PLAXIS

CONNECT Edition V21.00

Feature:

Convergence logging information for PLAXIS calculations

Last Updated: January 19, 2021

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1 Introduction

The PLAXIS calculation progress window provides access to a log file to help the user understand the convergence behavior of his analysis and debug the model if necessary. PLAXIS calculation kernel stores information in the PLAXIS project directory within a file named data.convd.rr# where # is the corresponding phase number (data.convf.rr# for flow problem). The file provides a summary of all convergence criteria for each for each step, attempt, and iteration of a given phase analysis. This diagnostic information is saved automatically for every analysis being run. If an analysis takes longer than expected or terminates prematurely, you can view the logging information in any editor to help determine the cause and to identify ways to correct the model eventually.

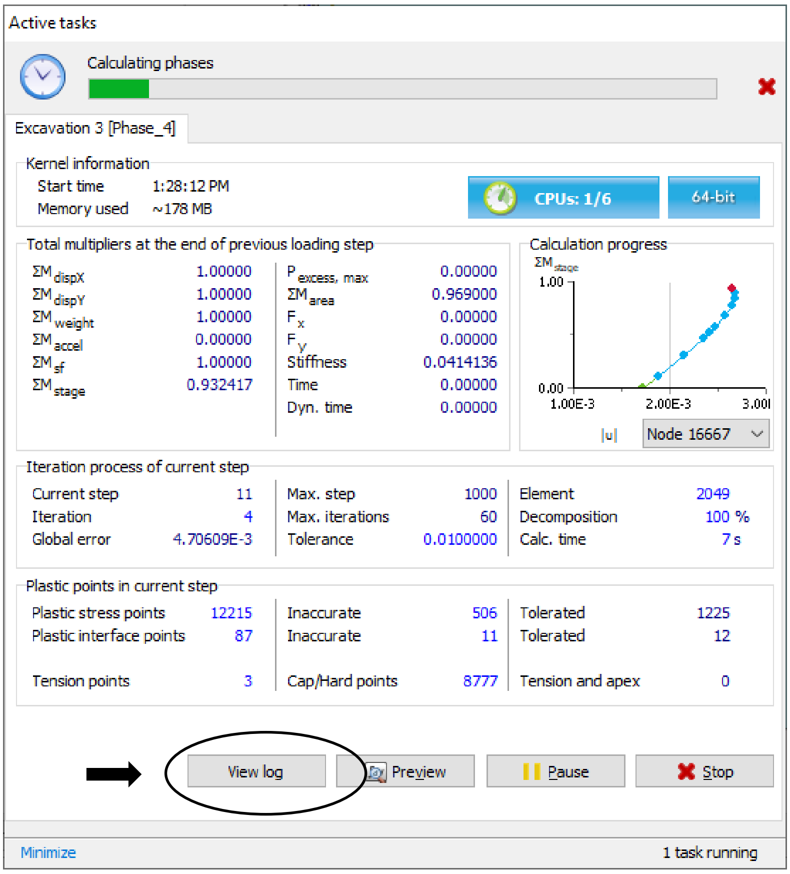

***Figure 1:** Accessing logging information from Calculation progress window*

The file can be opened either during the analysis of each phase set to be calculated through the View log button from the Calculation progress window (see Figure *1*) or from the Phase after calculation has been run (see *Figure 2*).

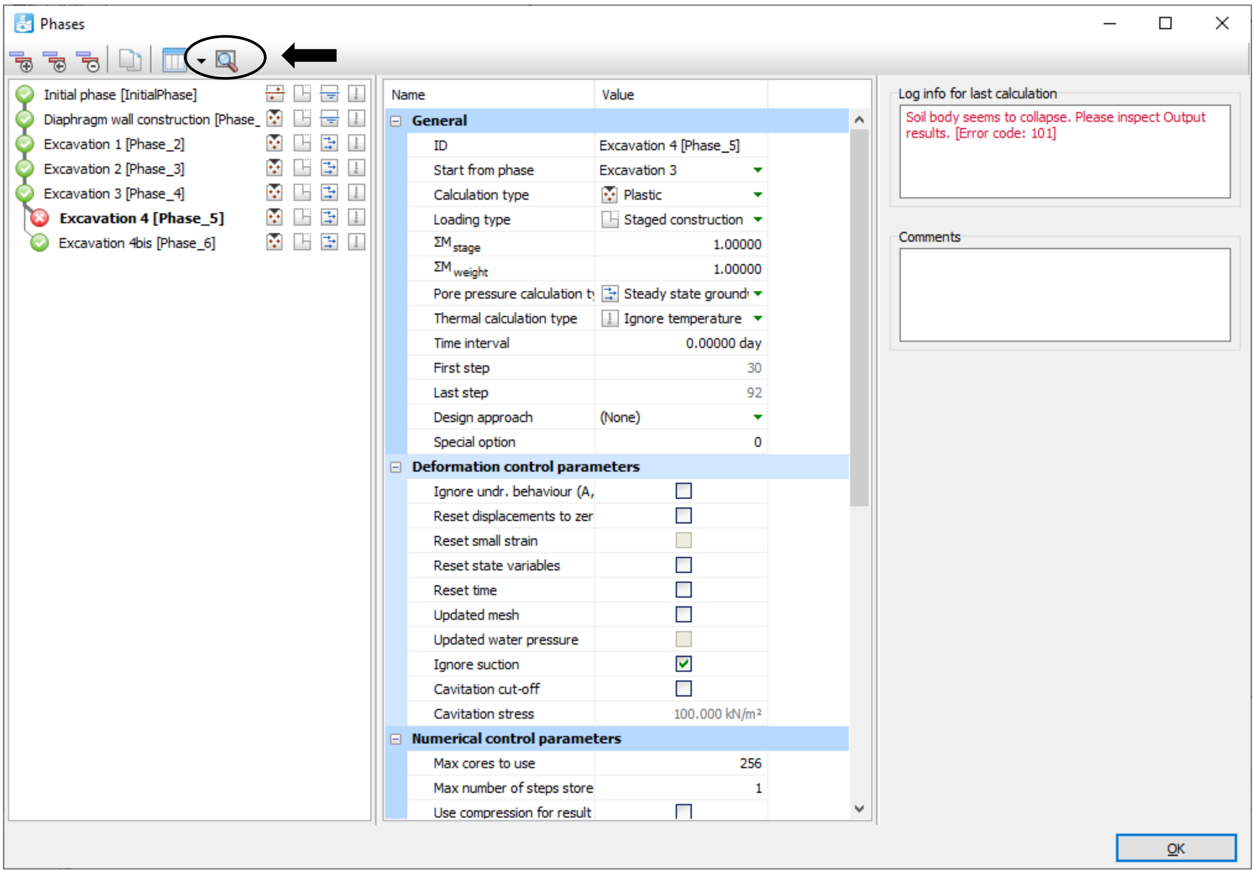

***Figure 2**: Accessing logging information from the Phase Explorer*

2 Structure of the convergence log file

An example of convergence log file is given in Figure 3.

The file always contains the following set of data:

- Information regarding the calculated Phase
  - Name
  - Number of active nodes
  - Number of active soil elements
  - Number of active structural elements (per type)
  - Number of active dof
- Calculation type and convergence criteria
- Step size control parameters
- List of steps
  - List of attempts
    - List of iterations
      - Current step size
      - Total number of iterations
      - Relevant convergence indicator values
      - Check against relevant convergence criteria
    - Step size reevaluation check
  - Step summary
    - Total time or relevant total multiplier value
    - Step convergence status
    - Locations of relevant maxima
- Phase summary

data.convd.rr

```
DATE 2020-10-27 TIME 13:36:06.225
Reference project: ...

... Lines skipped ...

 ========================================================
 ========================================================
 PHASE 1: "Diaphragm wall construction"

   17124 active of    17124 nodes
    2049 active of     2049 soil elements
     141 active of      141 interface elements
      65 active of       65 plate elements
   22279 active of    22279 dofs
```

```
========================================================
========================================================

CALCULATION TYPE                                 Plastic
CONVERGENCE CRITERIA
  Global force error tolerance                 1.000E-02
  Global moment error tolerance                1.000E-03
  Inaccurate soil plastic point tolerance      1.000E-01
  Integrated interface local error tolerance   1.000E-01
  Inaccurate interface plastic point tolerance 1.000E-01

STEP SIZE CONTROL PARAMETERS
  Maximum iterations per step                         60
  Desired minimum iterations per step                  6
  Desired maximum iterations per step                 15
  Maximum consecutive negative steps                   5
  Arc-length control                                  On

========================================================
STEP 1
========================================================

STEP 1 - ATTEMPT 1 - ITERATION 1
Current step size                              5.000E-01
Total number of iterations in current step             1
Global force error                             8.814E-04
Integrated soil local error                    7.134E-04
Global moment error                            1.258E-11
Soil plastic points
  Total                                            18065
  Inaccurate                                         541
Check convergence
  Minimum of 2 iterations per attempt not reached
  Converged attempt:                                  NO

STEP 1 - ATTEMPT 1 - ITERATION 2
Current step size                              4.265E-01
Total number of iterations in current step             2
Global force error                             4.733E-04
Integrated soil local error                    1.643E-04
Global moment error                            8.774E-12
Soil plastic points
  Total                                            18109
  Inaccurate                                          29
Check convergence
  Converged attempt:                                 YES

STEP 1 - SUMMARY
Reached sum-MStage                             4.265E-01
Step convergence achieved                            YES
Relative stiffness                             8.649E-01
Maximum out-of-balance force                  -3.617E+00
  At node     6782, dof 2
  Location ( 8.525E+00,-3.000E+01, 0.000E+00)
Maximum incremental displacement              -5.878E-04
  At node    16037, dof 2
```

```
   Location ( 8.600E+00,-5.597E-01, 0.000E+00)

... Lines skipped ...

========================================================
 ========================================================
 PHASE SUMMARY
 ========================================================
 ========================================================
 Prescribed ultimate state reached
```

Figure 3: *Convergence log file for plastic analysis*

3 Evaluating the accuracy of the solution

Appendix A presents a brief familiarization of the iterative convergence process in PLAXIS which highlights that a non-linear calculation can only be sufficiently accurate if the error in the residuals (or out-of-balance force) is less than the tolerated error. Moreover, depending on the type of problem modelled additional criteria are also considered by PLAXIS for enforcing both globally and locally closed-to-zero out-of-balances. This chapter presents an overview of all convergence criteria for either deformation or flow analyses.

3.1 Convergence criteria for deformation analysis

These criteria are applicable for the following type of analysis

- Plastic analysis
- Consolidation analysis
- Dynamic analysis
- Safety analysis
- Fully coupled analysis (deformation part)

This part describes the quantities being reported by the PLAXIS calculation kernel in the convergence log file with respect to reported convergence indicator values during the iterative process and looping of steps and iterations in an attempt to satisfy relevant convergence criteria for deformation analysis.

3.1.1 Global Error Checking

3.1.1.1 <u>Check on the force residuals</u>

A global error indicator for nodal forces is systematically calculated with the consideration of the total number of degrees of freedom

$$GlobalErrorForce = \frac{\|r\|_2}{CSP\left\|f_{ext}^{inact}\right\|_2 + \left\|f_{int}^{act}\right\|_2} = \frac{\|f_{int} - f_{ext}\|_2}{CSP\left\|f_{ext}^{inact}\right\|_2 + \left\|f_{int}^{act}\right\|_2}$$

Where $\|.\|_2$ is the quadratic norm.

And CSP (Current Stiffness Parameter) is defined as:

$$CSP = \frac{change\ of\ incremental\ elastic\ strain\ energy}{change\ of\ incremental\ total\ strain\ energy} = \frac{\int_{V_{act}} \Delta\varepsilon D^e \Delta\varepsilon\ dV}{\int_{V_{act}} \Delta\varepsilon\ \Delta\sigma\, dV}$$

The convergence criteria for force then reads:

$$GlobalErrorForce < ToleratedForceError$$

where

$$TolerancedForceError = ToleratedError$$

with $ToleratedError$ by default set to 0.01.

3.1.1.2 Check on the moment residuals (if applicable)

A global error indicator for moment in structural elements is calculated with respect to Moment for structural elements having rotational dof:

$$GlobalErrorMoment = \frac{\|m_{int} - m_{ext}\|_\infty}{m_{ref}}$$

where $\|.\|_\infty$ is the infinite norm and m_{ext} results on the moment contribution of each external force component. The reference moment m_{ref} is computed as follow:

$$m_{ref} = \sum_{el} \int |B_{beam}^T \sigma|_{moment}.dV$$

The convergence criteria for moment reads:

$$GlobalErrorMoment < ToleratedMomentError$$

where by default:

$$ToleratedMomentError = ToleratedError$$

This criterium only applies to structural elements with rotational degrees of freedom.

3.1.2 Local error indicators

As mentioned earlier PLAXIS also enforces several local convergence checks, local error indicators are the first of them

3.1.2.1 Inaccurate plastic points

3.1.2.1.1 Inaccurate plastic points concept

A point is defined as plastically inaccurate if:

$$PlasticLocalError > ToleratedPlasticLocalError$$

where $\mathit{PlasticLocalError}$ is calculated differently depending on the type of elements as explained later. This concept is considered for:

- soil elements,
- interface elements (including node-to-node interface of embedded beam except in PLAXIS 3D where inaccurate plastic points are counted separately for soil interface and EB interfaces).

Convergence is satisfied if

$$\textit{Numbers of inaccurate soil plastic points} < \textit{ToleratedInaccurateSoilPlasticPointPercentage*numbers of soil plastic points + 3}$$

and

$$\textit{Numbers of inaccurate interface plastic points} < \textit{ToleratedInaccurateInterfacePlasticPointPercentage*numbers of interface plastic points + 3}$$

where

$\mathit{ToleratedInaccurateSoilPlasticPointPercentage}$ and $\mathit{ToleratedInaccurateInterfacePlasticPointPercentage}$ are by default set to 0.1.

The check should be satisfied for each independent counting. Note that only inaccurate plastic points for soil and interface are being reported in the calculation progress window but in PLAXIS 2D inaccurate non-linear elastic points are also separately monitored for soil elements only (relevant for constitutive models with stress-dependent elastic stiffness). In PLAXIS 2D, convergence criteria then also consider:

$$\textit{Numbers of inaccurate non-linear elastic points} < \textit{ToleratedInaccurateSoilElasticPointPercentage*numbers of total linear elastic points + 3}$$

Where *numbers of total linear elastic points* include all elastic points (active and non-active).

Finally in PLAXIS 3D additional independent inaccurate plastic points counting are also accounted for the following structural elements:

- Beams
- Plates
- Anchors
- Geotextiles

3.1.2.1.2 Inaccurate plastic points for soil elements (both PLAXIS 2D and 3D)

For soil elements, PLAXIS calculates the following for each stress point:

$$SoilPlasticLocalError_j = \frac{\left\|\sigma_{eq,j} - \sigma_{c,j}\right\|}{\max\left(\tau_{max,j}, c, 1\ kPa\right)}$$

where: (see Figure 4)

$$\sigma_{c,j} = \sigma_0 + D^e\left(\Delta\varepsilon_j - \Delta\varepsilon_{p,j}\right) = \sigma_0 + D^e\left(\Delta\varepsilon_{e,j}\right)$$
$$\sigma_{eq,j} = \sigma_{c,j-1} + D^e\delta\varepsilon_j$$

The equilibrium stress is calculated based on the linearized soil behavior at FE model level whereas the constitutive stress is the ones computed by consideration of the real soil constitutive behavior at stress point.

Soil plastic stress points *j* are inaccurate if

$$SoilPlasticLocalError_j > ToleratedSoilPlasticLocalError$$

where by default

$$ToleratedSoilPlasticLocalError = ToleratedError$$

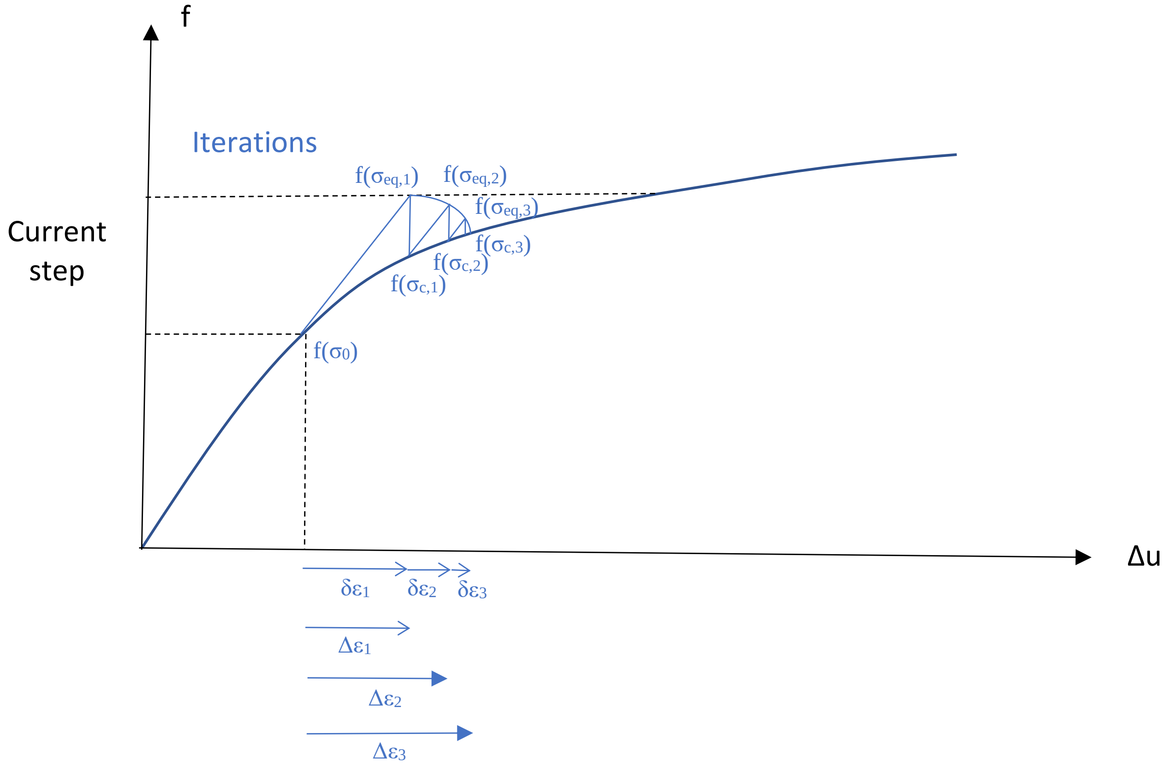

***Figure 4**: Equilibrium and constitutive stress concept*

In PLAXIS 3D convergence check on number of inaccurate soil plastic points are skipped if

$$GlobalErrorForce < 0.01 * ToleratedError$$

3.1.2.1.3 Particular case of inaccurate elastic points for soil elements in PLAXIS 2D

For PLAXIS 2D only, inaccurate non-linear elastic points are also monitored and defined as:

$$SoilElasticLocalError_j = \frac{\left\| \sigma_{eq,j} - \sigma_{c,j} \right\|}{\max\left(\tau_{max,j}, c, p_{ref}/200\right)}$$

Soil non-linear elastic stress points *j* are inaccurate if

$$SoilElasticLocalError_j > ToleratedSoilElasticLocalError$$

where by default

$$ToleratedSoilElasticLocalError = ToleratedError$$

3.1.2.1.4 Inaccurate plastic points for interfaces (both PLAXIS 2D and 3D)

For interface elements, PLAXIS also calculates the following quantity for each stress point:

$$InterfacePlasticLocalError_j = \frac{\left\| \tau_{eq,j} - \tau_{c,j} \right\|}{\max\left(\tau_{max,j}, c, 1\ kPa\right)}$$

Interface plastic points *j* are inaccurate if:

$$InterfacePlasticLocalError_j > ToleratedInterfacePlasticLocalError$$

where by default:

$$ToleratedInterfacePlasticLocalError = ToleratedError$$

One shall note that

- In PLAXIS 2D Inaccurate plastic points for interfaces also included embedded beam row coupling springs in its counting
- In PLAXIS 3D convergence check on number of inaccurate interface plastic points are skipped if:

$$GlobalErrorForce < 0.1 * ToleratedError$$

3.1.2.1.5 Inaccurate plastic points for beams (only PLAXIS 3D)

For beam elements, PLAXIS 3D calculates the following for each stress point:

$$BeamPlasticLocalError_j = \frac{\left\| \sigma_{eq,j}^{normal} - \sigma_{c,j}^{normal} \right\|}{\left\| \sigma_{eq,j}^{normal} \right\|}$$

Beam plastic points *j* are inaccurate if:

$$BeamPlasticLocalError_j > ToleratedBeamPlasticLocalError$$

where by default:

$$ToleratedBeamPlasticLocalError = ToleratedError$$

3.1.2.1.6 For plates (only PLAXIS 3D)

For plate elements, PLAXIS 3D calculates the following for each stress point:

$$PlatePlasticLocalError_j = \frac{\left\| F_{eq,j}^{in-plane} - F_{c,j}^{in-plane} \right\|}{\left\| F_{eq,j}^{in-plane} \right\|}$$

Plate plastic points *j* are inaccurate if:

$$PlatePlasticLocalError_j > ToleratedPlatePlasticLocalError$$

where by default:

$$ToleratedPlatePlasticLocalError = ToleratedError$$

3.1.2.1.7 For anchors (only PLAXIS 3D)

For anchor elements, PLAXIS 3D calculates the following for each stress point:

$$AnchorPlasticLocalError_j = \frac{\left\| F_{eq,j} - F_{c,j} \right\|}{\left\| F_{eq,j} \right\|}$$

Anchor plastic points j are inaccurate if:

$$AnchorPlasticLocalError_j > ToleratedAnchorPlasticLocalError$$

where by default:

$$ToleratedAnchorPlasticLocalError = ToleratedError$$

3.1.2.1.8 For geogrids (only PLAXIS 3D)

For plate elements, PLAXIS 3D calculates the local error quantities for each stress point. Local error calculation depends here whether the material is defined as isotropic

$$GeogridPlasticLocalError_j = \frac{\left\| F_{eq,j}^{in-plane} - F_{c,j}^{in-plane} \right\|}{\left\| F_{eq,j}^{in-plane} \right\|}$$

Or anisotropic

$$GeogridPlasticLocalError_{1,j} = \frac{\left\| \sigma_{eq,j}^{1} - \sigma_{c,j}^{1} \right\|}{\left\| \sigma_{eq,j}^{1} \right\|}$$
$$GeogridPlasticLocalError_{2,j} = \frac{\left\| \sigma_{eq,j}^{2} - \sigma_{c,j}^{2} \right\|}{\left\| \sigma_{eq,j}^{2} \right\|}$$

Geogrid plastic points *j* are inaccurate if:

$$GeogridPlasticLocalError_j > ToleratedGeogridPlasticLocalError$$

where by default:

$$ToleratedGeogridPlasticLocalError = ToleratedError$$

In PLAXIS 3D convergence check on number of inaccurate geogrid plastic points are ignored if

$$GlobalErrorForce < 0.01 * ToleratedError$$

3.1.2.2 Accuracy in the plastic zone (integrated local error) in PLAXIS 3D

The plastic global error also called the integrated local error is defined as:

$$PlasticGlobalError = \frac{\sum_{j=1,N} \int_{V_{p,j}} \left(PlasticLocalError_j \right) dV}{\sum_{j=1,N} \int_{V_p} dV}$$

where $PlasticLocalError_j$ has been defined in paragraph 3.1.2.1.1.

Convergence is satisfied if

$$PlasticGlobalError < ToleratedPlasticGlobalError$$

where by default:

$$ToleratedPlasticGlocalError = ToleratedError$$

The plastic error is calculated in PLAXIS 3D only for

- Soil elements for plastic points only with

$$SoilPlasticGlobalError < ToleratedSoilPlasticGlobalError$$

- Interfaces

$$InterfacePlasticGlobalError < ToleratedInterfacePlasticGlobalError$$

- Anchors

$$AnchorPlasticGlobalError < ToleratedAnchorPlasticGlobalError$$

- Geogrids

$$GeotextileGeogridGlobalError < ToleratedGeogridPlasticGlobalError$$

unless (3D geogrids only)

$$GlobalErrorForce < 0.01 * ToleratedError$$

3.1.3 Additional convergence criteria for embedded beams

Additional convergence criteria are being considered to check accuracy in couplings springs (or special interfaces) for embedded beams. Accuracy checks with that respect are different in PLAXIS 2D and PLAXIS 3D.

3.1.3.1 Convergence criteria for embedded beams in PLAXIS 2D

Inaccurate plastic points in coupling springs are counted together with inaccurate interfaces plastic points (no distinction between standard interfaces and special interfaces).

Additionally, a foot force error is computed as:

$$FootForceError = \frac{\sum_{p=1,N_{eb}} \left| F^{p}_{foot,eq} - F^{p}_{foot,c} \right|}{max\left(\sum_{p=1,N_{eb}} \left| F^{p}_{foot,c} \right|, 0.01 * \sum_{p=1,N_{eb}} \left| F^{p}_{foot,max} \right|, 1.0 \right)}$$

Convergence is satisfied if

$$FootForceError < ToleratedFootForceError$$

where by default

$$ToleratedFootForceError = 5 * ToleratedError$$

The concept of constitutive and equivalent forces for F_{foot} is also considered in the same fashion as for stresses in soil (Figure 4). The equilibrium foot force is calculated based on the linearized pile tip behavior whereas the constitutive force is the one computed by consideration of real constitutive behavior of coupling springs at embedded beam tip.

3.1.3.2 Convergence criteria for embedded beams in PLAXIS 3D

In PLAXIS 3D, an embedded beam error is computed for each individual embedded beam element *p* as

$$EmbeddedBeamInterfaceError_p = \frac{\left|F^p_{foot,eq} + \int_{Length} T^p_{skin,eq}\, dl - F^p_{foot,c} - \int_{Length} T^p_{skin,c}\, dl\right|}{max(F_{max}, 1.0)}$$

where

$$F_{max} = \max_{p=1,N_{eb}} \left(\left|F^p_{foot,eq} + \int_{Length} T^p_{skin,eq}\, dl\right|, \left|F^p_{foot,c} + \int_{Length} T^p_{skin,c}\, dl\right| \right)$$

The concept of constitutive and equivalent forces for F_{foot} and T_{skin} is also considered in the same fashion as for stress in soil (Figure 4). The equilibrium forces are calculated based on the linearized pile tip/skin behavior whereas the constitutive forces are the ones computed by consideration of real constitutive behavior of coupling springs along embedded beam length and at embedded beam tip.

This way inaccurate 3D embedded beams could be identified when

$$EmbeddedBeamInterfaceError_p > ToleratedEmbeddedBeamInterfaceError$$

where by default

$$ToleratedEmbeddedBeamInterfaceError = \begin{cases} ToleratedError \text{ if } GlobalError > ToleratedError \\ 5 \times ToleratedError \text{ if } GlobalError < 0.01 * ToleratedError \\ 2 \times ToleratedError \quad \text{otherwise} \end{cases}$$

The convergence criteria is relatively strict in PLAXIS 3D as no inaccurate embedded beam are allowed which reads:

$$\max_{p=1,N_{eb}} \left(EmbeddedBeamInterfaceError_p\right) < ToleratedEmbeddedBeamInterfaceError$$

3.2 Convergence criteria for flow analysis

These criteria are applicable for the following type of analysis

- Steady state flow (groundwater and/or thermal)
- Transient flow (groundwater and/or thermal)
- Fully coupled analysis (groundwater flow part)

This part describes the quantities being reported by the PLAXIS calculation kernel in the convergence log file with respect to reported convergence indicator values during the iterative process and looping of steps and iterations in an attempt to satisfy relevant convergence criteria for flow analysis.

3.2.1 Global Error Checking

3.2.1.1 Flow error

A global error indicator for nodal flux is systematically calculated with the consideration of the total number of degrees of freedom

$$GlobalFlowError = \frac{\sum \| q_{int} - q_{ext} \|_{closed\ nodes}}{max\left(\sum \| q_{ext}^{out} \|_{open\ nodes}, \sum \| q_{ext}^{in} \|_{open\ nodes}\right)}$$

Note that fluxes are computed differently in groundwater flow than in heat flow (Darcy flux *GWFlowError* vs heat flux *ThFlowErr*).

Convergence is satisfied if:

$$GlobalFlowError < TolerGlobalFlowError$$

Where $TolerGlobalFlowError$ is by default set to 0.01. This condition is enforced for each (pseudo) time step.

3.2.1.2 Change of heat storage for thermal flow calculation

For heat flow PLAXIS is also considering additional checks

$$AverageHeatStorageChange < ToleratedHeatStorageChange$$

where

$$AverageHeatStorageChange = \frac{\| Q_{new} - Q_{old} \|}{\| Q_{old} \|}$$

and for which the calculation of the heat storage *Q* takes into consideration the sensible heat of each constituent (soil, water, ice) along with possible latent heat due to phase change.

In PLAXIS 2D we have:

$$TolerateHeatStorageChange = 10.TolerGlobalFlowError$$

3.2.1.3 Unsaturated behaviour convergence check for groundatwer flow analysis

In case of unsaturated behavior in groundwater flow analysis, additional checks are being performed as the hydraulic conductivity and storativity matrices are computed at the beginning of each time step assuming particular values for the degree-of-saturation and relative permeability. A time step is judged valid if:

$$AverageSaturationChange < ToleratedSaturationChange$$

and

$$AverageKrChange < ToleratedKrChange$$

where

$$AverageSaturationChange = \frac{\|Sat_{new} - Sat_{old}\|}{\|Sat_{old}\|}$$

and

$$AverageKrChange = \frac{\|K_{rel,new} - K_{rel,old}\|}{\|K_{rel,old}\|}$$

In PLAXIS, we have

$$ToleratedKrChange = ToleratedSaturationChange = TolerGlobalFlowError$$

Some additional checks are also being considered to prevent excessive changes of the relative permeability and/or degree of saturation in the unsaturated zone which reads:

$$AverageSaturationChange < ExcessiveSaturationChange$$

and

$$AverageKrChange < ExcessiveKrChange$$

with

$$ExcessiveKrChange = ExcessiveSaturationChange = 10 * TolerGlobalFlowError$$

If excessive changes of degree-of-saturation or relative permeability are being detected, then a new step attempt will be taken with a smaller time step size (downscaling factor is 1.5)

Finally, after any increase of the step size (upscaling due to number of iterations lower than minimum desired), a sanity check is being performed (namely the step size upscaling validity). If the resulting change of right after leads to:

$$AverageSaturationChange > ExcessiveSaturationChange$$

and

$$AverageKrChange > ExcessiveKrChange$$

Then the upscaling is ignored, and the step size is scaled back to its previous value.

3.2.2 Local error indicators for groundwater flow calculation

As for deformation analysis, flow analysis in PLAXIS also enforces additional local convergence check. Those are related to monitoring inaccurate nodes for specific boundary conditions for groundwater flow calculation

3.2.2.1 Inaccurate nodes for specific boundary conditions

3.2.2.1.1 Inaccurate seepage nodes

Seepage nodes can be open or closed. If water wants to flow out, the pore pressure should be zero. If the water tries to flow in, the node should be closed. In PLAXIS, a seepage node which change status between two consecutive time steps is set as inaccurate.

The following convergence criteria is then enforced:

*Numbers of inaccurate seepage nodes < max (int(ToleratedInaccurateSeepageNodesPercentage*numbers of seepage nodes), 1)*

where:

ToleratedInaccurateSeepageNodesPercentage = 0.001

3.2.2.1.2 Inaccurate well (extraction) nodes

If the water head in the well nodes goes below minimum specified head and if the active pore pressure in the same node is positive, then the pore pressure in the well node is set to P_{max}. A well node which change status between two consecutive time steps is set as inaccurate.

The corresponding convergence criteria on inaccurate well nodes reads:

Numbers of inaccurate well nodes < 1

3.2.2.1.3 Inaccurate drain nodes

If inflow is being detected in a drain node then it is switched to closed node. A drain node which change status between two consecutive time steps is set as inaccurate.

The corresponding convergence criteria on inaccurate drain nodes reads:

$$Numbers\ of\ inaccurate\ drain\ nodes < 1$$

3.2.2.1.4 Inaccurate ponding nodes

If the head to get prescribed inflow is exceeded the nodes BC is changed to prescribed head A well node which change status between two consecutive time steps is set as inaccurate.

The corresponding convergence criteria on inaccurate drain nodes reads:

$$Numbers\ of\ inaccurate\ ponding\ nodes < (int(ToleratedInaccuratePondingNodesPercentage*numbers\ of\ seepage\ nodes),\ 1)$$

where:

$$ToleratedInaccuratePondingNodesPercentage = 0.01$$

3.2.2.2 Maximum allowable number of iterations for checking status change

The check on node status for each of the boundary condition types will done over a Maximum allowable number of iterations. Beyond the boundary conditions status check will be ignored. The maximum **allowable number of iterations for checking status change are:**

- For seepage nodes: 10
- For drain nodes: 10
- For well nodes: 10
- For ponding nodes: 2

3.2.3 Particular case of steady-state calculation

A steady state calculation is also solved using an implicit time stepping scheme (pseudo time step). |For each pseudo-time step, the global error indicator for nodal flux must satisfied the global flow error convergence criteria as given in paragraph 3.2.1.1:

$$GlobalFlowError < TolerGlobalFlowError$$

One shall note that in PLAXIS the $TolerGlobalFlowError$ is internally set to 0.01 for steady state flow analysis and cannot be explicitly set by user.

For steady state heat flow analysis, convergence criteria regarding change of heat storage for thermal flow calculation has described in paragraph 3.2.1.2 is also enforced

For steady state groundwater flow calculation, the same convergence criteria as considered in transient analysis with respect to unsaturated behavior are also being enforced (see paragraph 3.2.1.3).Moreover, additional specific flow checks are being considered and which are only considered for steady state groundwater flow analysis.

3.2.3.1 Local GW flow error

An additional condition on flow error at local level should be satisfied during steady-state flow analysis:

$$LocalFlowError < TolerLocalFlowError$$

where

$$LocalFlowError = \frac{\left(\max_{closed\ nodes} |q_{ex} - q_{in}|\right)^2}{max\left(\sum \|q_{ext}^{out}\|_{open\ nodes}, \sum \|q_{ext}^{in}\|_{open\ nodes}\right)^2}$$

In PLAXIS one has:

$TolerLocalFlowError$= 10. $TolerGlobalFlowError$

3.2.3.2 Average pore pressure change

The following steady state regime condition should be satisfied as well:

$$AveragePorePressureChange < TolerAveragePorePressureChange$$

where

$$AveragePorePressureChange = \frac{\sum_{i=1,Nnodes}\left({p_{new,i}}^2 - {p_{old,i}}^2\right)}{\sum_{i=1,Nnodes}\left({p_{old,i}}^2\right)}$$

and

$$TolerAveragePorePressureChange = 0.005$$

with new and old subscripts referring to two consecutive pseudo-time steps

3.2.3.3 Maximum pore pressure change

3.2.3.3.1 Concept of inaccurate pore pressure nodes

The number of inaccurate pore pressure nodes are counted during a steady state groundwater flow analysis and a flow node *i* is inaccurate if

$$SSErrND(i) > TolerSSGWFNodeError$$

where

$$SSErrND(i) = \frac{{p_{new,i}}^2 - {p_{old,i}}^2}{{p_{old,i}}^2}$$

By default, in PLAXIS we have:

$$TolerSSGWFNodeError = \ 100.TolerAveragePorePressureChange$$

3.2.3.3.2 Maximum pore pressure change criteria

A check on the maximum pore pressure change (the most inaccurate pore pressure node) will be enforced"

$$MaximumPorePressureChange < TolerMaximumPorePressureError$$

Where

$$MaximumPorePressureChange = \max_{i=1,Nnodes} \left(\frac{{p_{new,i}}^2 - {p_{old,i}}^2}{{p_{old,i}}^2} \right)$$

By default

$$TolerMaximumPorePressureError = 10 * TolerAveragePorePressureChange$$

However, if the number of steps is larger than 10 and either

- The number of inaccurate pore pressure nodes *nSSErrNod* satisfies*:*

  *nSSErrNod < ToleratedInaccurateNodesPercentageSSFlow **
  total number of flow nodes (active and inactive)

  ToleratedInaccurateNodesPercentageSSFlow = 0.01 (default)

- Or the global flow error $GlobalFlowError$ satisfies:

$$GlobalFlowError < 0.01.TolerGlobalFlowError$$

Then the criteria on maximum pore pressure change will be disregarded

3.2.3.4 Inaccurate nodes for specific boundary conditions

Inaccurate nodes are also considered during steady state groundwater flow calculation using the exact same definition as given in paragraph 3.2.2.1. This only specificity of steady state calculation with respect to transient is in the definition of the maximum allowable number of iterations for checking status change.

For steady-state calculation, the maximum allowable numbers of iterations for checking status change are reduced and equal to:

- For seepage nodes: 2
- For drain nodes: 2
- For well nodes: 2
- For ponding nodes: 2

Appendix: Understanding the Iterative Convergence Process in PLAXIS

When using finite element analysis, the term “convergence” is often used. Mesh convergence is often the most important issue that needs to be addressed. By mesh convergence one means how small the elements need to be to ensure that the results of the finite element analysis are not affected by changing the size of the mesh. Usually this is handled by the user by performing mesh sensitivity analysis until the FEA solution is no longer affected by any further mesh refinement.

Additionally, in nonlinear problems, convergence in the iteration procedure also needs to be considered. This is what the convergence logging information provided by PLAXIS is all about. It provides a set of information that allow the user to monitor the progress of the analysis run.

Quasi-Newton Raphson method

In nonlinear problems the governing balance equations must be solved iteratively by means of the Newton-Raphson method. The iterative process as schematized in Figure 5 considers the update of the incremental displacement as

$$\delta u_i = K_i^{-1}\left(f_{ext} - f_{int,i}\right)$$

The process is repeated iteratively until the out-of-balance force $\left(f_{ext} - f_{int,i}\right)$ becomes small compared the applied force itself f_{ext}. A disadvantage of this method is that the stiffness matrix K has to be set up at every iteration and the time-consuming decomposition of the matrix (to compute its inverse) has to be performed every iteration as well.

This is for this reason that PLAXIS is rather using a Quasi-newton method because the computation cost of finding a new stiffness matrix inverse at every iteration. The Quasi-Newton method essentially uses the information of previous solution vectors and out-of-balance force vectors during the increment to achieve a better approximation (see Figure 6) . Unlike Regular Newton-Raphson, the Quasi-Newton method does not set up a completely new stiffness matrix every iteration and is computationally significantly more efficient

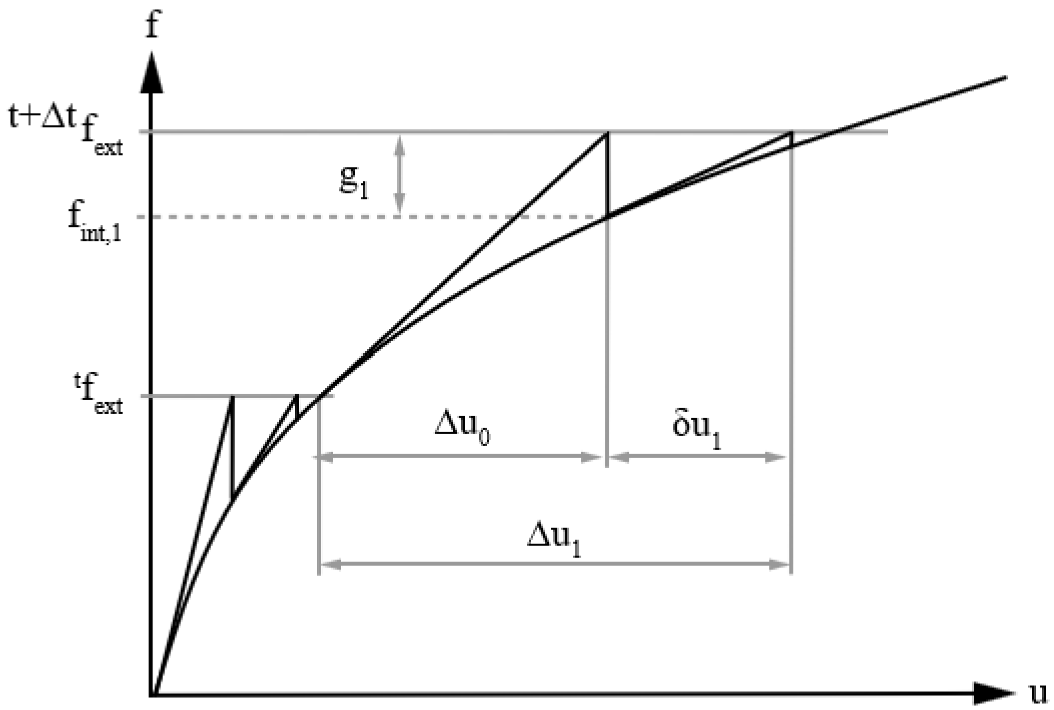

Figure 5: *Iteration process*

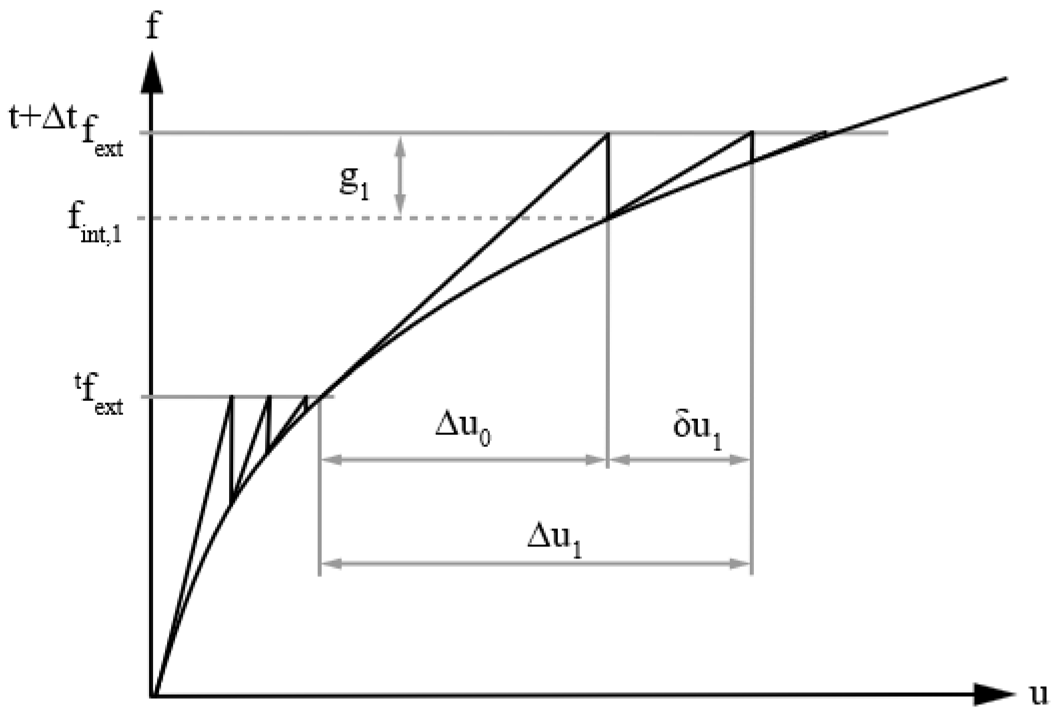

Figure 6*: Quasi-Newton iteration*

Field equations and convergence criteria

Depending on the physical nature of the problem PLAXIS is solving different model global errors (out-of-balance) are computed as shown below

Physical problem	Field variable	Conjugate flux
Soil stress analysis	Displacement **u**	Force **F**
Structural stress analysis	Rotation **Φ**	Moment **M**
Groundwater flow analysis	Pore water pressure p	GW Flow q_{GW}
Heat transfer analysis	Temperature T	Heat flow q_T

The global error should always be less than the tolerated error:

$$GlobalError < ToleratedError$$

The tolerated error can be provided by the user and usually a default value of 0.01 (1%) is considered. In PLAXIS, the tolerated error is the same for each criterion checked upon.
Coupled problems requires the satisfaction of the convergence criteria of each field independently except for consolidation analysis for which only soil stress and eventually structural stress if structural elements have been activated in such consolidation phase.

Due to the high non-linearity nature of the physical problem being model, the obtention on accurate non-linear solution requires the consideration of additional local criteria convergence the nature and value of which depends on the type of problems being considered as described in the main part of the documentation.