Structural reliability analysis of deep excavations

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Introduction

The Finite Element Method is nowadays widely used in structural design, both for the Servicibility Limit State (SLS) and also for the Ultimate Limit State (ULS). Especially the ULS design rules are based on the idea of ensuring sufficient structural reliability, which is usually expressed in a maximum admissible failure probability. This is commonly achieved by prescribing design rules and establishing partial safety factors. These load and resistance factors are calibrated for a wide range of typical cases with typical dimensions. For more extraordinary cases it could be that the application of these concepts leads to extremely conservative or possibly also to unconservative designs. The presented approach enables us to use the optimization potential for these case. Furthermore, the determination of failure probabilities is a substantial and indispensable element in modern risk-based design strategies (see fig. 1).

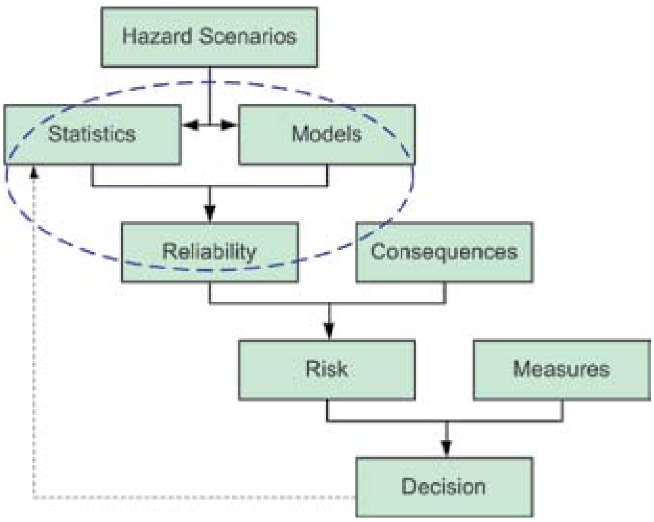

Figure 1. Risk-based Design Concepts

In this article we present a way of determining the structural reliability respectively the failure probability by means of probabilisitic calculations directly. To this end Plaxis is coupled with the generic probabilistic toolbox 'ProBox', developed by TNO Built Environment and Geosciences. ProBox enables us to carry out a reliability analysis using Plaxis in a fully automatized manner. We will also show that, in contrast to common prejudices, probabilistic analysis does not necessarily require thousands or millions of calculations as e.g. the Monte Carlo method, if we use more advanced and more efficient reliability techniques.

Structural Reliability

The task of the engineer in structural design is to ensure that the resistance (R) of the structure is larger than the load (S) it is exhibited to. Both quantities usually imply several variables, e.g. soil parameters, geometrical dimensions or forces. The magnitude of most of these quantities is uncertain. To ensure the safety of a structure it is common nowadays for most types of structures to apply partial safety factors to the load and resistance variables (LRFD: Load and Resistance Factor Design). This design approach is meant and calibrated to ensure a certain minimum reliability level, i.e. that the probability of structural failure is sufficiently low.

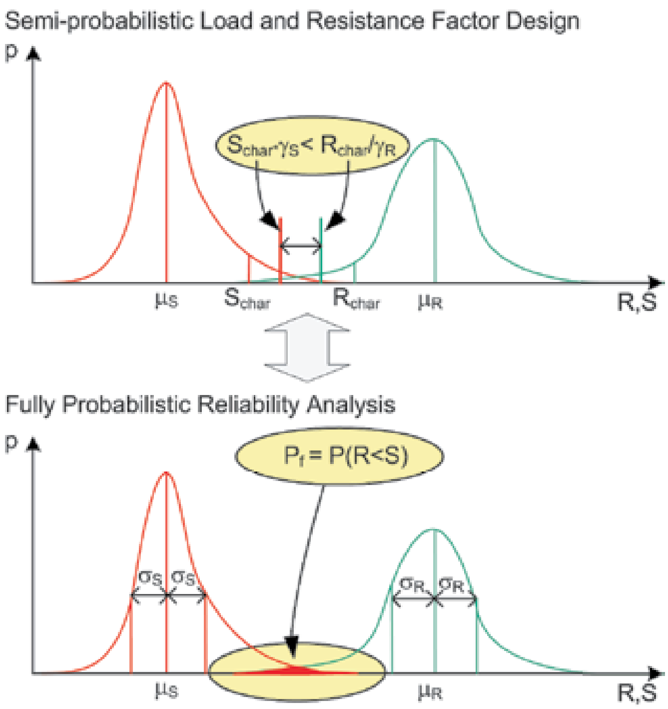

Figure 2. Partial Safety Factors vs. Fully Probabilistic

An essential task in structural reliability analysis is thus the determination of failure probabilities. To this end the first thing to do is the definition of failure. This failure definition does not necessarily have to mean structural collapse, but an unwanted event or state of the structure. In general, failure is defined as the load exceeding the strength. An example for an excavation with a sheet pile retaining wall would be the that the bending moment exceeds the elastic or plastic moment of the sheet pile, respectively that the stresses in the pile exceed the yield strength or the ultimate strength of the steel.

For the analysis we have to define a limit state function (Z). This function is the mathematical description of our failure definition. It implies all relevant load and resistance variables. A negative Z-value (Z<0) corresponds to failure, whereas a positive value (Z>0) to the desired state. A simple example for a limit state function is

Z = R - S

where R is the structural resistance and S is the load. Consequently, when the load

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exceeds the strength (S>R -> Z<0) we obtain failure.

To obtain the failure probability we have to use furthermore the statistical information of the variables. In essence, we integrate the probability density over the failure domain:
The reliability is the converse of the probability of failure. It is often expressed in terms

$$P_f = \iint\limits_{Z<0} f_{R,S}(r,s)\,dr\,ds$$

of the reliability index:
where Φ^{-1} is the inverse cumulative normal distribution. For low values of β one can

$$\Phi\beta = -\Phi^{-1}\{P_f\}$$

approximate the failure probability by $P_f = 10^{\beta}$.

Functionality of Probox

The program ProBox allows us to carry out this complex operation of determining the failure probability with advanced and efficient methods of high accuracy (level II and level III), amongst which:

- FORM / SORM
- Crude Monte Carlo
- Directional Sampling / DARS
- Increased Variance Sampling
- Numerical Integration

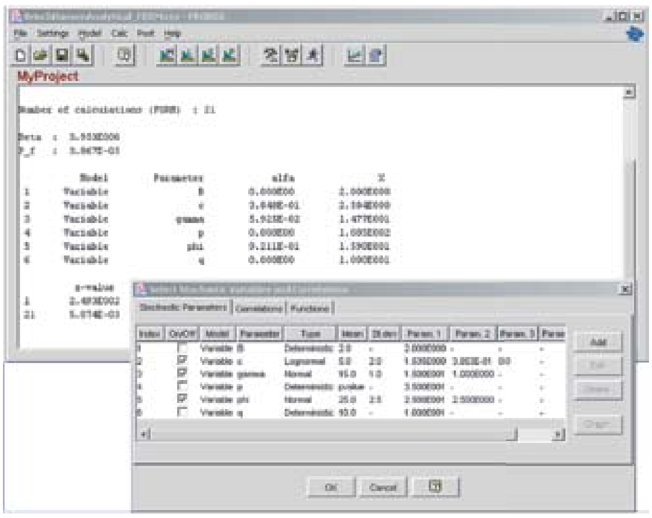


Figure 3. Screenshot Probox

For the model input statistics several distribution types can be used, like e.g. normal, lognormal or extreme value distributions. The correlations among the variables can be accounted for in form of a correlation matrix.

The strength of ProBox is the possibility of using external models for reliability analysis. For example the influence of corrosion on a sheet pile structure can be analyzed by using a corrosion model for the strength reduction part, whilst the load on the wall would be determined by Plaxis.

ProBox has already been used in combination with:

- FEM-codes (DIANA, PLAXIS, Catpro)
- Matlab
- Excel
- other stand-alone applications (Sobek, Ozone, etc.)
- user-defined dll's (Fortran, C, C++, Java)

The most relevant results of an analysis with ProBox are the probability of failure P_f, the reliability coefficient and the influence coefficients expressing the influence of each stochastic variable on the analyzed limit state β. The results can also be visualized in form of scatter plots, histograms or line plots.

Coupling Probox - Plaxis

The reliability analysis is fully controlled by ProBox. Plaxis is used to evaluate the limit state function for a parameter combination which is determined by the reliability algorithm. In other words, the Plaxis analysis allows us to decide wether a certain parameter combination would lead to structural failure or not.

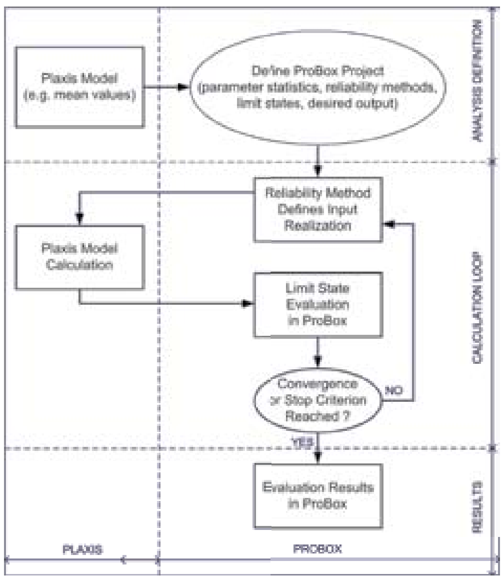
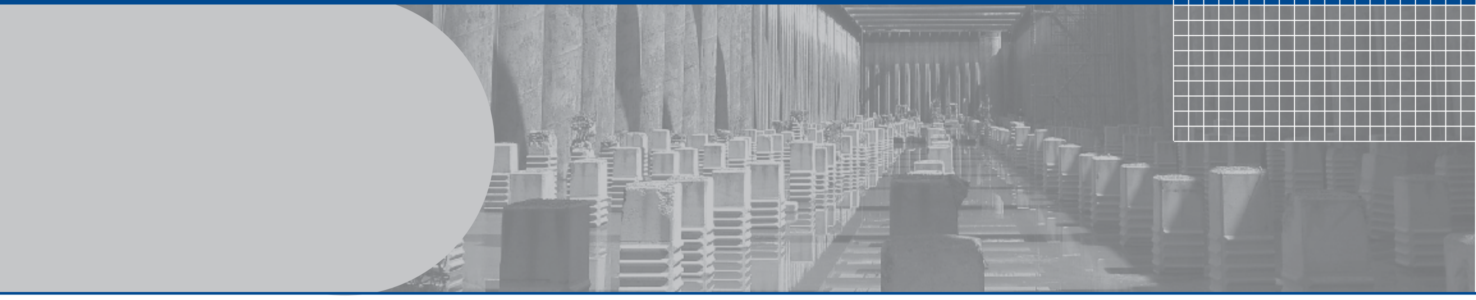


Figure 4. Coupling Scheme ProBox-Plaxis



The scheme in figure 4 illustrates how the coupling between ProBox and Plaxis works. In first instance one has to build the structural model in Plaxis. In ProBox you assign statistical properties to material properties like e.g. soil parameters or load characteristics that reflect their uncertainty. The reliability algorithm defines for each limit state function evaluation the parameter combination that has to be evaluated with the Plaxis model. The according Plaxis input files are manipulated, before the calculation is started. After the FEM-analysis ProBox reads the relevent results from the output files. This procedure is repeated until the convergence or stop criteria of the chosen reliability method are reached.

Example

The following example of a sheet pile wall with one anchor layer in soft soil will illustrate the presented ideas.

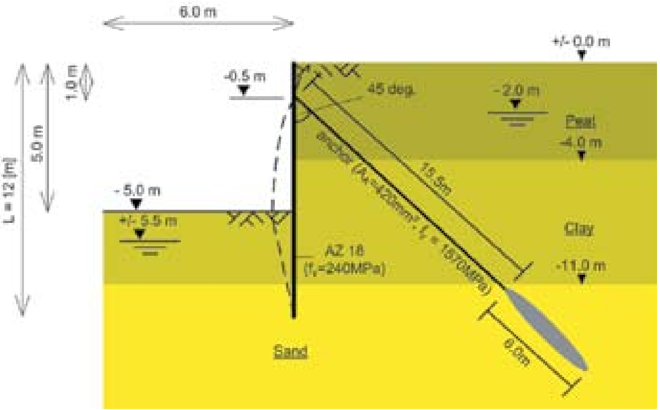


Figure 5. Example Geometry

The soil properties and the distributions of the soil parameters are listed in table 1. Based on the problem geometry and these soil parameters a deterministic design was made, based on the Dutch technical recommendations for sheet pile structures (CUR 166). The obtained structural dimensions are also indicated in figure 5.

The choice of distribution functions in table 1 is partially based on avoiding illposed problems. A Lognormal distribution cannot assume values smaller than 0 and a Beta distribution has a lower and an upper limit and is therefore well suited for parameters such as the Poisson ratio.

In this example we want to determine the probability of failure of the sheet pile itself. The simplest way to do so would be to determine the probability that the design moment M_d is exceeded. the according limit state function would be:

$$\mathbf{Z = M_d - M = W_{el} * f_y - M}$$

where W_{el} is the elastic section modulus, f_y is the yield strength and M is the bending moment calculated in Plaxis. One could also use a plastic moment, when plastic hinges are allowed.

This expression can be refined by accounting for the axial forces in the wall as well. In this case we can determine the probability that the yield strength f_y is exceeded using the limit state function:

$$\mathbf{Z = f_y - \sigma = f_y - (M/W_{el} + F/A)}$$

where the stress σ is composed of the bending moment M, the section modulus W_{el},

Peat, medium		COV	Mean	STD	Lower Bound	Upper Bound	Distribution	Unit
saturated volumetric weight	γ_{sat}	5%	13.1	0.65	-	-	Normal	[kN/m³]
cohesion	c	20%	7.5	1.5	0.0	-	Lognormal	[kPa]
friction angle	φ'	10%	23.9	2.39	-	-	Normal	[deg]
dilation angle	ψ	-	0.0	-	-	-	deterministic	[deg]
Young's modulus	E	25%	850	212	-	-	Normal	[kPa]
Poisson ratio	ν	10%	0.35	0.035	0.0	0.5	Beta	[-]
interface strength	R_{inter}	20%	0.6	0.12	0.0	1.0	Beta	[-]
Clay, medium		**COV**	**Mean**	**STD**	**Lower Bound**	**Upper Bound**	**Distribution**	**Unit**
saturated volumetric weight	γ_{sat}	5%	18.5	0.93	-	-	Normal	[kN/m³]
cohesion	c	20%	14.9	2.98	0.0	-	Lognormal	[kPa]
friction angle	φ'	10%	20.9	2.09	-	-	Normal	[deg]
dilation angle	ψ	-	0.0	-	-	-	deterministic	[deg]
Young's modulus	E	25%	3400	850	-	-	Normal	[kPa]
Poisson ratio	ν	10%	0.35	0.035	0.0	0.5	Beta	[-]
interface strength	R_{inter}	20%	0.6	0.12	0.0	1.0	Beta	[-]
Sand, dense		**COV**	**Mean**	**STD**	**Lower Bound**	**Upper Bound**	**Distribution**	**Unit**
saturated volumetric weight	γ_{sat}	-	19.0	-	-	-	deterministic	[kN/m³]
cohesion	c	-	7.5	-	-	-	deterministic	[kPa]
friction angle	φ'	10%	35.0	3.50	-	-	Normal	[deg]
dilation angle	ψ	-	φ'-5	-	-	-	deterministic	[deg]
Young's modulus	E	-	125,000	-	-	-	deterministic	[kPa]
Poisson ratio	ν	-	0.3	-	-	-	deterministic	[-]
interface strength	R_{inter}	-	0.7	-	-	-	deterministic	[-]

Table 1. Soil Parameter Distributions

Number of Calculations (FORM): 99

β: 4.207
P_f: 1.293×10^{-5}

Variable X_i	Infl. factor α_{X_i}	Design Point X_i*
E_clay	+ 0.878	1,328 [kPa]
E_peat	+ 0.114	733.2 [kPa]
ν_clay	- 0.320	0.396 [-]
ν_peat	+ 0.107	0.335 [-]
γ_sat_clay	- 0.037	18.64 [kN/m³]
γ_sat_peat	- 0.174	13.58 [kN/m³]
ϕ_clay	+ 0.037	20.48 [deg]
ϕ_peat	- 0.000	23.78 [deg]
ϕ_sand	+ 0.064	33.90 [deg]
c_clay	+ 0.079	13.69 [kPa]
c_peat	- 0.002	7.36 [kPa]
R_inter_clay	+ 0.240	0.474 [-]
R_inter_peat	- 0.007	0.608 [-]

Table 2. Results Reliability Analysis Sheet Pile Failure

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the axial force F and the cross sectional area A. This expression has the additional advantage that e.g the influence of corrosion can be accounted for via changes in the geometrical properties of the sheet pile W_{el} and A easily.

The results of a reliability analysis, in which the relevant soil properties were treated as stochastic quantities, are listed in table 2. The Analysis was carried out with a FORM algorithm. 99 evaluations of the limit state function were carried out, i.e. the Plaxis model was evaluated 99 times. For this relatively simple model this resulted in a calculation time of only approximately 30 minutes. The results were furthermore checked against 'exact' level three calculations, which gave similar results. These were carried out with Directional Sampling and required 655 model evaluations. Considering the low failure probability a Crude Monte Carlo simulation would not be feasible within reasonable time, since the required number of Plaxis calculations would be in the order of 10^{+7}.

The target reliability index of class II structure designed with the CUR 166 recommendations is $\beta = 3.4$. Using this approach we calculate a higher reliability ($\beta = 4.2$). The structural design is thus conservative for this limit state. There might be room for optimization. The influence coefficients in table 2 and figure 6 give us information about the importance of the parameters involved. There are two contributions in this influence measure, the sensitivity of the model to a certain variable and the amount of uncertainty in the same parameter. A positive value of indicates a positive influence on the limit state (and the reliability index), whereas a negative value indicates a negative influence. The design point is the most likely combination of parameters leading to failure (highest probability density).

For this specific example the stiffness parameters of the clay layer apparently dominate the load on the wall. The influence factors α^2 from figure 6 give information about the contribution of the variables to the total uncertainty. Considering the definition of the influence factor that means that e.g. decreasing the uncertainty in these stiffness properties by additional soil investigation could increase the reliability considerably.

Figures 7 and 8 show the principal effective stresses in the design point. The design point is the parameter combination with highest probability density that leads to failure. Especially from figure 8 can be concluded that the shear strength in the soil behind the wall is mobilized to a very low degree. The problem is thus still fully elastic (The calculations were carried out with the Mohr-Coulomb model.). That explains why the strength parameters of the soil play a minor role for this limit state.
This was just an example of results of a reliability analysis and possible conclusions for the optimization of the problem. The outcomes of such an analysis contain a lot of useful information that can be used either for design refinements or in risk-based design approaches.

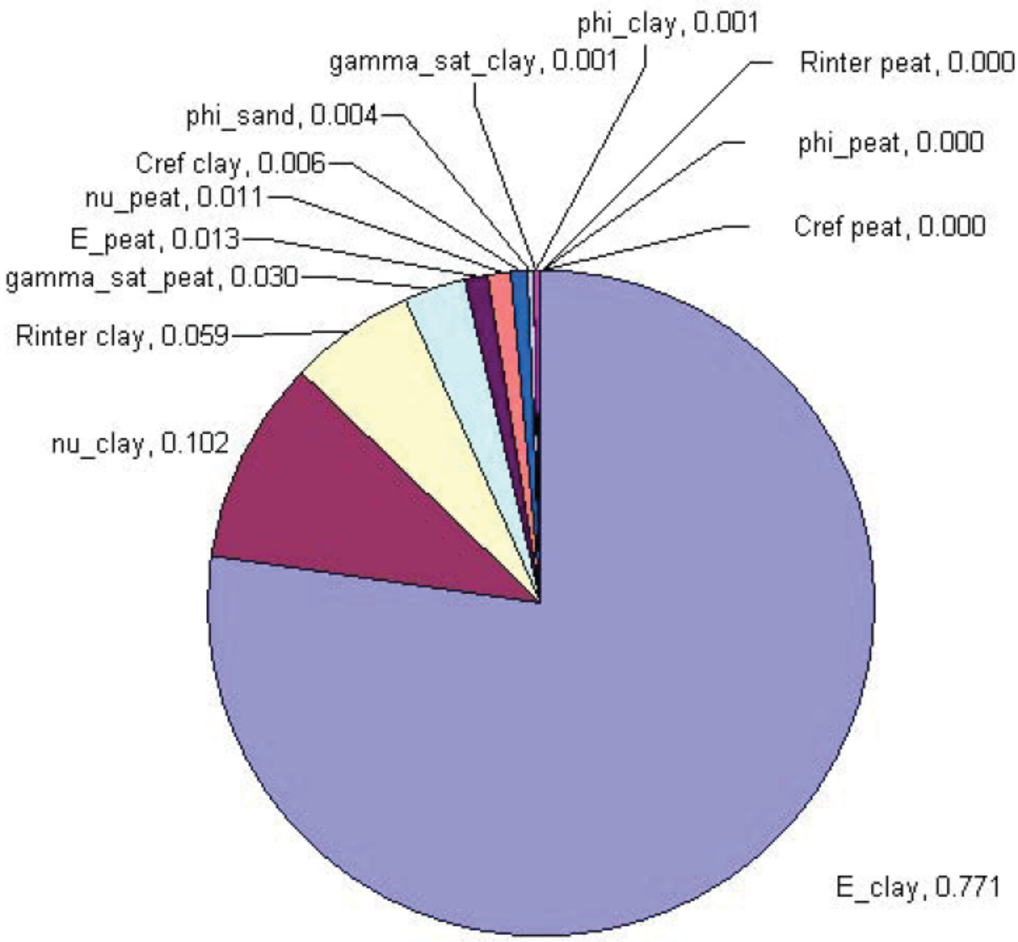
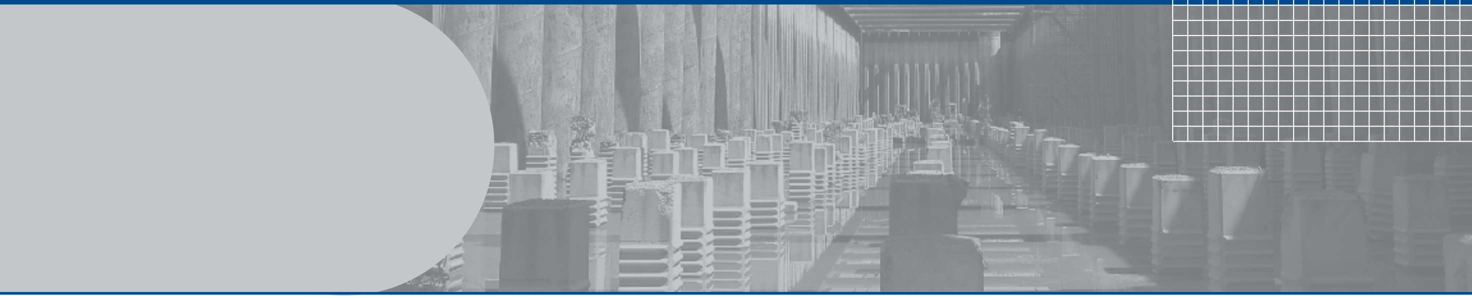


Figure 6. Influence Factors α^2



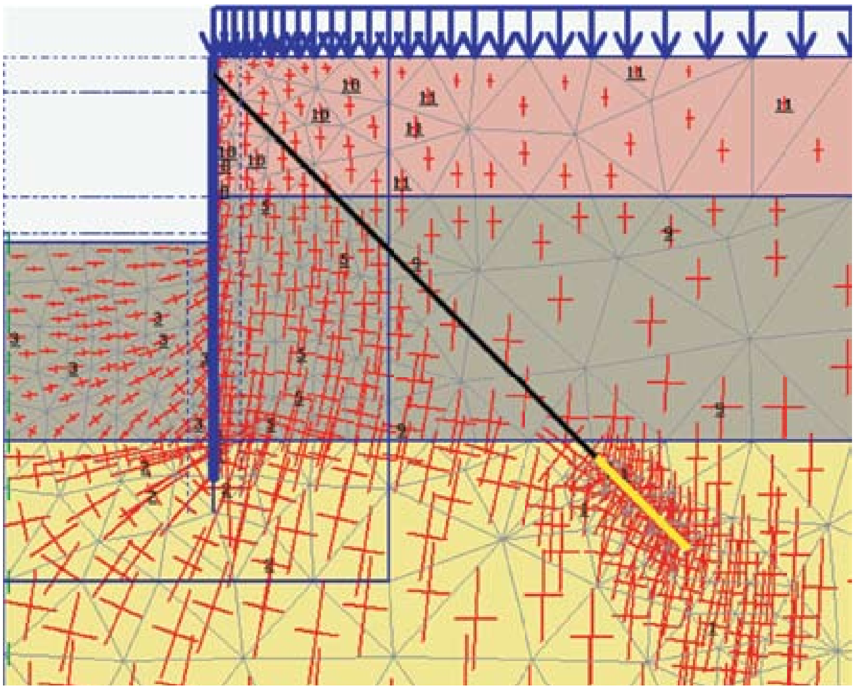


Figure 7. Principal Effective Stresses (Design Point Sheet Pile Failure)

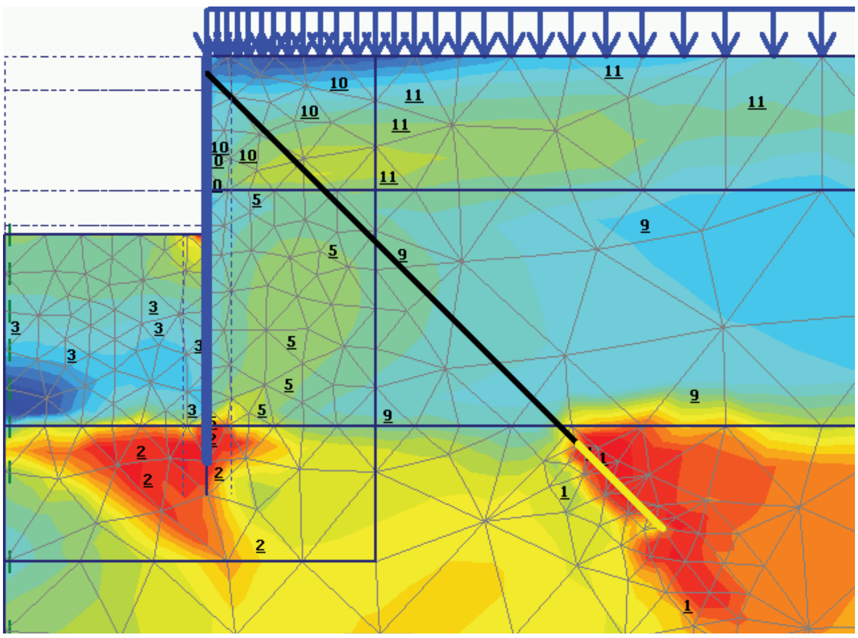


Figure 8. Relative Shear (Design Point Sheet Pile Failure)

Conclusions

Fully probabilistic reliability analysis can be carried out with the presented framework with reasonable modeling and computational effort.

This type of analysis allows us to calculate the reliability of a structure directly. This information can be used for optimization purposes, in risk-based design concepts and for the calibration of partial safety factors in design codes.

- The influence coefficients as result of the analysis provide useful information for optimization purposes and also for the physical understanding of the model behavior close to failure.

Acknowlegdements

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