

# Performance of the GHS model in excavation projects

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## Introduction

It is observed in simulations of excavation projects using the Hardening Soil model (or HS small model) that the soil below the bottom of the excavation behaves less stiff than in reality. The reason is that the stiffness of soil is stress dependent and excavation releases the stress on top, however, in reality the stiffness will not reduce so much due to the pre-consolidation stress. Therefore, in the GHS model (generalized hardening soil model) the stress dependency formulation of the Hardening Soil small model has been altered in order to account for the pre-consolidation stress.

## Theory

The Generalized Hardening Soil model (GHS) is, basically, a more modular version of the original Hardening Soil small model. It has several switches which allow the users to change the configurations of strain and stress dependency.

### Strain dependency

Adding strain dependency to the HS model gives the HS small model. In the GHS model the users are able to switch on/off the strain dependent stiffness, which can be seen as a change between the HS model and HS small model. The strain dependency is controlled by the “Strain Dependent Stiffness” in the material properties input sheet. The strain dependent stiffness is not activated when the value is set to 0, which is the situation of the HS model; while change the value to 1, the situation is what is used in the HS small model.

### Stress dependency

There are two switches related to the stress dependency in the GHS model, which are the “Stress Dependent Stiffness” and the “Stress Dependent Formula”. The former controls whether and when the stress dependent stiffness is used; the latter defines which stress dependency relation to be used once the stress dependent stiffness is activated. To be more detailed, there are three options for the “Stress Dependent Stiffness” switch, namely:

- Option 0: Use constant  $E_{ur}$  all through the calculation, the value equals to the input unloading/reloading stiffness. This option is similar to  $m = 0$ .
- Option 1: Update  $E_{ur}$  for every calculation phase based on stress dependency formula determined by the “Stress Dependent Formula” switch, based on the stresses at the beginning of the calculation phase. This option is used, for example, in a Safety calculation, in which an update of stiffness during the phase is not required.

- Option 2: Update  $E_{ur}$  for every calculation step based on stress dependency formula determined by the “Stress Dependent Formula” switch, based on the stresses at the beginning of every calculation step.

There are also three options for the “Stress Dependent Formula” switch, namely:

- Option 0: The stress dependency is based on  $\sigma_3$  and strength parameters, which is:

$$E_{ur} = E_{ur}^{ref} \left( \frac{\sigma_3 + c \cdot \cot \phi}{\sigma^{ref} + \cot \phi} \right)^m$$

This is the same formula as used in the HS model.

- Option 1: The stress dependency is based on  $\sigma_3$  and pre-consolidation stress, which is:

$$E_{ur} = E_{ur}^{ref} \left( \frac{(\sigma_3 + p_c)/2}{p^{ref}} \right)^m$$

- Option 2: The stress dependency is based on mean effective stress and pre-consolidation stress, which is:

$$E_{ur} = E_{ur}^{ref} \left( \frac{(p' + p_c)/2}{p^{ref}} \right)^m$$

In the case of Option 1 or 2, a minimum value of the numerator of  $p^{ref}/100$  is used.

An overview of combinations of stress dependent stiffness switches is given in the Table 1.

SD formula \ SD stiffness		0	1	2
		Stress dependency based on the original formula of the HS model	Stress dependency based on $\sigma_3$ and mean stress	Stress dependency based on preconsolidation stress and mean stress
0	Constant $E_{ur}$ being equal to the input value	$E_{ur}$	$E_{ur}$	$E_{ur}$
1	Stiffness updates within each calculation phase	$E_{ur} = E_{ur}^{ref} \left( \frac{\sigma_3 + c \cdot \cot \phi}{\sigma^{ref} + \cot \phi} \right)^m$	$E_{ur} = E_{ur}^{ref} \left( \frac{(\sigma_3 + p_c)/2}{p^{ref}} \right)^m$	$E_{ur} = E_{ur}^{ref} \left( \frac{(p' + p_c)/2}{p^{ref}} \right)^m$
2	Stiffness updates within each calculation step	$E_{ur} = E_{ur}^{ref} \left( \frac{\sigma_3 + c \cdot \cot \phi}{\sigma^{ref} + \cot \phi} \right)^m$	$E_{ur} = E_{ur}^{ref} \left( \frac{(\sigma_3 + p_c)/2}{p^{ref}} \right)^m$	$E_{ur} = E_{ur}^{ref} \left( \frac{(p' + p_c)/2}{p^{ref}} \right)^m$

Table 1 The function of the stress dependent stiffness switches

Since other stiffnesses  $E_{50}$ ,  $E_{oed}$  and  $G_0$ , follow the same stress-dependency rule as  $E_{ur}$ , it should be noted that the stress dependency also influences the other stiffnesses. Moreover, it should be noted that in the new stress-dependent stiffness formulations (Option 1 and 2), the resulting  $E_{50}^{ref}$  (as can be observed from a simulation of a standard drained triaxial test at a cell pressure equal to the reference stress) is a bit higher than the input value of  $E_{50}^{ref}$ . The increase is larger for Option 2 than it is for Option 1.

## Plasticity model

The switch of the “Plasticity Model” is used to switch among different plastic yield functions. The explanation of the switcher options are listed in Table 2. It is obvious that if the value of the “Plastic Model” is set to 4, than the original HS model is reached.

Options	Plastic yield function
1	Mohr Coulomb model
2	Shear hardening + MC failure criterion
3	Cap hardening + MC failure criterion
4	Shear hardening + Cap hardening + MC failure criterion

Table 2 Options for the switch "Plasticity Model"

## Results

There are several combinations of the switchers to play with for various purposes. In this report, we mainly tested the application of using pre-consolidation-related stress dependency formula to overcome the problem mentioned in the Introduction, which is to obtain realistic unloading stiffness in excavation projects.

The comparison is between the original HS small model and the GHS model. The input material parameters for the two models are intend to be identical except for the “stress dependency formula”, hence, the switcher options are:

Stress Dependent Stiffness	2
Strain Dependent Stiffness	1
Plasticity Model	4
Stress Dependency Formula	2

Table 3 Options of the switches in this report

By applying the options above, the GHS model is actually equivalent to the HS small model with stress dependency determined by the pre-consolidation stress.

### Validation of the GHS model by the Tutorial Chapter 3

The chapter 3 of the tutorial manual of PLAIXS 2D is used for the validation of the GHS model. The sand and loam layers modeled by hardening soil model are replaced by GHS model with identical input parameters except for the stress-dependency option. The input parameters for the GHS model is listed in Table 4 and Table 5.

Identification		Loam_GHS	
Material model		User-defined	
Drainage type		Drained	
$\gamma_{\text{unsat}}$	kN/m <sup>3</sup>	17	Unsaturated unit weight
$\gamma_{\text{sat}}$	kN/m <sup>3</sup>	19	Saturated unit weight
Dilatancy cut-off		No	
e_init		0,5	Initial void ratio
E_50	kN/m <sup>2</sup>	1,20E+04	Reference secant stiffness
E_oed	kN/m <sup>2</sup>	8000	Reference tangent stiffness
E_ur	kN/m <sup>2</sup>	3,60E+04	Reference unloading/reloading stiffness
power (m)		0,8	Power for stress-level dependency of stiffness
$\vartheta$	°	29	Friction angle
$\psi$	°	0	Dilatancy angle
c	kN/m <sup>2</sup>	5	Cohesion
$\gamma_{0.7}$		1,00E-05	Shear strain at which $G_s=0,722G_0$
G_0^ref	kN/m <sup>2</sup>	1,50E+04	Reference shear modulus at very small strains ( $\epsilon < 10^{-6}$ )
v_ur		0,2	Poisson's ratio for unloading and reloading
P_ref	kN/m <sup>2</sup>	100	Reference stress
R_f		0,9	Failure ratio
$\sigma_t$	kN/m <sup>2</sup>	0	Tensile strength
failure (0:MC or 1:M-N)		0	Switcher for failure criteria (Mohr-Coulomb or Matsuoka—Nakai). We advise to use 0 for the MC criterion.
OCR		1	Overconsolidation ratio
POP		25	Preconsolidation pressure
k_0		0	$K_0$ , but will be overwritten by the $K_{0,x}$ at the "initial" tabsheet
k_0^nc		0,5152	$K_0$ value for normal consolidation pressure

v_u		0	Undrained Poisson's ratio, when the value is 0 the default value 0,495 is assigned.
M (internal)		0	These 3 values are automatically calculated by the executable "hscapitr.exe" when 0 are used.
K_s/K_c (internal)		0	
G_50^ref (internal)		0	
Stress Dependent Stiffness		2	See Table 1 and Table 2
Strain Dependent Stiffness		1	
Plasticity Model		4	
Stress Dependency Formula		2	
E_oed^ref	kN/m <sup>2</sup>	8000	Reference tangent stiffness for the interface, should be calculated based on the strength reduction factor.
c_ref	kN/m <sup>2</sup>	5	Reference cohesion for the interface, should be calculated based on the strength reduction factor.
φ (phi)	°	29	Friction angle of the interface, should be calculated based on the strength reduction factor.
ψ (psi)	°	0	Dilatancy angle of the interface, should be in consistent with the surrounding soil
UD-Power		0	User-defined power for stress dependency of stiffness of interface
UD-P^ref	kN/m <sup>2</sup>	100	User-defined reference stress of interface
K_0 determination		Automatic	Value of K0 for the initial phase
Data set		USDA	Flow parameters
Model		Van Genuchten	
Type		Loam	

Table 4 Input parameters of the loam layer

Identification		GHS_sand_2
Material model		User-defined
Drainage type		Drained
γ_unsat	kN/m <sup>3</sup>	17
γ_sat	kN/m <sup>3</sup>	20
Dilatancy cut-off		No
e_init		0,5
E_50	kN/m <sup>2</sup>	3,00E+04
E_oed	kN/m <sup>2</sup>	3,00E+04
E_ur	kN/m <sup>2</sup>	9,00E+04

power (m)		0,5
$\vartheta$	°	34
$\psi$	°	4
c	kN/m <sup>2</sup>	0
$\gamma_{0.7}$		1,50E-04
G_0^ref	kN/m <sup>2</sup>	3,75E+04
e_init		0,5
v_ur		0,2
P_ref	kN/m <sup>2</sup>	100
R_f		0,9
$\sigma_t$	kN/m <sup>2</sup>	0
failure (0:MC or 1:M-N)		0
OCR		1
POP		0
k_0		0
k_0^nc		0,4408
v_u		0
M (internal)		0
K_s/K_c (internal)		0
G_50^ref (internal)		0
Stress Dependent Stiffness		2
Strain Dependent Stiffness		1
Plasticity Model		4
Stress Dependency Formula		2
E_oed^ref	kN/m <sup>2</sup>	3,00E+04
c_ref	kN/m <sup>2</sup>	1,00E-03
$\phi$ (phi)	°	25,3
$\psi$ (psi)	°	0
UD-Power		0,5
UD-P^ref	kN/m <sup>2</sup>	100
K_0 determination		Manual
K_0,x		0,4408

Table 5 Input parameters of the sand layer

Figure 1 shows the influence of different stress dependency configuration, where we can see that the uplift of the bottom of excavation reduces significantly (42.8%) when the stress dependency option 2 is applied.

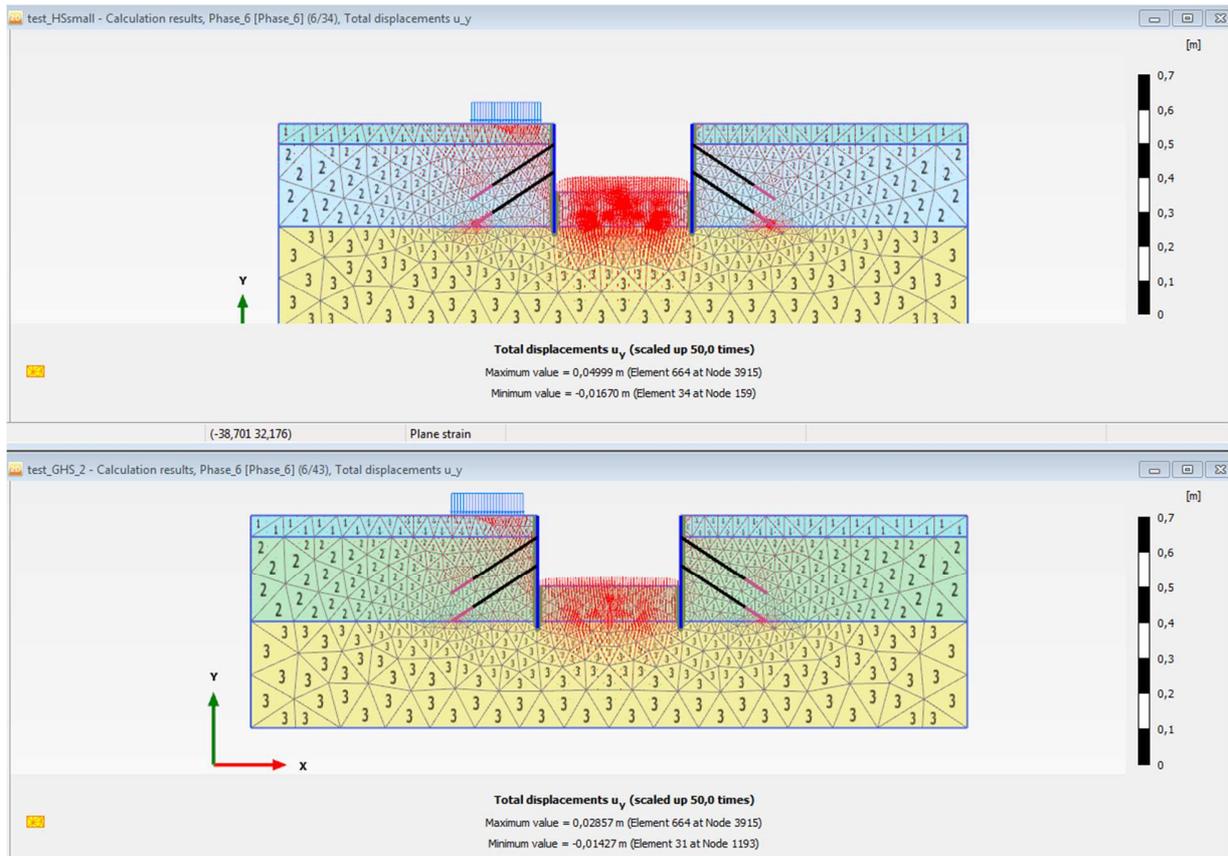


Figure 1 Comparison of the results from HS small model and GHS model

## Validation of the GHS model by the Tutorial Chapter 2

Identification		Sand_GHS
Drainage type		Drained
$\gamma_{\text{unsat}}$	kN/m <sup>3</sup>	17
$\gamma_{\text{sat}}$	kN/m <sup>3</sup>	20
Dilatancy cut-off		No
e_init		0,5
e_min		0
e_max		999
E_50	kN/m <sup>2</sup>	4,00E+04
E_oed	kN/m <sup>2</sup>	4,00E+04
E_ur	kN/m <sup>2</sup>	1,20E+05
power (m)		0,5
$\vartheta$	°	32
$\psi$	°	2
c	kN/m <sup>2</sup>	0
$\gamma_{0.7}$		1,00E-05
G_0^ref	kN/m <sup>2</sup>	5,00E+04

e_init		0,5
v_ur		0,2
P_ref	kN/m <sup>2</sup>	100
R_f		0,9
σ_t	kN/m <sup>2</sup>	0
failure (0:MC or 1:M-N)		0
OCR		1
POP		0
k_0		0
k_0^nc		0,4701
v_u		0
M (internal)		0
K_s/K_c (internal)		0
G_50^ref (internal)		0
Stress Dependent Stiffness		2
Strain Dependent Stiffness		1
Plasticity Model		4
Stress Dependency Formula		2
E_oed^ref	kN/m <sup>2</sup>	4,00E+04
c_ref	kN/m <sup>2</sup>	1,00E-05
φ (phi)	°	32
ψ (psi)	°	2
UD-Power		0,5
UD-P^ref	kN/m <sup>2</sup>	100
K_0 determination		Automatic
K_0,x		0,4701

Table 6 Input parameters for the GHS model

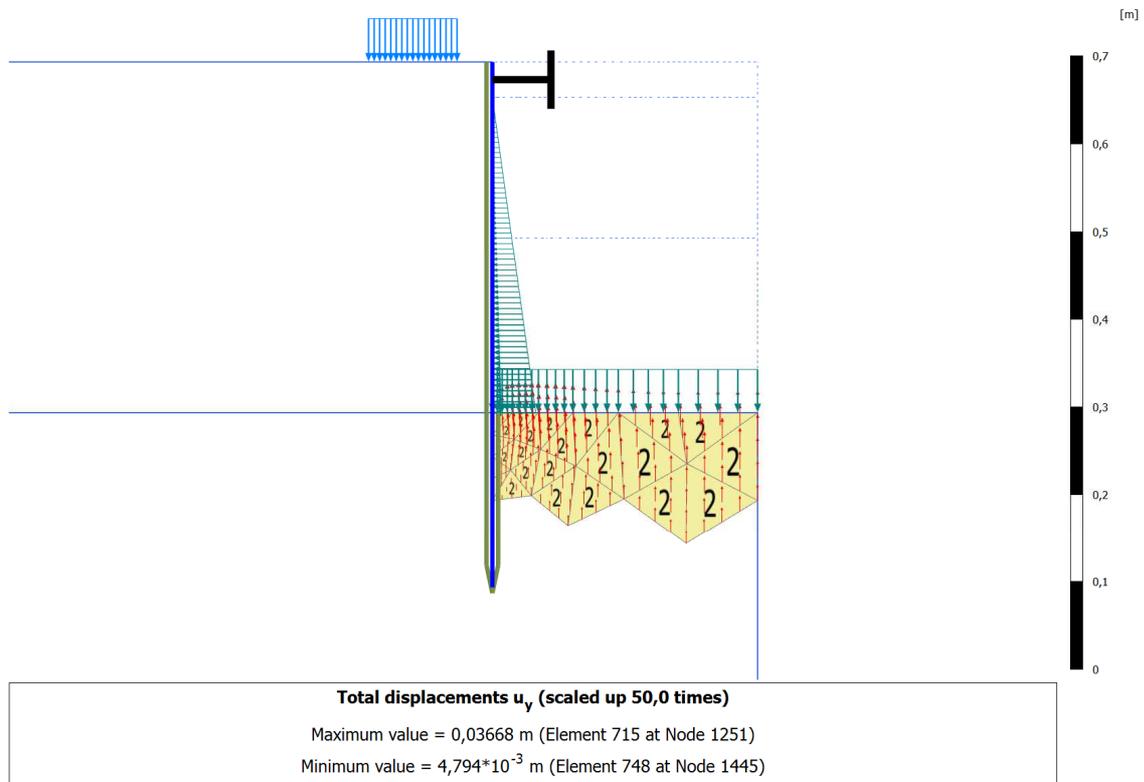
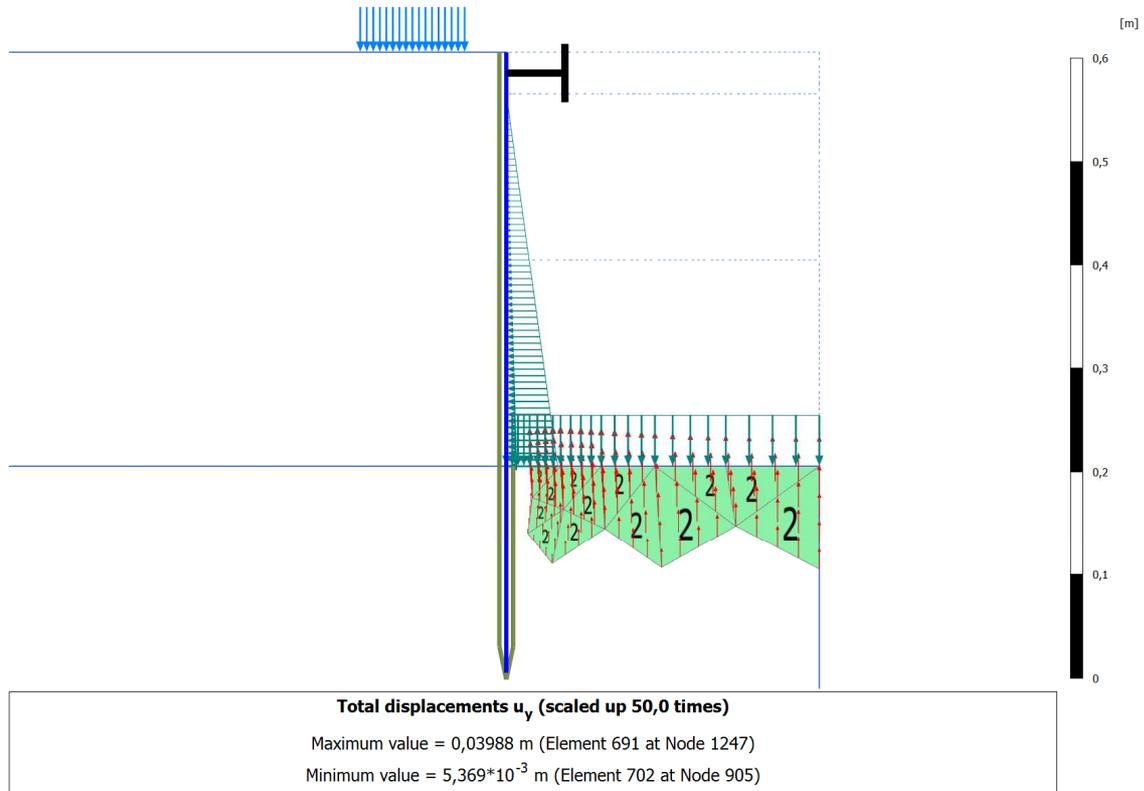


Figure 2 Vertical displacement of the bottom of the excavation from the GHS model



**Figure 3 Vertical displacement of the bottom of excavation from the HS small model**

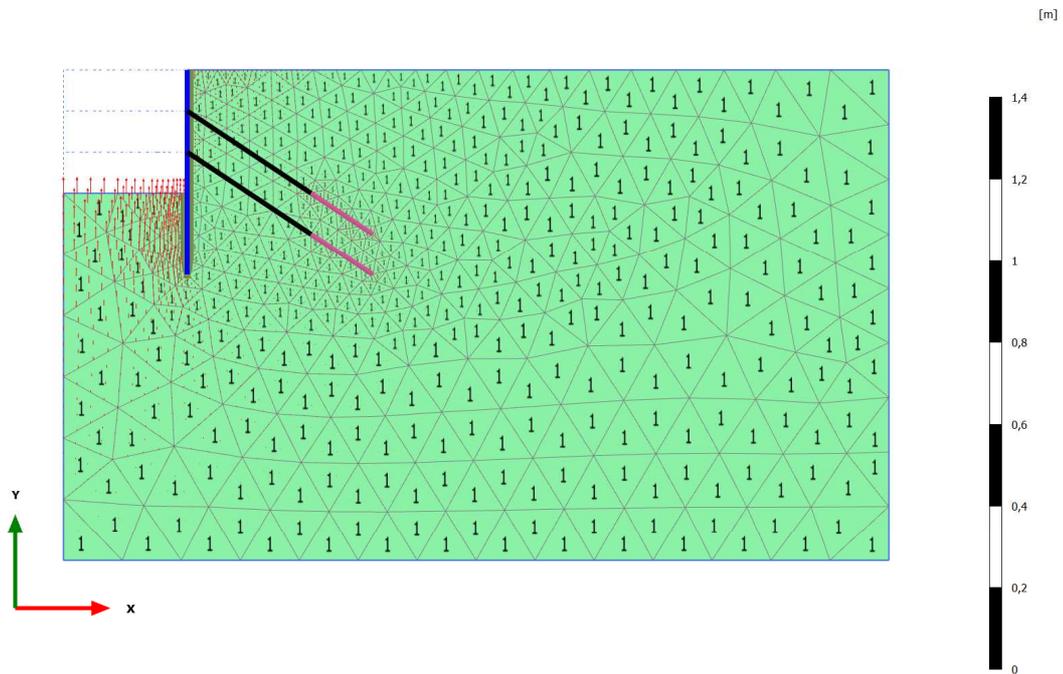
Comparison of Figure 2 and Figure 3 shows the influence of the stress dependency configuration. The results predicted by the GHS model show less uplift (approximately 8%).

### Validation of the GHS model by the other tied-back excavation case

Identification		Sand_GHS
Drainage type		Drained
$\gamma_{\text{unsat}}$	kN/m <sup>3</sup>	18
$\gamma_{\text{sat}}$	kN/m <sup>3</sup>	18
Dilatancy cut-off		No
e_init		0,5
e_min		0
e_max		999
Rayleigh $\alpha$		0
Rayleigh $\beta$		0
E_50	kN/m <sup>2</sup>	2,00E+04
E_oed	kN/m <sup>2</sup>	2,00E+04
E_ur	kN/m <sup>2</sup>	8,00E+04
power (m)		0,5

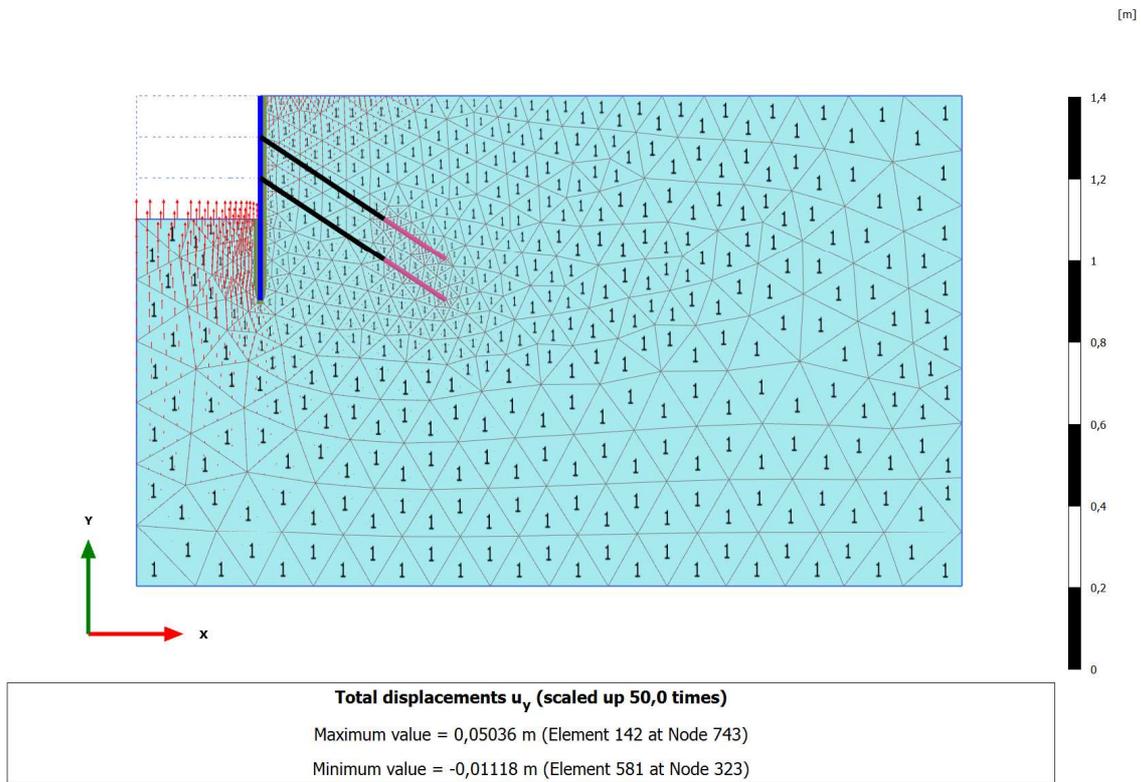
$\theta$	°	35
$\psi$	°	5
c	kN/m <sup>2</sup>	1
$\gamma_{0.7}$		1,50E-04
$G_0^{\text{ref}}$	kN/m <sup>2</sup>	1,00E+05
e_init		0,5
v_ur		0,2
P_ref	kN/m <sup>2</sup>	100
R_f		0,9
$\sigma_t$	kN/m <sup>2</sup>	0
failure (0:MC or 1:M-N)		0
OCR		1
POP		0
k_0		0
$k_0^{\text{nc}}$		0,4264
v_u		0
M (internal)		0
$K_s/K_c$ (internal)		0
$G_{50}^{\text{ref}}$ (internal)		0
Stress Dependent Stiffness		2
Strain Dependent Stiffness		1
Plasticity Model		4
Stress Dependency Formula		2
$E_{\text{oad}}^{\text{ref}}$	kN/m <sup>2</sup>	2,00E+04
c_ref	kN/m <sup>2</sup>	0,6
$\phi$ (phi)	°	22,79
$\psi$ (psi)	°	5
UD-Power		0,5
UD-P <sup>ref</sup>	kN/m <sup>2</sup>	100
$K_0$ determination		Manual
$K_{0,x}$		0,4264

Table 7 Input parameters



**Total displacements  $u_y$  (scaled up 50,0 times)**  
 Maximum value = 0,04058 m (Element 43 at Node 759)  
 Minimum value =  $-7,486 \cdot 10^{-3}$  m (Element 581 at Node 323)

Figure 4 Results from the GHS model



**Figure 5 Results from the HS small model**

Comparison of Figure 2 and Figure 3 shows the influence of the stress dependency configuration. The results predicted by the GHS model show less uplift (approximately 19.4%).

### Validation of the GHS model by the tutorial chapter 5

This is a tunnel excavation project. The geometry of the model is shown in Figure 6. There are 4 layers of soil in the model, in this validation the bottom layer (deep sand) originally modelled by the hardening soil small model is going to be replaced by the GHS model.

The excavation leads to stress releases; therefore it is expected to have different displacements in x direction. As shown in Figure 7 and Figure 8, the horizontal displacement predicted by the GHS model is 11.6% smaller than the hardening soil small model.

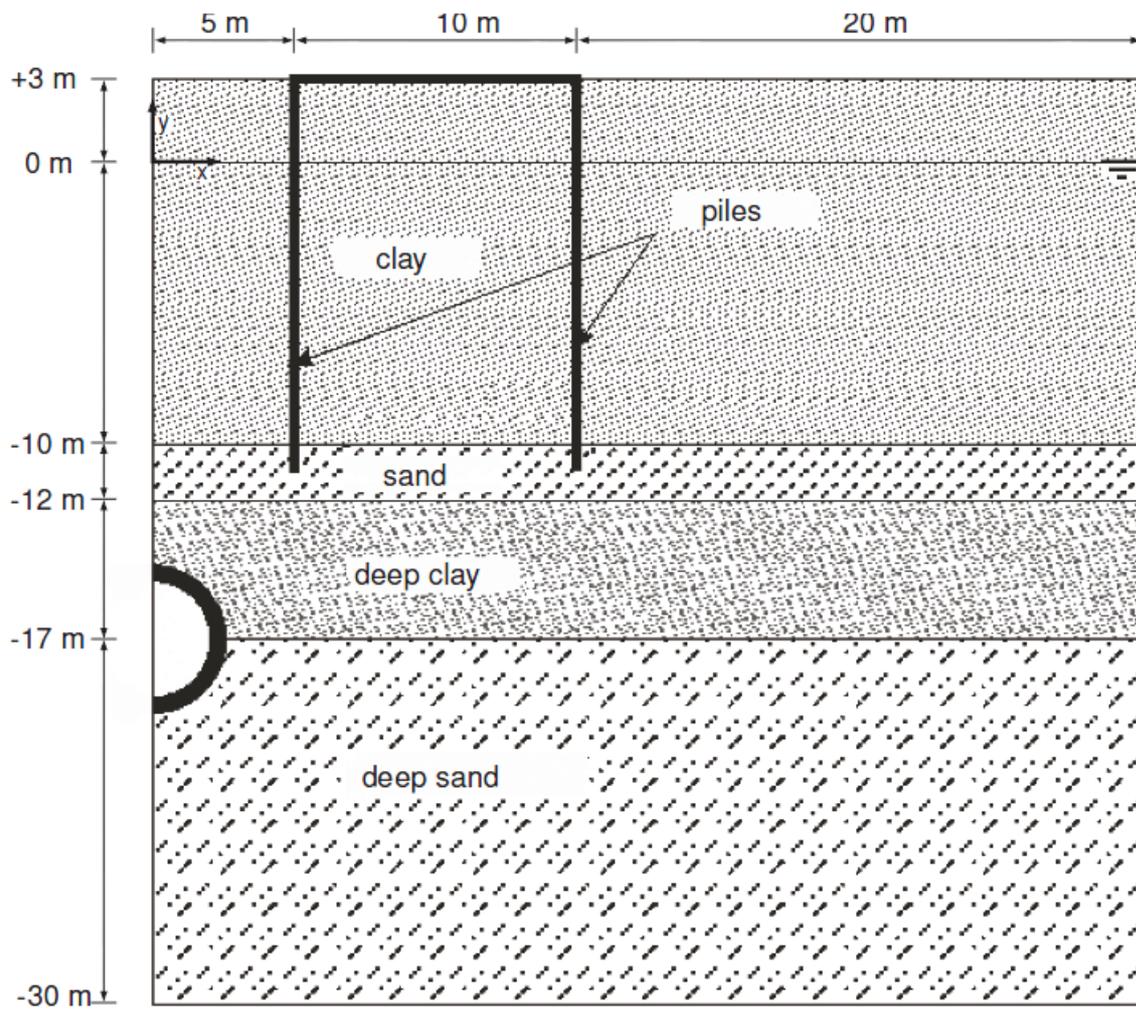
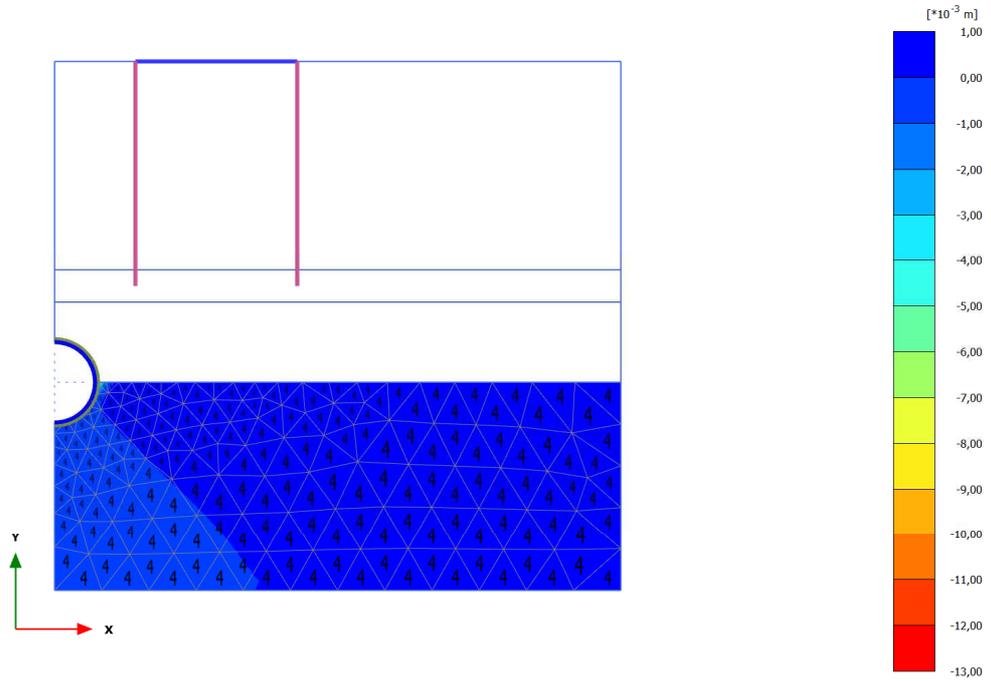
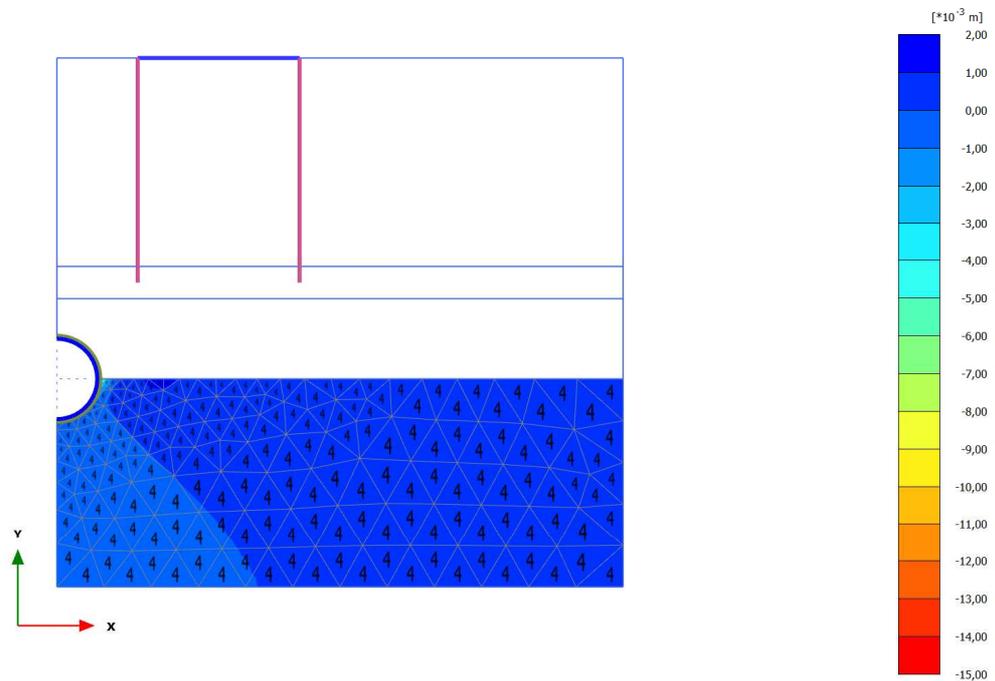


Figure 6 Geometry of the tunnel project



**Total displacements  $u_x$**   
Maximum value =  $0,9203 \cdot 10^{-3}$  m (Element 1154 at Node 6375)  
Minimum value =  $-0,01263$  m (Element 1176 at Node 6063)

Figure 7 Results from the GHS model



**Total displacements  $u_x$**   
 Maximum value =  $1,041 \cdot 10^{-3}$  m (Element 1159 at Node 6369)  
 Minimum value = -0,01464 m (Element 1176 at Node 6063)

Figure 8 Results from the HS small model