The role of numerical analysis in the study of the behaviour of hard-rock pillars

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The role of numerical analysis in the study of the behaviour of hard-rock pillars

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Abstract. Rock pillars can be defined as the in-situ rock between two or more underground openings. However, one aspect that is seldom considered in analysis of hard rock pillars, and indirectly in synthetic rock mass models to determine rock mass strength, is the actual stress level and stress path imposed on the pillar due to the excavation sequence and the location of the pillars within the mine lay out. In this paper we propose to use numerical stress analysis to determine how stresses vary across the excavated pillars in a typical room-and-pillar mine lay out, and thus generate a spatially variable determination of pillar stability. Because of the computational difficulty associated with hybrid modelling of realistic discrete fracture networks, synthetic rock mass modelling is commonly carried out using a 2D approach. By comparing 3D and 2D results for selected cross-sections across, we believe the results of the analysis will provide the opportunity to better constrain the stability implications of a 2D approach to pillar design and synthetic rock mass modelling.

1. Introduction

Rock pillars can be defined as the in-situ rock between two or more underground openings. Over the past decades hard rock pillars have been the subject of many studies that have culminated in the formulation of empirical pillar strength equations [e.g., 1 to 7]. In the past 20 years we have also observed the focus of the research shifting from empirical observations to numerical analysis [e.g., 8; 9, 10, 11, 12]. Numerical models allow to include scenarios that transcend those encountered in a mine, and to study the failure process in more details. However, despite increased modelling ability, full-scale forward modelling of mine systems remains problematic. Numerical models either trade the lack of explicitly considering fracturing and kinematics processes for a 3D mine system and the assumption of the rock mass being an equivalent continuum media, or they do consider explicit simulations of fracturing and kinematics processes but limited to 2D/3D analysis of limited system, as in the cause of numerical simulations of pillar strength.

To this date the introduction of a 3D mine system model being able to incorporate fracture mechanics principles at the level required to capture failure both globally and locally, while at the same time including a detailed representation of the rock mass structural character, remains a challenge. Modelling thus requires the real problem be idealised and simplified. In this paper we attempt to define the impact that such a simplification may have on design consideration, by including a comparison between 2D and 3D modelling approaches, and single system analysis (isolated pillar) versus mine systems (mine layout).

1.1. A philosophical discussion on the use of numerical modelling in rock engineering

As discussed in Elmo et al. [13, 14], many of the details of rock mass behaviour are unknown and unknowable; therefore, geomechanical models are constrained by the impossibility to reduce the uncertainty associated with data collection (geological uncertainty), modelling (parameter and model uncertainty) and persons (human uncertainty). As shown in figure 1, we cannot completely reduce uncertainty and the potential permeation of cognitive biases in the modelling process [15]; accordingly, a complex numerical model does not necessarily provide more accurate predictions than a simple one, and, as discussed by [16], a model is not and cannot be a perfect imitation of reality.

Nonetheless, we must acknowledge that numerical modelling plays an important role in the design of rock engineering systems, which often involve complex processes that cannot be fully described by empirical methods. To be successful, modelling strategies need to be driven by motivations, questions, objectives, and mechanisms. On a qualitative basis (figure 1), a rock mass could be classified into three groups: (i) massive rock mass; (ii) blocky to very blocky rock mass; and (iii) very blocky to disintegrated rock mass. Some regions of the rock mass could be treated as a continuum media, whilst discontinuum analysis would be better suited to capture the key role of natural fractures.

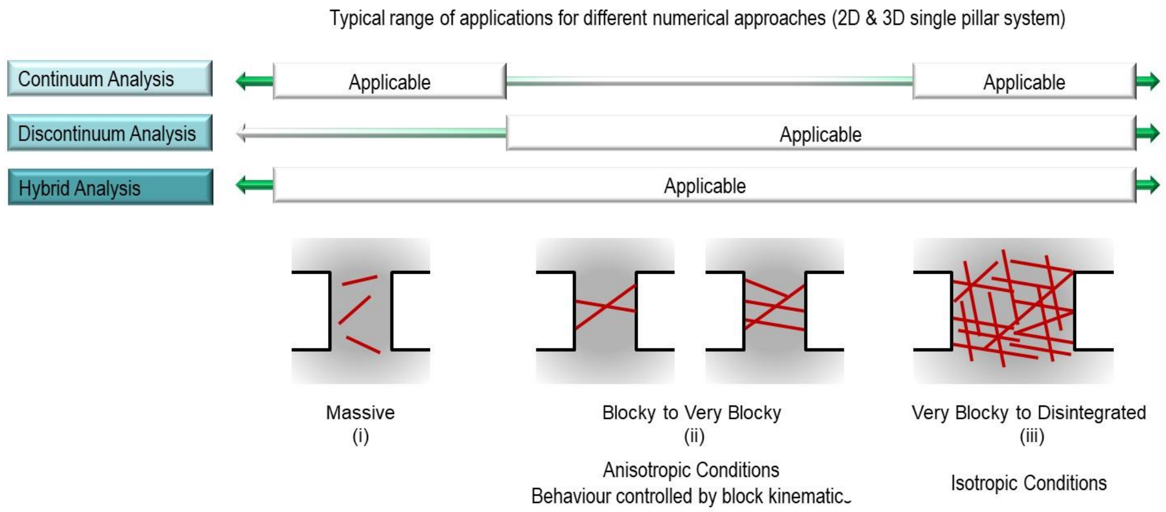

Figure 1. Qualitative classification of numerical strategies relative to the structural character of the rock mass.

The concepts of continuum and discontinuum are, however, not absolute but relative and problem specific, depending on the problem scale [17]. The choice of a modelling strategy (Finite Element continuum models - FEM - vs. Discrete Element discontinuum models - DEM - vs. Hydrid FEM/DEM models) must be driven by questions and mechanisms, and the motivation of the model should be to further our understanding of the processes involved [18]. Those become the metric that should be used to define the effectiveness of a model. Modelling should be guided by precise objectives, and engineers and practitioners would first need to establish which process need to be considered explicitly and which can be represented in an average way [19]. The question of whether we should use 2D or 3D numerical analysis, FEM vs. DEM vs. other forms of numerical analysis (hybrid models) generally produces opposite and dividing opinions, which, in the authors opinions, tend to culminate in a very unproductive debate. This discussion, albeit philosophical, is important to explain the motivation of the numerical analysis presented in the following sections. The objective is not a back analysis of an existing mine, rather a comparison of different numerical strategies and empirical methods.

2. Strength of hard rock pillars

Stress distribution in pillars is a function of the pillar geometry [8], ratio of σ_1 to σ_3 within the pillars [7]. However, as reported by [20], confinement in a pillar also depends on the ratio between the far-field horizontal stress k; the effects of k could be ignored only for pillars with width to height ratio of less than 1. Pillar strength is defined as the ultimate stress, and it generally estimated using empirical formulae based on survey data from actual mining conditions [21]. However, empirical methods fail to consider specific failure mechanisms and limitations associated with their intrinsic derivation from specific material properties (size, shape and stress conditions, presence of natural fractures). Numerical simulations offer a potentially useful means of overcoming some of the limits of the empirical methods (9). A summary of the most common empirical pillar strength formulae is provided by (8). Pillar formulae can be expressed according to the following general expression:

$$\sigma_p = K\left(A + B\frac{W^{\alpha}}{H^{\beta}}\right) \tag{1}$$

where σ_p is the pillar strength, K represents the strength of a unit cube of the rock mass, and W and H are the pillar width and pillar height, respectively. The constants *A*, *B*, α and β are derived empirically. Shape effect formulae use empirical constants α and β equal to 1, meaning that pillar strength is independent of the pillar volume. Size effect formulae assume *A* equal to zero and *B* equal to 1, while the constants α and β are not equal, meaning that pillar strength will decrease as pillar volume increases (for pillars of the same shape). In this paper we will refer to three different pillar formulae:

$$\sigma_p = K\left(0.778 + 0.222\frac{W}{H}\right) \tag{2}$$

$$\sigma_p = K\left(\frac{W^{0.46}}{H^{0.66}}\right) \tag{3}$$

$$\sigma_p = 0.44\sigma_c(0.68 + 0.52\kappa) \tag{4}$$

Equations (2) and (3) are derived from [6] and [2], respectively. In Equation (4), derived from [7] *k* represents a mine pillar friction term calculated from the average minor/major stress ratio within the pillar core. All these equations generally refer to pillars that are square in shape and they have been determined empirically based on a combination of numerical analysis (elastic modelling) and field observations of pillars conditions. When later plotting these equations as a function of the pillars' width-to-height ratio (*W:H*), we assumed that *W* remains constant and *H* increases, or vice versa.

Under the assumption of a 2D plane strain analysis of a series of mine pillars, those would represent rib-pillars, i.e., pillars with length in the out of plane direction that is significantly larger than the pillar width. For a given pillar width *W*, rib pillars would be able to carry a much larger load than equivalent square pillars. Therefore, under the same gravitational stress conditions and same geological conditions, a rib-pillar would be characterised by a much lower average induced stress across its mid-section compared to a square pillar. These aspects raise important questions, including:

- Is it correct to use these empirical strength equations to determine the factor of safety of hard rock pillars if the imposed stresses are determined using 2D stress analysis?
- If not, can we obtain reasonable estimation of factor of safety by comparing 2D stress analysis of mine induced stresses to pillar strengths determined using 2D synthetic rock mass models?

To answer those questions, Section 3 below presents the results of a 2D/3D stress analysis based on an actual room-and-pillar layout system.

3. Case study

We have used published data from an abandoned room-and-pillar mine in the United Kingdom as the basis of the discussion about the differences between 2D and 3D design approaches for hard rock pillars (18). Middleton mine (Derbyshire, UK) was a classic square room-and-pillar mining operation with drift access working mostly under a cover of about 150 m. As-built pillars dimensions were 14 m x 14 m dimensions in plan with rooms 14 m wide. Rooms were created in single pass operations (as built height of 7 m) with double height rooms created in suitable ground. The Middleton mine orebody consisted of a strong, thickly bedded crystalline limestone. Material properties and rock mass characteristics are summarised in table 1.

Table 1. Material properties for the limestone orebody at Middleton mine.

Depth (m)	UCS (MPa)	E (GPa)	v (-)	Stress ratio k (-)	RMR (-)
150	48	27	0.3	0.5	70 - 75

3.1. FEM Analysis

Three different modelling scenarios have been used to assess the general stability along with the stress state for the three selected pillars (figure 2a):

- 3D Model – General assessment for the three selected pillars (figure 3b)
- 2D Model Sec. A-A – Comparison with stresses in 3D for the Pillar 1
- 2D Model Sec. B-B – Comparison with stresses in 3D for Pillars 1, 2 and 3

The analysis has been conducted using the 2D and 3D versions of FEM code PLAXIS (23). Mine configurations with 7 m high and 14 m high pillars have been simulated. Mine advance was simulated in 7-stages for each mine lift in the 3D model, whilst in the 2D model the excavation of each room was simulated in two phases (50% deconfinement for each phase; the value was chosen based on tests at lower intervals of deconfinement showing that negligible plasticity had occurred).

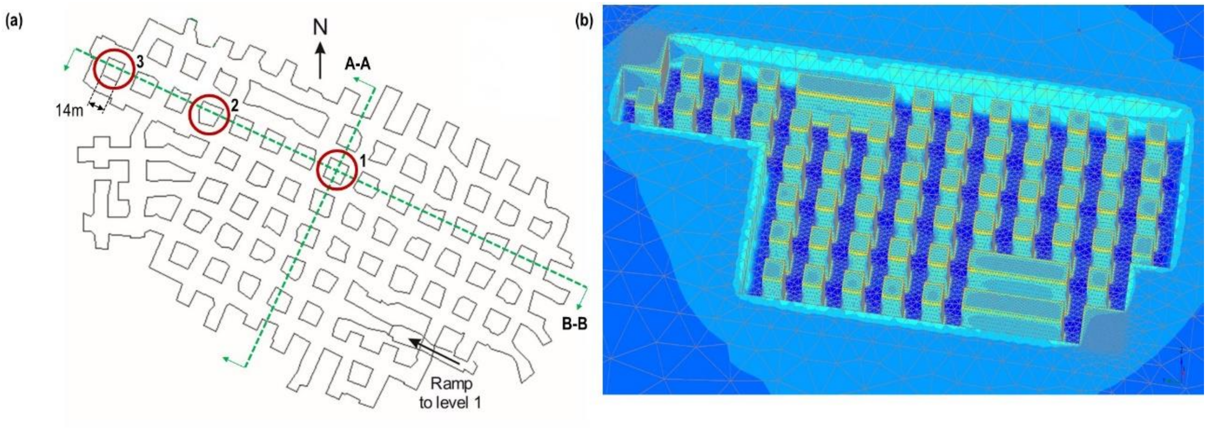

Figure 2: (a) mine lay-out and selected pillars use in the analysis; and (b) 3D stress model created in PLAXIS.

The results are reported in table 2, indicating the ratio between the maximum principal max stress and the uniaxial compressive strength (σ_1/UCS) for Pillars 1, 2 and 3 shown in figure 2. The values of σ_1 have been determined by averaging along 4 orthogonal linear cross sections at mid-height of the 3D pillars. Note that the elevation of the linear cross sections is 146.5 m and 150 m for the models with 7 m high and 14 m high pillars, respectively. Figure 3 compares the pillar stress measured in the 3D and 2D FEM models to the pillar strength Equations presented in Section 2. It is apparent how the determination of the factor of safety (FoS) of the pillars would be a function of the different pillar strength equation assumed in the design process. The value of the constant K (strength of a unit cube of rock mass) has been calculated using the scaling law relationship by [23] with exponent of 0.22, to yield a value of 26.4 MPa.

Table 2. Ratios between average σ_1 and UCS for pillars 1, 2 and 3 in figure 3.

PLAXIS Results	W:H	σ_1 3D Model (MPa)	σ_1 2D Model (MPa)	(σ_1/UCS) 3D	(σ_1/UCS) 2D
Pillar 1 7m AA	2	11.61	7.22	0.24	0.15
Pillar 1 7m BB	2	11.61	7.49	0.24	0.16
Pillar 2 7m BB	2	10.62	7.39	0.22	0.15
Pillar 3 7m BB	2	9.04	7.09	0.19	0.15
Pillar 1 14m AA	1	11.99	7.01	0.25	0.15
Pillar 1 14m BB	1	12.11	7.45	0.25	0.16
Pillar 2 14m BB	1	10.62	7.36	0.22	0.15
Pillar 3 14m BB	1	8.71	6.83	0.18	0.14

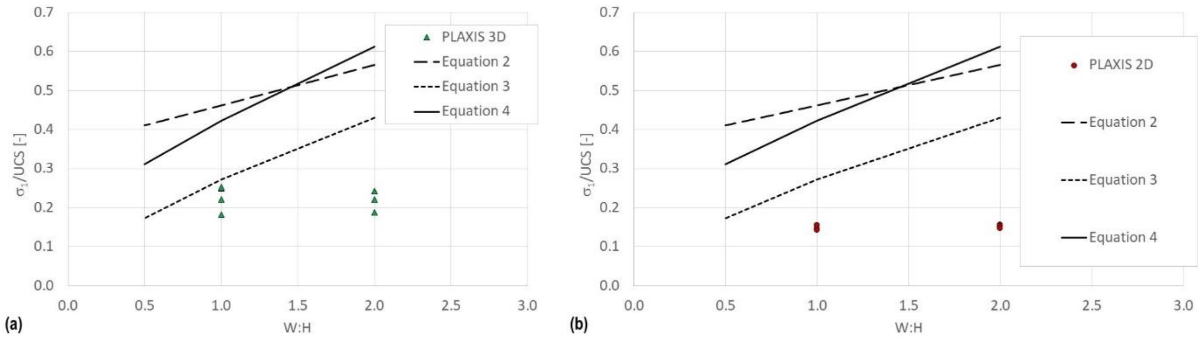

Figure 3: Comparison between 3D and 2D FEM results with empirical pillar strength formulae ((a) and (b), respectively).

The results of the 3D models support the field observations in terms of stability of the pillars. For instance, we know that at Middleton mine the pillars with a height dimension of 7 m (W:H of 2) were generally stable, with over-breaking mostly due to blasting induced damage. Likewise, field conditions for the 14 m high pillars (W:H of 1) appeared to also indicate a FoS greater than 1. Furthermore, when comparing the FoS calculated using Equation (3) (most conservative approach) for all the pillars across the mine layout, it is apparent how a typical room-and-pillar design could be improved by optimising the dimensions of the pillars to reflect their actual location. For instance, figure 4 clearly shows how the FoS for the different pillars would progressively increase away from the center of the panel layout.

Conversely, the 2D FEM results in table 2 indicate an average pillar stress for Pillars 1, 2 and 3 that are approximately 30% to 35% lower than the equivalent 3D pillar stresses. 2D results appear to be less sensitive to the actual location of the pillars, i.e., whether the pillar is close or away from the mine abutments. It would not be truly correct to compare 2D stress results to the strength values determined using empirical equations, since the latters are based on visual observations of actual 3D pillars. As shown in figure 5, ratios of pillar stress at failure to UCS for simulated 2D pillars at Middleton mine [17] show a relatively large scatter due to the different fracturing degree used in the models by [17]. Note that those were hybrid FEM-DEM models explicitly simulating failure of the pillar as a combination of intact rock failure and sliding along natural discontinuities.

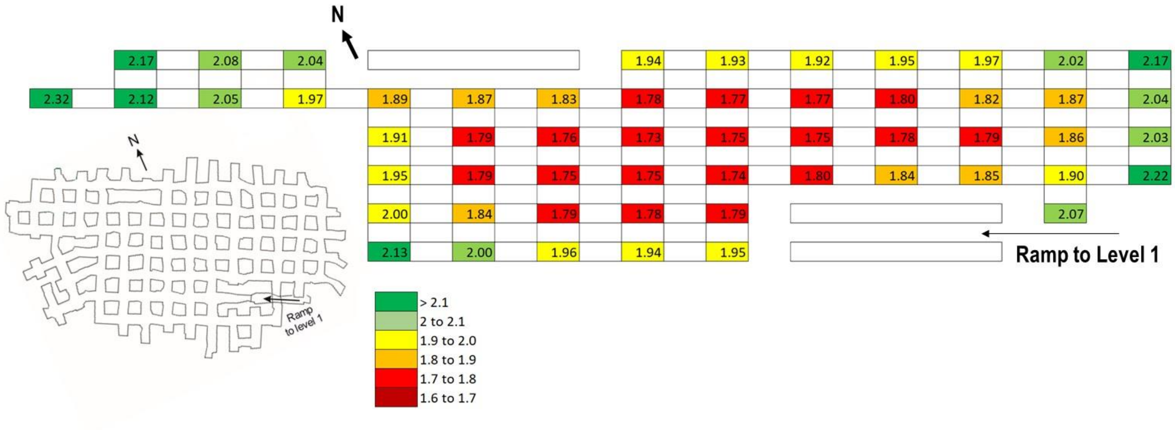

Figure 4. FoS calculated using Equation (3) and the 3D stress values obtained from the PLAXIS model for all the pillars in the mine lay-out.

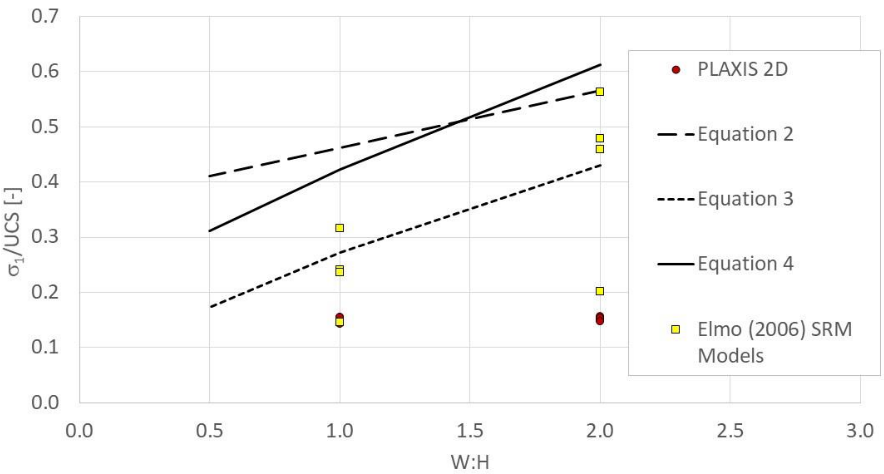

Figure 5. Comparison of 2D PLAXIS results to empirical pillar strength formulae and simulated 2D pillar strength for Middleton mine published in [17].

Table 3 compares the FoS determined by averaging the stresses measured in the 3D and 2D models for pillars with W:H of 2 and 1, respectively, and the strength values obtained using Equations (2) to (4); for example, considering the 3D models with W:H of 2, the an average pillar stress (Pillars 1, 2 and 3) is 10.7 MPa, and using a strength value of 27.1 MPa (Equation 1) we obtain a FoS of 2.5. For the 2D case, we have calculated the FoS using strength values of 20.4 MPa and 11.3 MPa from [17] for the pillars with W:H of 2 and 1, respectively.

The results presented in table 3 would confirm the earlier statement about the limitations of using 3D pillar formulae in combination with 2D stress values. Highlighted FoS values are assumed to be comparable, suggesting that using 3D stress models and the empirical formula (4) would yield equivalent FoS values to those calculated using 2D stress analysis and the empirical formula (3). Alternatively, one may use 2D stress analysis and pillar strength values estimated using 2D FEM-DEM analysis.

Table 3. Comparison of factor of safety (FoS) determined using different empirical strength formulae and 3D and 2D derived pillar stresses.

FoS determined averaging pillar stresses for the 3 pillars (ignoring pillar location)	3D FEM W:H = 2	3D FEM W:H = 1	2D FEM W:H = 2	2D FEM W:H = 1
FoS Equation 2	2.5	2.1	*3.7*	*3.1*
FoS Equation 3	*1.9*	*1.2*	**2.8**	**1.8**
FoS Equation 4	**2.7**	**1.9**	*4.0*	*2.8*
Elmo (2006)			**2.8**	**1.6**

4. Conclusions

This paper presented a discussion on the impact that modelling implication (3D vs. 2D) may have on the design of hard rock pillars. The results clearly show that under selected circumstances (pillar analysis), the FoS calculated using 2D stress analysis and 3D empirical pillar formulas may greatly overestimate the stress of the pillars. For instance, for the case of 14 m wide and 7 m high pillars (W:H of 2) values of FoS in the range of 2.8 to 4.0 were calculated using 2D average pillar stresses and common empirical pillar formulae. Conversely, FoS values in the range of 1.9 to 2.7 were determined using results from an actual 3D mine scale model. When comparing 2D stresses and 2D derived pillar strength values, then we obtained a more reasonable FoS of 2.8 that better agree with what observed for the 3D models. More importantly, the results would suggest that input strength values for 2D analyses should be obtained using 2D based synthetic rock mass models.

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