PLAXIS

PLAXIS The PM4Sand model 2018

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1 INTRODUCTION

The PM4Sand model successfully simulates the material behaviour of sands in dynamic loading, including the pore pressure generation, liquefaction and post-liquefaction phenomena. It is a very attractive model for the industry due to a small number of parameters to be calibrated. They are mostly related to the usually available data in the design practice (e.g. D_R, SPT, CPT, V_s values).

This report summarises the implementation of the PM4Sand model version 3.1 (Boulanger & Ziotopoulou, 2017) in the PLAXIS finite element code. The model is defined in the basic framework of the stress-ratio controlled, critical state compatible, bounding surface plasticity model for sand by Dafalias & Manzari (2004). Many modifications were added to the Dafalias & Manzari (2004) model in order to more accurately simulate stress-strain responses that are important to geotechnical earthquake engineering practice. The developments are described in the manuals (version 1: (Boulanger, 2010); version 2: (Boulanger & Ziotopoulou, 2012); version 3: (Boulanger & Ziotopoulou, 2015); version 3.1: (Boulanger & Ziotopoulou, 2017)).

2 MODEL FORMULATION

2.1 BASIC STRESS AND STRAIN QUANTITIES

The model is defined in terms of the effective stresses. The prime symbol is dropped for convenience in all of the terms that follow in the report. The model is defined in the plane stress conditions. This formulation allows the model to be used in 2D plane strain numerical models, where the out-of-plane stress is ignored by the global finite element equations. The effective stress tensor $\boldsymbol{\sigma}$, the mean effective stress p, the deviatoric stress tensor $\boldsymbol{s}$ and the deviatoric stress ratio tensor $\boldsymbol{r}$ used in the model formulation have the following form:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} \tag{2.1}$$

$$p = \frac{\sigma_{xx} + \sigma_{yy}}{2} \tag{2.2}$$

$$\boldsymbol{s} = \boldsymbol{\sigma} - p\boldsymbol{I} = \begin{pmatrix} \sigma_{xx} - p & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} - p \end{pmatrix} \tag{2.3}$$

$$\boldsymbol{r} = \frac{\boldsymbol{s}}{p} = \begin{pmatrix} \dfrac{\sigma_{xx} - p}{p} & \dfrac{\sigma_{xy}}{p} \\ \dfrac{\sigma_{xy}}{p} & \dfrac{\sigma_{yy} - p}{p} \end{pmatrix} \tag{2.4}$$

The strains are split into the volumetric ε_v and deviatoric part $\boldsymbol{e}$ as follows:

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} \tag{2.5}$$

$$\boldsymbol{e} = \varepsilon - \frac{\varepsilon_v}{3}\boldsymbol{I} = \begin{pmatrix} \varepsilon_{xx} - \dfrac{\varepsilon_v}{3} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} - \dfrac{\varepsilon_v}{3} \end{pmatrix} \tag{2.6}$$

In addition to stresses σ_{xx}, σ_{yy} and σ_{xy} the model computes also the value of out of plane normal stress σ_{zz} based on the linear elastic assumption in the out of plane direction as:

$$d\sigma_{zz} = \nu(d\sigma_{xx} + d\sigma_{yy}) \tag{2.7}$$

where ν is the Poisson's ratio.

It should be noted that the values of σ_{zz} can be viewed in PLAXIS Output but they do not have any practical relevance nor any influence on the model behaviour.

2.2 CRITICAL STATE SOIL MECHANICS FRAMEWORK

The model is defined according to the Critical State Soil Mechanics Framework (Schofield & Wroth, 1968). It uses the relative state parameter index ξ_R (Konrad, 1988) instead of the state parameter ψ (Been & Jefferies, 1985). ξ_R is defined as:

$$\xi_R = D_{R,cs} - D_R \tag{2.8}$$

where D_R is the current relative density and $D_{R,cs}$ is the relative density on the critical state line at the current mean effective stress p. $D_{R,cs}$ is defined by the critical state line in the $D_R - p$ plane as follows:

$$D_{R,cs} = \frac{R}{Q - \ln\left(100\frac{p}{p_A}\right)} \tag{2.9}$$

where p_A is the atmospheric pressure while Q and R are Bolton's parameters (Bolton, 1986). The values of Q and R were shown by Bolton to be about 10.0 and 1.0, respectively, for quartzitic sands. An example of the critical state line in the $D_R - p$ plane with the parameters Q=10 and R=1.5 is shown in Figure 2.1.

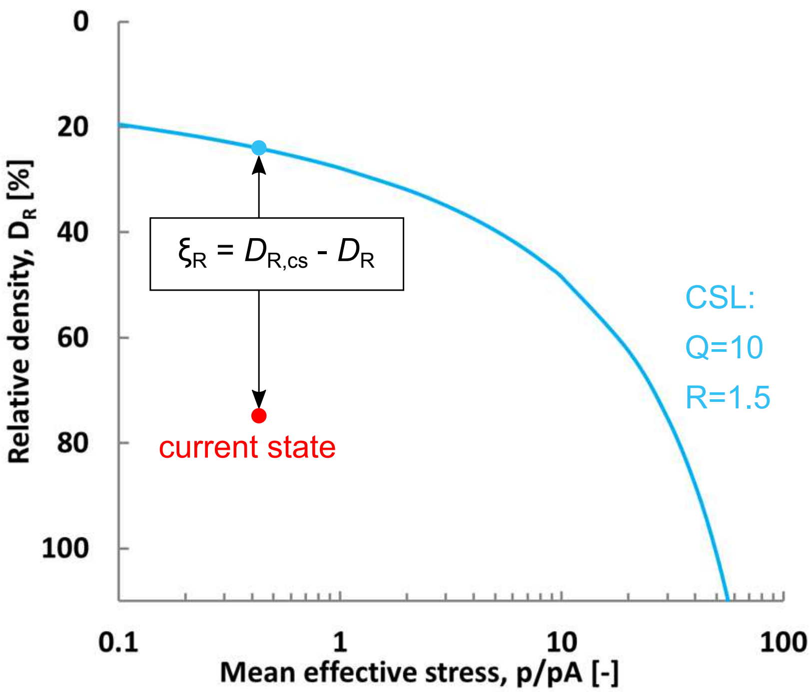


Figure 2.1 The critical state line (CSL) in the $D_R - p/p_A$ plane with the parameters Q=10 and R=1.5

2.3 BOUNDING, DILATANCY, CRITICAL AND YIELD SURFACES

The model uses the bounding, dilatancy and the critical surfaces, following the model of Dafalias & Manzari (2004). In the triaxial setting, the surfaces are defined with the stress ratios M^b, M^d and M, which depend on the relative state index ξ_R according to the following relationships:

$$M^b = M\exp(-n^b\xi_R) \tag{2.10}$$

$$M^d = M\exp(n^d\xi_R) \tag{2.11}$$

$$M = 2\sin(\varphi_{c\nu}) \tag{2.12}$$

n^b and n^d are the model parameters defining the computation of M^b and M^d in relation to M. $\varphi_{c\nu}$ is the critical state (constant volume) effective friction angle (also a model parameter). As the model is sheared, ξ_R approaches the value of zero while M^b and M^d both approach the value of M. In Figure 2.2 the schematic representation of changing of bounding and dilatancy stress ratios M^b and M^d in relation to the relative state index ξ_R in the triaxial plane is shown for the denser and looser than critical states.

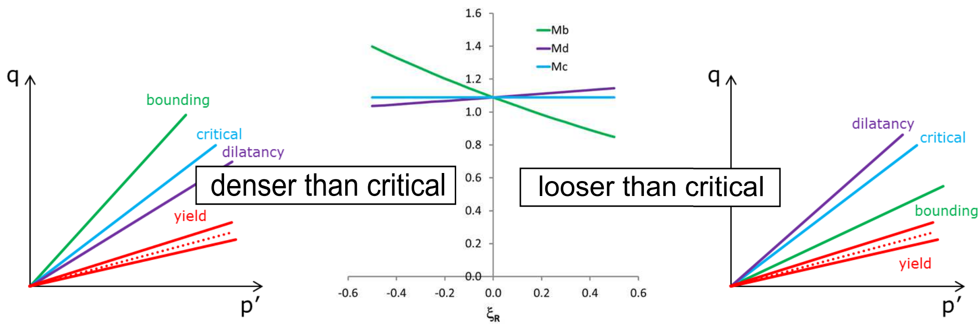


Figure 2.2 The schematic representation of changing of bounding and dilatancy stress ratios M^b and M^d according to the relative state index ξ_R in the triaxial plane. The yield surface stress ratios are also shown along with the yield surface axis.

The yield surface is formulated as a small cone in the stress space with the following expression:

$$f = \sqrt{(\boldsymbol{s} - p\boldsymbol{\alpha}) : (\boldsymbol{s} - p\boldsymbol{\alpha})} - \sqrt{\frac{1}{2}}pm = 0 \tag{2.13}$$

where the back-stress ratio tensor $\boldsymbol{\alpha}$ denotes the position of the yield surface in the deviatoric stress ratio space and m is the size of the yield surface with the predefined value of 0.01.

In the general formulation of the model, the bounding and dilatancy surfaces are defined in terms of the image back-stress ratios $\boldsymbol{\alpha}^b$ and $\boldsymbol{\alpha}^d$ as

$$\boldsymbol{\alpha}^b = \sqrt{\frac{1}{2}}\,[M^b - m]\boldsymbol{n} \tag{2.14}$$

$$\boldsymbol{\alpha}^d = \sqrt{\frac{1}{2}}\,[M^d - m]\boldsymbol{n} \tag{2.15}$$

where $\boldsymbol{n}$ is the deviatoric unit normal to the yield surface defined as

$$\boldsymbol{n} = \frac{\boldsymbol{r} - \boldsymbol{\alpha}}{\sqrt{\frac{1}{2}}\,m} \tag{2.16}$$

and $\boldsymbol{r}$ is the deviatoric stress ratio tensor defined as

$$\boldsymbol{r} = \frac{\boldsymbol{s}}{p} \tag{2.17}$$

The schematic of the yield, dilatancy and bounding surfaces, tensor $\boldsymbol{n}$ and the image back-stress ratios in $r_{yy} - r_{xy}$ plane are shown in Figure 2.3.

The distance between the yield surface axis $\boldsymbol{\alpha}$ and the image back-stress ratio $\boldsymbol{\alpha}^b$ defines the plastic modulus K_p, while the distance between $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^d$ defines the amount of dilatancy or contractancy via the quantity D (K_p and D are further explained in the subsections below). The full explanation of the role of $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^b$ and $\boldsymbol{\alpha}^d$ can be found in Boulanger & Ziotopoulou (2017).

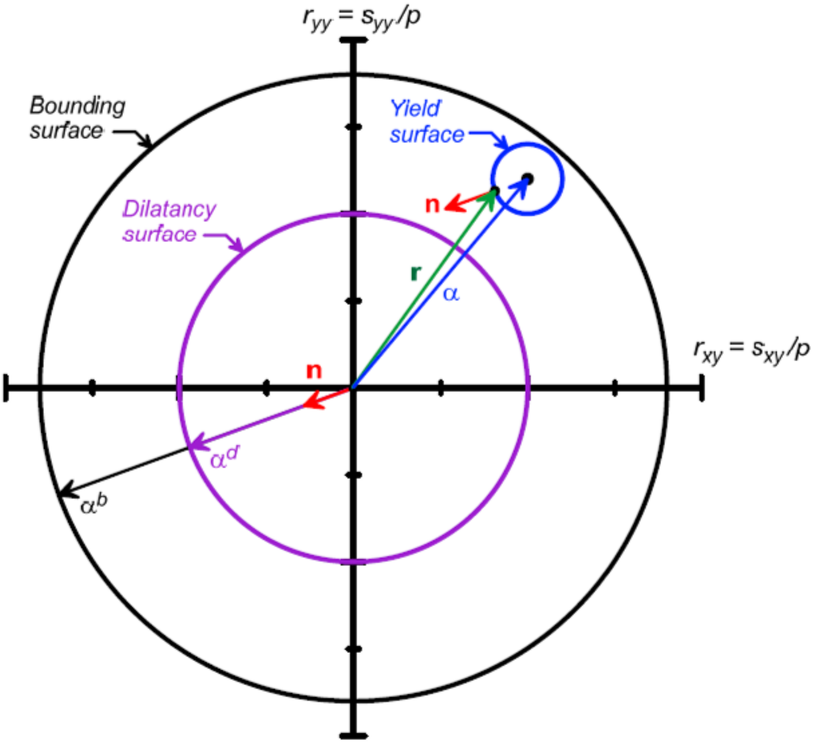


Figure 2.3 The schematic of the yield, dilatancy and bounding surfaces, the tensor $\boldsymbol{n}$ and the image back-stress ratios in $r_{yy} - r_{xy}$ plane. (after Boulanger & Ziotopoulou (2017))

2.4 STRESS REVERSAL AND INITIAL BACK-STRESS RATIO TENSORS

According to the bounding surface formulation by Dafalias (1986) which was also adopted in Dafalias & Manzari (2004), the model keeps track of the initial back-stress ratio $\boldsymbol{\alpha}_{in}$, which is updated at the reversal in loading direction. The reversal in loading direction is identified whenever the following condition holds

$$(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \boldsymbol{n} < 0 \tag{2.18}$$

At the reversal, the initial stress ratio $\boldsymbol{\alpha}_{in}$ is updated to the current one $\boldsymbol{\alpha}$. To overcome the high stiffness at a small load reversal, the initial stress ratio $\boldsymbol{\alpha}_{in}$ is subdivided into three initial stress ratios, namely the apparent ($\boldsymbol{\alpha}_{in}^{app}$), true ($\boldsymbol{\alpha}_{in}^{true}$) and the previous initial stress ratio ($\boldsymbol{\alpha}_{in}^{p}$). The initial stress ratios take part in the expressions of dilatancy (D) and plastic modulus (K_p) which will be shown later. All the details about the mechanism of tracking the stress reversals can be found in Boulanger & Ziotopoulou (2017).

2.5 ELASTIC PART OF THE MODEL

The elastic volumetric and deviatoric strain increments are calculated as:

$$d\varepsilon_v^{el} = \frac{dp}{K} \tag{2.19}$$

$$d\boldsymbol{e}^{el} = \frac{d\boldsymbol{s}}{2G} \tag{2.20}$$

where G is the elastic shear modulus and K the elastic bulk modulus. The elastic shear modulus is dependent on the mean effective stress, stress ratio and fabric as follows:

$$G = G_0 p_A \sqrt{\frac{p}{p_A}} \left(1 - C_{SR,0}\left(\frac{M}{M^b}\right)^{m_{SR}}\right) \left(\frac{1 + \frac{z_{cum}}{z_{max}}}{1 + \frac{z_{cum}}{z_{max}} C_{GD}}\right) \tag{2.21}$$

G_0 is a parameter taking into account the small strain shear modulus, while the parameters $C_{SR,0}$ and m_{SR} impose the stress ratio effects according to Yu & Richart Jr. (1984). $C_{SR,0}$ and m_{SR} are set to 0.5 and 4 internally. This lets the effect of stress ratio on elastic modulus be small at small stress ratios, but lets the effect increase to a 60% reduction when the stress ratio is on the bounding surface Boulanger & Ziotopoulou (2017). z_{cum} is the cumulative value of absolute changes of the fabric tensor $\boldsymbol{z}$ calculated as:

$$dz_{cum} = |d\boldsymbol{z}| \tag{2.22}$$

and z_{max} is the parameter computed at the time of the model initialisation according to the initial relative state index ξ_{R0} as:

$$z_{max} = 0.7 \exp(-6.1\xi_{R0}) \leq 20 \tag{2.23}$$

C_{GD} is the factor controlling the shear modulus degradation dependent on the cumulative plastic deviatoric strains. The maximum degradation approaches a factor of $1/C_{GD}$. The value of C_{GD} is set internally to 2.0. The elastic bulk modulus K is related to the shear modulus G by the following relationship:

$$K = \frac{2(1+\nu)}{3(1-2\nu)} G \tag{2.24}$$

where ν is a constant Poisson's ratio. The recommended value of ν is 0.3 and can be changed by the user.

2.6 PLASTIC COMPONENTS OF THE MODEL

2.6.1 PLASTIC STRAINS, LOADING INDEX AND STRESS INCREMENT

The increment of the plastic volumetric and deviatoric strain is calculated with the following expressions:

$$d\varepsilon_v^{pl} = \langle L \rangle \, D \tag{2.25}$$

$$d\boldsymbol{e}^{pl} = \langle L \rangle \, \boldsymbol{n} \tag{2.26}$$

where L is the loading index, $\langle\rangle$ MacCauley brackets that set negative values to zero and D is the dilatancy which will be defined in a later section. The loading index is calculated according to the following:

$$L = \frac{2G\boldsymbol{n} : d\boldsymbol{e} - \boldsymbol{n} : \boldsymbol{r} K d\epsilon_v}{K_p + 2G - KD\boldsymbol{n} : \boldsymbol{r}} \tag{2.27}$$

By using the calculated loading index, the stress increment can be calculated as:

$$d\boldsymbol{\sigma} = 2Gd\boldsymbol{e} + Kd\varepsilon_v \boldsymbol{I} - \langle L \rangle \, (2G\boldsymbol{n} + KD\boldsymbol{I}) \tag{2.28}$$

2.6.2 HARDENING-SOFTENING RULE AND THE PLASTIC MODULUS

The evolution of the back-stress ratio $\boldsymbol{\alpha}$ corresponding to the axis of the yield surface is used according to Dafalias & Manzari (2004) as:

$$d\boldsymbol{\alpha} = \langle L \rangle \, \frac{2}{3} h(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) \tag{2.29}$$

where h is the hardening coefficient. The plastic modulus K_p is defined as follows:

$$K_p = Gh_0 \frac{\sqrt{(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \boldsymbol{n}}}{[\exp\left((\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}) : \boldsymbol{n}\right) - 1] + C_{\gamma 1}} C_{rev} \tag{2.30}$$

$$C_{rev} = \frac{(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}) : \boldsymbol{n}}{(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{true}) : \boldsymbol{n}} \qquad \text{for } (\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{p}) : \boldsymbol{n} \leq 0$$

$$C_{rev} = 1 \qquad \text{otherwise}$$

where h_0 is the parameter adjusting the ratio between plastic and elastic moduli. The value of h_0 is internally set according to the following relationship:

$$h_0 = \frac{(0.25 + D_{R0})}{2} \geq 0.3 \tag{2.31}$$

where D_{R0} is the initial relative density, i.e. the first input parameter of the model. The constant $C_{\gamma 1}$ serves to avoid the division by zero and is internally set to $h_0/200$. From the Eq. (2.30) it can be seen that the plastic modulus K_p is proportional to G and to the distance of the back-stress ratio $\boldsymbol{\alpha}$ to the bounding back-stress ratio $\boldsymbol{\alpha}^b$ and inversely proportional to the distance of the back-stress ratio from the initial back-stress ratio $\boldsymbol{\alpha}_{in}^{app}$. The plastic modulus relationship was revised by Boulanger & Ziotopoulou (2017) in comparison to the Dafalias & Manzari (2004) model in order to provide an improved approximation of empirical relationships for secant shear modulus and equivalent damping ratios during drained strain-controlled cyclic loading. More details on the subject can be found in Boulanger & Ziotopoulou (2017).

2.6.3 PLASTIC VOLUMETRIC STRAINS - THE DILATION PART

The plastic volumetric dilation occurs when $(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n} < 0$. In that case the dilatancy D is negative and calculated according to the following expression (without the fabric effects which will be described in a later section):

$$D = A_{d0}[(\boldsymbol{\alpha}_d - \boldsymbol{\alpha}) : \boldsymbol{n}] \tag{2.32}$$

From Eq. (2.32) it can be recognised that the dilatancy is proportional to the constant A_{d0} and the distance of the back-stress ratio $\boldsymbol{\alpha}$ to the dilatancy back-stress ratio $\boldsymbol{\alpha}^d$. The constant A_{d0} is related to the dilatancy relationship proposed by Bolton (1986):

$$\phi_{pk} - \phi_{cv} = -0.8\psi \tag{2.33}$$

where ϕ_{pk} is the peak angle of shearing resistance, ϕ_{cv} the angle of shearing resistance at the constant volume and ψ the dilatancy angle. Taking into account the Eq. (2.33), the constant A_{d0} is defined as:

$$A_{d0} = \frac{1}{0.4} \frac{\arcsin\left(\frac{M^b}{2}\right) - \arcsin\left(\frac{M}{2}\right)}{M^b - M^d} \tag{2.34}$$

Further details on the formulation of dilatancy in the dilatant regime can be found in Boulanger & Ziotopoulou (2017).

2.6.4 PLASTIC VOLUMETRIC STRAINS - THE CONTRACTION PART

Whenever $(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n} > 0$, the plastic volumetric contraction occurs and the dilatancy D is positive and calculated according to the following expression:

$$D = A_{dc}[(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \boldsymbol{n} + C_{in}]^2 \frac{(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n}}{(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n} + C_D} \tag{2.35}$$

where A_{dc} is calculated as:

$$A_{dc} = \frac{A_{d0}}{h_p} \tag{2.36}$$

by taking the A_{d0} term from the dilatant part and dividing it by h_p,which depends on the parameter h_{p0} and the current relative state parameter index ξ_R as:

$$h_p = h_{p0} \exp(-0.7 + 7(0.5 - \xi_R)^2) \quad \text{for } \xi_R \leq 0.5 \tag{2.37}$$
$$h_p = h_{p0} \exp(-0.7) \quad \text{for } \xi_R > 0.5$$

The parameter h_{p0} can be varied during the calibration process to get the desired cyclic resistance ratios. The C_{in} term depends on fabric and is described in a later section. The value of the constant C_D is internally set to 0.16. The refined Eq. (2.35) in comparison with the Dafalias & Manzari (2004) model improved the slope of the cyclic resistance ratio (CRR) versus number of equivalent uniform loading cycles for undrained cyclic element tests. The effect of the overburden stress on the cyclic resistance (K_σ) was taken into account by the Eq. (2.37). Further explanation of the dilatancy in the contractant regime can be found in Boulanger & Ziotopoulou (2017).

2.6.5 FABRIC EFFECTS

The effects of prior straining on the model response have been taken into account by using the fabric-dilatancy tensor $\boldsymbol{z}$ that was introduced by Dafalias & Manzari (2004). The evolution of $\boldsymbol{z}$ is defined as:

$$d\boldsymbol{z} = -\frac{c_z}{1 + \left\langle \dfrac{z_{cum}}{2z_{max}} - 1 \right\rangle} \frac{\left\langle -d\varepsilon_v^{pl} \right\rangle}{D} (z_{max}\boldsymbol{n} + \boldsymbol{z}) \tag{2.38}$$

The tensor $\boldsymbol{z}$ evolves with the plastic deviatoric strains that occur during dilation only. It can be seen from the Eq. (2.38) that the rate of evolution of $\boldsymbol{z}$ decreases with increasing values of z_{cum}. In this way during the undrained cyclic loading the shear strains progressively accumulate rather than lock-up into a repeating stress-strain loop. The model tracks additional quantities regarding the fabric history, such as z_{peak}, $\boldsymbol{z}_{in}$ and p_{zp}. Using aforementioned quantities many issues of the model behaviour have been taken into account:

- effects of sustained static shear stresses,
- fabric effects for various drained versus undrained loading conditions,
- degree of stress rotation and its effect on plastic modulus,
- erasure of fabric formed during liquefaction in reconsolidation stages and
- effect of prior strain history at loading of the mean effective stress that is smaller or larger than the mean effective stress when fabric was formed.

More details on tracking of fabric and its effects on model response can be found in Boulanger & Ziotopoulou (2017).

2.6.6 FABRIC EFFECTS ON PLASTIC MODULUS AND DILATANCY

For brevity, the final plastic modulus and dilatancy expressions considering also the fabric effects will be given only in the short form without the explanations of the meaning of terms. All the explanations and full forms of expressions can be found in Boulanger & Ziotopoulou (2017).

The plastic modulus Eq. (2.30) is multiplied with the fabric terms as following:

$$K_p = Gh_0 \frac{\sqrt{(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \boldsymbol{n}}}{\left[\exp\left((\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}) : \boldsymbol{n}\right) - 1\right] + C_{\gamma 1}} C_{rev} \cdot \frac{C_{k\alpha}}{1 + C_{K_p} \left(\frac{z_{peak}}{z_{max}}\right) \left\langle (\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \boldsymbol{n} \right\rangle \sqrt{1 - C_{zpk2}}} \tag{2.39}$$

Effects of fabric on plastic volumetric dilation have been incorporated through the introduction of the additional rotated dilatancy surface and then splitting the dilatancy expression into two parts regarding the position to the rotated dilatancy surface. The A_{d0} term from Eq. (2.32) has been transformed into the term A_d of the following form:

$$A_d = \frac{A_{d0} C_{zin2}}{\left(\frac{z_{cum}^2}{z_{max}}\right)\left(1 - \frac{\langle -\boldsymbol{z} : \boldsymbol{n} \rangle}{\sqrt{2} z_{peak}}\right)^3 (C_\varepsilon)^2 C_{pzp} C_{pmin} C_{zin1} + 1} \tag{2.40}$$

The fabric terms are present in C_{zin1}, C_{zin2} and C_{pzp} terms through the quantities z_{peak}, z_{max}, z_{cum}, and $\boldsymbol{z}_{in}$ while C_{pmin} as well as C_{pzp} include also the dependence on the value of mean effective stress. Effects of fabric on plastic volumetric contraction have been put through the modification of the term A_{dc} in Eq. (2.36). The fabric dependent expression for A_{dc} is defined as:

$$A_{dc} = \frac{A_{d0}(1 + \langle \boldsymbol{z} : \boldsymbol{n} \rangle)}{h_p C_{dz}} \tag{2.41}$$

The term C_{dz} includes the fabric dependence through the quantities z_{peak}, z_{max} and z_{cum}.

All the details about the term formulations as well as explanations of the logic behind can be found in Boulanger & Ziotopoulou (2017).

2.6.7 POST-SHAKING RECONSOLIDATION

The development of volumetric strains during post-liquefaction reconsolidation of sand is difficult to numerically model using the conventional separation of strains into elastic and plastic part in the constitutive model, since a large portion of the post-liquefaction reconsolidation strains are due to sedimentation effects (Boulanger & Ziotopoulou, 2017). Generally the numerically predicted post-liquefaction reconsolidation strains are an order of magnitude smaller than observed in experimental studies (e.g. Boulanger & Ziotopoulou (2013); Howell, Rathje & Boulanger (2014)).

To more accurately take into account the post-liquefaction reconsolidation strains, Boulanger & Ziotopoulou (2017) proposed the reduction of elastic moduli by a reduction factor F_{sed} that is also implemented in PLAXIS as follows:

$$G_{post-shaking} = F_{sed}\, G \tag{2.42}$$

$$K_{post-shaking} = F_{sed}\, K \tag{2.43}$$

The factor F_{sed} is calculated as:

$$F_{sed} = F_{sed,min} + (1 - F_{sed,min})\left(\frac{p}{20p_{sed}}\right)^2 \leq 1 \tag{2.44}$$

$$p_{sed} = p_{sed_0} + \left(\frac{z_{cum}}{z_{cum} + z_{max}}\right)\left\langle 1 - \frac{M^{cur}}{M^d}\right\rangle^{0.25} \tag{2.45}$$

$$F_{sed,min} = 0.03\exp(2.6D_{R0}) \leq 0.99 \tag{2.46}$$

$$p_{sed_0} = \frac{p_A}{5} \tag{2.47}$$

where the constant $F_{sed,min}$ represents the smallest value F_{sed} can attain and is dependent on the initial relative density D_{R0}. Parameter p_{sed_0} is the mean effective stress up to which reconsolidation strains are enhanced.

The user is advised to create two copies of the same PM4Sand material, one having *PostShake* equal to 0 and the other *PostShake* equal to 1. The dynamic analysis phase should be divided into 2 phases, the first phase covering the strong shaking motion with appropriate *Dynamic time interval* and the second phase covering the remaining weak shaking with the remaining *Dynamic time interval*. Please note not to use the *Reset time* option in the second phase to avoid starting the analysis from the beginning of the input signal. In the first phase the PM4Sand material with *PostShake* equal to 0 should be used and in the second phase the PM4Sand material with *PostShake* equal to 1.

2.7 INSTALLATION OF THE MODEL

The model has been compiled as a PLAXIS User-defined model in the form of a dynamic link library (dll). The name of the library is `pm4sand64.dll`. The library (i.e. the dll file) must be placed into the subfolder `udsm` in the PLAXIS installation folder.

In the PLAXIS Input program the user has to choose the *User-defined* material as shown in Figure 2.4. Then in the *Parameters* tab the *DLL file* `pm4sand64.dll` along with the *Model in DLL* PM4Sand as shown in Figure 2.5 has to be chosen. Afterwards, the model will be ready to be used along with other defined material models.

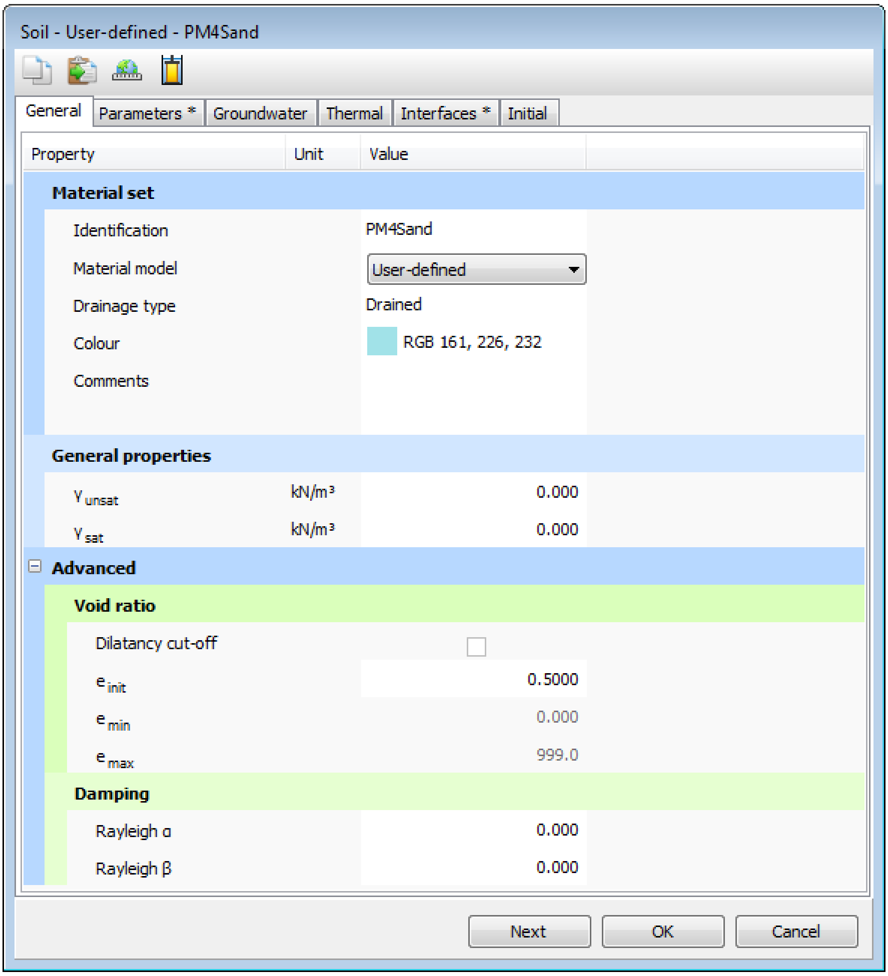


Figure 2.4 User-defined material choice as a *Material model*

2.8 MODEL PARAMETERS

The model has 13 input parameters that can be modified by the user. All of the parameters can be set in the Material input menu as shown in Figure 2.5.

The model parameters are grouped into two categories:

- a primary set of 4 parameters (i.e. D_{R0},G_0,h_{p0} and p_A) that are most important for model calibration and
- a secondary set of 9 parameters (i.e. e_{max}, e_{min}, n^b, n^d, φ_{cv}, ν, Q, R and PostShake) that may be modified from the recommended default values in special circumstances.

Boulanger & Ziotopoulou (2017) have provided the default values that are supposed to generally produce reasonable agreement with the trends in typical design correlations.

The default values are given for each of the secondary parameters in the following subsections and must be entered by the user, even if the user wants to use the default

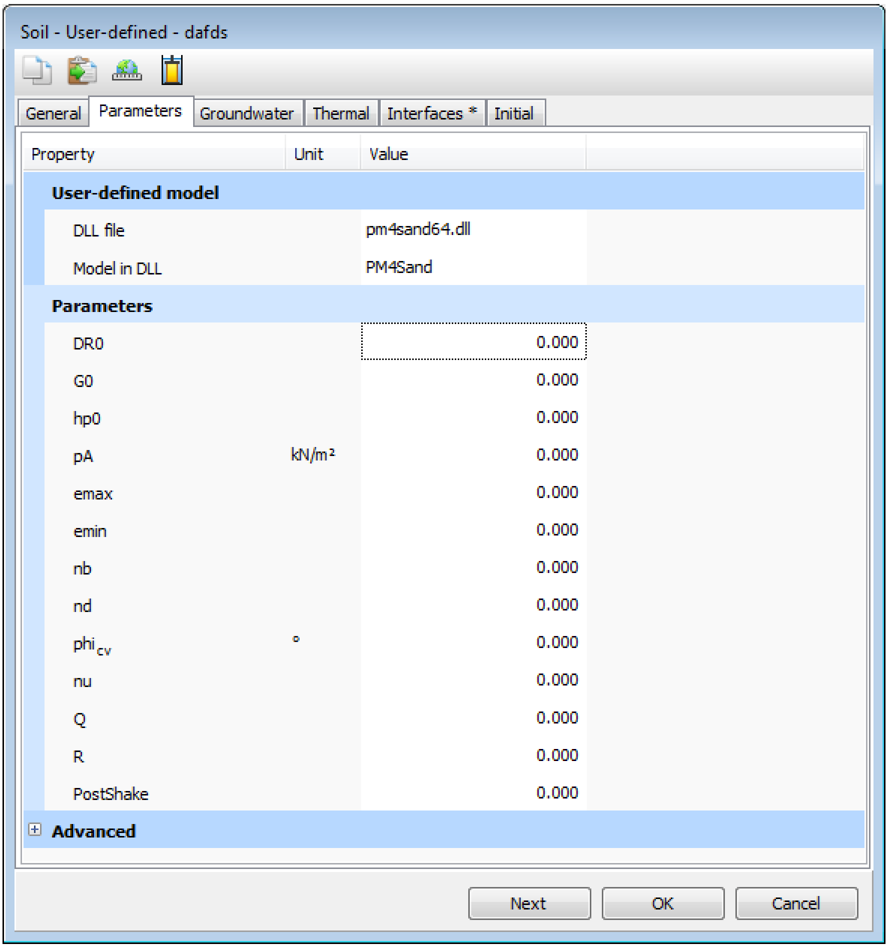


Figure 2.5 Parameters of PLAXIS User defined PM4Sand model in the *Material data set* window

values because the value of each parameter of the User-defined model is initially set to 0.0.

In the following subsections, the meaning of each of the input parameters will be discussed along with the recommended procedures and values to be used as stated by Boulanger & Ziotopoulou (2017).

2.8.1 RELATIVE DENSITY D_R (DR0) (-)

The relative density parameter D_R controls the dilatancy and stress-strain response characteristics. Its value can be estimated by correlation to penetration resistances of SPT and CPT tests. A common form for SPT correlation is:

$$D_R = \sqrt{\frac{(N_1)_{60}}{C_d}} \tag{2.48}$$

The value of C_d can be taken according to Idriss & Boulanger (2008) recommendations. They reviewed published data and past relationships and then adopted the value of

C_d = 46 in the development of their liquefaction triggering correlations. Regarding the use of the CPT penetration resistance the user is advised to also use the Idriss & Boulanger (2008) recommendations by the following expression:

$$D_R = 0.465\left(\frac{q_{c1N}}{C_{dq}}\right)^{0.264} - 1.063 \tag{2.49}$$

for which they adopted C_{dq} = 0.9.

The above recommendations are stated in Boulanger & Ziotopoulou (2017). The authors also comment that the input value D_R is best considered an "apparent relative density", rather than a strict measure of relative density from conventional laboratory tests. The value of D_R influences the response of the model through D_R correlations and the relative state parameter index ξ_R. Therefore, there may be situations where the user may choose to adjust the input D_R up or down relative to the above relationships to improve the calibration to some other relationship or data.

2.8.2 SHEAR MODULUS COEFFICIENT G_0 (G0) (-)

The shear modulus coefficient G_0 controls the elastic (small strain) shear modulus G as:

$$G = G_0 p_A \sqrt{\frac{p}{p_A}} \tag{2.50}$$

G_0 should be calibrated to fit estimated or measured V_s values, according to

$$G = \rho(V_s)^2 \tag{2.51}$$

or alternatively fit to values of V_s that are estimated by correlation to penetration resistances. Boulanger & Ziotopoulou (2017) used the $V_{s1} - (N_1)_{60}$ correlations from Andrus & Stokoe (2000) (Figure 2.6) with a slight modification extrapolating to very small $(N_1)_{60}$ in the following form:

$$V_{s1} = 85[(N_1)_{60} + 2.5]^{0.25} \tag{2.52}$$

Alternatively, the expression covering a range of typical densities can be used to directly calculate the parameter G_0 as proposed by Boulanger & Ziotopoulou (2017) in the following form:

$$G_0 = 167\sqrt{(N_1)_{60} + 2.5} \tag{2.53}$$

2.8.3 CONTRACTION RATE PARAMETER h_{p0} (hp0) (-)

The contraction rate parameter h_{p0} adjusts the contractiveness of the model and hence enables the calibration to specific values of cyclic resistance ratio (CRR). This parameter is meant to be calibrated last after the values of other parameters have been assigned. The user can use PLAXIS Soil test facility to perform uniform cyclic direct simple shear tests to calibrate the parameter.

The liquefaction triggering correlation by Idriss & Boulanger (2008) from Figure 2.7 can

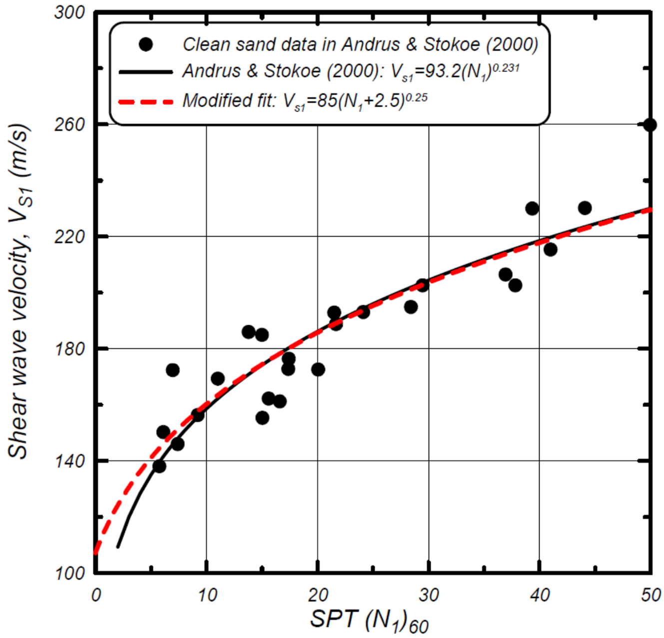


Figure 2.6 Correlation between overburden-corrected shear wave velocity and SPT penetration resistances in clean sands (after Andrus & Stokoe (2000) and modifications by Boulanger & Ziotopoulou (2017)).

be used to get the needed *CRR* value of the material (as has been done for the examples in the report by Boulanger & Ziotopoulou (2017)) if the cyclic laboratory data of the material is not available. This relationship provides the target CRR values for an effective overburden stress of 100kPa and an earthquake magnitude of M = 7.5. Looking at the earthquake magnitude of 7.5, the corresponding number of uniform cycles at 65% of the peak stress is equal to 15 (Figure 2.8). Boulanger & Ziotopoulou (2017) assumed that this corresponds approximately to *CRR* of 15 uniform loading cycles with the liquefaction triggering criterion of causing a peak shear strain of 3% in direct simple shear loading. For other earthquake magnitudes than M=7.5, the user is advised to use the $N_{M=7.5} - M$ relation from Figure 2.9 to read the value N_M at a desired magnitude M. Then the magnitude scaling factor *MSF* should be calculated according to the following relationship:

$$MSF = \left(\frac{N_{M=7.5}}{N_M}\right)^b \tag{2.54}$$

where b is the slope of $CRR - N$ lines, equal to b = 0.34 for sands, $N_{M=7.5}$ is the number of equivalent stress cycles at M = 7.5 and N_M is the number of equivalent stress cycles at the desired magnitude M. With the known *MSF*, the target CRR_M at the desired magnitude can be evaluated as:

$$CRR_M = MSF\ CRR_{M=7.5} \tag{2.55}$$

Instead of using the Eq. (2.55), the magnitude scaling factors can be read from the graphs in Figure 2.9. More details on the above procedure of calculating MSF and CRR_M can be found in Idriss & Boulanger (2008). In the case of the availability of cyclic laboratory $CRR - N$ curves of the modelling material, the user can calibrate the parameter h_{p0} according to points on those curves. Generally, decreasing the value of h_{p0} gives more contractant behaviour and consequently a lower CRR.

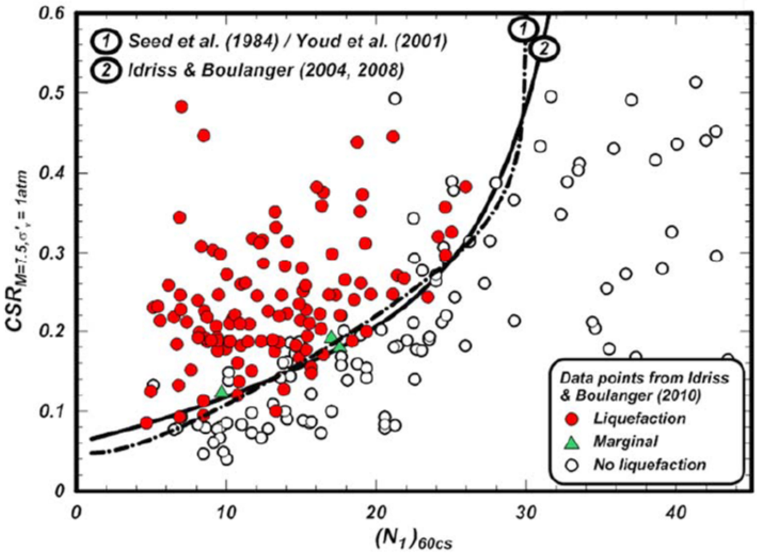


Figure 2.7 Correlations for cyclic resistance ratio (CRR) from SPT data (after Idriss & Boulanger (2010))

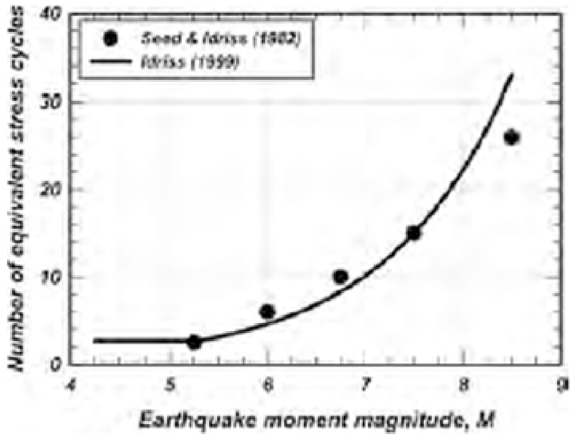


Figure 2.8 Mean number of equivalent uniform cycles at 65% of the peak stress versus earthquake magnitude (after Idriss & Boulanger (2008))

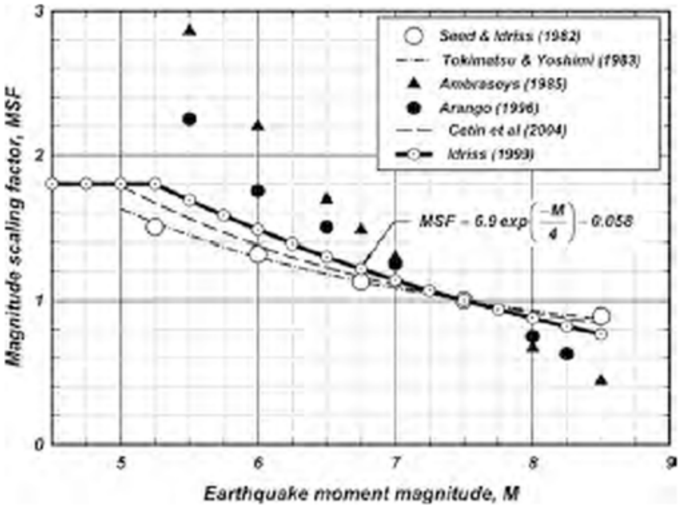


Figure 2.9 Magnitude scaling factors values proposed by various researchers (after Idriss & Boulanger (2008))

2.8.4 ATMOSPHERIC PRESSURE p_A (pA) (kPa) (DEFAULT: 101.3kPa)

The atmospheric pressure p_A should be specified by the user in the unit set being used for the analysis. In order to be consistent with the original formulation the recommended value is 101.3 kPa.

2.8.5 MAXIMUM AND MINIMUM VOID RATIO e_{max} and e_{min} (emax and emin) (-) (DEFAULT: 0.8; 0.5)

The maximum and minimum (e_{max},e_{min}) void ratios influence the computation of the relative state index ξ_R and the relationship between volume changes and the relative state index. The default values recommended by Boulanger & Ziotopoulou (2017) are 0.8 and 0.5, respectively.

2.8.6 BOUNDING SURFACE PARAMETER n^b (nb) (-) (DEFAULT: 0.5)

The bounding surface parameter n^b controls the relative position of the bounding surface to the critical state surface dependent on the relative state index ξ_R (Eq. (2.10)). The default value is 0.5. This affects the dilatancy and thus also the peak effective friction angles. It should be noted that M^b for looser than critical states (i.e. $\xi_R > 0$) is computed using the value of $n^b/4$.

2.8.7 DILATANCY SURFACE PARAMETER n^d (nd) (-) (DEFAULT: 0.1)

The dilatancy surface parameter n^d controls the value of the stress-ratio at which the contraction transitions to the dilation and vice versa. This transition is often referred to as phase transformation. The default value of n^d is 0.1. The value of 0.1 produces a phase transformation angle slightly smaller than φ_{cv} angle, which is consistent with the experimental data (Boulanger & Ziotopoulou, 2017). Similarly to the bounding surface parameter n^b the value of M^d for loose of critical states (i.e. $\xi_R > 0$) is computed using the value of $4n^d$.

2.8.8 CRITICAL STATE FRICTION ANGLE φ_{cv} (phi$_{cv}$) (^{0}) (DEFAULT: 33)

The critical state friction angle φ_{cv} defines the position of the critical state surface (i.e. the value of M stress ratio from Eq. (2.12)). The default value is 33 degrees.

2.8.9 POISSON'S RATIO ν (nu) (-) (DEFAULT: 0.3)

The Poisson's ratio ν. The default value is 0.3.

2.8.10 CRITICAL STATE LINE PARAMETERS Q AND R (Q, R) (-) (DEFAULT: 10; 1.5)

The parameters Q and R define the critical state line, see Eq. (2.9) and Figure 2.1. The default values for quartzitic sands per recommendations of Bolton (1986) are 10 and 1. Boulanger & Ziotopoulou (2017) use a slight increase in R to a value of 1.5 to lower the critical state line to better approximate typical results for direct simple shear loading.

2.8.11 POST SHAKE SWITCH (PostShake) (-) (DEFAULT: 0)

The post shake switch is used to activate the reduction of elastic stiffness to simulate the post-shaking reconsolidation. According to Boulanger & Ziotopoulou (2017) the user should activate this feature only after the end of strong shaking. To properly simulate this in PLAXIS the copy of PM4Sand material in use should be created with the value of *PostShake* parameter equal to 1.0. Then the materials with *PostShake* equal to 0 and 1 should be interchanged as discussed in Section 2.6.7.

2.8.12 OTHER MODEL PARAMETERS

The values of the remaining model parameters of the original PM4Sand model are equal to the default values or are calculated from the index properties according to procedures described in Boulanger & Ziotopoulou (2017).

2.9 STATE PARAMETERS

The model uses around 30 state parameters as well as additional tracking variables. 3 state parameters are meant to be used by the user and therefore also have the clear identification, namely:

- σ_{v0} (sigv0): the initial vertical stress. It is reinitialised at the beginning of each phase to compute the realistic ru and ru,max values.
- r_u (ru): the pore-pressure ratio according to the vertical stress, calculated as:

$$r_u = 1 - \frac{\sigma_\nu}{\sigma_{\nu 0}} \tag{2.56}$$

- $r_{u,max}$ (ru,max): the maximum value of the pore-pressure ratio r_u in the current phase

2.10 ADVICE ON THE USE OF THE MODEL

- The model can only be used in 2D plane-strain analyses, because the out-of-plane stress σ_{zz} is ignored in the model formulation. The model still returns σ_{zz} using the linear elastic assumption for this component of stress following Eq. (2.7).
- The user is advised to use the manual time stepping with small time steps (substeps) otherwise the situation can arise with too big strain increments applied to the model which may result in time consuming stress integration and also a global divergence.
- If the free field boundaries are used and are in contact with the liquefied material the stress concentration can occur. Therefore the user is advised to use a drained zone preferably of another material type close to the free field boundaries, as shown in Figure 2.10.

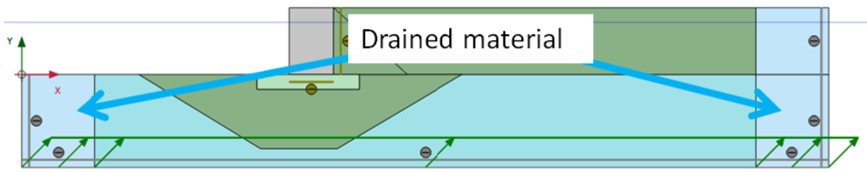


Figure 2.10 The drained material at the left and right free field boundary.

- Due to the plane-stress model formulation the only applicable test conditions in the PLAXIS Soil Test facility are the direct simple shear (DSS) and cyclic direct simple shear (CDSS) tests.
- In order to reduce the non-realistic concentrations of excess pore pressures and stresses, the user is advised to use the cavitation cut-off set at 100kPa during the undrained dynamic analysis. This setting can be chosen in the *Phases* window under the *Deformation control parameters*.

A better spread of pore-water pressure and less mesh dependence is made possible with the use of Dynamic analysis with Consolidation (i.e. Biot dynamics), which has been available since PLAXIS 2D 2018 version.

- The state variables can be explicitly initialised for a particular phase in the *Phases* window under the *Deformation control parameters* by enabling the option *Reset state variables*. Note that, this would reinitialise the state variables of all constitutive models.
- Due to the known less accurate simulation capabilities of the model in static loading conditions for the case if the model was calibrated for the dynamic loading, the recommended procedure of using the model in the calculations is to use other relevant material in place of the PM4Sand material in order to get the static (gravity, staged construction) phases of the calculations. At the dynamic phase, the material should be changed to PM4Sand. In this way the model internal variables will be initialised according to stress state of the previous static phase prior to the dynamic loading.

3 EXERCISE 1: SIMULATION OF CSR-N RESPONSE WITH THE PM4SAND MODEL

The aim of the excercise:

The aim of this exercise is to simulate all the points on two published CSR-N curves generated by the original PM4Sand model (Boulanger & Ziotopoulou, 2015) by the use of cyclic direct simple shear simulations in PLAXIS SoilTest facility. The tests are stress-controlled and in undrained conditions.

The chosen material is Ottawa sand at D_R = 65%. Ziotopoulou (2017) calibrated two material sets that were used in the LEAP project analyses (i.e. Case A and Case B material sets). The parameters used for cases A and B are given in Tables 3.1 and 3.2, while the only difference between the two material sets is the contraction rate parameter h_{p0} as given in Table 3.3.

Table 3.1 The calibrated parameters

Parameter	Value
D_{R0} [-]	0.65
G_0 [-]	240
e_{max} [-]	0.81
e_{min} [-]	0.4915

Table 3.2 The parameters with default values

Parameter	Value
p_A [kPa]	101.3
n^b [-]	0.5
n^d [-]	0.1
φ_{cv} [^{0}]	33
ν [-]	0.3
Q [-]	10
R [-]	1.5
PostShake [-]	0

The contraction rate parameter h_{p0} is taking the following 2 values:

Table 3.3: The values of the contraction rate parameter h_{p0}

Parameter	Case A	Case B
h_{p0}	0.05	0.2

In Figure 3.1 the target CRR-N plots are shown from undrained cyclic stress-controlled DSS simulations using the original PM4Sand model (as reported by Ziotopoulou (2017)). Additionally, plots from simulations performed by the PM4Sand model implemented in PLAXIS by using the SoilTest facility are shown. These points (i.e. blue dots) represent the result of this exercise.

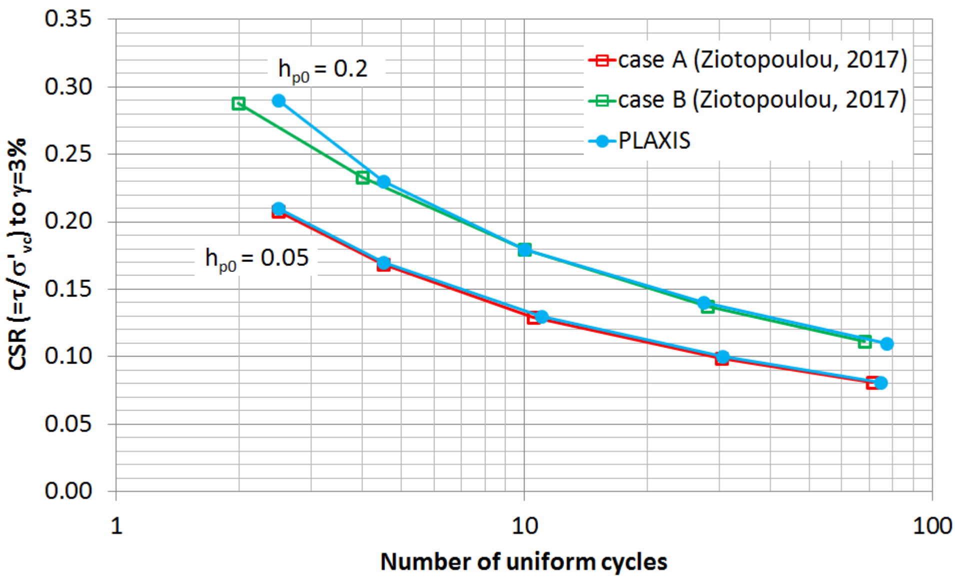


Figure 3.1 CSR-N relationships for case A and case B material sets. (modified from Ziotopoulou (2017)

The numbers used to draw the PLAXIS points on the plots are given in Tables 3.4 and 3.5, for each material set respectively.

Table 3.4 CSR-N values at single amplitude shear strain γ=3% for Case A material

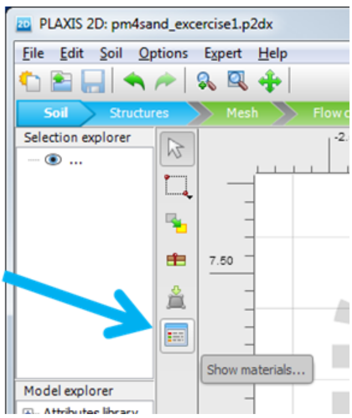
Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
1	0.21	21	2.5
2	0.17	17	4.5
3	0.13	13	11
4	0.1	10	30.5
5	0.081	8.1	74.5

Table 3.5 CSR-N values at single amplitude shear strain γ=3% for Case B material

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
1	0.29	29	2.5
2	0.23	23	4.5
3	0.18	18	10
4	0.14	14	27.5
5	0.11	11	77

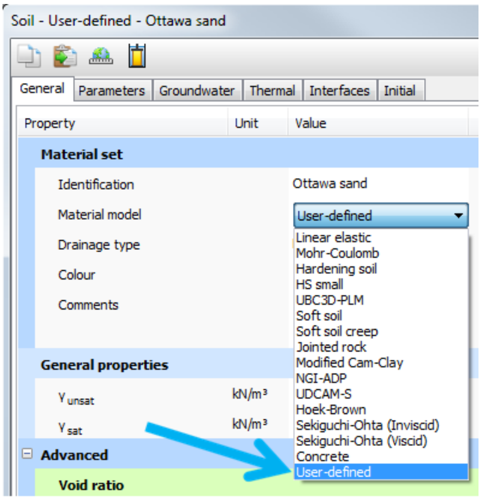
Step by step guide through the excercise

1. Start PLAXIS Input.
2. Start a new project.
3. In the *Project properties* window, click *Next*.
4. In the *Model* tab sheet of the *Project properties* window, make sure that *Units* are set to [m], [kN] and [day], then click *OK*.
5. In the PLAXIS 2D Input window, click the *Materials* button in the toolbar.

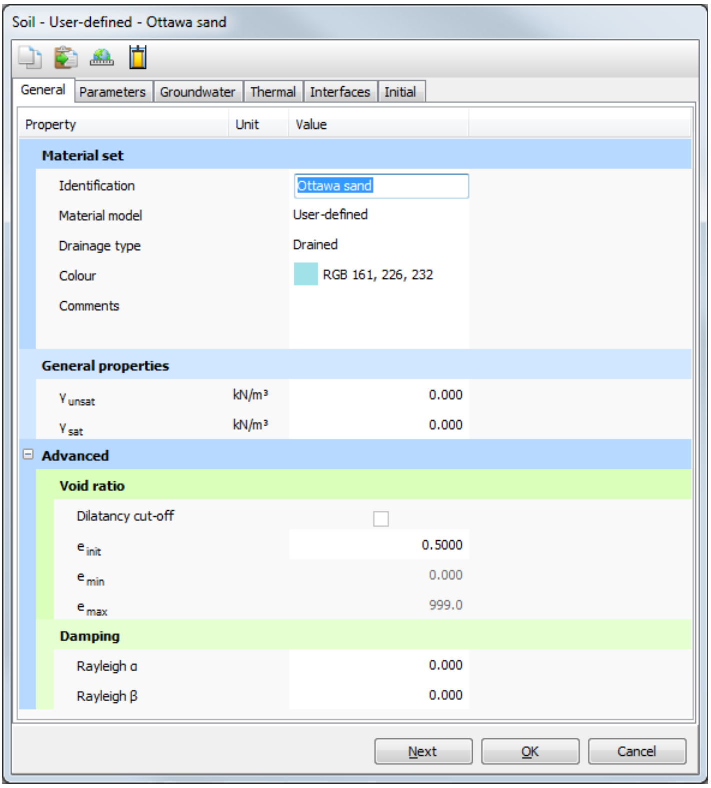


6. We will create one PM4Sand material set. In the *Material sets* window, click *New...*.
7. In the *General* tab sheet of the *Soil* window, enter an *Identification* for the data set

like "Ottawa sand" and then select *User-defined* as the Material model (see the picture below). Other settings on this tab sheet are not relevant for this exercise.



The *General* tab sheet should look like this:

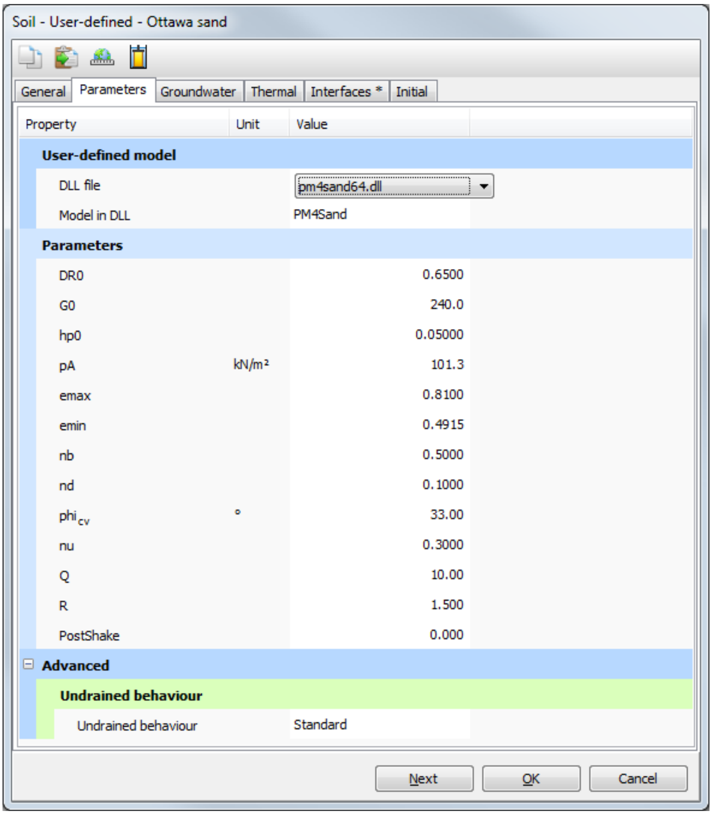


8. Click *Next* to get to the *Parameters* tab sheet.
9. In the *Parameters* tab sheet choose the *DLL file* pm4sand64.dll and *Model in DLL* PM4Sand.
   All the material parameters of the model will be shown with the values of 0.0.

10. Enter the following values of the parameters:

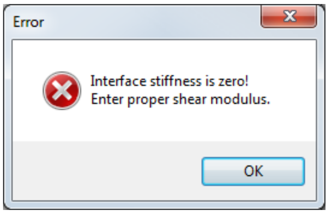
Parameter	Value
DR0 [-]	0.65
G0 [-]	240
hp0 [-]	0.05
pA [kPa]	101.3
emax [-]	0.81
emin [-]	0.4915
nb [-]	0.5
nd [-]	0.1
phi_{cv} [°]	33
nu [-]	0.3
Q [-]	10
R [-]	1.5
PostShake [-]	0

The *Parameters* tab sheet should look like in the following figure:

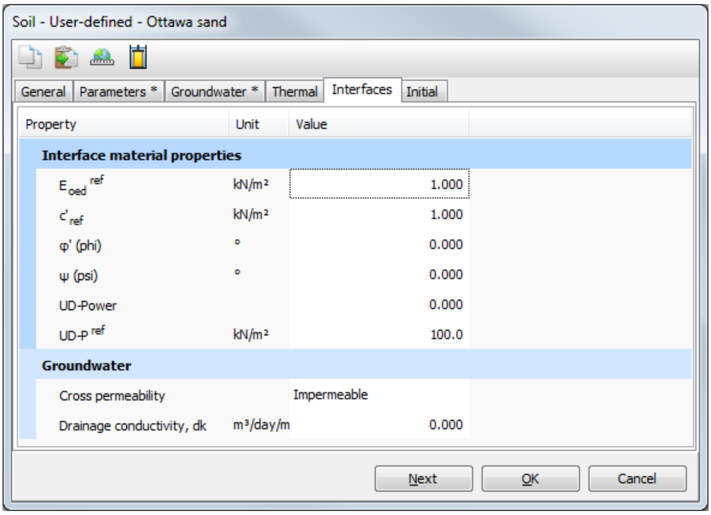


11. Go to the *Interfaces* tab sheet and enter imaginary values of 1 in the first two fields

(i.e. E_{oed}^{ref} and c'_{ref}). The interfaces will not be used in this exercise, but the numbers other than zeros are needed for PLAXIS to accept the data as a consistent data set. Otherwise, the following warning will be shown and the creation of the material will not be finished:

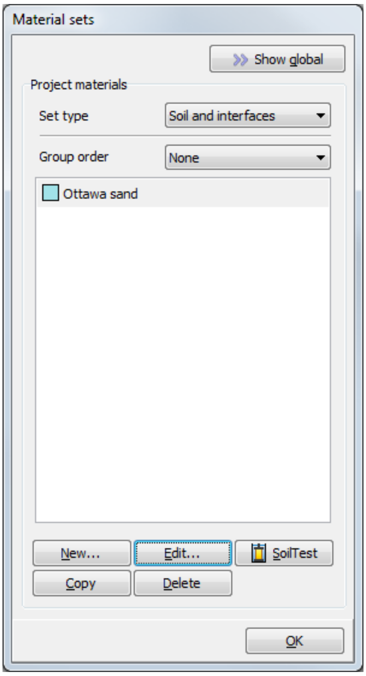


The *Interfaces* tab sheet should look like the following:

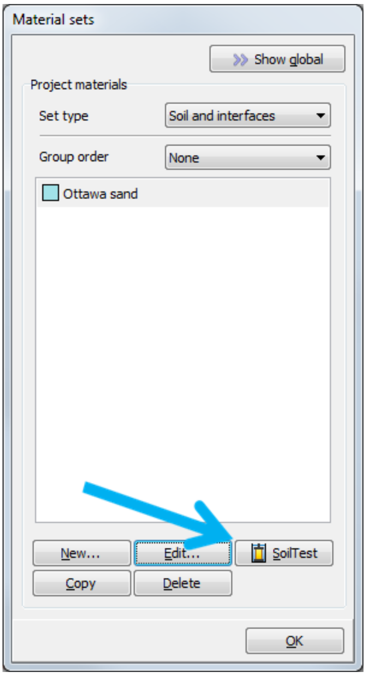


12. Click *OK* to finish the creation of the material set Ottawa sand.

    The *Material sets* window will look like this:

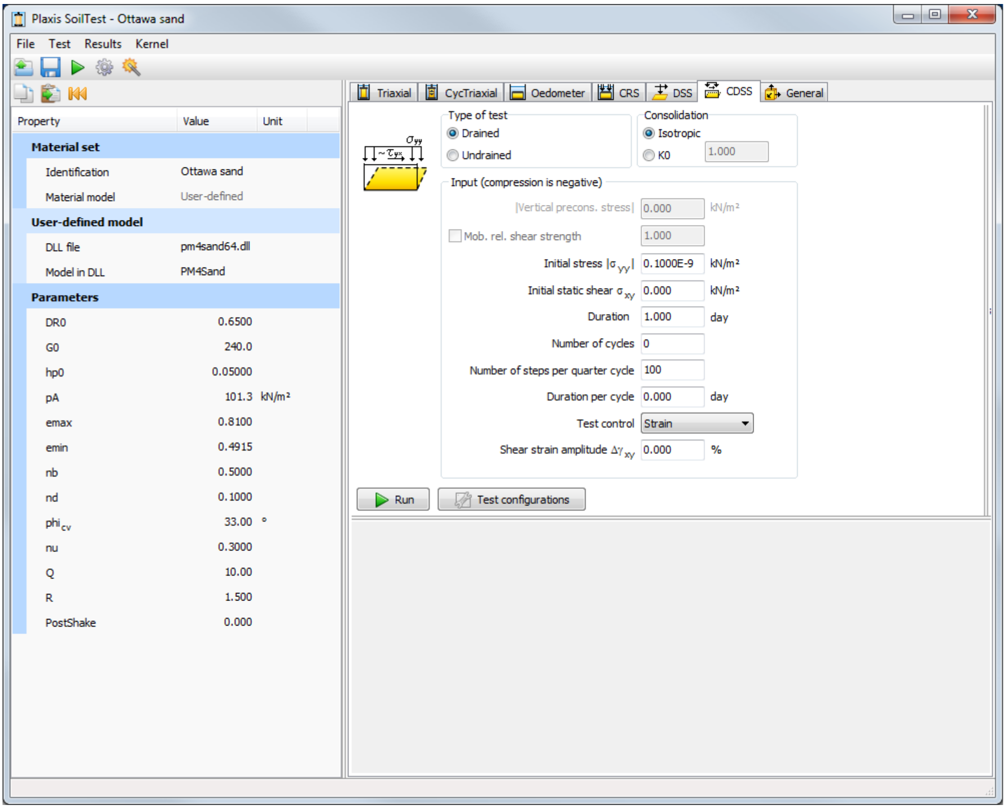


13. Click on the *Ottawa sand* material set and afterwards click on the *SoilTest* button as shown in the following figure:



The PLAXIS SoilTest facility will be opened.

14. In the PLAXIS *SoilTest* window choose the *CDSS* tab sheet.

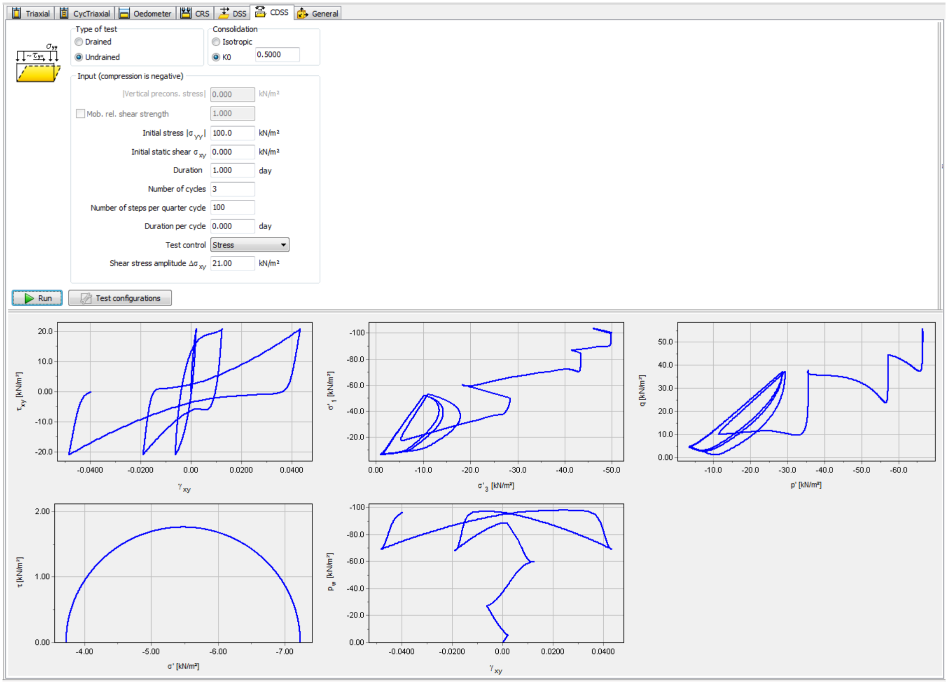


15. Enter the following test conditions in the fields:

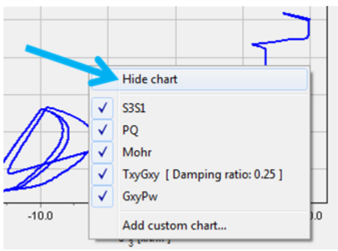
Parameter	Value
Type of test	Undrained
Consolidation / K_0	0.5
Initial stress $\|\sigma_{yy}\|$	100 kN/m^2
Initial static shear σ_{xy}	0 kN/m^2
Duration	Not relevant (1 day)
Number of cycles	3
Number of steps per quarter cycle	100
Duration per cycle	Not relevant (0 day)
Test control	Stress
Shear stress amplitude $\Delta\sigma_{xy}$	21 kN/m^2

Make sure that the K_0 value is set to 0.5, otherwise the liquefaction resistance will be higher than the target values from Figure 3.1.

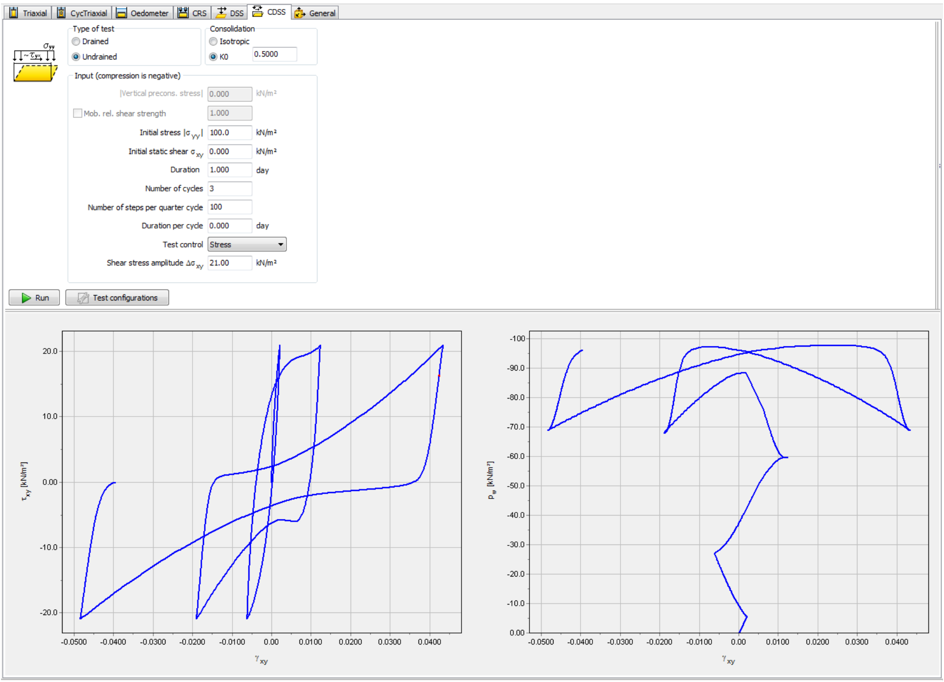
16. Press the *Run* button. The simulation will run and plot the results when finished, as in the following figure:



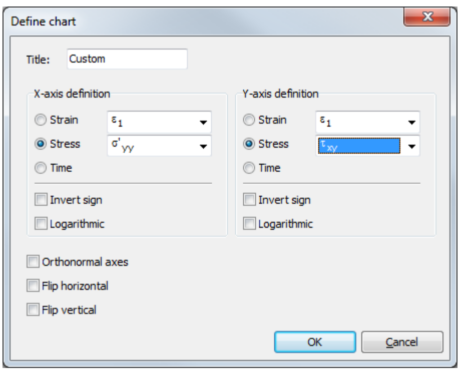
17. Remove the non-relevant plots and keep only the $\tau_{xy} - \gamma_{xy}$ and $p_w - \gamma_{xy}$ plots. Do this by right clicking on them and selecting *Hide chart*.



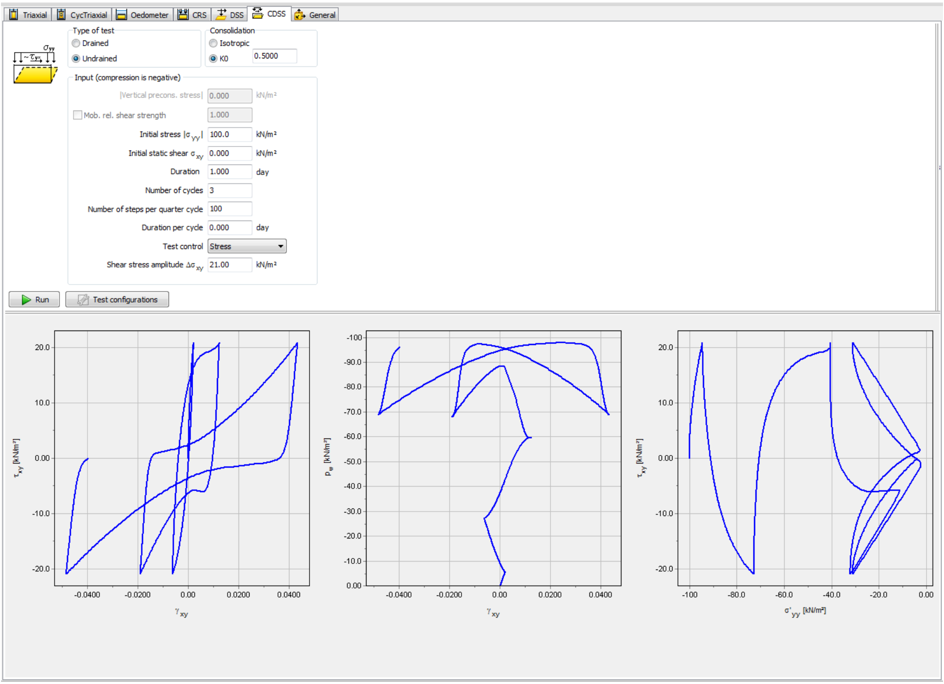
The *SoilTest* window will look like the following:



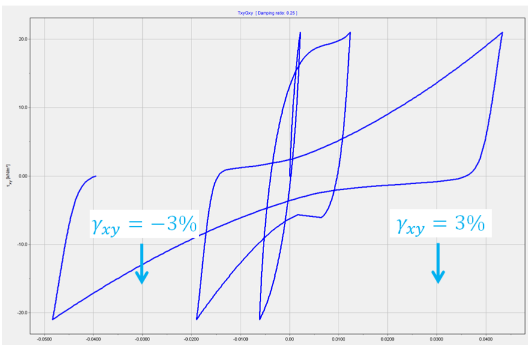
18. Add the $\tau_{xy} - \sigma'_{yy}$ plot by right clicking on the plot area and selecting *Add custom chart...* as:



Now the three characteristic plots are present in the window:



From the $\tau_{xy} - \gamma_{xy}$ plot below it can be seen that the liquefaction triggering condition $|\gamma_{xy}| > 3\%$ is reached at approximately 2.5 cycles.

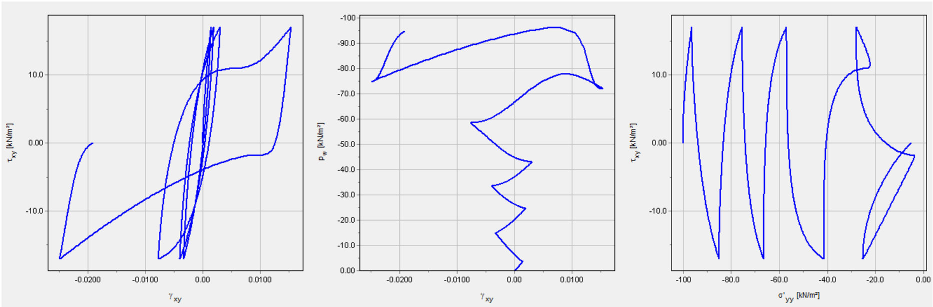


Therefore, the first point in Table 3.4 can be confirmed to have N=2.5 cycles to liquefaction:

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
1	0.21	21	2.5

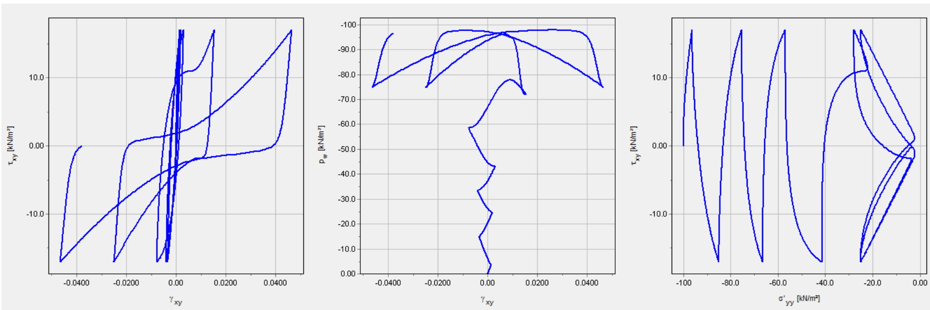
The point 2 from Table 3.4 is calculated by using $\Delta\sigma_{xy}$ =17 kN/m^2 and Number of cycles at 4 and 5.

If 4 cycles are chosen the following result is shown:



in which case liquefaction has not been reached yet, while $|\gamma_{xy}|$< 3%.

If 5 cycles are run, the following result is obtained:



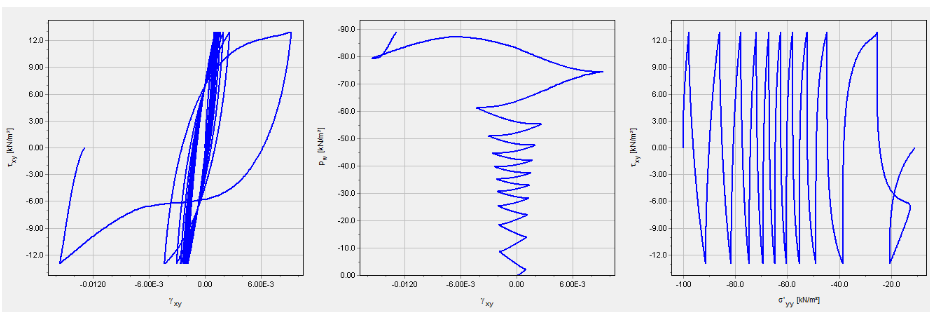
Therefore the result for point 2 is approximately 4.5 uniform cycles.

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
2	0.17	17	4.5

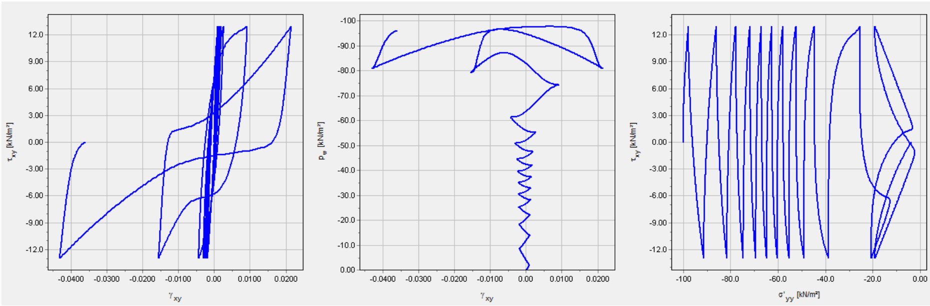
Point 3:

The point 3 from Table 3.4 can be calculated by using $\Delta\sigma_{xy}$ =13 kN/m^2 and Number of cycles at 10 and 11.

If 10 cycles are chosen, the following result is obtained:



While for 11 cycles the result is:



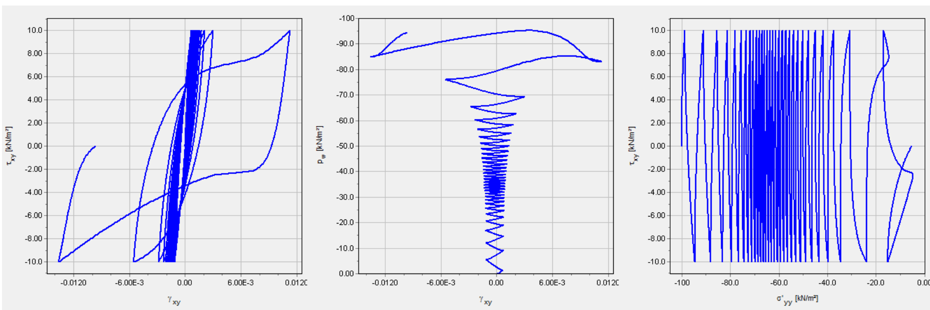
Therefore the result for the point 3 is N=11:

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
3	0.13	13	11

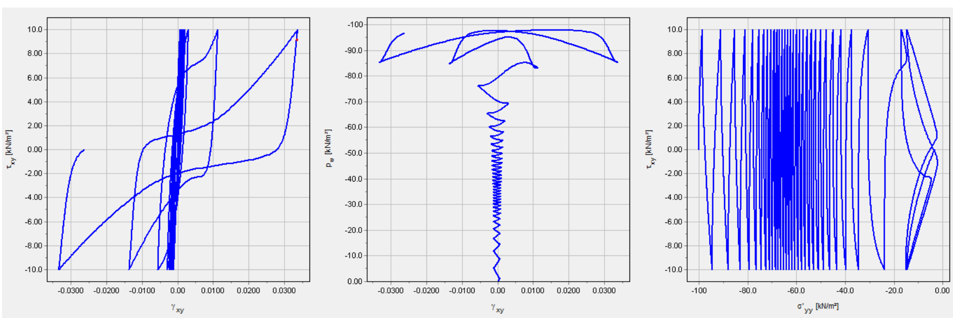
Point 4:

The point 4 from Table 3.4 can be calculated by using $\Delta\sigma_{xy}$ =10 kN/m^2 and Number of cycles at 30 and 31.

If 30 cycles are run, the following result is obtained:



while if 31 cycles are run, the result is:



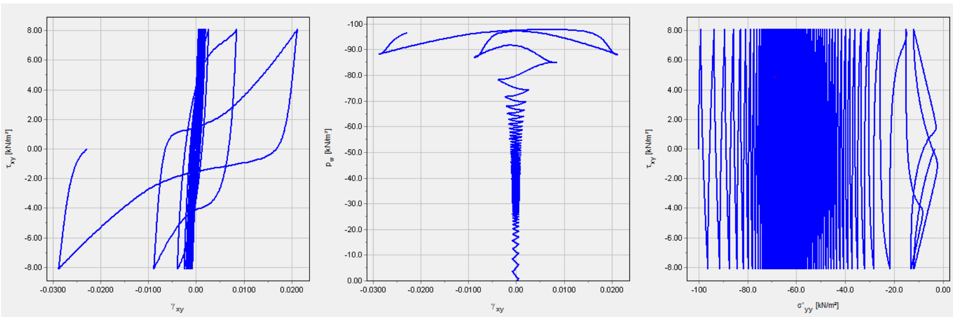
Therefore the final result for the point 4 is N=30.5:

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
4	0.10	10	30.5

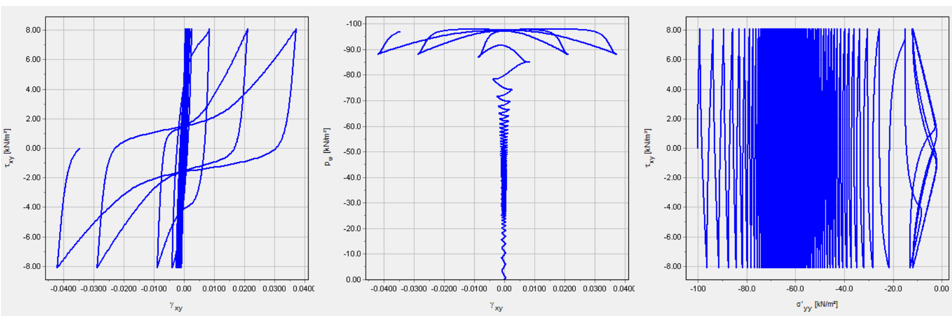
Point 5:

The point 5 from Table 3.4 can be calculated by using $\Delta\sigma_{xy}$ =8.1 kN/m^2 and Number of cycles from 74 to 75.

If 74 cycles are chosen, the following result is obtained:



And if 75 cycles are chosen, the result is:

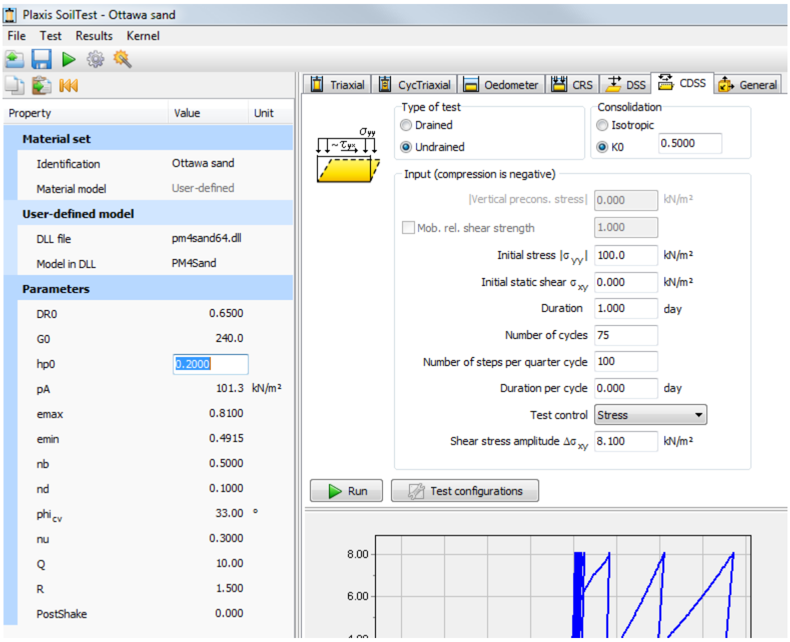


Therefore the final result for the point 5 is approximately N=74.5:

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
5	0.081	8.1	74.5

Now the simulations will be performed with the higher contraction rate parameter h_{p0}.The effect of higher h_{p0} is in increasing the cyclic strength of the material. Therefore, the CSR-N curve in Figure 3.1 for the material with h_{p0}=0.2 lies above the CRS-N for the material with h_{p0}=0.05.

19. Insert the value 0.2 next to the hp0 parameter field (see figure below).

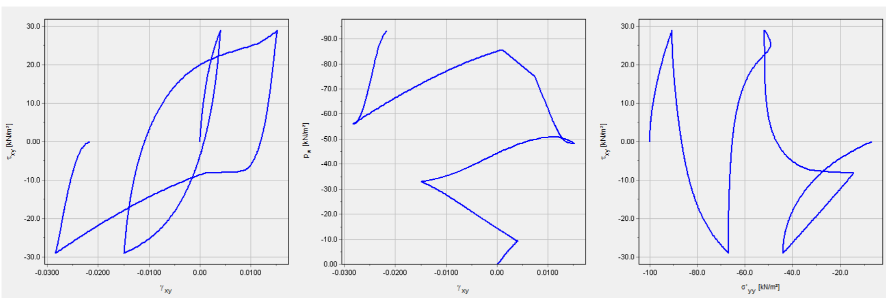


In the following pages, all the five points from the CSR-N curve for Case B will be simulated.

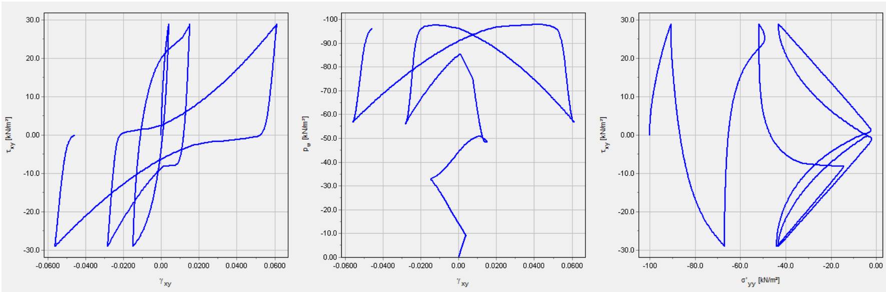
Point 1:

The point 1 from Table 3.5 can be calculated by using $\Delta\sigma_{xy}$=29 kN/m^2 and Number of cycles at 2 and 3.

If 2 cycles are simulated, the following result is obtained:



while in the case of 3 cycles, the following result is obtained:

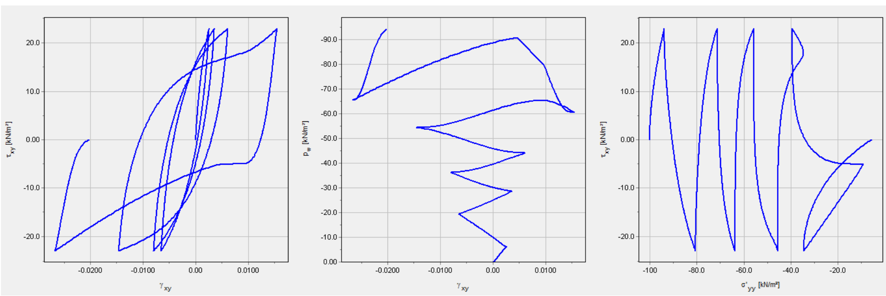


The final result for the point 1 is approximately N=2.5:

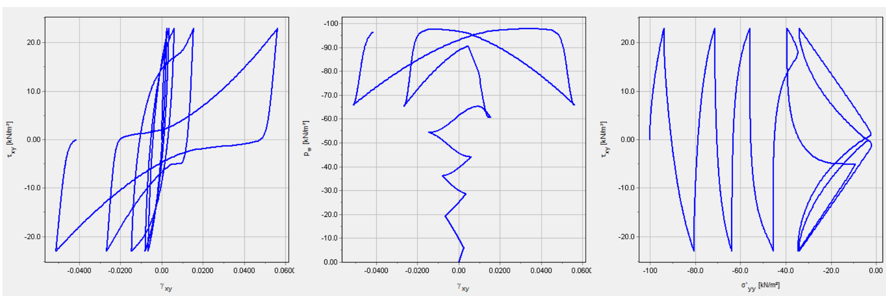
Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
1	0.29	29	2.5

The point 2 from Table 3.5 can be calculated by using $\Delta\sigma_{xy}$=23 kN/m^2 and Number of cycles at 4 and 5.

The result for 4 cycles is:



and the result for 5 cycles is:

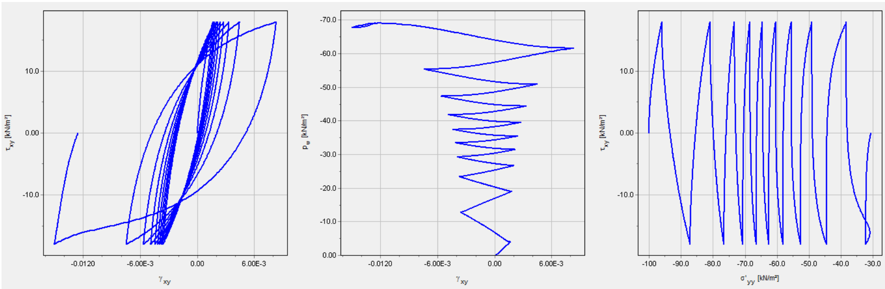


Therefore, the final result for the point 2 is approximately N=4.5:

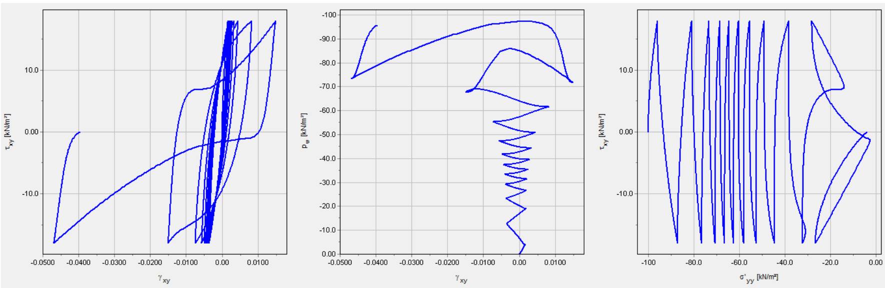
Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
2	0.23	23	4.5

The point 3 from Table 3.5 can be calculated by using $\Delta\sigma_{xy}$=18 kN/m^2 and Number of cycles at 9 and 10.

The result for 9 cycles is:



And the result for 10 cycles is:

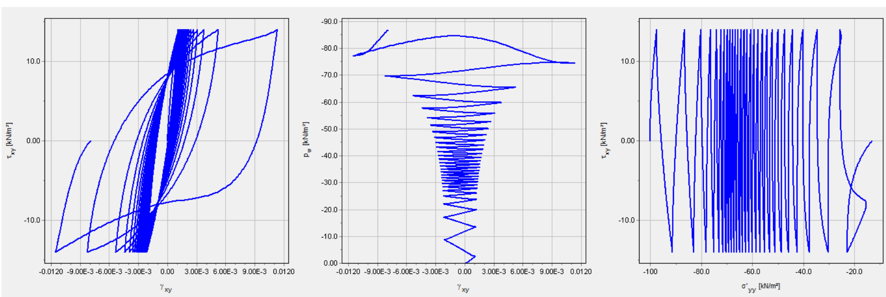


The final result for the point 3 is N=10:

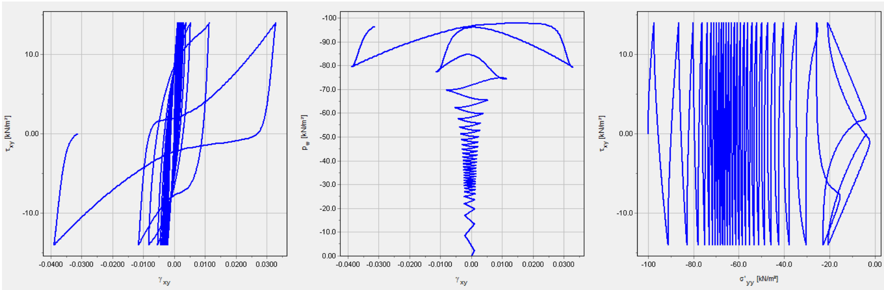
Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
3	0.18	18	10

The point 4 from Table 3.5 can be calculated by using $\Delta\sigma_{xy}$=14 kN/m^2 and Number of cycles at 27 and 28.

The result for 27 cycles is:



and the result for 28 cycles is:

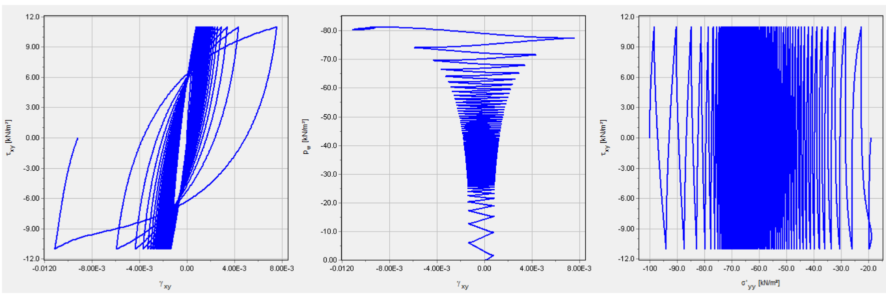


Therefore, the final result for the point 4 is N=27.5:

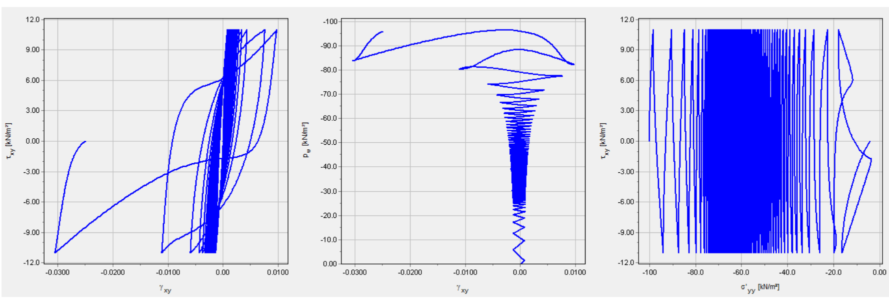
Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
4	0.14	14	27.5

The final point from Table 3.5 can be calculated by using $\Delta\sigma_{xy}$=11 kN/m^2 and Number of cycles at 76 and 77.

The result for 76 cycles is:



and for 77 cycles is:



Therefore, the final result for the point 5 is N=77:

Point	CSR[-]	Shear Stress amplitude $\Delta\sigma_{xy}$ [kPa]	Number of uniform cycles N at γ = 3% [-]
5	0.11	11	77

4 EXERCISE 2: 1D WAVE PROPAGATION ANALYSIS WITH THE PM4SAND MODEL

The aim of the exercise:
The exercise aims to perform the dynamic numerical analysis using PLAXIS to predict the onset of liquefaction in the sandy layer modelled with the PM4Sand model.

The soil stratigraphy (Figure 4.1) consists of an overconsolidated clay layer of medium compressibility that extends from the ground surface to 5m depth, followed by 10m of the sand layer with D_R = 55% and 25m of clay, until the bedrock is reached. The water table is assumed to be coincident with the ground surface level.

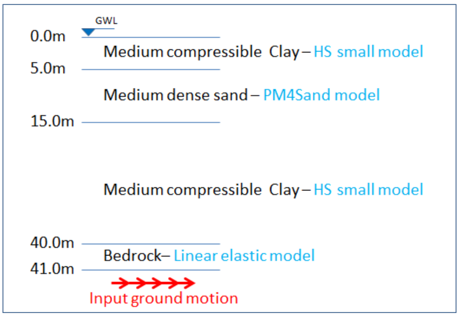


Figure 4.1 The soil stratigraphy used for the exercise

The clay material is modelled using the HS small model, while the sand material is modelled using the PM4Sand model. The bedrock layer of 1m thickness is modelled with the linear elastic material. The values of the material parameters for HS small model are given in Table 4.1 and material parameters for the bedrock in Table 4.2.

Table 4.1: HS small parameters of clay

Parameter	Symbol	Value	Unit
Drainage type	-	Undrained (A)	-
Unsaturated unit weight	γ_{unsat}	19	kN/m^3
Saturated unit weight	γ_{sat}	21	kN/m^3
Rayleigh damping coefficient	α	0.096	-
Rayleigh damping coefficient	β	0.00079	-
Secant stiffness in standard drained TX test	E_{50}^{ref}	9000	kN/m^2
Tangent stiffness for primary oedometer loading	E_{oed}^{ref}	9000	kN/m^2
Unloading-reloading stiffness	E_{ur}^{ref}	27000	kN/m^2

Power for stress-level dependency of stiffness	m	1	-
Cohesion	c_{ref}'	30	kN/m^2
Friction angle	φ'	26	$\circ$
Dilatancy angle	ψ	0	$\circ$
Shear strain at which $G_s = 0.722 G_0$	$\gamma_{0.7}$	0.0007	-
Shear modulus at very small strains	G_0^{ref}	60000	kN/m^2
Poisson's ratio	ν_{ur}'	0.2	-
Reference stress	p_{ref}	100	kN/m^2
Normally consolidated earth pressure at rest	K_0^{nc}	0.5616	-
Cohesion increment	c_{inc}'	0	$kN/m^2/m$
Reference coordinate	y_{ref}	0	m
Failure ratio	R_f	0.9	-
Tension cut-off	-	True	-
Tensile strength	σ_t	0	kN/m^2
Over-consolidation ratio	OCR	2	-

Table 4.2: Linear elastic parameters of bedrock

Parameter	Symbol	Value	Unit
Drainage type	-	Drained	-
Unsaturated unit weight	γ_{unsat}	22	kN/m^3
Saturated unit weight	γ_{sat}	22	kN/m^3
Young's modulus	E'	$8 \cdot 10^6$	kN/m^3
Poisson's ratio	ν'	0.2	-

The values of the parameters for the PM4Sand model are given in Table 4.3. Only the primary parameters are given in Table 4.3, while the secondary have the default values given in Table 4.4.

Table 4.3: PM4Sand primary parameters of sand

Parameter	Symbol	Value	Unit
Drainage type	-	Undrained (A)	-
Unsaturated unit weight	γ_{unsat}	14	kN/m^3
Saturated unit weight	γ_{sat}	18	kN/m^3
Rayleigh damping coefficient	α	0.096	-
Rayleigh damping coefficient	β	0.00079	-
Initial relative density (D_{R0})	Dr0	0.55	-
Shear modulus coefficient (G_0)	G0	677	-
Contraction rate (h_{p0})	hp0	0.40	-

Table 4.4: PM4Sand secondary (default) parameters

Parameter	Symbol	Value	Unit
Atmospheric pressure (p_A)	pA	101.3	kPa
Maximum void ratio ($e_m ax$)	emax	0.8	-
Minimum void ratio ($e_m in$)	emin	0.5	-
Bounding surface position according to $\xi_R(n^b)$	nb	0.5	-
Dilatancy surface position according to $\xi_R(n^d)$	nd	0.1	-
Critical state friction angle (ϕ_{cv})	phi_{cv}	33	$^\circ$
Poisson's ratio	nu	0.3	-
Critical state line parameter	Q	10	-
Critical state line parameter	R	1.5	-
Post-shaking reconsolidation	PostShake	0	-
Earth pressure coefficient	K_0	**0.5**	-

Step by step guide through the exercise:

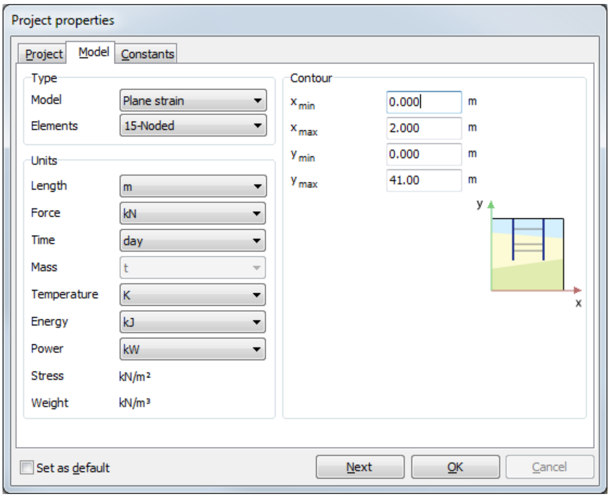
1. Start PLAXIS Input.

2. Start a new project.
3. In the *Project properties* window, click *Next* to open the *Model* tab sheet.
4. Define the dimensions of the calculation domain as:

x_{min}	0.0m
x_{max}	2.0m
y_{min}	0.0m
y_{max}	41.0m

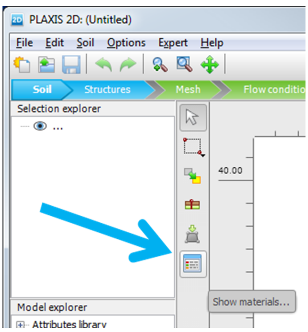
5. Make sure that *Units* are set to [m], [kN] and [day]. Then click *OK*.

   The Model tab sheet should look like this:

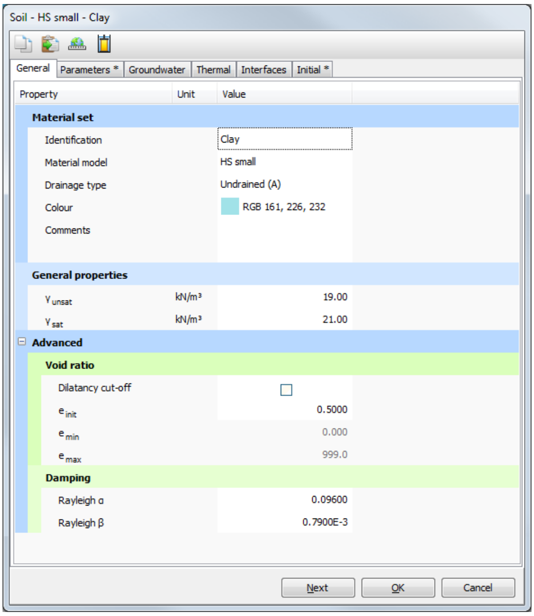


4.1 CREATION OF THE MATERIALS

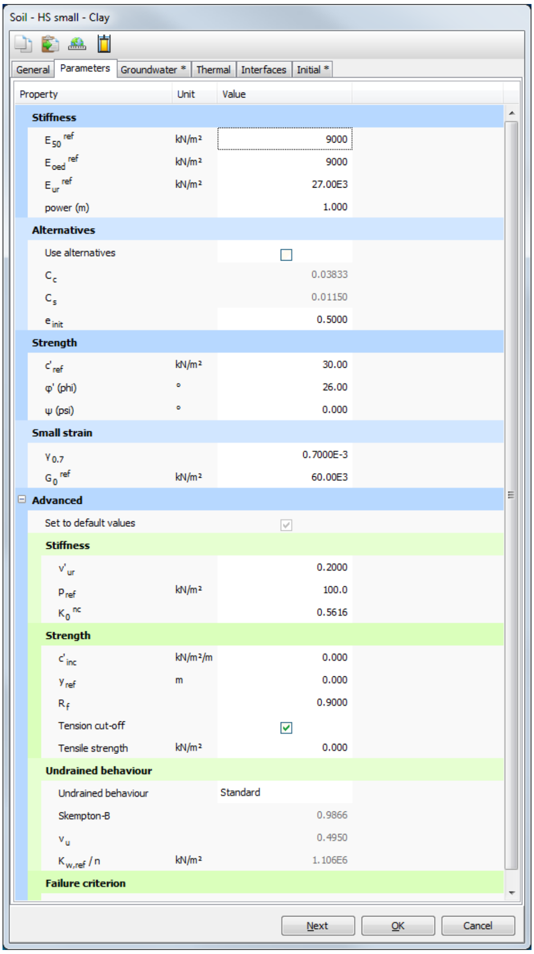
1. Open the *Material sets* window by clicking on *Show materials...* button.



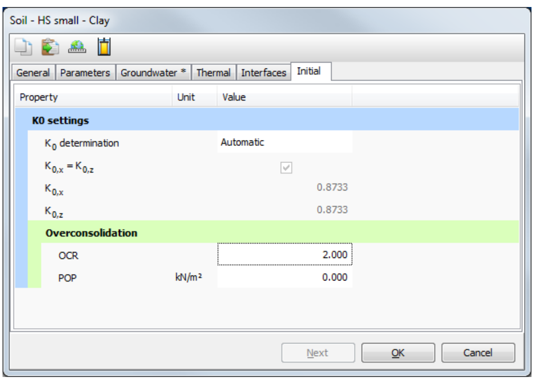
2. Create the new material for the clay layers. Type *Identification* as *Clay*. Choose *Material model HS small* and choose *Undrained (A)* drainage type. Enter the unsaturated and saturated unit weights ($\gamma_{unsat} = 19kN/m^3$ and $\gamma_{sat} = 21kN/m^3$) and Rayleigh damping ratios ($\alpha = 0.096$, $\beta = 0.00079$). The *General* tab sheet for *Clay* material should look like the following:



3. Go to the *Parameters* tab sheet of HS small modeland fill in the values. The filled tab sheet should look like the following:

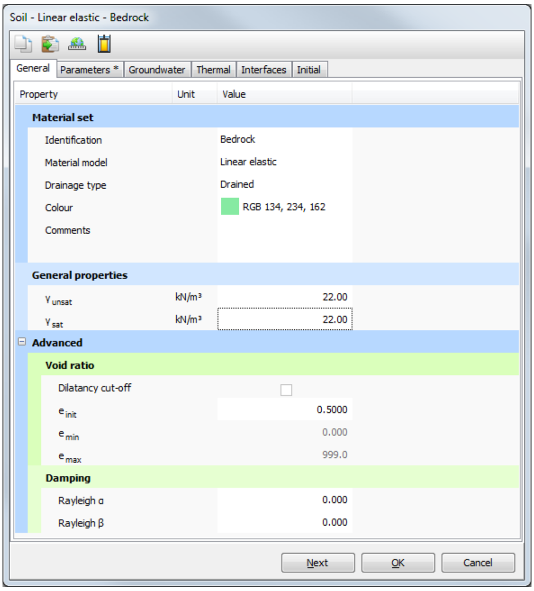


4. Go to the *Initial* tab sheet and insert the value of over-consolidation ratio *OCR* to be equal to 2 as in the following figure:

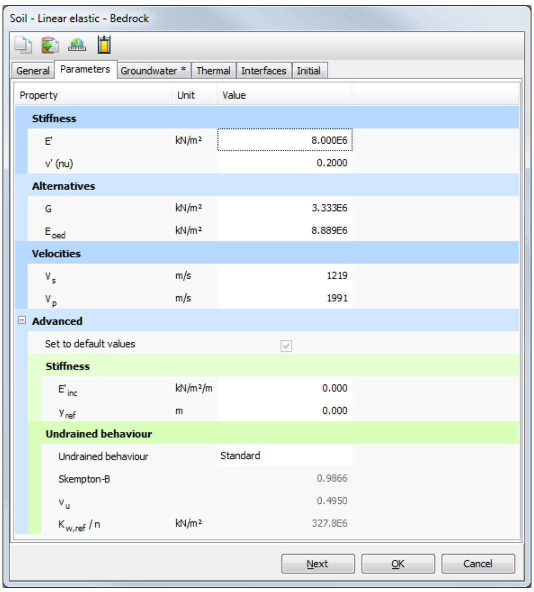


5. The creation of the *Clay* material is finished. Click *OK*.

6. Add a new material and name it *Bedrock*. Choose the *Linear elastic* material model, drainage type as *Drained* and insert the unit weights ($\gamma_{unsat} = 22kN/m^3$ and $\gamma_{sat} = 22kN/m^3$). The General tab should look lke the following:

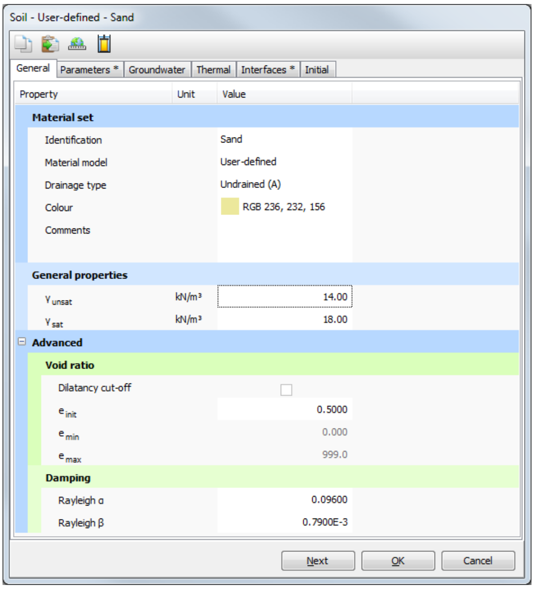


7. Go to the *Parameters* tab sheet and *E*' and ν' values. The tab sheet should look like the following:

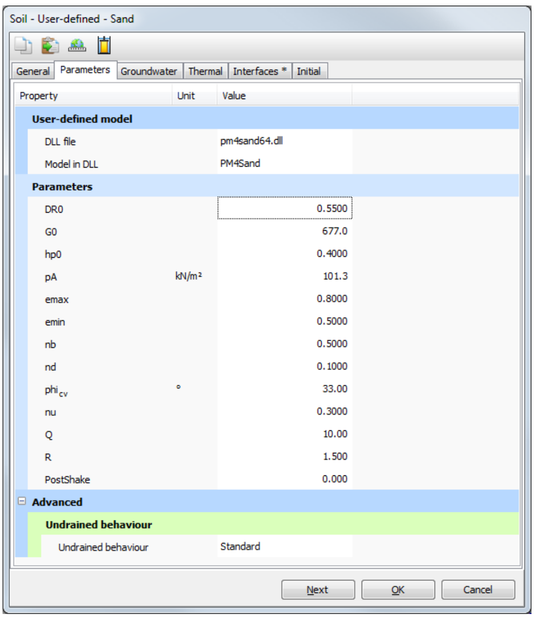


8. Press *OK* to finish the creation of the *Bedrock* material.

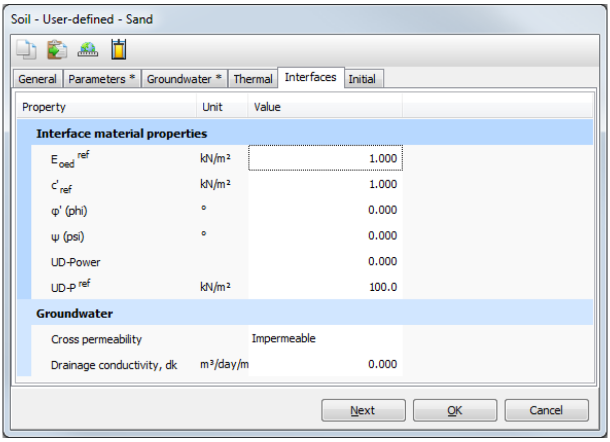
9. Create a new material and name it *Sand*. Choose the *User-defined* material model, choose *Undrained (A)* behaviour and fill in the unit weights (γ_{unsat} = 14kN/m^3 and γ_{sat} = 18kN/m^3) and Rayleigh damping ratios (α = 0.096, β = 0.00079). The *General* tab sheet should look like the following:



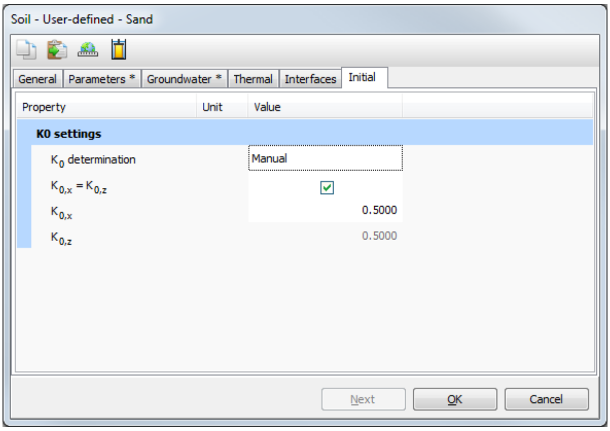
10. Go to the *Parameters* tab sheet. Choose DLL file to be pm4sand64.dll and Model in DLL to PM4Sand. All the parameters of the PM4Sand model will appear. Fill them in. The *Parameters* tab sheet should look like the following:



11. Go to *Interfaces* tab sheet and insert value 1 into E_{oed}^{ref} and c'_{ref} fields to make the model accepted as valid by PLAXIS, even though the interfaces will be not used for this model. The *Interfaces* tab should look like:



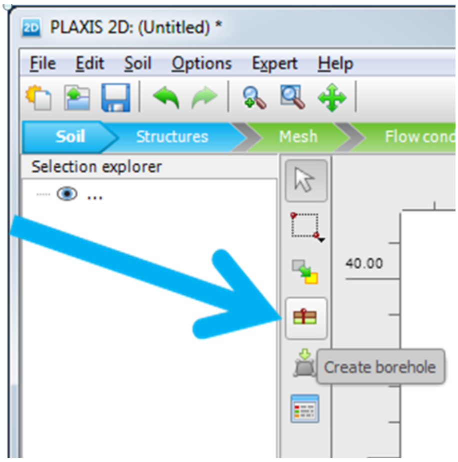
12. Go to the *Initial* tab and choose *Manual K_0 determination* and K_0 value to 0.5. The *Initial* tab should look like the following:



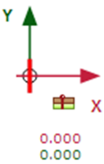
13. Click *OK* to finish the creation of the *Sand* material with PM4Sand model.

4.2 CREATION OF THE LAYERS

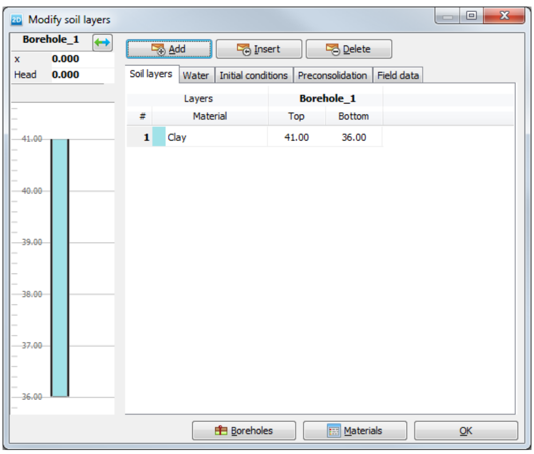
1. Create the borehole by clicking at the borehole icon in PLAXIS Input toolbar.



And click on the origin of the model:

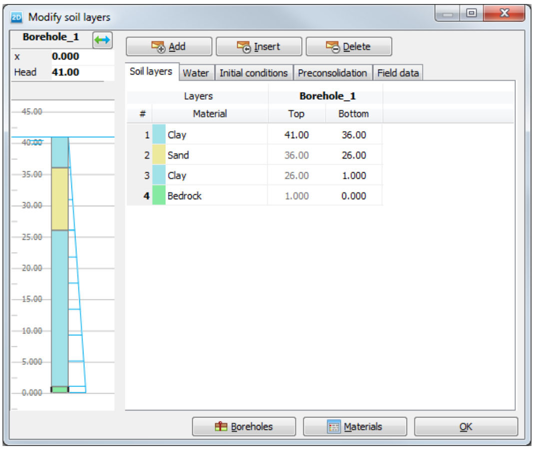


2. The *Modify soil layers* window will appear. Define the stratigraphy as sketched in Figure 4.1. Click the *Add* button to add a layer. Assign the *Clay* material for the first layer and the *Top* and *Bottom* coordinates to 41.0*m* and 36.0*m*. The *Modify soil layers* window should look like this:



3. Add the next layer by clicking on the *Add* button. Assign the *Sand* material to it and the *Bottom* coordinate to 26.0m.
4. Add another layer. Assign the *Clay* material to it and the *Bottom* coordinate to 1.0m.
5. Add the last layer. Assign the *Bedrock* material to it and the *Bottom* coordinate to 0.0m.
6. Finally, assign the groundwater head to 41.0m in the *Head* field. This means that the

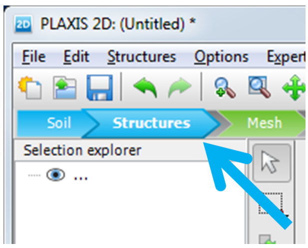
groundwater level is assumed to be at the surface of the model. The complete information in the *Modify soil layers* window should look like the following:



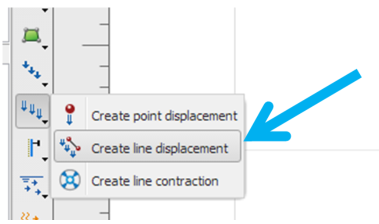
7. Click *OK* to finish the definition of layers.

4.3 DEFINITION OF THE EARTHQUAKE GROUND MOTION

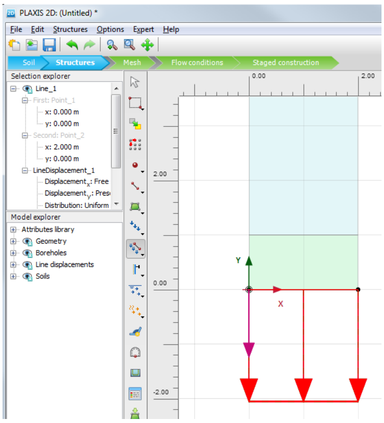
1. Go to the *Structures* mode in PLAXIS Input window by clicking on *Structures* button in the main toolbar as shown in the following figure:



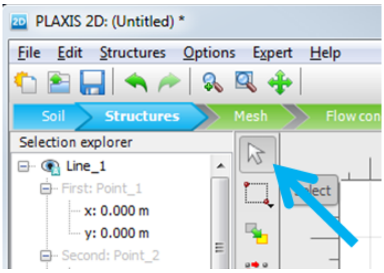
2. Zoom into the bottom of the model by using the mouse wheel (dragging and turning) and click on *Create line displacement* as shown below:



3. Left click at the point (0.000, 0.000) and (2.000, 0.000), (and then right click to finish the insertion) to create line displacement at the bottom of the model as shown below:



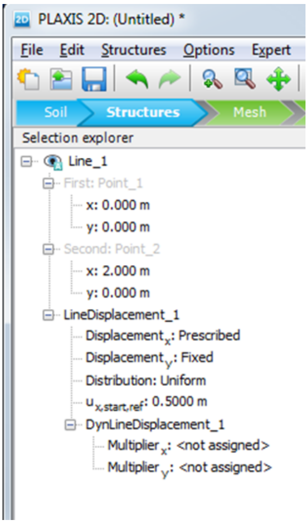
4. Click on the *Select* button to deactivate the line displacement creation, as shown below:



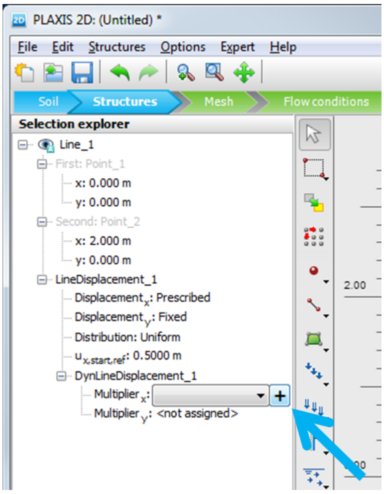
5. Click on the *Line displacement* at the bottom of the model and modify the definition of it in the left panel as follows:

   - Displacement$_x$: Prescribed
   - Displacement$_y$: Fixed
   - Distribution: Uniform
   - $U_{x,start,ref}$: 0.5000 m

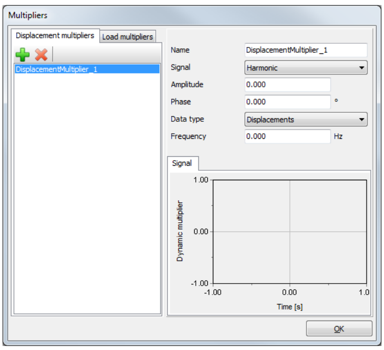
   Considering that the boundary condition at the base of the model will be defined using a compliant base, the input signal has to be taken as half of the outcropping motion. Therefore the factor 0.5m is used as $U_{x,start,ref}$. The left panel should look like the following:



6. Now the earthquake loading history will be assigned. Click next to the Multiplier$_x$ and at the + button, as shown below:

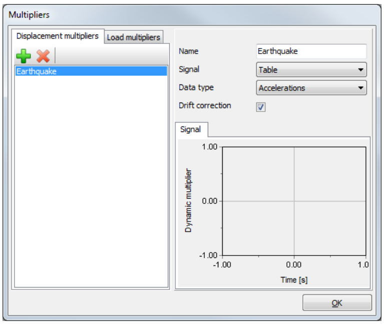


7. The *Multipliers* window will open.

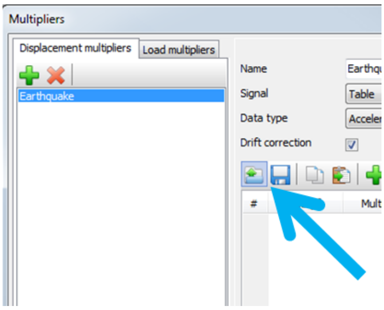


8. Enter the name *Earthquake* and choose *Signal* as *Table*, *Data type* as *Accelerations*

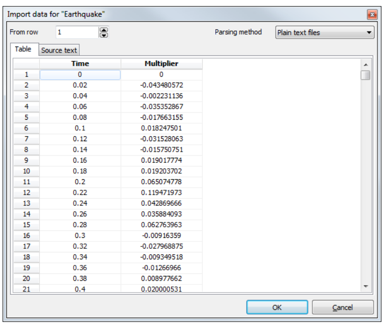
and *Drift correction* to *On*. The *Multipliers* window should look as shown below.



9. Click on the button to import the input motion as follows.

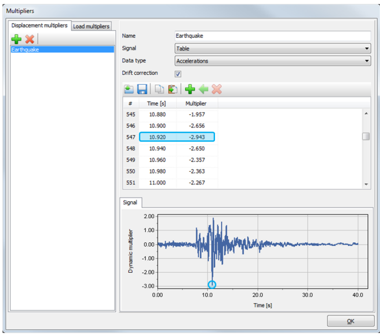


10. The *Open* window will appear. Choose the file amax3_amax03g.txt with accelerogram data. The *Import data* for Earthquake window will appear with the imported 2 columns of data (i.e. *Time* and *Multiplier*). The *Import data* window is shown below.

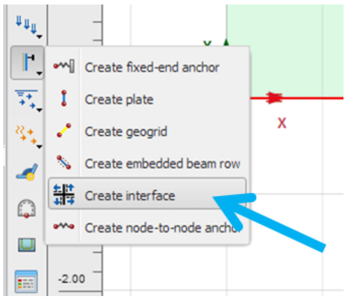


	Time	Multiplier
1	0	0
2	0.02	-0.043480572
3	0.04	-0.002231136
4	0.06	-0.035352867
5	0.08	-0.017663155
6	0.1	0.018247501
7	0.12	-0.031528063
8	0.14	-0.015750751
9	0.16	0.019017774
10	0.18	0.019203702
11	0.2	0.065074778
12	0.22	0.119471973
13	0.24	0.042869666
14	0.26	0.035884093
15	0.28	0.062763963
16	0.3	-0.00916359
17	0.32	-0.027968875
18	0.34	-0.009349518
19	0.36	-0.01266966
20	0.38	0.008977662
21	0.4	0.020000531

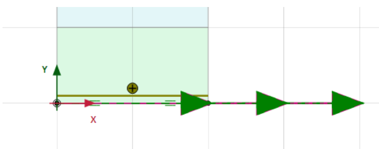
11. Press *OK* to close the *Import data* window. The imported data will be shown again in the *Multipliers* window together with the *Signal* plot. It can be seen that the maximum absolute value of the dynamic multiplier is 2.94 at 10.92*s* corresponding to approximately the peak horizontal acceleration of $3m/s^2$ (i.e. $0.3g$). A moment magnitude M_w equal to 6.9 characterises the provided accelerogram. The *Multipliers* window is shown below, together with the marked acceleration at 2.94*s*.



12. Press *OK* to finish the definition of earthquake loading.
13. To model a compliant base, it is required to specify an interface at the bottom of the model. Therefore click at the *Create interface* button in the toolbar as shown in the following figure.

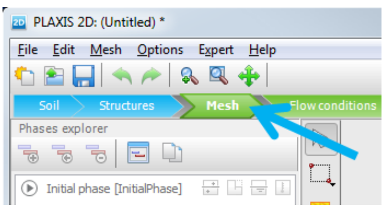


14. Create the interface, starting at the point (2.000, 0.000) towards the point (0.000, 0.000). In this way, the interface will be created with the properties of the bedrock side.
15. After finishing, the model at the bottom should look like in the following figure.

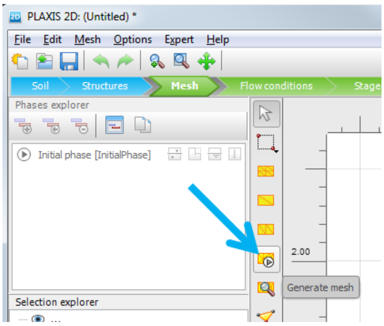


4.4 CREATION OF THE FINITE ELEMENT MESH

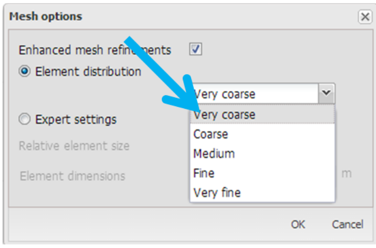
1. Click on the *Mesh* button of the main PLAXIS Input toolbar as shown in the figure:



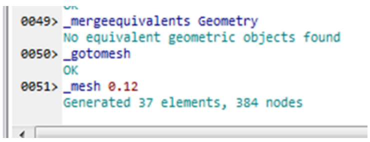
2. Click on the *Generate mesh* button as shown in the figure:



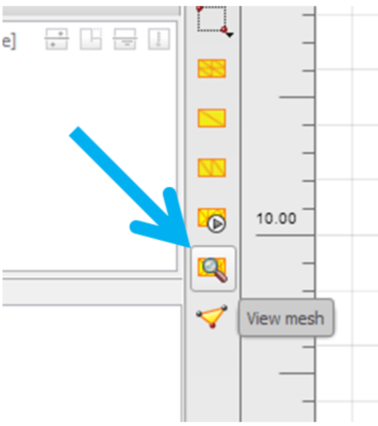
3. The *Mesh options* window will appear. Select the *Element distribution* option to *Very coarse* as shown below:



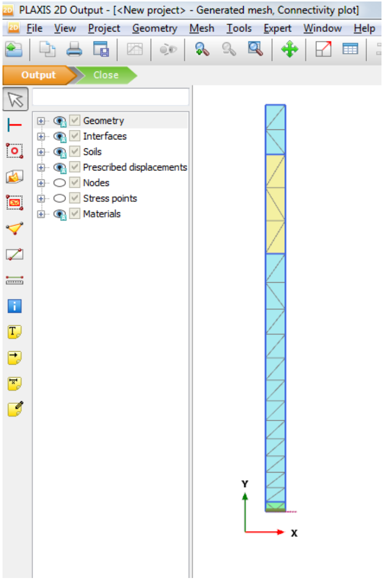
4. Then click *OK* to generate the mesh. 37 finite elements are generated which is indicated in the bottom panel as follows:



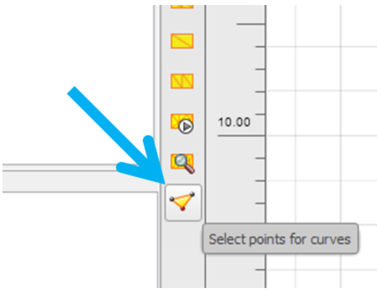
5. The created finite element mesh can be viewed by clicking on the *View mesh* button as shown below:



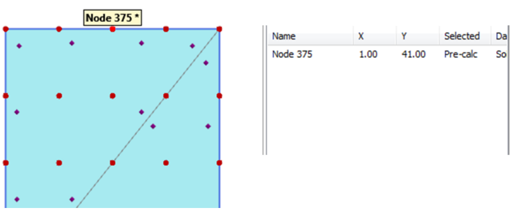
6. The PLAXIS Output will open with the displayed generated mesh as shown in the following figure:



7. Click at the *Close* button in the main toolbar to get back to PLAXIS Input.
8. Lastly, we will define the Gauss points and nodes for plotting the curves. Click at the *Select points for curves* button as shown below:

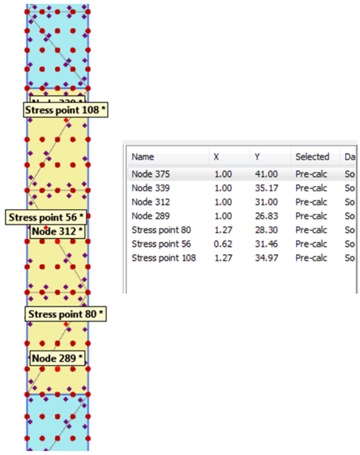


9. The PLAXIS Output will open again with the connectivity plot and visible Gauss points and nodes. Zoom in with the mouse wheel and drag the model with the mouse wheel pressed. Click on the node (1.00, 41.00) at the top of the model (nodes are marked as red points). It will get marked with the letter A and be added to the database of points as shown below:



10. Select three nodes in the sand layer at positions (1.00, 35.17), (1.00, 31.00) and (1.00, 26.83). Select also three Gauss points in the sand layer (Gauss points are marked as purple points), positioned at (1.27, 34.97), (0.62, 31.46) and (1.27, 28.30). The selected points in the sand layer are shown as follows, together with the table of

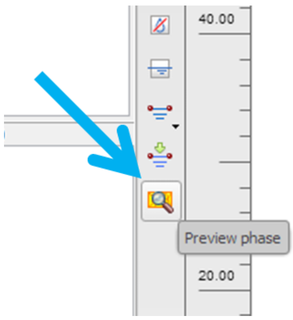
all selected points:



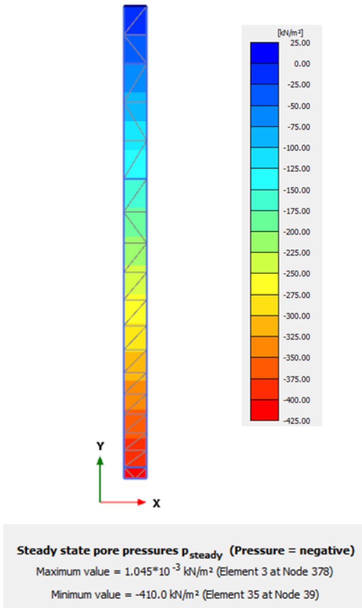
11. Click on the *Update* button to get back to the PLAXIS Input.

4.5 FLOW CONDITIONS

1. Click on the *Flow conditions* button on the main PLAXIS Input toolbar.
2. The groundwater level has already been defined in the borehole. The initial steady state pore-water pressures can be seen by clicking on *Preview phase* button as shown below:



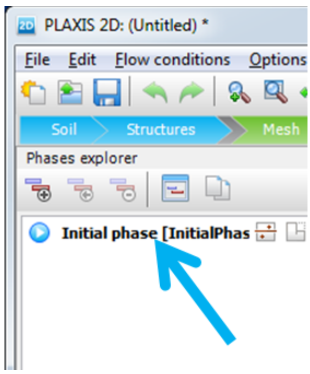
3. The PLAXIS Output opens with the Steady-state pore water pressures in the form of shadings diagram as shown in the following figure:



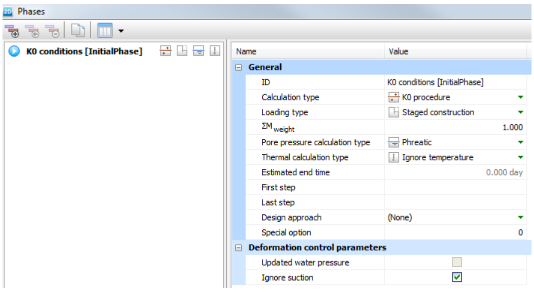
4. Press *Close* button at the main toolbar to get back to the PLAXIS Input program.

4.6 DEFINITION OF CALCULATION STAGES, CALCULATION SETTINGS, THE DEFORMATION BOUNDARY CONDITIONS AND THE COMPLIANT BASE BOUNDARY

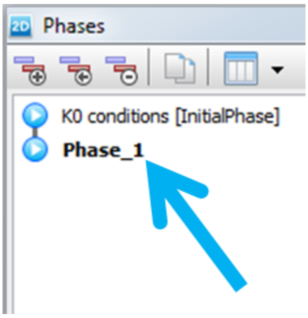
1. Click on *Staged construction* button in the main PLAXIS Input toolbar.
2. Double click on the Initial phase at the bottom of the left panel as shown below:



3. The *Phases* window will open. You can change the ID of the phase to *K0 conditions*. Make sure that the *Calculation type* is set to *K0 procedure* as shown in the following figure:

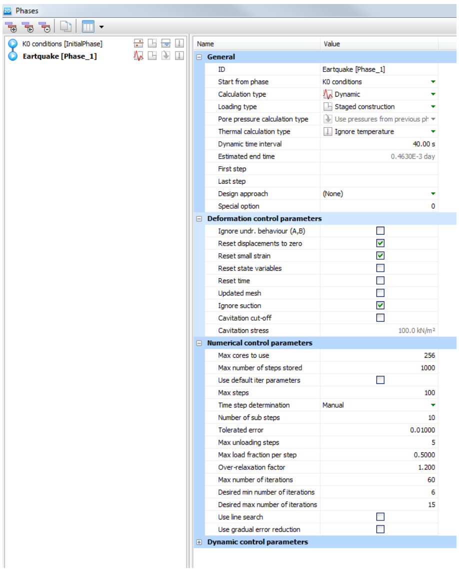


4. Add another phase by clicking on the + button on the toolbar of the *Phases* window.
5. Another phase has been added to the panel. Click on it as shown below:



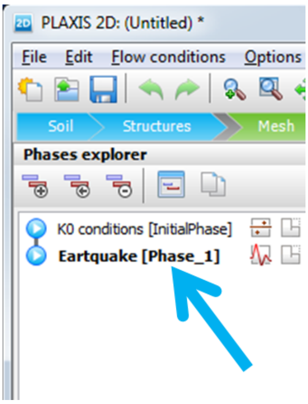
6. Make the following changes to the phase settings:
   - Type Earthquake as its ID.
   - Change the *Calculation type* to *Dynamic*.
   - Assign the *Dynamic time interval* to 40.0s.
   - Set the *Max number of steps stored* to 1000.
   - Deselect the *Use default iter parameters*.
   - Set the *Time step determination* to *Manual*.
   - Set the *Max steps* to 1000.
   - Set the *Number of sub-steps* to 10.

   Leave the other settings at the default values. The *Phases* window for the Earthquake phase should look like in the following figure:

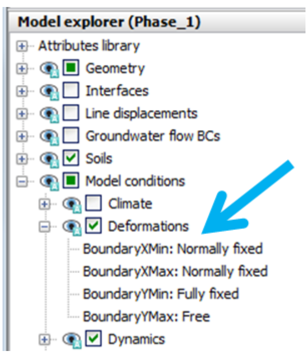


7. When the settings are prepared, click *OK* to close the *Phases* window.

8. First make sure that the Earthquake phase is active by clicking on it in the panel, as shown below:

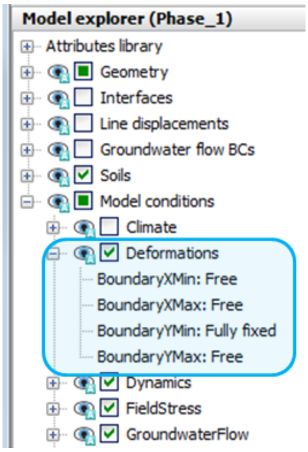


9. The deformation boundary conditions can be defined by expanding *Model conditions* -> *Deformations* in the *Model explorer* panel as shown below:



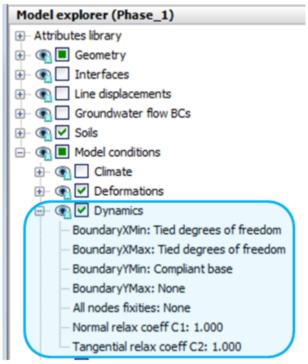
10. Set the deformation boundary conditions to the following values:
    - *BoundaryXMin* to *Free*.
    - *BoundaryXMax* to *Free*.
    - *BoundaryYMin* and *BoundaryYMax* will be left to the default choices, namely *Fully fixed* and *Free*.

    The correctly set deformation boundary conditions are shown in the following figure:



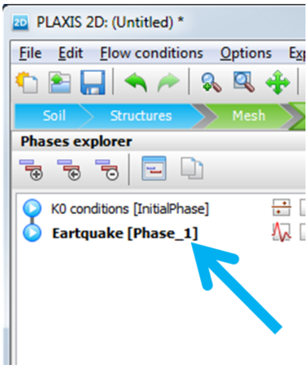
11. To perform a 1D wave propagation analysis, the *Tied degrees of freedom* will be used on the left and right vertical boundaries of the problem domain and *Compliant base conditions* at the bottom. This can be done by clicking and expanding the *Model conditions* -> *Dynamics* option in the *Model explorer* panel. The following options must be set in the *Dynamics* subitems:
    - *BoundaryXMin* to the *Tied degrees of freedom*.
    - *BoundaryXMax* to the *Tied degrees of freedom*.
    - *BoundaryYMin* to the *Compliant base*.

    Other settings will be left to their default values. The *Model explorer* panel should look as follows:



4.7 ACTIVATION OF THE EARTHQUAKE LOADING

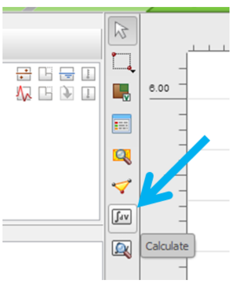
1. Make sure that the Earthquake phase is active by clicking at it in the *Phases explorer* panel as shown below:



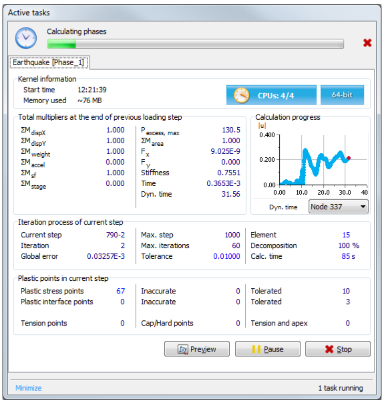
2. Go to the *Selection explorer* panel and activate the LineDisplacement_1_1. The *Selection explorer* panel should look as shown below and the arrows indicating the prescribed displacements will be coloured red:

4.8 RUNNING THE CALCULATION

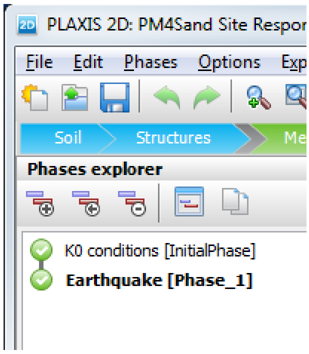
1. Now the model is prepared to run the calculation. In order to start the calculation, press the *Calculate* button in the PLAXIS Input toolbar as shown below:



2. The *Active tasks* window will appear and both phases will be calculated as shown below:

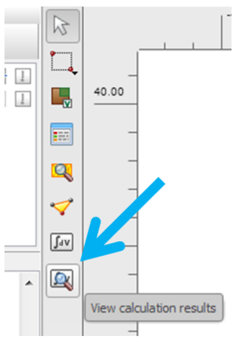


3. If the calculation is successfully finished, the tick marks in front of phase labels in the *Phases explorer* will be in green colour as shown in the following figure:

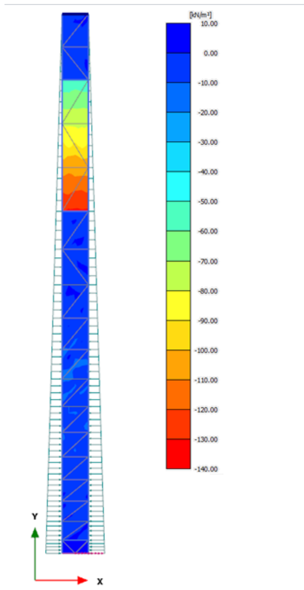


4.9 VIEWING THE RESULTS OF THE CALCULATION

1. To view the calculation results click on the *View calculation results* button on the PLAXIS Input toolbar as shown below, and the PLAXIS Output will open:

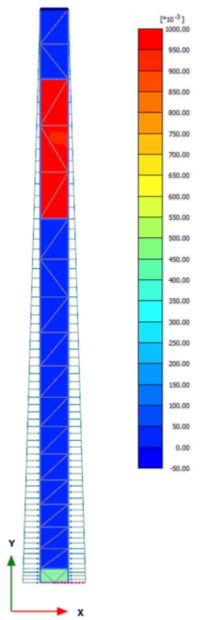


2. The deformed mesh at the end of the Earthquake phase is shown. By clicking at the main menu *Stresses* -> *Pore pressures* -> p_{excess}, the excess pore pressure at the end of the analysis are shown, as shown below:



Excess pore pressures p_{excess} (Pressure = negative) (Time 40.00 s)
Maximum value = 5.489 kN/m² (Element 33 at Node 74)
Minimum value = -132.1 kN/m² (Element 10 at Node 282)

3. By selecting *Stresses* -> *State parameters* -> *User-defined parameters* -> *[PM4Sand] ru*, max the maximum pore pressure ratio of the analysis in the *Sand* material is shown as below:

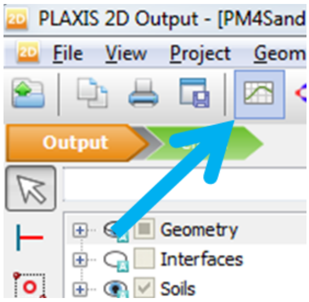


[PM4Sand] ru,max (Time 40.00 s)
Maximum value = 0.9846 (Element 10 at Node 280)
Minimum value = 0.000 (Element 1 at Node 345)

It can be observed that the whole Sand layer is completely liquefied.

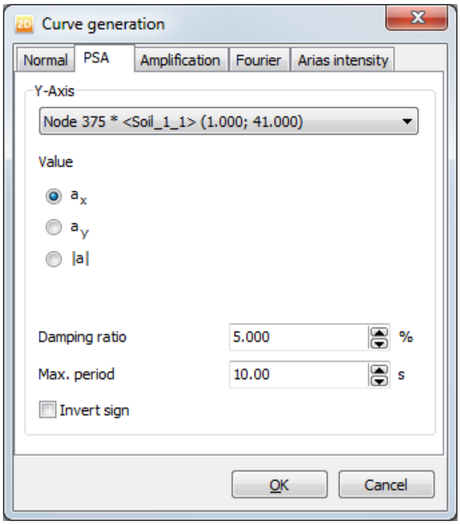
4. At the end of this exercise, the power spectrum in acceleration can be calculated and

visualised. In PLAXIS Output click at the *Curves manager* button (in the main toolbar) as shown below:



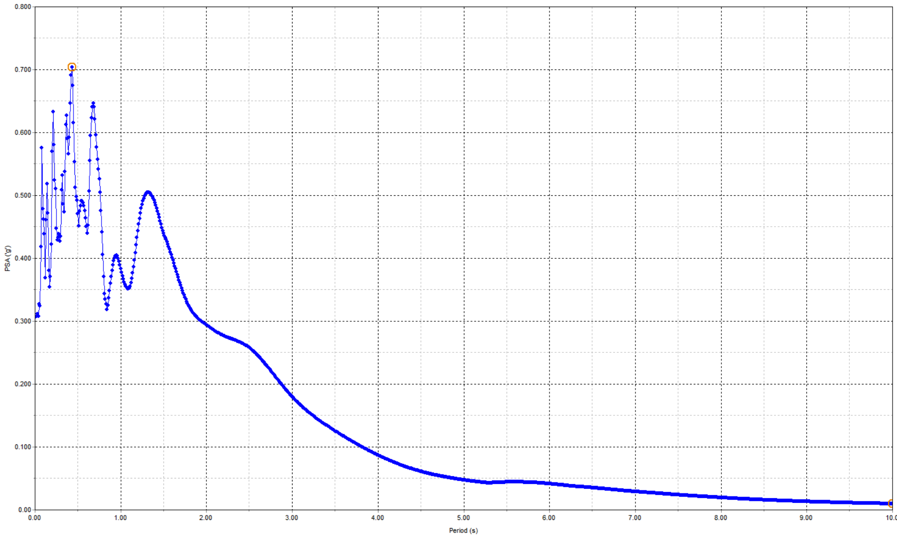
5. The *Curves manager* window opens. Click on the *New* button.

6. The *Curve generation* window will open. Choose the tab sheet PSA and select a node at location (1.00, 41.00). It is the node at the ground surface of the model. Leave the settings of the *Curve generation* window as follows:

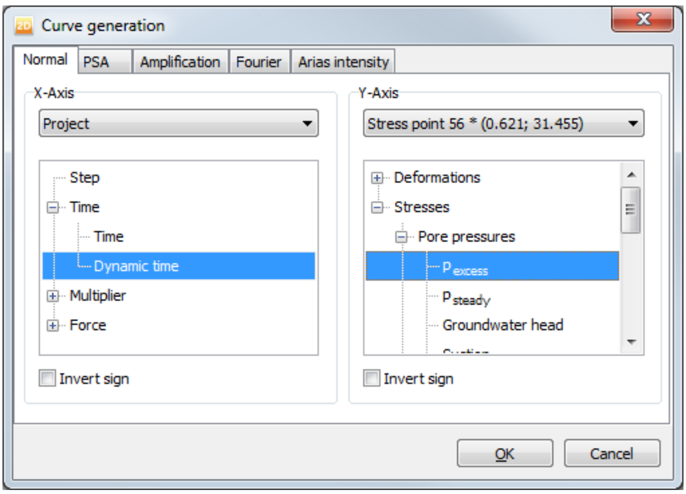


7. Press *OK* and select the Earthquake phase in the *Select phases* window and press *OK*.

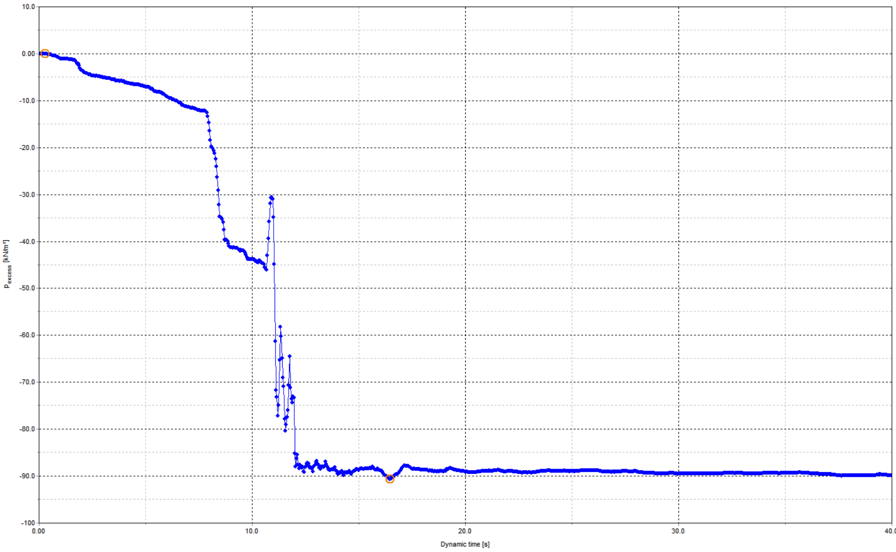
8. The PSA will be calculated and displayed as shown below:



9. The development of the excess pore-pressure can be plotted by going to the *Curves manager* again, clicking the *New* button. In the *Curve generation* window the dynamic time can be chosen for the X-axis and the excess pore-pressure for the Y-axis as is shown below for the stress point at position (0.621, 31.455):



10. The resulting excess pore-pressure development plot is shown below:


10.0
0.00
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-90.0
-100
p_{excess} [kN/m²]
0.00
10.0
20.0
30.0
40.0
Dynamic time [s]

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