft (1990).

$$R_1 = \frac{0.148}{h_b} \qquad [\ 230\ ]$$

where:

h_b	=	*geometric mean bubbling pressure (cm)*

Following is an estimator equation for n, derived by relating n to the fractal dimension, D, using a nonlinear regression.

$$n = 1.86 D^{5.34} \qquad r = 0.91 \qquad [\ 231\ ]$$

where:

D	=	*fractal dimension of soil porosity as derived by Tyler and Wheatcraft (1990).*

The porosity fractal dimension can be estimated as:

$$D = 2 - \lambda \qquad [\ 232\ ]$$

λ	=	*Brooks and Corey (1964) pore-size distribution index*

The pore-size distribution index can be predicted using the following equation presented in Rawls et al., (1991).

$$\begin{aligned}\lambda = \mathrm{Exp}(&-0.784 + 0.018\ PS - 1.062\ PO - 0.00005\ PS^2 - 0.003\ PC^2 + \\ &1.111\ PO^2 - 0.031\ PS\ PO + 0.0003\ PS^2\ PO^2 - 0.006\ PC^2\ PO^2 - \\ &0.000002\ PC^2 - 0.008\ PC^2\ PO - 0.007\ PC\ PO^2)\end{aligned} \qquad [\ 233\ ]$$

where:

PS	=	*percent silt,*
PO	=	*percent organic, and*
PC	=	*percent clay*

5.2.12 NAVFAC (1974) Estimation

The Naval Facilities Engineering Command design manual DM7 (NAVFAC, 1974) provides a chart to determine saturated k_{sat} for clean sand and gravel as a function of void ratio, e and d_{10}. Chapuis (2004) proposed an equation to represent the chart.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > NAVFAC ksat

Formulation:

$$k_{sat}(cm/s) = 10^{1.291e - 0.6435} \left[d_{10}(mm) \right] 10^{0.5504 - 0.2937e} \qquad [\ 234\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		Saturated coefficient of permeability (cm/s)
e		void ratio
d_{10}		diameter of the 10% passing particle- size in mm

Required input: Void ratio, and grain-size distribution (d_{10})
Applicable soil types: clean sands and gravels

Modified fields:

Dialogue Field Name	Description
NAVFAC ksat	estimated saturated coefficient of permeability [cm/s]

5.2.13 Chapuis (2004) Estimation

Chapuis (2004) developed an equation to determine k_{sat} based on Hazen (1911) and NAVFAC (1974). The equation provides a good prediction of k_{sat} for uniform sands and gravels (Cu < 12) through two grain-size parameters, void ratio e and d_{10}.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Chapuis

Formulation:

$$k_{sat}(cm/s) = 2.4622\left[d_{10}\frac{e^3}{1+e}\right]^{0.7825} \quad [\ 235\ ]$$

where:

d_{10} = *diameter of the 10% passing particle- size in mm.*

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		saturated coefficient of permeability (cm/s)
e		void ratio
d_{10}		diameter of the 10% passing particle- size in mm

Required input: Void ratio, and grain-size distribution (d_{10})
Applicable soil types: clean sands and gravels

Modified fields:

Dialogue Field Name	Description
Chapuis ksat	estimated saturated coefficient of permeability [cm/s]

5.2.14 Fair-Hatch (1959) Estimation (Allan and Cherry, 1979)

The following summary is taken from:
Allan, F. A. and Cherry, J. A. (1979). "Groundwater." Prentice-Hall, Englewood Cliffs.

Menu location: SVFlux > Hydraulic Conductivity > ksat Options > Fair-Hatch ksat

Formulation:

$$k_{sat} = \left(\frac{\rho g}{\mu}\right)\left[\frac{n^3}{(1-n)^2}\right]\left[\frac{1}{m\left(\frac{\theta}{100}\sum\frac{P}{d_m}\right)^2}\right] \quad [\ 236\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_{sat}		Saturated coefficient of permeability (cm/hr)
ρ		density of fluid
μ		fluid viscosity
n		porosity
m		packing factor
θ		sand shape factor
P		percentage of sand held between adjacent sieves

d_m		geometric mean of the rated sizes of adjacent sieves

Required input: Fluid density and viscosity, porosity, sand shape factor, percentage of sand held between adjacent sieves, geometric mean of the rated sizes of adjacent sieves

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
FairHatch ksat	estimated saturated coefficient of permeability (m/s)
FairHatch_SandShape	Sand Shape Factor used by Fair-Hatch equation in the estimation of the saturated coefficient of permeability

5.3 PERMEABILITY FUNCTION (UNSATURATED)

Earth structures such as soil covers, tailing impoundments, earth dams and other waste management structures are routinely designed and constructed using unsaturated permeability functions. The amount of seepage that may occur through these structures under saturated and unsaturated conditions is important for the proper design of such structures. The coefficient of a permeability function (and water storage function) is a key property required in the design of near-ground-surface earth structures.

Various types of soils are used in the construction of earth structures. The saturated coefficient of permeability can vary more than 10 orders of magnitude when considering soils that range from gravel to clay. The unsaturated coefficient of permeability of a single soil type can also vary widely as the suction in the soil is changed. Studies have shown that the mass of water flow, pore-water pressures and hydraulic heads are directly proportional to the coefficient of permeability values for unsaturated soils. For geotechnical and geo-environmental structures comprised of unsaturated soils, the knowledge of the coefficient of permeability, pore-water pressures and hydraulic heads are of primary interest.

Theoretical concepts and tools for the study of unsaturated soil behavior, including the coefficient of permeability based on experimental procedures are available (Fredlund and Rahardjo 1994). However, experimental procedures for the measurement of the coefficient of permeability of unsaturated soils are time consuming, difficult and hence costly. In the last few decades, it has become quite routine for the engineers in consulting firms and organizations to use seepage models to quantify the flow behavior under saturated-unsaturated conditions. Several advancements have been made in the prediction of the engineering behavior of unsaturated soils in the recent years. Powerful computer tools are available today to model the unsaturated soils behavior.

Once the saturated permeability function of a soil is known, the coefficient of permeability of a soil can be assumed to be a relatively unique function of soil suction. The function appears to be relatively unique as long as the volume change of the soil is negligible or reversible. A number of pedo-transfer functions (PTFs) have been implemented within SVSOILS to allow calculation of the entire permeability soil property function. A description of each of these calculation methods is presented in the following sections.

The following sections present the development of the theory used in the estimation of permeability functions for unsaturated soils. There are numerous methods that have been presented for the estimation of the saturated coefficient of permeability. The following methods were selected for their simplicity and popularity in engineering practice.

Kunze et al. (1968) Estimation
Fredlund et al., (1994) Estimation
Campbell (1973) Estimation
van Genuchten Estimation
Brooks and Corey Estimation
Modified Campbell Estimation
Mualem Estimation
Leong and Rahardjo Estimation

The required information for predicting unsaturated coefficient of permeability is the saturated coefficient of permeability and the soil-water characteristic curve (i.e., generally the drying or desorption SWCC). These equations are quite commonly used in geotechnical engineering.

5.3.1 Kunze, Uehara and Graham (1968) Estimation, (KCAL)

Kunze et al., (1968) is implemented into the SVSOILS software to allow estimation of the permeability curve. The Kunze et al., (1968) method (also referred to as the KCAL method) is presented below.

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Kunze (KCAL) Estimation

Formulation: Algorithm
Required input: Saturated coefficient of permeability and van Genuchten fit of the soil-water characteristic curve
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Kunze Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Kunze Error	difference between the fit and laboratory values in terms of R^2

Kunze et al., (1968) equation is based on a statistical approach and it provides reasonable predictions of the coefficient of permeability function for unsaturated soils. The original equation proposed by Kunze et al., (1968) is modified to use SI units and the term matric suction instead of pore-water pressure head:

$$k_w(\theta_i)=\frac{k_s}{k_{sc}}\frac{T_s^2\ g}{2\mu_w}\frac{\theta_s^p}{n^2}\sum_{j=i}^{m}\left[(2j+1-2i)\psi_j^{-2}\right] \qquad [\ 237\ ]$$

where:

$k_w(\theta_i)$ = *the calculated coefficient of permeability for a specified volumetric water, θ_i, corresponding to the i^{th} interval,*
i = *the interval number that increases with the decreasing water content (for example, i = 1 identifies the first interval that closely corresponds to the saturated water content θ_s, and i = m identifies the last interval corresponding to the lowest water content, θ_L, on the laboratory soil-water characteristic curve),*
j = *counter from i to m,*
k_{sc} = *calculated saturated coefficient of permeability,*
T_s = *the surface tension of water,*
ρ_w = *the water density,*
g = *the gravitational acceleration,*
μ_w = *the absolute viscosity of water,*
p = *a constant that accounts for the interaction of pores of various sizes,*
m = *the total number of intervals between the saturated volumetric water content w_L on the laboratory soil-water characteristic curve,*
n = *the total number of intervals computed between the saturated volumetric water content, θ_s and zero water content (i.e., w = 0) (note that $n = m[\theta_s/\theta_s - \theta_L)$; $m< n$; and $m = n$ when $w_L = 0$), and*
ψ_j = *the suction (kPa) corresponding to the midpoint of j^{th} interval (Figure 2).*

Figure 44 illustrates the calculation procedure associated with the calculation of the coefficient of permeability function. The volumetric water content and suction relationship is divided into *n* equal water-content increments. Inherent in the analysis is the assumption that the soil does not change volume as soil suction is changed.

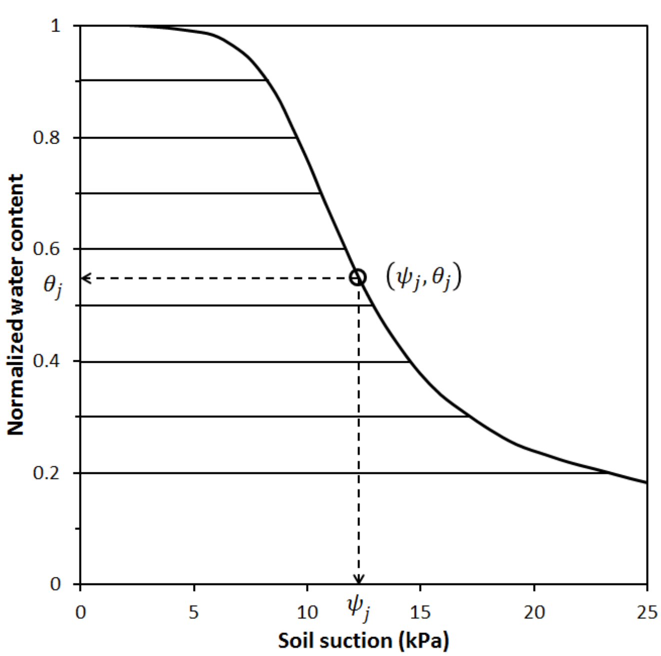


Figure 44 A typical soil-water characteristic curve used to predict the permeability function. (θ_j, is the midpoint of the j^{th} water-content interval; ψ_j, suction corresponding to θ_j).

The calculation of the coefficient of permeability, $k(\theta_i)$ at a specific volumetric water content θ_i involves the summation of soil suction values that correspond to the water contents at and below θ_i. The matching factor, (k_s/k_{sc}), based on the saturated coefficient of permeability is necessary to provide a more accurate fit for the unsaturated coefficient of permeability. The shape of the permeability function is determined by the terms inside the summation-sign portion of the equation, which in turn is obtained from the soil-water characteristic curve.

5.3.1.1 Example problem for computation of the coefficient of permeability using Kunze et al., (1968) equation

The soil-water characteristic curve is divided into *m* equal intervals (i.e., 20) of volumetric water content as shown in Figure 45. The maximum volumetric water content is 0.388 and minimum volumetric water content is 0.102. The first volumetric water content corresponds to saturated conditions. Each volumetric water midpoint corresponds to a particular suction, ψ. The midpoints are numbered starting at point 1 (i.e., *i* equal to 1) to point 20 (i.e., *i* equal to *m*). The permeability function is predicted using equation [237].

The saturated coefficient of permeability, k_{sat}, is independently measured in the laboratory, and in this example has a value of 5.83x10^{-8} m/s (Gonzalez and Adams, 1980). The permeability values, $k_w(q_w)$, are computed by substituting soil suction values with corresponding midpoints into equation [237]. A comparison between the computed and measured values, $k_w(q_w)$ is shown in Figure 46. More detailed information with respect to the calculations is available in Fredlund and Rahardjo (1993).

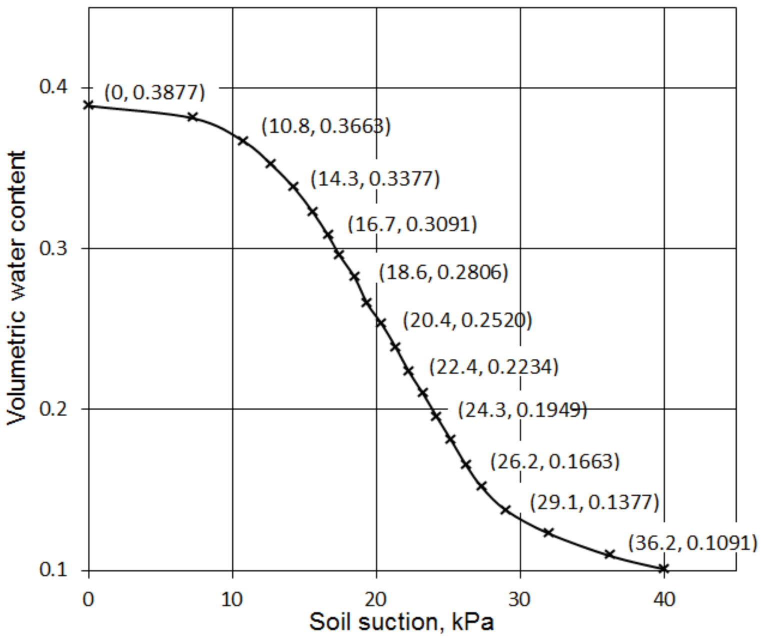


Figure 45 Calculation of the coefficient of permeability function using the soil-water characteristic curve for a fine sand (modified after Gonzalez and Adams, 1980)

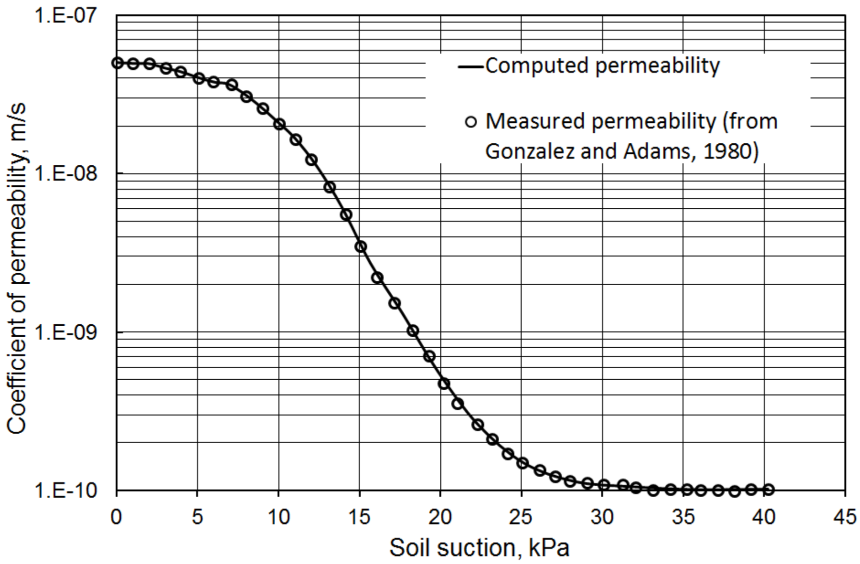


Figure 46 Comparisons between the computed and measured coefficient of permeability (from Fredlund and Rahardjo, 1993))

5.3.2 Fredlund, Xing and Huang (1994) Estimation

Fredlund et al., (1994) presented a modification of the Mualem (1976) integration as a method of estimating the coefficient of permeability of a soil as a function of soil suction. The integration is complex and a closed-form solution is not available. SVSOILS performs the integration and will output points onto a graph or as *x-y* data. The calculated points along the permeability function can then be best-fit with another appropriate mathematical equation (e.g., Gardner, 1964).

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Fredlund and Xing Estimation

Formulation:

$$k_r(\psi)=\frac{\displaystyle\int_{\ln(\psi)}^{b}\frac{\theta(e^y)-\theta(\psi)}{e^y}\theta'(e^y)dy}{\displaystyle\int_{\ln(\psi_{aev})}^{b}\frac{\theta(e^y)-w\theta}{e^y}\theta'(e^y)dy} \quad [\ 238\ ]$$

Solution method: Integration by Simpsons rule
Required input: Saturated coefficient of permeability and Fredlund and Xing (1994) fit of the soil-water characteristic curve
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Fredlund Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Fredlund Drying Error	difference between the fit and laboratory values in terms of R^2

Most equations proposed for the prediction of the coefficient of permeability use the soil-water characteristic curve data over a limited soil suction range (Brooks and Corey, 1964; Mualem 1976; and van Genuchten, 1980). The residual water content, θ_r, is the water content below which water flow appears to occur mainly in the vapor form.

Kunze et al., (1968) investigated the effect of using a partial soil-water characteristic curve for the prediction of coefficient of permeability. They found that the accuracy of the calculated permeability function was significantly improved when the entire soil-water characteristic curve was used.
Fredlund et al., (1994) proposed an integral form of the equation for the calculation of the coefficient of permeability. The proposed equation uses the soil-water characteristic curve data for the entire suction range of 0 to 1,000,000 kPa and does not require the use of the term residual water content, θ_r in the equation. This equation has benefits for modeling applications, but

it must also be realized that the vapor flow may enforce a lower limit on the coefficient of permeability. In structures such as soil covers, the coefficient of permeability value may be of interest at large soil suction values (i.e., greater than 3000 kPa).

The equation suggested by Fredlund et al., (1994) for calculation of the coefficient of permeability is given below:

$$k_r(\psi)=\frac{\displaystyle\int_{\ln(\psi)}^{b}\frac{\theta(e^y)-\theta(\psi)}{e^y}\theta'(e^y)dy}{\displaystyle\int_{\ln(\psi_{AEV})}^{b}\frac{\theta(e^y)-w\theta}{e^y}\theta'(e^y)dy} \qquad [\ 239\ ]$$

where:

b = *ln(1,000,000),*
y = *dummy variable of integration representing the logarithm of suction,*
ψ = *soil suction, given a function of volumetric water content, and*
AEV = *air-entry value.*

Fredlund et al., (1994) equation uses the Fredlund and Xing (1994) SWCC equation (i.e., equation [129]) for fitting the soil-water characteristic curve data over the entire range of soil suctions. The Fredlund and Xing (1994) equation has been found to fit the soil-water characteristic data well for all types of soils and suction ranges (Benson et al., 1997; and Leong and Rahardjo, 1997). More details with respect to equation [239] are available in Fredlund et al., (1994).

5.3.2.1 Example problem for computation of the coefficient of permeability using Fredlund et al., (1994) equation

The soil-water characteristic curve data for Guelph loam in drying and wetting stages is shown in Figure 47 (Elrick and Bowman, 1964). Fredlund and Xing (1994) equation (i.e., equation [129]) is used to best-fit the laboratory SWCC data over the entire range of soil suction values.

The coefficient of permeability is calculated using equation [239] and the results are shown in Figure 48. There is a close correlation between the calculated and measured values of the coefficient of permeability.

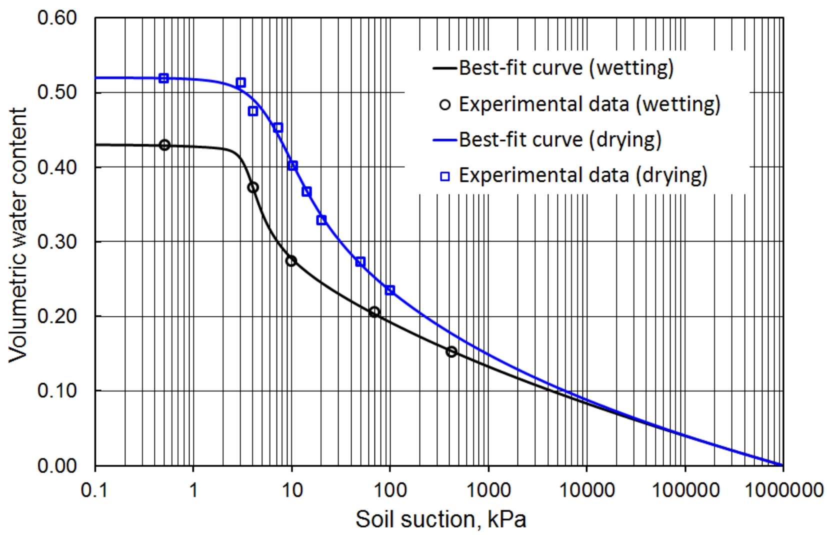


Figure 47 Best-fit curves to the laboratory data for Guelph loam (data from Elrick and Bowman, 1964).

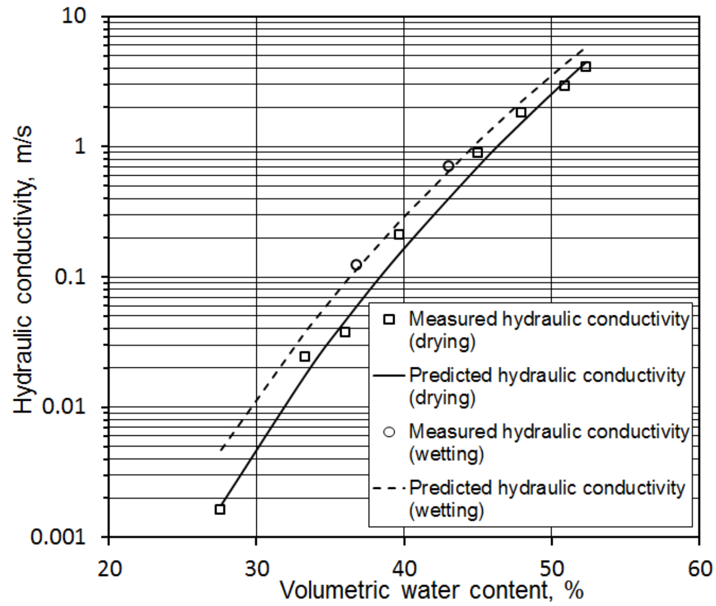


Figure 48 Comparisons of predicted coefficient of permeability with the measured data for Guelph loam (data from Elrick and Bowman, 1964)

5.3.3 Campbell (1973) Estimation

Campbell (1973) presents a method of estimating the permeability curve that can be used in conjunction with a number of methods for representing the soil-water characteristic curve. The Campbell (1973) method as implemented in the SVSOILS uses the Fredlund and Xing (1994) fit of the soil-water characteristic curve as the basis for the permeability function estimation.

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Campbell Estimation

Formulation:

$$k_w = k_s \left(\frac{\theta_s}{\theta} \right)^{2+\frac{2}{b}} \qquad [\ 240\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability at any particular suction
k_{sat}		saturated coefficient of permeability determined by the ***Campbell linked ksat*** field
θ_s		saturated volumetric water content
θ		volumetric water content at any particular suction as given by the soil-water characteristic curve
b	Campbell p	parameter used to vary the Campbell (1973) estimation

Required input: Saturated coefficient of permeability, campbell *p* parameter and Fredlund and Xing (1994) fit of soil-water characteristic curve

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Campbell Predicted	indicates if the estimation algorithm has been successfully executed on the current data
Campbell Error	difference between the fit and laboratory values in terms of R^2

Several investigators provided statistical models for constant volume porous media to predict the coefficient of permeability of unsaturated soils using the soil-water characteristic curve and the saturated coefficient of permeability (Childs and Collis-

George, 1950; Marshall, 1958; Millington and Quirk, 1959). Reasonably close correlations have been observed between the measured and predicted coefficient of permeability values using these equations.

Childs (1969) has shown that the coefficient of permeability can be represented by the following equation:

$$k_w = M \int_0^R \int_0^R r^2 F(r) dr \, F(r) dr \qquad [\ 241\ ]$$

where:

- *r* = *pore radius,*
- *M* = *constant to be determined,*
- *R* = *radius of the largest water filled pore, and*
- *F(r)* = *pore-size distribution function.*

The pore size distribution function can be defined such that the total porosity is:

$$f = \int_0^\infty F(r) dr \qquad [\ 242\ ]$$

The pore radii, in turn, can be related to the water content of the porous body through use of the soil-water characteristic curve and the capillarity equation. The soil-water characteristic curve over a limited range of suction can be reasonably well predicted using the relationship below:

$$\psi = \psi_e \left(\frac{\theta}{\theta_s} \right)^{-b} \qquad [\ 243\ ]$$

where:

- ψ_e = *air-entry value of the soil,*
- θ_s = *saturated volumetric water content, and*
- *b* = *fitting parameter.*

Campbell (1974) extended the capillarity equation and equations [241] through [243] above, and proposed a function for predicting the coefficient of permeability as:

$$k_w = k_s \left(\frac{\theta_s}{\theta} \right)^{2+\frac{2}{b}} \qquad [\ 244\ ]$$

More details of the derivation are available in Campbell (1974).

Equation [244] along with Millington and Quirk (1961) equation was used for comparing the calculated and measured coefficient of permeability for five different soils. The agreement between calculated and measured coefficient of permeability appears to be reasonable when using Campbell's equation. The value of *b* varied from 0.14 to 12.5 for the five soils tested.

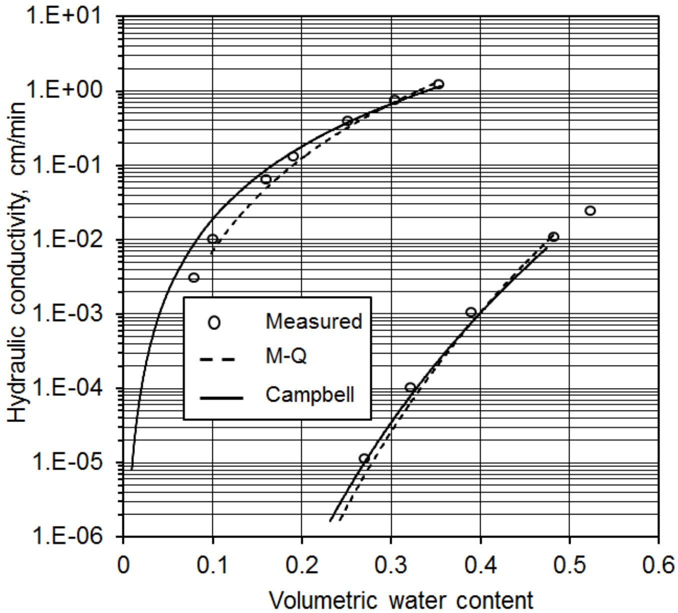


Figure 49 Coefficient of permeability as a function of water content for Botany sand and Guelph loam showing measured values (dots), calculated values using the Millington and Quirk method (dashed line), and calculated values using Campbell (1974) equation (data from Campbell, 1974)

5.3.4 van Genuchten (1980) Estimation

Several investigators (e.g., Brooks and Corey (1964) and Mualem (1976)) have proposed closed-form equations for predicting the coefficient of permeability of unsaturated soils based on the Burdine (1953) theory. The Brooks and Corey (1964) equation may not converge rapidly when used in numerical simulations of seepage in saturated-unsaturated soils. The Mualem (1976) equation is in integral form and enables the derivation of a closed-form analytical equation when provided with a suitable equation for the soil-water characteristic curve.

The equation proposed by van Genuchten (1980) for best-fitting the soil-water characteristic curve is flexible, continuous and has a continuous slope. The closed-form equation proposed for estimating the coefficient of permeability can be used for saturated-unsaturated soils flow modeling.

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > van Genuchten and Mualem Estimation

Formulation:

$$k_w(\psi) = k_{sat}\left[\frac{\left(1-(\alpha\psi)^{nm}\left[1+(\alpha\psi)^n\right]^{-m}\right)^2}{\left[1+(\alpha\psi)^n\right]^{m/2}}\right] \qquad [\ 245\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase
k_{sat}		saturated coefficient of permeability of the water phase determined by the ***van Genuchten Linked ksat*** field
α	avg	van Genuchten soil-water characteristic curve fitting parameter
n	nvg	van Genuchten soil-water characteristic curve fitting parameter
m	mvg	van Genuchten soil-water characteristic curve fitting parameter
ψ		soil suction

Required input: Saturated coefficient of permeability and van Genuchten and Mualem fit of the soil-water characteristic curve

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
van Genuchten Predicted	indicates if the fit algorithm has been successfully executed on the current data
van Genuchten Error	difference between the fit and laboratory values in terms of R^2

The van Genuchten's equation (1980) for fitting the soil-water characteristic curve data is as follows:

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{\left[1 + \left(\alpha \psi^n\right)\right]^m} \qquad [\ 246\ ]$$

where:

θ = *volumetric water content,*
θ_s = *saturated volumetric water content,*
θ_r = *residual volumetric water content, and*
α, *n and m* = *fitting parameters.*

van Genuchten (1980) suggested the use of the volumetric water content at 1,500 kPa as a residual value. For many soils this value may be a reasonably good approximation. An alternate analytical procedure is also suggested for estimating the residual water content.

Four independent parameters *(w_r, w_s, α, and n)* can be estimated from the soil-water characteristic curve data. This information is used in estimating the coefficient of permeability function for an unsaturated soil. The permeability function derived by van Genuchten (1980) is given below:

$$k_r(\psi) = \frac{\left(1 - (\alpha\psi)^{n-1}\left[1 + (\alpha\psi)^n\right]^{-m}\right)^2}{\left[1 + (\alpha\psi)^n\right]^{\frac{m}{2}}} \qquad [\ 247\ ]$$

Another permeability function was proposed based on the Burdine (1953) model. The equation is given below:

$$k_r(\psi) = \frac{1 - (\alpha\psi)^{n-2}\left[1 + (\alpha\psi)^n\right]^{-m}}{\left[1 + (\alpha\psi)^n\right]^{2m}} \qquad [\ 248\ ]$$

Figure 50 provides a comparison between the predicted and measured values of the soil-water characteristic curve along the drying and wetting paths and also the variation of permeability coefficient with respect to suction for Geulph loam (from van Genuchten, 1980). The equations proposed by van Genuchten provide close fits for most types of soils, with the exception of clayey soils.

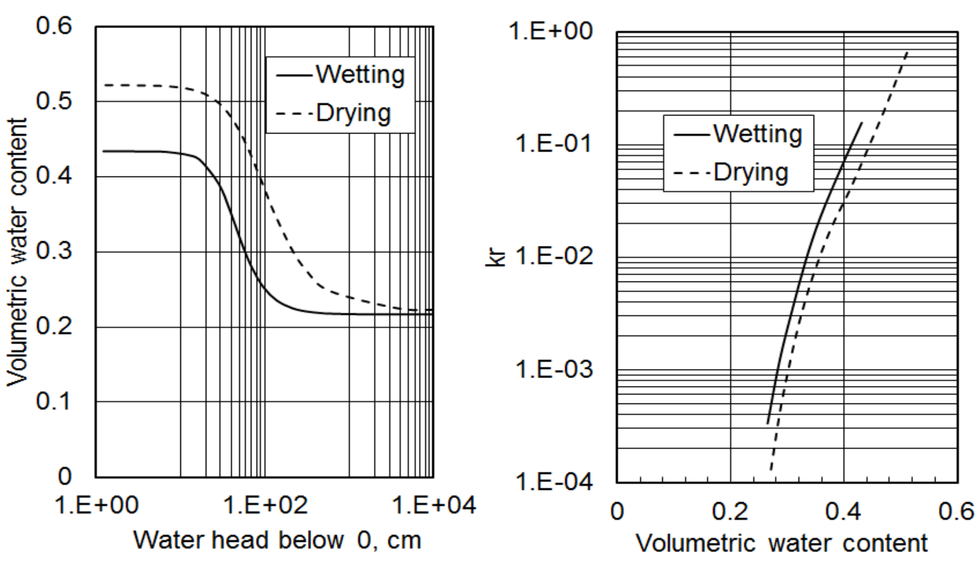


Figure 50 Comparison between the predicted (continuous solid lines) and measured values (solid circles) of the soil-water characteristic curve along drying and wetting paths and the variation of coefficient of permeability with respect to suction (data from van Genuchten, 1980)

5.3.5 Brooks and Corey (1964) Estimation

Brooks and Corey (1964) propose a permeability function for predicting the unsaturated coefficient of permeability. The estimation is based on a bi-linear fitting of the soil-water characteristic curve. The proposed equations for estimation of the saturated-unsaturated permeability function for a soil is as follows:

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Brooks and Corey Estimation

Formulation:

$$k_w = k_{sat}\left(\frac{\psi_b}{\psi}\right)^{2+3\lambda} \quad \text{for suction, } \psi > \psi_b \qquad [\ 249\ ]$$

$$k_w = k_{sat} \quad \text{for suction, } \psi \leq \psi_b \qquad [\ 250\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase
k_{sat}		saturated coefficient of permeability of the water phase
ψ_b	ac	Brooks and Corey (1964) soil-water characteristic curve fitting parameter
λ	nc	Brooks and Corey (1964) soil-water characteristic curve fitting parameter
ψ		soil suction

Required input: Saturated coefficient of permeability and a fit of the soil-water characteristic curve by the Brooks and Corey (1964) equation.

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Corey Predicted	indicates if the fit algorithm has been successfully executed on the current data
Corey Error	difference between the fit and laboratory values in terms of R^2

The Brooks and Corey (1964) equation that best-fits the soil-water characteristic curve data takes the form of a power-law relationship.

$$\Theta = \left(\frac{\psi_b}{\psi}\right)^{\lambda} \quad \text{for suction, } \psi \geq \psi_b \qquad [\ 251\ ]$$

where:

Θ = *normalized water content,*
ψ_b = *air-entry value,*
ψ = *suction, and*
λ = *pore-size distribution index.*

The normalized volumetric water content (or effective degree of saturation) is defined as:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \qquad [\ 252\ ]$$

where:

θ_S = *saturated volumetric water content, and*

θ_r = *residual volumetric water content.*

Equation [251] is suitable for fitting the laboratory data of soil-water characteristic curves of coarse soils that have a low air-entry value. Brooks and Corey (1964) also suggested a procedure for estimating the residual water content.

Brooks and Corey (1964) permeability function is based on the model of a porous medium developed by Burdine (1953), Kozeny (1927) and Wyllie and Gardner (1958). The function was derived based on the recommended functions shown below:

$$k_w = k_{sat} \qquad for \qquad \psi \leq \psi_b \qquad [\ 253\ ]$$

$$k_w = k_{sat}\,\theta^{\delta} \qquad for \qquad \psi > \psi_b \qquad [\ 254\ ]$$

where:

k_w = *coefficient of permeability with respect to the water phase for the soil saturation (i.e., S = 100 %), and*
δ = *empirical constant.*

The empirical constant, δ, in turn is related to the pore-size distribution index and is given by the relationship:

$$\delta = \frac{2+3\lambda}{\lambda} \qquad [\ 255\ ]$$

Soils with a wide range of pore sizes have a small value of λ. Figure 51 presents typical λ values for various soils that have been obtained from various soil-water characteristic curves.

The coefficient of water permeability with respect to degrees of saturation can be computed using equation [253] and equation [254]. The relative water phase coefficient of permeability, k_{rw} *(%)* can be estimated using the relationship given below:

$$k_{rw} = \frac{k_w\,(100)}{k_{sat}} \qquad [\ 256\ ]$$

Typical laboratory results for a sandstone expressed in terms of the relative permeability are shown in Figure 52. Brooks and Corey (1964) model is simple and can be used with a reasonable degree of success, particularly for coarse-grained soils such as sands and gravels.

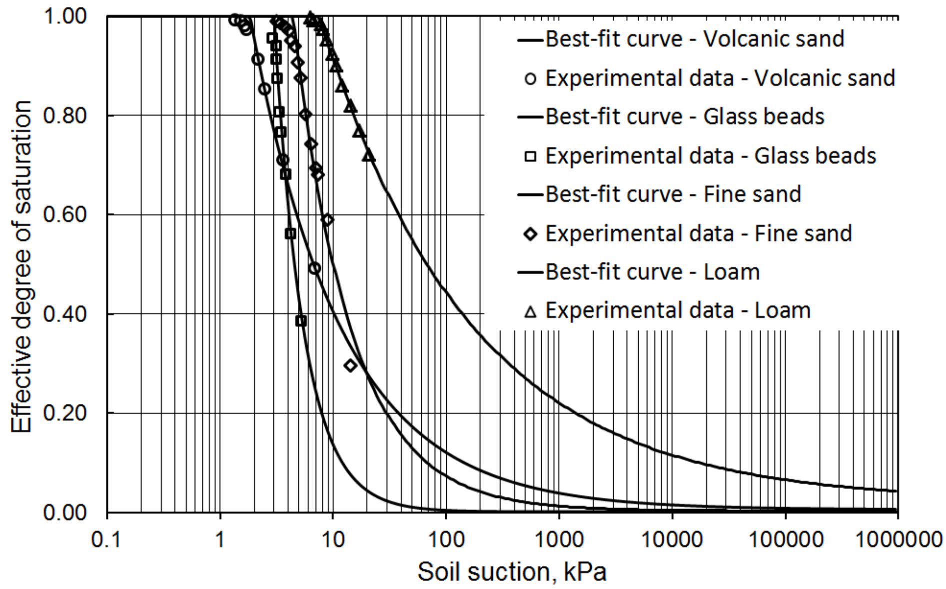


Figure 51 Typical soil-water characteristic curves for various soils (modified after Brooks and Corey, 1964).

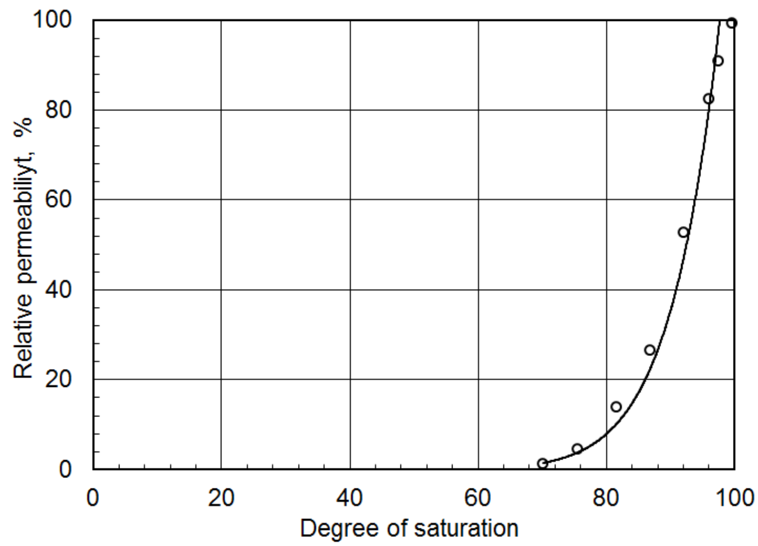


Figure 52 Relative permeability of water as a function of the degree of saturation during drainage (from Brooks and Corey, 1964).

5.3.6 Modified Campbell (1973) Estimation

The modified Campbell (1973) equation is implemented in SVSOILS to provide a coefficient of permeability equation that levels off at high suction values. This modification is in keeping with the theory that the coefficient of permeability of an unsaturated soil becomes essentially constant in the vicinity of residual suction. The point of residual suction is assumed to be near the point at which the water phase in an unsaturated soil becomes discontinuous. The Campbell (1974) equation was modified to produce an equation that would level to a limiting value near residual suction conditions. The modified equation as implemented into the SVSOILS software is presented below (M.D. Fredlund, 1996).

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Modified Campbell Estimation

Formulation:

$$k_w(\psi) = (k_{sat} - k_{\min})\left[\left[1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\left[\ln\left[\exp(1) + \left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right]\right]^p + k_{\min} \qquad [\ 257\ ]$$

Defintions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase
k_{sat}		saturated coefficient of permeability of the water phase determined by the ***MCampbell Linked ksat*** field
k_{min}		calculated minimum coefficient of permeability
p	MCampbell p	parameter used to control the modified Campbell (1973) estimation of coefficient of permeability
a_f	af	Fredlund & Xing (1994) soil-water characteristic curve fitting parameter
n_f	nf	Fredlund & Xing (1994) soil-water characteristic curve fitting parameter
m_f	mf	Fredlund & Xing (1994) soil-water characteristic curve fitting parameter
h_r	hr	Fredlund & Xing (1994) soil-water characteristic curve fitting parameter
ψ		soil suction

Required input: Saturated coefficient of permeability and a fit of the soil-water characteristic curve by the Fredlund and Xing (1994) equation.

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
MCampbell Predicted	indicates if the fit algorithm has been successfully executed on the current data
MCampbell Error	difference between the fit and laboratory values in terms of R^2

Campbell (1974) presents the following equation to model the reduction in the coefficient of permeability as a soil dries.

$$k_w = k_{sat}\Theta^p(\psi) \qquad [\ 258\ ]$$

where:

k_w = *permeability at any level of suction,*
k_{sat} = *saturated coefficient of permeability,*
ψ = *soil suction (kPa),*
Θ = *normalized volumetric water content or* θ_w/θ_s *represented with any equation (i.e., van Genuchten, Fredlund and Xing, etc.),*
p = *power factor to adjust the prediction.*

A modification was made to the Campbell (1974) equation before it was implemented into SVSOILS. This modification adjusts the Campbell equation such that the function flattens once a minimum permeability has been reached.

It is assumed that the coefficient of permeability remains relatively constant once the water phase in the soil becomes discontinuous. Water flow in the soil is then primarily the result of vapor diffusion through the air in the soil. The Campbell (1974) equation was modified to model this phenomenon as shown below:

$$k_w(\psi) = (k_{sat} - k_{\min})\Theta^p(\psi) + k_{\min} \qquad [\ 259\ ]$$

where:

k_{min} = *minimum permeability.*

The above equation allows the coefficient of permeability versus soil suction function to level off after a selected minimum permeability value. The minimum permeability value is assumed in SVSOILS to occur at one log cycle of suction higher than the suction corresponding to residual water content. A close agreement has been observed between the laboratory and calculated permeability data.

The Campbell (1974) procedure implemented into SVSOILS involves the development of an algorithm using the soil-water characteristic curve and the saturated coefficient of permeability to predict the coefficient of permeability of a soil at all levels of suction.

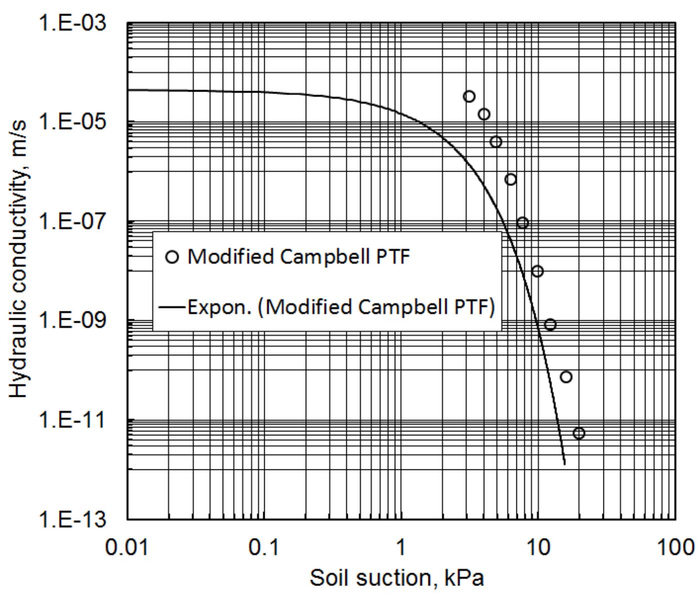


Figure 53 Comparison between predicted and laboratory (data for a Sand sample reported by Murray, 2000)

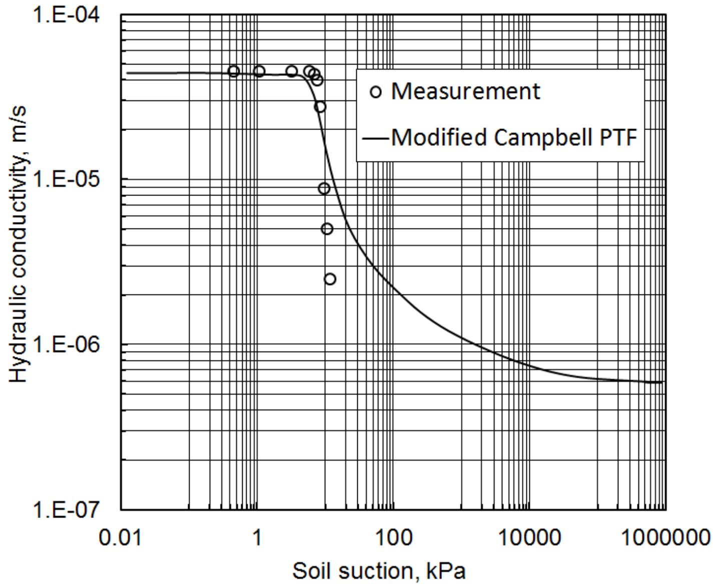


Figure 54 Comparison between laboratory and predicted (data for a Touchet Silt Loam reported by Leij et al., 1996)

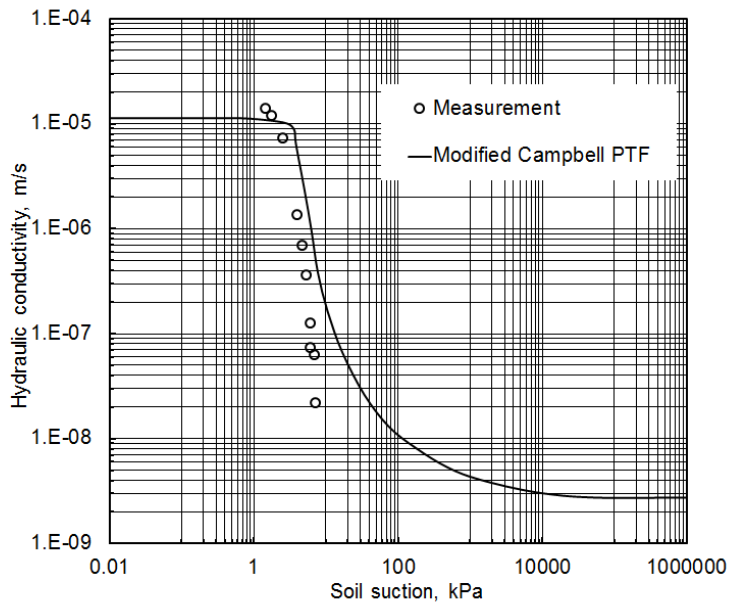


Figure 55 Comparison between laboratory and predicted conductivity (data for a Sand sample reported by Leij et al., 1996)

5.3.7 Mualem (1976) Estimation

Mualem (1976) suggested a simple statistical model to predict the unsaturated coefficient of permeability using the soil-water characteristic curve and the measured saturated coefficient of permeability. The Mualem model has been implemented into the SVSOILS software.

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Mualem Estimation

Formulation:

$$k_w(\psi) = k_{sat}\left[\left(\frac{n}{\psi}\right)^{\lambda}\right]^{n+2+2/\lambda} \qquad [\ 260\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase
k_{sat}		saturated coefficient of permeability of the water phase determined by the ***Mualem Linked ksat*** field
n	ac	Brooks and Corey (1964) soil-water characteristic curve fitting parameter
λ	nc	Brooks and Corey (1964) soil-water characteristic curve fitting parameter
ψ		soil suction

Required input: Saturated coefficient of permeability and a Brooks and Corey (1964) fit of the soil-water characteristic curve.

Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Mualem Predicted	indicates if the fit algorithm has been successfully executed on the current data
Mualem Error	difference between the fit and laboratory values in terms of R^2

Mualem (1986) provided an extensive summary of the statistical models for calculating the coefficient of permeability from the soil-water characteristic curve.

The following three common assumptions are characteristics of the statistical models:

- The flow system in a porous medium is simulated as a set of interconnected, randomly distributed pores with a frequency distribution, *f(r)*. The areal distribution is equal to *f(r)* and is the same for any cross-section.
- The Hagen-Poiseuille equation is valid at the singular pore level.
- The soil-water characteristic curve is considered to be related to the pore size distribution function using the capillary law.

Burdine (1953) suggested that the relative coefficient of permeability, k_r can be determined using the equation below:

$$k_r(\theta) = S_e^2 \frac{\int_{\theta=0}^{\theta} \frac{d\theta}{\psi^2}}{\int_{\theta=0}^{\theta_{sat}} \frac{d\theta}{\psi^2}} \qquad [\ 261\ ]$$

Mualem (1976) analyzed the Burdine (1953) model and other models such as the Childs and Collis-George (1950) model and suggested another relationship. To derive this relationship, Mualem (1976) considered soil as a porous media. Two imaginary parallel slabs normal to the flow direction and with a distance of *dx* were considered, were *dx* is the same order of pore radii. The following assumptions were also made:

- There is by-pass between the slab pores, and
- The pore configuration can be replaced by a pair of capillary elements whose lengths are proportional to radii (i.e., $l_1/l_2 = r/\rho$). The coefficient of permeability is then proportional to $r_e^2 = r_\rho$. A correlation factor was suggested to simulate the partial correlation between r and ρ. A tortuosity factor was also used. These factors were assumed to be a power function of the effective degree of saturation, S_e.

The effective degree of saturation, S_e is defined as below:

$$S_e = \frac{(\Theta - \Theta_r)}{(\Theta_{sat} - \Theta_r)} \qquad [\ 262\ ]$$

where:

Θ = *actual water content,*
Θ_r = *residual water content.*

The soil-water characteristic curve is not always measured for a large range of soil suction values. Consequently, it is not possible to define the residual water content from the laboratory data. Mualem (1976) suggested an analytical procedure for extrapolating the limited laboratory data and estimating the residual water content, Θ_r.

Based on these assumptions Mualem's derived the equation for predicting the coefficient of permeability as given below:

$$k_r(\theta)=S_e^n\left[\frac{\int_{\theta=0}^{\theta}\frac{d\theta}{\psi}}{\int_{\theta=0}^{\theta_{sat}}\frac{d\theta}{\psi}}\right]^2 \qquad [\ 263\ ]$$

The value of *n* in the above equation may be positive or negative and *n* is related to the pore sizes and tortuosity of soil. Studies have shown that *n* equal to 0.5 provides better fits for correlations between the measured and predicted values of coefficient of permeability.

The Mualem (1976) equation (i.e. equation [263]) is simple and easy to apply. For $\psi(\theta)$ given in analytical form, $k_r(\theta)$ can be derived explicitly (Mualem, 1976). In other words, equations such as Brooks and Corey (1964), Farrel and Larson (1972) can be substituted into equation [263] and closed-form relationships can be obtained for predicting the coefficient of permeability.

$$k_r(S_e)=S_e^{\,n+2+2/\lambda} \qquad [\ 264\ ]$$

Forty-five sets of measured data of coefficient of permeability were compared with using predicting procedures suggested by Averjanov (1950); Millington and Quirk (1961) and Wylie and Garner (1958) and Mualem (1976) model. The Mualem (1976) model provided an improved prediction of the permeability functions when compared to other models.

Mualem (1976) studies have been extended by various investigators to propose simpler permeability functions.

5.3.8 Leong and Rahardjo (1997) Estimation

Leong and Rahardjo (1997) propose a permeability function for predicting the unsaturated coefficient of permeability. The estimation is based on the fit of the soil-water characteristic curve with the Fredlund and Xing (1994) equation. The equation proposed for the estimation of the unsaturated permeability of a soil is as follows:

Menu location: SVFlux > Hydraulic Conductivity > Unsaturated Hydraulic Conductivity > Leong and Rahardjo Estimation

Formulation:

$$k_w(\psi)=k_{sat}\left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right]^p \qquad [265]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability or permeability of the water phase
k_{sat}		saturated coefficient of permeability of the water phase determined by the ***Leong Linked ksat*** field
p	Leong p	parameter used to control the Leong and Rahardjo (1997) estimation of coefficient of permeability
a_f	af	Fredlund and Xing (1994) soil-water characteristic curve fitting parameter
n_f	nf	Fredlund and Xing (1994) soil-water characteristic curve fitting parameter
m_f	mf	Fredlund and Xing (1994) soil-water characteristic curve fitting parameter
ψ		soil suction

Required input: Saturated coefficient of permeability, Leong *p* parameter, and either Fredlund and Xing (1994) or Fredlund (2000) bimodal fit of the soil-water characteristic curve.
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Leong Predicted	indicates if the fit algorithm has been successfully executed on the current data
Leong Error	difference between the fit and laboratory values in terms of R^2

The best-fitted permeability function was used by Leong and Rahardjo (1997) for comparing the predicted and measured coefficients of permeability. The study included several soil types and included both wetting and drying processes. A good fit to all experimental data was obtained. It was shown that if the exponent *p* was known for a given soil, the coefficient of permeability could be obtained indirectly from the soil-water characteristic curve. Otherwise, *p* can be determined by curve-fitting the permeability data. The value of *p* varied from 4.32 to 52.1 for the soils studied.

5.4 PERMEABILITY VERSUS VOID RATIO

SVSOILS provides the Taylor (1948) estimation method for calculating the relationship between the coefficient of permeability and void ratio.

5.4.1 Taylor (1948) Estimation

Taylor (1948) has provided a cubic equation that can be used to mathematically model the relationship between the coefficient of permeability and void ratio. Details of the equation are as follows.

Menu location: SVFlux > Ksat vs Void Ratio > Taylor Estimation

Formulation:

$$k_w(e) = \frac{C\,e^3}{(1+e)} \qquad [\ 266\ ]$$

Definitions:

Equation Variable	Dialogue Field Name	Description
k_w		coefficient of permeability
C	Taylor Coefficient	Taylor coefficient
e		void ratio

Fitting method: Least squares nonlinear regression
Required input: Taylor coefficient field
Applicable soil types: All soils

Modified fields:

Dialogue Field Name	Description
Taylor Predicted	indicates if the fit algorithm has been successfully executed on the current data
Taylor Error	difference between the fit and laboratory values in terms of R^2

5.4.1.1 Taylor (1948) Coefficient

This algorithm is provided to back-calculate the Taylor Coefficient field used in the Taylor PTF. The algorithm uses the laboratory saturated coefficient of permeability, ksat, value shown on the Hydaulic Conductivity dialog as well as the insitu void ratio presented in the Volume Mass dialog. The coefficient is back-calculated according to the following equation.

$$C = k_{sat}\,\frac{(1+e_o)}{e_o^3} \qquad [\ 267\ ]$$

where:

C	=	*Taylor coefficient,*
k_{sat}	=	*laboratory saturated coefficient of permeability as entered in the permeability form, and*
e_o	=	*initial void ratio.*

6 STATISTICAL CALCULATIONS

Statistical functions are useful when used in conjunction with soils data to calculate confidence limits, check correlations, and determine if relationships exist between soil parameters. SVSOILS allows free-form statistics to be calculated based on any field preset in the database. SVSOILS also allows statistical functions to be calculated based on any particular subset of the data.

SVSOILS implements many basic statistical functions used in the field of geostatistics. Statistical tools may be selected under the **Material > Grain-size > Statistics** menu.

It is not the purpose of this user manual to provide a complete description of geostatistics. The functions provided in SVSOILS are described in geostatistics textbooks.

6.1 UNIVARIATE STATISTICS

The unvariate form allows statistical description of a single field in the database. Frequency diagrams or histograms, probit plots, and an auto regression function provides the user with a number of methods of analyzing data.

6.1.1.1 Frequency Diagrams or Histograms

One of the most common and useful presentations of data sets is the histogram. A histogram illustrates how often observed values fall within certain intervals or classes. The histogram in SVSOILS currently develops a histogram based on ten equal divisions between the minimum and maximum of the selected field. The minimum and maximums are calculated as two standard deviations each direction from the mean.

6.1.1.2 Histograms and Normal Distribution

Plotted alongside the histogram is a normal distribution function. The normal or Gaussian distribution function is calculated based on the average and standard deviation of the selected field. It is often interesting to know how close a variable distribution comes to being Gaussian. The normal probability plot helps decide this question. Also see the description of the probit plot.

SVSOILS generates histograms on both an arithmetic and log scale. The logarithmic scale is useful for evaluating the lognormal distribution of such soil properties as saturated permeability.

6.1.1.3 Probit Plot

The probit plot, in addition to the normal distribution, gives an indication of whether the field is normally distributed. A perfect normal distribution will plot as a straight line on a probit plot. Probit plots are useful for checking for the presence of **multiple** populations. While small 'bumps' in the plots do not necessarily indicate multiple populations, they represent changes in the characteristics of the cumulative frequencies over different intervals.

6.2 STATISTICS THEORY

There are three general categories of summary statistics: measures of location, measures of spread, and measures of shape. Measures of locations and measures of spread will be covered in this manual.

Measures of location give us information about where various parts of the distributions lie. The mean, the median and the mode can give us some idea where the center of the distribution lies. Measures of spread are used to describe the variability of data values.

MEAN: The mean, m, as calculated in SVSOILS is the arithmetic average of the data values.

$$m = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad [\ 268\ ]$$

The number of data is n and x_i ... are the data values.

MEDIAN: The median, M, is the midpoint of the observed values if they are arranged in increasing order. Half of the values are below the median and half of the values are above the median. Once the data are ordered so that $x_1 \le x_2 \le ... \le x_n$, the median can be calculated from one of the following equations:

$$M = x\frac{n+1}{2} \quad \text{if } n \text{ is odd,} \qquad [\ 269\ ]$$

$$M = \frac{\left(x\frac{n}{2} + x\frac{n}{2}n \right)}{2} \text{ if } n \text{ is even} \quad [\ 270\ ]$$

Both the mean and the medium are measures of the location of the center of the distribution. The mean is quite sensitive to erratic high values.

MODE: The mode is the value that occurs most frequently. The class with the tallest bar on the graph gives a quick idea where the mode is.

MINIMUM: The smallest value is the data set is the minimum.

MAXIMUM: The largest value in the data set is the maximum.

6.3 MEASURES OF SPREAD

VARIANCE: The variance, σ^2, is given by:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^2 \quad [\ 271\ ]$$

It is the average squared difference of the observed values from their mean.

STANDARD DEVIATION: The standard deviation, σ, is simply the square root of the variance. It is often used instead of the variance and its units are the same as the unit of the variables being described.

6.4 BIVARIATE DESCRIPTION

While univariate tools can be used to describe the distribution of individual variables, we get a very limited view, however, if we analyze more than one variable at a time. Many important and interesting features of soils data sets are the relationships between variables. While SVSOILS implements the basic bivariate functions, it is considered beyond the scope of this user's manual to provide a complete description of bivariate statistics.

6.5 SCATTER PLOTS

The most common display of bivariate data is the scatter plot, which is an *x & y* graph of the data on which the *x*-coordinate corresponds to the value of one variable and the *y*-coordinate to the value of the other variable. The scatter plot is automatically generated when the bivariate calculations are executed.

6.6 LINEAR REGRESSION

A strong relationship between two variables can help us predict one variable if the other is known. The simplest recipe for this type of prediction is linear regressions in which we assume that the dependence of one variable on the other causes a relationship which can be described by the equation of a straight line:

$$y = a\,x + b \quad [\ 272\ ]$$

SVSOILS will attempt to fit a linear regression through bivariate data. The linear regression fit will give an indication of whether two fields are linearly related.

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