PLAXIS

CONNECT Edition V22.02

User Defined Soil Models - PM4Sand: A Sand Plasticity model for Earthquake Engineering

Bentley®
Advancing Infrastructure

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1 Introduction

The PM4Sand model successfully simulates the material behaviour of sands in dynamic loading, including the pore pressure generation, liquefaction and post-liquefaction phenomenon. It is a very attractive model for the industry due to a small number of parameters to be calibrated. They are mostly related to the usually available data in the design practice (e.g. D_R , SPT, CPT, V_S values).

This report summarises the implementation of the PM4Sand model version 3.1 (Boulanger & Ziotopoulou, 2017 (on page 79)) in thePLAXIS finite element code. The model is defined in the basic framework of the stress-ratio controlled, critical state compatible, bounding surface plasticity model for sand by Dafalias and Manzari (2004) (on page 79). Many modifications were added to the Dafalias and Manzari (2004) (on page 79) model in order to more accurately simulate stress-strain responses that are important to geotechnical earthquake engineering practice. The developments are described in the manuals (version 1: (Boulanger, 2010 (on page 79)); version 2: (Boulanger & Ziotopoulou, 2012 (on page 79)); version 3: (Boulanger & Ziotopoulou, 2015 (on page 79)); version 3.1: (Boulanger & Ziotopoulou, 2017 (on page 79))).

2 Model formulation

2.1 Basic stress and strain quantities

The model is defined in terms of the effective stresses. The prime symbol is dropped for convenience in all of the terms that follow in the report. The model is defined in the plane stress conditions. This formulation allows the model to be used in 2D plane strain numerical models, where the out-of-plane stress is ignored by the global finite element equations. The effective stress tensor $\boldsymbol{\sigma}$, the mean effective stress p, the deviatoric stress tensor $\boldsymbol{s}$ and the deviatoric stress ratio tensor $\boldsymbol{r}$ used in the model formulation have the following form as shown in Eq. [1], Eq. [2], Eq. [3] and Eq. [4].

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{yy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} \quad Eq.\ [1]$$

$$p = \frac{\sigma_{xx} + \sigma_{yy}}{2} \quad Eq.\ [2]$$

$$\boldsymbol{s} = \boldsymbol{\sigma} - p\boldsymbol{I} = \begin{pmatrix} \sigma_{xx} - p & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} - p \end{pmatrix} \quad Eq.\ [3]$$

$$\boldsymbol{r} = \frac{\boldsymbol{s}}{p} = \begin{pmatrix} \frac{\sigma_{xx} - p}{p} & \frac{\sigma_{xy}}{p} \\ \frac{\sigma_{xy}}{p} & \frac{\sigma_{yy} - p}{p} \end{pmatrix} \quad Eq.\ [4]$$

The strains are split into volumetric ε_v and the deviatoric part $\boldsymbol{e}$ as follows shown in Eq. [5] and Eq. [6].

$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} \quad Eq.\ [5]$$

$$\boldsymbol{e} = \varepsilon - \frac{\varepsilon_V}{3}\boldsymbol{I} = \begin{pmatrix} \varepsilon_{xx} - \frac{\varepsilon_v}{3} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} - \frac{\varepsilon_v}{3} \end{pmatrix} \quad Eq.\ [6]$$

In addition to stresses σ_{xx}, σ_{yy} and σ_{xy} the model computes also the value of out of plane normal stress σ_{zz} based on the linear elastic assumption in the out of plane direction as shown in Eq. [7].

$$d\sigma_{zz} = \nu(d\sigma_{xx} + d\sigma_{yy}) \quad Eq.\ [7]$$

where

ν = Poisson's ratio.

It should be noted that the values of σ_{zz} can be viewed in PLAXIS Output but they do not have any practical relevance nor any influence on the model behaviour.

2.2 Critical state Soil Mechanics framework

The model is defined according to the Critical State Soil Mechanics Framework (Schofield & Wroth, 1968 (on page 79)). It uses the relative state parameter index ξ_R (Konrad, 1988 (on page 79)) instead of the state parameter ψ (Been & Jefferies, 1985 (on page 79)). ξ_R is defined as shown in Eq. [8]:

$$\xi_R = D_{R,cs} - D_R \quad Eq.\ [8]$$

where

D_R = Current relative density.

$D_{R,cs}$ = Relative density on the critical state line at the current mean effective stress p.

$D_{R,cs}$ is defined by the critical state line in the $D_R - p$ plane as shown in Eq. [9]:

$$D_{R,cs} = \frac{R}{Q - \ln\left(100\frac{p}{p_A}\right)} \quad Eq.\ [9]$$

where

p_A = Atmospheric pressure.

Q and R = Bolton's parameters (Bolton, 1986 (on page 79)).

The values of Q and R were shown by Bolton to be about 10.0 and 1.0, respectively, for quartzitic sands. An example of the critical state line in the $D_R - p$ plane with the parameters Q=10 and R=1.5 is shown in Figure 1 (on page 6).

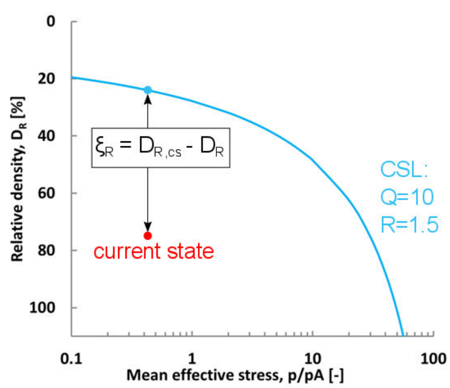


Figure 1: The critical state line (CSL) in the $D_R - p\ /\ p_A$ plane with the parameters Q=10 and R=1.5

2.3 Bounding, Dilatancy, Critical and Yield surfaces

The model uses the bounding, dilatancy and the critical surfaces, following the model of Dafalias & Manzari (2004) (on page 79). In the triaxial setting, the surfaces are defined with the stress ratios M^b, M^d and M, which depend on the relative state index ξ_R according to the following relationships:

$$M^b = M\exp\left(-n^b\xi_R\right) \quad Eq.\ [10]$$

$$M^d = M\exp\left(n^d\xi_R\right) \quad Eq.\ [11]$$

$$M = 2\sin\left(\varphi_{cv}\right) \quad Eq.\ [12]$$

where

n^b and n^d	=	Model parameters defining the computation of M^b and M^d in relation to M.
φ_{cv}	=	Critical state (constant volume) effective friction angle (also a model parameter).

As the model is sheared, ξ_R approaches the value of zero while M^b and M^d both approach the value of M. In Figure 2 (on page 7), the schematic representation of changing of bounding and dilatancy stress ratios M^b and M^d in relation to the relative state index ξ_R in the q-p plane is shown for the denser and looser than critical states.

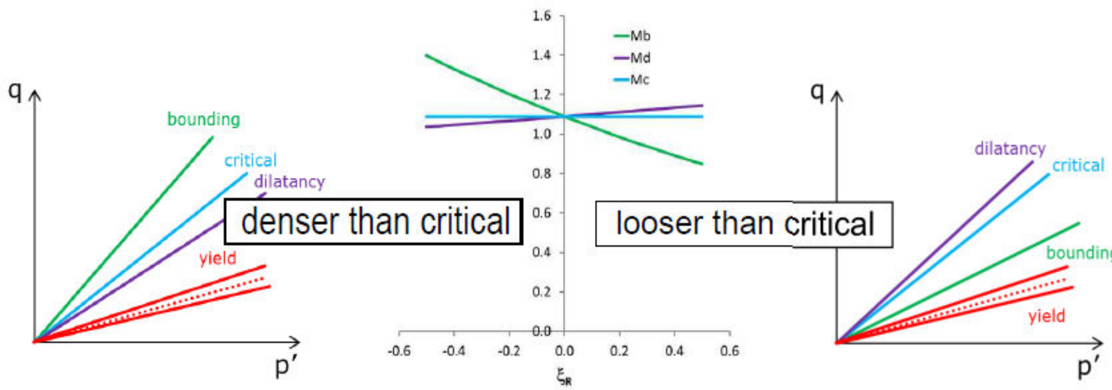


Figure 2: Schematic representation of changing of bounding and dilatancy stress ratios M^b and M^d according to the relative state index ξ_R in the q-p plane. The yield surface stress ratios are also shown along with the yield surface axis

The yield surface is formulated as a small cone in the stress space with the following expression in Eq. [13]:

$$f = \sqrt{(\mathbf{s} - p\boldsymbol{\alpha}) : (\mathbf{s} - p\boldsymbol{\alpha})} - \sqrt{\frac{1}{2}}pm = 0 \quad Eq.\ [13]$$

where

$\boldsymbol{\alpha}$	=	Back-stress ratio tensor which denotes the position of the yield surface in the deviatoric stress ratio space.
m	=	Size of the yield surface with the predefined value of 0.01.

In the general formulation of the model, the bounding and dilatancy surfaces are defined in terms of the image back-stress ratios $\boldsymbol{\alpha}^b$ and $\boldsymbol{\alpha}^d$ as expressed in Eq. [14] and Eq. [15] respectively.

$$\boldsymbol{\alpha}^b = \sqrt{\tfrac{1}{2}}[M^b - m]\boldsymbol{n} \quad Eq.\ [14]$$

$$\boldsymbol{\alpha}^d = \sqrt{\tfrac{1}{2}}[M^d - m]\boldsymbol{n} \quad Eq.\ [15]$$

where

$\boldsymbol{n}$ = Deviatoric unit normal to the yield surface defined as shown in Eq. [16].

$$\boldsymbol{n} = \frac{\boldsymbol{r} - \boldsymbol{\alpha}}{\sqrt{\tfrac{1}{2}}m} \quad Eq.\ [16]$$

where

$\boldsymbol{r}$ = Deviatoric stress ratio tensor

$$\boldsymbol{r} = \frac{\boldsymbol{s}}{p} \quad Eq.\ [17]$$

The schematic of the yield, dilatancy and bounding surfaces, tensor $\boldsymbol{n}$ and the image back-stress ratios in $r_{yy} - r_{xy}$ plane are shown in Figure 3 (on page 8).

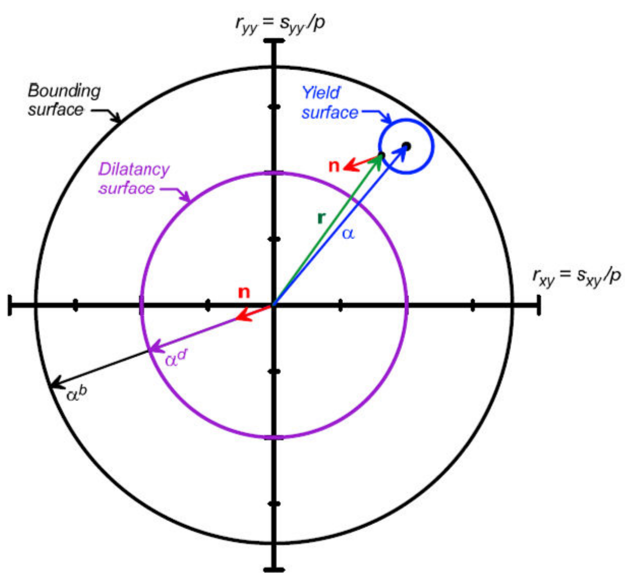


Figure 3: The schematic of the yield, dilatancy and bounding surfaces, the tensor $\boldsymbol{n}$ *and the image back-stress ratios in* $r_{yy} - r_{xy}$ *plane. (After Boulanger and Ziotopoulou (2017))*

The distance between the yield surface axis $\boldsymbol{\alpha}$ and the image back stress ratio $\boldsymbol{\alpha}^b$ defines the plastic modulus K_p, while the distance between $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^d$ defines the amount of dilatancy or contractancy via the quantity D (K_p and D are further explained in the subsections below). The full explanation of the role of $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^b$ and $\boldsymbol{\alpha}^d$ can be found in Boulanger and Ziotopoulou (2017) (on page 79).

2.4 Stress reversal and initial back-stress ratio tensors

According to the bounding surface formulation by Dafalias (1986) (on page 79), which was also adopted in Dafalias and Manzari (2004) (on page 79), the model keeps track of the initial back-stress ratio $\boldsymbol{\alpha}_{in}$, which is updated at the reversal in loading direction. The reversal in loading direction is identified whenever the following condition holds as shown in Eq. [18].

$$\left(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}\right) : \boldsymbol{n} < 0 \qquad Eq.\ [18]$$

At the reversal, the initial stress ratio $\boldsymbol{\alpha}_{in}$ is updated to the current one $\boldsymbol{\alpha}$. To overcome the high stiffness at a small load reversal, the initial stress ratio $\boldsymbol{\alpha}_{in}$ is subdivided into three initial stress ratios, namely the apparent $\left(\boldsymbol{\alpha}_{in}^{app}\right)$, true $\left(\boldsymbol{\alpha}_{in}^{true}\right)$ and the previous initial stress ratio $\left(\boldsymbol{\alpha}_{in}^{p}\right)$. The initial stress ratios take part in the expressions of dilatancy (D) and plastic modulus (K_p) which will be shown later. All the details about the mechanism of tracking the stress reversals can be found in Boulanger & Ziotopoulou (2017) (on page 79).

2.5 Elastic part of the model

The elastic volumetric and deviatoric strain increments are calculated as:

$$d\varepsilon_v^{el} = \frac{d\ p}{K} \qquad Eq.\ [19]$$

$$d\boldsymbol{e}^{el} = \frac{d\boldsymbol{s}}{2G} \qquad Eq.\ [20]$$

where

G	=	Elastic shear modulus
K	=	Elastic bulk modulus

The elastic shear modulus is dependent on the mean effective stress, stress ratio and fabric as shown in Eq. [21]:

$$G = G_0\, p_A \sqrt{\frac{p}{p_A}} \left(1 - C_{SR,0}\left(\frac{M}{M^{\,b}}\right)^{m_{SR}}\right)\left(\frac{1 + \frac{z_{cum}}{z_{\max}}}{1 + \frac{z_{cum}}{z_{\max}}C_{GD}}\right) \qquad Eq.\ [21]$$

G_0 is a parameter taking into account the small strain shear modulus, while the parameters $C_{SR,0}$ and m_{SR} impose the stress ratio effects according to Yu & Richart Jr. (1984) (on page 79). $C_{SR,0}$ and m_{SR} are set to 0.5 and 4 internally. This lets the effect of stress ratio on elastic modulus be small at small stress ratios, but lets the effect increase to a 60% reduction when the stress ratio is on bounding surface Boulanger & Ziotopoulou (2017) (on page 79). z_{cum} is the cumulative value of absolute changes of the fabric tensor $\boldsymbol{z}$ calculated as:

$$dz_{cum} = \sqrt{\frac{d\boldsymbol{z} : d\boldsymbol{z}}{2}} \qquad Eq.\ [22]$$

z_{max} is the parameter computed at the time of the model initialisation according to the initial relative state index ξ_{R0} as:

$$z_{\max} = 0.7 \exp\left(-6.1\xi_{R0}\right) \leq 20 \qquad \text{Eq. [23]}$$

C_{GD} is the factor controlling the shear modulus degradation dependent on the cumulative plastic deviatoric strains. The maximum degradation approaches a factor of $1/C_{GD}$. The value of C_{GD} is set internally to 2.0. The elastic bulk modulus K is related to the shear modulus G by the following relationship in Eq. [24]:

$$K = \frac{2(1+\nu)}{3(1-2\nu)}G \qquad \text{Eq. [24]}$$

where

ν = Constant poisson's ratio

The recommended value of ν is 0.3 and can be changed by the user.

2.6 Plastic components of the Model

2.6.1 Plastic strains, Loading index and Stress increment

The increment of the plastic volumetric and deviatoric strain is calculated with the following expressions:

$$d\varepsilon_v^{pl} = \langle L \rangle D \qquad \text{Eq. [25]}$$

$$d\boldsymbol{e}^{pl} = \langle L \rangle \boldsymbol{n} \qquad \text{Eq. [26]}$$

where

L = Loading index

$\langle\rangle$ = Macaulay brackets that set negative values to zero and D is the dilatancy which will be defined in a later section.

The loading index is calculated as shown in Eq. [27].

$$L = \frac{2G\boldsymbol{n} : d\boldsymbol{e} - \boldsymbol{n} : \boldsymbol{r} K d\epsilon_v}{K_p + 2G - KD\boldsymbol{n} : \boldsymbol{r}} \qquad \text{Eq. [27]}$$

By using the calculated loading index, the stress increment can be calculated as in Eq. [28]:

$$d\boldsymbol{\sigma} = 2Gd\boldsymbol{e} + Kd\epsilon_v\boldsymbol{I} - \langle L \rangle (2G\boldsymbol{n} + KD\boldsymbol{I}) \qquad \text{Eq. [28]}$$

2.6.2 Hardening-Softening rule and the Plastic modulus

The evolution of the back-stress ratio α corresponding to the axis of the yield surface is used according to Dafalias & Manzari (2004) (on page 79) as shown in Eq. [29].

$$d\boldsymbol{\alpha} = \langle L \rangle \frac{2}{3} h \left(\boldsymbol{\alpha}^{b} - \boldsymbol{\alpha}\right) \quad Eq.\ [29]$$

where

h = Hardening coefficient.

The plastic modulus K_p is defined as shown inEq. [30]:

$$K_p = G h_0 \frac{\sqrt{(\boldsymbol{\alpha}^{b} - \boldsymbol{\alpha}) : \boldsymbol{n}}}{\left[\exp\left((\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}) : \boldsymbol{n}\right) - 1\right] + C_{\gamma 1}} C_{rev} \quad Eq.\ [30]$$

$$For \left(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{p}\right) : \boldsymbol{n} \le 0 \quad C_{rev} = \frac{(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}) : \boldsymbol{n}}{(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{true}) : \boldsymbol{n}}$$

$$otherwise \quad C_{rev} = 1$$

where

h_0 = Parameter adjusting the ratio between plastic and elastic moduli.

The value of h_0 is internally set according to the following relationship:

$$h_0 = \frac{\left(0.25 + D_{R0}\right)}{2} \ge 0.3 \quad Eq.\ [31]$$

where

D_{R0} = Initial relative density i.e. the first input parameter of the model.

The constant $C_{\gamma 1}$ serves to avoid the division by zero and is internally set to $h_0/200$. From the Eq. [30] it can be seen that the plastic modulus K_p is proportional to G and to the distance of the back-stress ratio $\boldsymbol{\alpha}$ to the bounding back-stress ratio $\boldsymbol{\alpha}^{b}$ and inversely proportional to the distance of the back-stress ratio from the initial back-stress ratio $\boldsymbol{\alpha}_{in}^{app}$. The plastic modulus relationship was revised by Boulanger & Ziotopoulou (2017) (on page 79) in comparison to the Dafalias & Manzari (2004) (on page 79) model in order to provide an improved approximation of empirical relationships for secant shear modulus and equivalent damping ratios during drained strain-controlled cyclic loading. More details on the subject can be found in Boulanger & Ziotopoulou (2017) (on page 79).

2.6.3 Plastic volumetric strains- The dilation part

Plastic volumetric dilation occurs when $\left(\boldsymbol{\alpha}^{d} - \boldsymbol{\alpha}\right) : \boldsymbol{n} < 0$. In that case the dilatancy D is negative and calculated according to the following expression (without the fabric effects which will be described in a later section).

$$D = A_{d0}\left[(\boldsymbol{\alpha}^{d} - \boldsymbol{\alpha}) : \boldsymbol{n}\right] \quad Eq.\ [32]$$

From Eq. [32] it can be recognised that the dilatancy is proportional to the constant A_{d0} and the distance of the back-stress ratio $\boldsymbol{\alpha}$ to the dilatancy back-stress ratio $\boldsymbol{\alpha}^{d}$. The constant A_{d0} is related to the dilatancy relationship proposed by Bolton (1986) (on page 79):

$$\varphi_{pk} - \varphi_{cv} = -0.8\psi \quad Eq.\ [33]$$

where

φ_{pk} = Peak angle of shearing resistance.

φ_{cv}	=	Angle of shearing resistance at the constant volume.
ψ	=	Dilatancy angle.

Taking into account the Eq. [33], the constant A_{d0} is defined as shown in Eq. [34]:

$$A_{d0} = \frac{1}{0.4} \frac{\arcsin\left(\frac{M^b}{2}\right) - \arcsin\left(\frac{M}{2}\right)}{M^b - M^d} \qquad Eq.\ [34]$$

Further details on the formulation of dilatancy in the dilatant regime can be found in the Boulanger & Ziopoulou (2017) (on page 79).

2.6.4 Plastic volumetric strains-The contraction Part

Whenever $(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n} > 0$, plastic volumetric contraction occurs and the dilatancy D is positive and calculated according to the following expression as shown in Eq. [35].

$$D = A_{dc}\left[(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \boldsymbol{n} + C_{in}\right]^2 \frac{(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n}}{(\boldsymbol{\alpha}^d - \boldsymbol{\alpha}) : \boldsymbol{n} + C_D} \qquad Eq.\ [35]$$

where A_{dc} is calculated as shown in Eq. [36]:

$$A_{dc} = \frac{A_{d0}}{h_p} \qquad Eq.\ [36]$$

By taking A_{d0} term from the dilatant part and dividing it by h_p, which depends on the parameter h_{p0} and the current relative state parameter index ξ_R as:

$$h_p = h_{p0} \exp\left(-0.7 + 7(0.5 - \xi_R)^2\right) \qquad for\ \xi_R \le 0.5$$
$$h_p = h_{p0} \exp(-0.7) \qquad for\ \xi_R > 0.5 \qquad Eq.\ [37]$$

The parameter h_{p0} can be varied during the calibration process to get the desired cyclic resistance ratios. The C_{in} term depends on fabric and is described in a later section. The value of the constant C_D is internally set to 0.16. The refined Eq. [35] in comparison with the Dafalias and Manzari (2004) (on page 79) model improved the slope of the cyclic resistance ratio (CRR) versus number of equivalent uniform loading cycles for undrained cyclic element tests. The effect of the overburden stress on the cyclic resistance (K_σ) was taken into account by the Eq. [37]. Further explanation of the dilatancy in the contractant regime can be found in Boulanger & Ziotopoulou(2017) (on page 79).

2.6.5 Fabric effects

The effects of prior straining on the model response have been taken into account by using the fabric-dilatancy tensor $\boldsymbol{z}$ that was introduced by Dafalias & Manzari (2004) (on page 79). The evolution of $\boldsymbol{z}$ is defined as:

$$d\boldsymbol{z} = -\frac{c_z}{1+\left\langle \frac{z_{cum}}{2z_{\max}} - 1 \right\rangle} \frac{\left\langle -d\varepsilon_v^{pl} \right\rangle}{D} \left(z_{\max}\boldsymbol{n} + \boldsymbol{z} \right) \qquad \textit{Eq. [38]}$$

The tensor $\boldsymbol{z}$ evolves with the plastic deviatoric strains that occur during dilation only. It can be seen from Eq. [38] that the rate of evolution of $\boldsymbol{z}$ decreases with increasing values of z_{cum}. In this way during the undrained cyclic loading the shear strains progressively accumulate rather than lock-up into a repeating stress-strain loop. The model tracks additional quantities regarding the fabric history, such as z_{peak}, $\boldsymbol{z}_{in}$ and p_{zp}. Using aforementioned quantities many issues of the model behaviour have been taken into account:

- Effects of sustained static shear stresses,
- Fabric effects for various drained versus undrained loading conditions,
- Degree of stress rotation and its effect on plastic modulus,
- Erasure of fabric formed during liquefaction in reconsolidation stages and
- Effect of prior strain history at loading of the mean effective stress that is smaller or larger than the mean effective stress when fabric was formed.

More details on tracking of fabric and its effects on model response can be found in Boulanger & Ziotopoulou (2017) (on page 79).

2.6.6 Fabric effects on plastic modulus and dilatancy

For brevity, the final plastic modulus and dilatancy expressions considering also the fabric effects will be given only in the short form without the explanations of the meaning of terms. All the explanations and full forms of expressions can be found in Boulanger & Ziotopoulou (2017) (on page 79).

The plastic modulus Eq. [30] is multiplied with the fabric terms as shown in Eq. [39]:

$$K_p = Gh_0 \frac{\sqrt{\left(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}\right) : \boldsymbol{n}}}{\left[\exp\left(\left(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}^{app}\right) : \boldsymbol{n}\right) - 1\right] + C_{\gamma 1}} C_{rev} \cdot \frac{C_{ka}}{1 + C_{Kp}\left(\frac{z_{peak}}{z_{\max}}\right)\left\langle \left(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}\right) : \boldsymbol{n} \right\rangle \sqrt{1 - C_{zpk2}}} \qquad \textit{Eq. [39]}$$

Effects of fabric on plastic volumetric dilation have been incorporated through the introduction of the additional rotated dilatancy surface and then splitting the dilatancy expression into two parts regarding the position to the rotated dilatancy surface. The A_{d0} term from Eq. [32] has been transformed into the term A_{d0}^* of the following form:

$$A_{d0}^* = \frac{A_{d0} C_{zin2}}{\left(\frac{z_{cum}^2}{z_{\max}}\right)\left(1 - \frac{\langle -\boldsymbol{z} : \boldsymbol{n} \rangle}{\sqrt{2} z_{peak}}\right)^3 (C_\varepsilon)^2 C_{pzp} C_{p\min} C_{zin1} + 1} \qquad \textit{Eq. [40]}$$

The fabric terms are present in C_{zin1}, C_{zin2} and C_{pzp} terms through the quantities z_{peak}, z_{max}, z_{cum} and $\boldsymbol{z}_{in}$ while C_{pmin} as well as C_{pzp} include also the dependence on the value of mean effective stress. Effects of fabric on plastic volumetric contraction have been put through the modification of the term A_{dc} in Eq. [36]. The fabric dependent expression for A_{dc} is defined as:

$$A_{dc} = \frac{A_{d0}^{*}(1 + \langle \boldsymbol{z} : \boldsymbol{n} \rangle)}{h_p C_{dz}} \qquad \textit{Eq. [41]}$$

The term C_{dz} includes the fabric dependence through the quantities z_{peak}, z_{max}, z_{cum}.

2.6.7 Post-shaking reconsolidation

The development of volumetric strains during post-liquefaction reconsolidation of sand is difficult to numerically model using the conventional separation of strains into elastic and plastic part in the constitutive model, since a large portion of the post-liquefaction reconsolidation strains are due to sedimentation effects (Boulanger & Ziotopoulou (2013) (on page 79)). Generally the numerically predicted post-liquefaction reconsolidation strains are an order of magnitude smaller than observed in experimental studies (e.g Boulanger & Ziotopoulou (2013) (on page 79); Howell, Rathje & Boulanger(2014) (on page 79)).

To more accurately take into account the post-liquefaction reconsolidation strains, Boulanger & Ziotopoulou (2017) (on page 79) proposed the reduction of elastic moduli by a reduction factor F_{sed} that is also implemented in PLAXIS as follows:

$$G_{post-shaking} = F_{sed} G \qquad \textit{Eq. [42]}$$

$$K_{post-shaking} = F_{sed} K \qquad \textit{Eq. [43]}$$

The factor F_{sed} is calculated as:

$$F_{sed} = F_{sed,\min} + (1 - F_{sed,\min})\left(\frac{p}{20 p_{sed}}\right)^2 \le 1 \qquad \textit{Eq. [44]}$$

$$p_{sed} = p_{sed_0} + \left(\frac{z_{cum}}{z_{cum} + z_{\max}}\right)\left\langle 1 - \frac{M^{cur}}{M^{d}}\right\rangle^{0.25} \qquad \textit{Eq. [45]}$$

$$F_{sed,\min} = 0.03\exp\left(2.6 D_{R0}\right) \le 0.99 \qquad \textit{Eq. [46]}$$

$$p_{sed_0} = \frac{p_A}{5} \qquad \textit{Eq. [47]}$$

where

$F_{sed,min}$ = The smallest value F_{sed} can attain and is dependent on the initial relative density D_{R0}.

The parameter p_{sed_0} is the mean effective stress up to which reconsolidation strains are enhanced.

The user is advised to create two sets of the same PM4Sand material, one having *PostShake* equal to 0 and the other *PostShake* equal to 1. The dynamic analysis phase should be divided into 2 phases, the first phase covering the strong shaking motion with appropriate *Dynamic time interval* and the second phase covering the remaining weak shaking with the remaining *Dynamic time interval*. When a single dynamic multiplier is used for both strong and weak shaking motion phases the *Reset time* option should not be activated in the second phase to avoid starting the analysis from the beginning of the input signal. In the first phase the PM4Sand material with *PostShake* equal to 0 should be used and in the second phase the PM4Sand material with *PostShake* equal to 1.

2.7 Installation of the model

The model has been compiled as a PLAXIS User-defined model in the form of a dynamic link library (dll). The name of the library is *pm4sand64.dll*. The library (i.e. the dll file) must be placed into the subfolder *udsm* in the PLAXIS installation folder.

In the PLAXIS Input program the user has to choose *User-defined* for the option *Soil model* as shown in Figure 4 (on page 15). Then in the *Mechanical* tab the *DLL file pm4sand64.dll* along with the *Model* in *DLL PM4Sand* as shown in Figure 5 (on page 16) have to be chosen. Afterwards, the model will be ready to be used along with other defined material models.

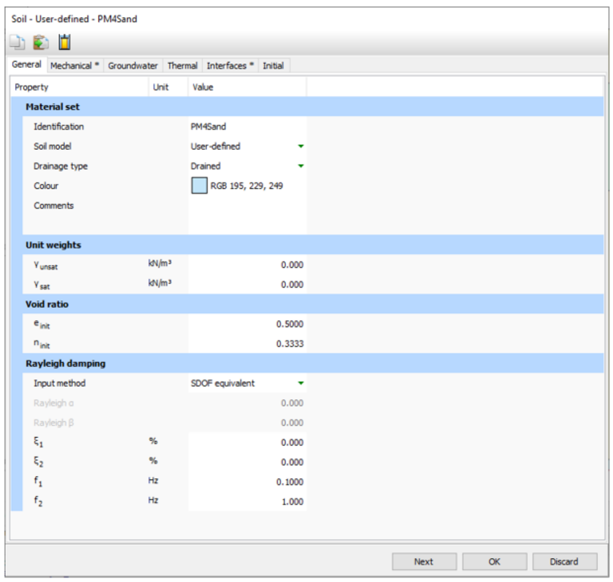


Figure 4: Material model set as User defined

2.8 Model parameters

The model has 13 input parameters that can be modified by the user. All of the parameters can be set in the Material input menu as shown in Figure 5 (on page 16).

The model parameters are grouped into two categories:

- A primary set of 4 parameters (i.e. D_{R0}, G_0, h_{p0} and p_A) that are most important for model calibration and
- A secondary set of 9 parameters (i.e. e_{max}, e_{min}, n^b, n^d, φ_{cv}, ν, Q, R and PostShake) that may be modified from the recommended default values in special circumstances.

Boulanger & Ziotopoulou (2017) (on page 79) have provided the default values that are supposed to generally produce reasonable agreement with the trends in typical design correlations.

The default values are given for each of the secondary parameters in the following subsections and must be entered by the user, even if the user wants to use the default values (note that the values of all parameters are initially set to zero, refer to Figure 5 (on page 16).).

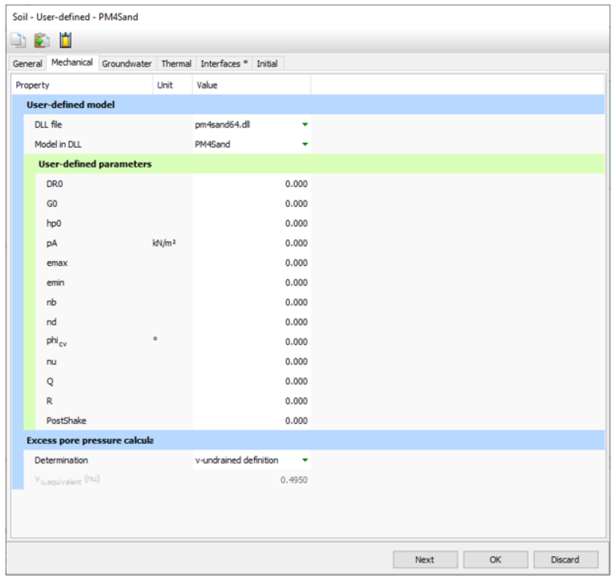


Figure 5: Parameters of PLAXIS User defined PM4Sand model in the Material data set window

In the following subsections, the meaning of each of the input parameters will be discussed along with the recommended procedures and values to be used as stated by Boulanger & Ziotopoulou (2017) (on page 79).

2.8.1 Relative density D_R (DR0) (-)

The relative density parameter D_R controls the dilatancy and stress-strain response characteristics. Its value can be estimated by correlations with penetration resistances of SPT or CPT tests. A common form for SPT correlation is shown in Eq. [48]:

$$D_R = \sqrt{\frac{(N_1)_{60}}{C_d}} \qquad \textit{Eq. [48]}$$

Where the value of C_d can be taken according to Idriss & Boulanger (2008) (on page 79) recommendations. They reviewed published data and past relationships and then adopted the value of C_d = *46* in the development of their liquefaction triggering correlations. Regarding the use of the CPT penetration resistance the user is advised to also use the Idriss & Boulanger (2008) (on page 79) recommendations by the following expression shown in Eq. [49]:

$$D_R = 0.465\left(\frac{q_{c1N}}{C_{dq}}\right)^{0.264} - 1.063 \qquad \textit{Eq. [49]}$$

For which they adopted $C_{dq} = 0.9$.

The above recommendations are stated in Boulanger & Ziotopoulou (2017) (on page 79). The authors also comment that the input value D_R is best considered an "apparent relative density", rather than a strict measure of relative density from conventional laboratory tests. The value of D_R influences the response of the model through D_R correlations and the relative state parameter index ξ_R. Therefore, there may be situations where the user may choose to adjust the input D_R up or down relative to the above relationships to improve the calibration to some other relationship or data.

2.8.2 Shear modulus coefficient G_0 (G0) (-)

The shear modulus coefficient G_0 controls the elastic (small strain) shear modulus G as shown in Eq. [50]:

$$G = G_0\, p_A \sqrt{\frac{p}{p_A}} \qquad Eq.\ [50]$$

G_0 should be calibrated to fit estimated or measured V_s values, according to

$$G = \rho (V_s)^2 \qquad Eq.\ [51]$$

or alternatively fit to values of V_s that are estimated by correlation to penetration resistances. Boulanger & Ziotopoulou (2017) (on page 79) used the $V_{s1} - (N_1)_{60}$ correlations from Andrus & Stokoe (2000) (on page 79) (Figure 6 (on page 18)) with a slight modification for very small values of $(N_1)_{60}$ in the form shown in Eq. [52]:

$$V_{s1} = 85\left[(N_1)_{60} + 2.5\right]^{0.25} \qquad Eq.\ [52]$$

Alternatively, the expression covering a range of typical densities can be used to directly calculate the parameter G_0 as proposed by Boulanger & Ziotopoulou (2017) (on page 79) in the following form shown in Eq. [53]:

$$G_0 = 167\sqrt{(N_1)_{60} + 2.5} \qquad Eq.\ [53]$$

2.8.3 Contraction rate parameter h_{p0} (hp0) (-)

The contraction rate parameter h_{p0} adjusts the contractiveness of the model and hence enables the calibration to specific values of cyclic resistance ratio (CRR). This parameter is meant to be calibrated at last after the values of other parameters have been assigned. The user can use PLAXIS Soil test facility to simulate uniform cyclic direct simple shear tests to calibrate the parameter.

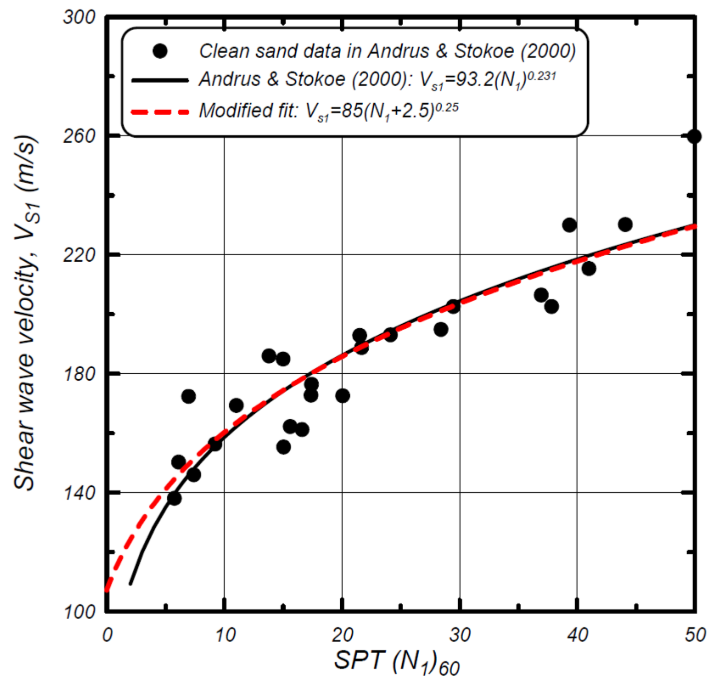


Figure 6: Correlation between overburden-corrected shear wave velocity and SPT penetration resistances in clean sands (after Andrus & Stokoe (2000) and modifications by Boulanger & Ziotopoulou (2017)) .

The liquefaction triggering correlation by Idriss & Boulanger (2008) (on page 79) from Figure 7 (on page 19) can be used to get the needed *CRR* value of the material (as has been done for the examples in the report by Boulanger & Ziotopoulou (2017) (on page 79)) if cyclic laboratory data of the material is not available. This relationship provides the target *CRR* values for an effective overburden stress of 100kPa and an earthquake magnitude of *M* = 7.5. Looking at the earthquake magnitude of 7.5, the corresponding number of uniform cycles at 65% of the peak stress is equal to 15 (Figure 8 (on page 19)). Boulanger & Ziotopoulou (2017) (on page 79) assumed that this corresponds approximately to *CRR* of 15 uniform loading cycles with the liquefaction triggering criterion of causing a peak shear strain of 3% in direct simple shear loading. For other earthquake magnitudes than M=7.5, the user is advised to use the $N_{M=7.5} - M$ relation from Figure 9 (on page 20) to read the value N_M at a desired magnitude *M*. Then the magnitude scaling factor MSF should be calculated according to the relationship shown in Eq. [54]:

$$MSF = \left(\frac{N_{M=7.5}}{N_M}\right)^b \qquad \textit{Eq. [54]}$$

where

b	=	The slope of CRR-N lines, equal to b = 0.34 for sands.
$N_{M=7.5}$	=	The number of equivalent stress cycles at magnitude M=7.5.
N_M	=	The number of equivalent stress cycles at the desired magnitude M.

With the known MSF, the target CRR_M at the desired magnitude can be evaluated as:

$$CRR_M = MSF \;\; CRR_{M=7.5} \quad \textit{Eq. [55]}$$

Instead of using the Eq. [55], the magnitude scaling factors can be read from Figure 9 (on page 20). More details on the above procedure of calculating MSF and CRR_M can be found in Idriss & Boulanger (2008) (on page 79). In the case of the availability of cyclic laboratory CRR - N curves of the modelling material, the user can calibrate the parameter h_{p0} according to points on those curves. Generally, decreasing the value of h_{p0} gives more contractant behaviour and consequently a lower CRR.

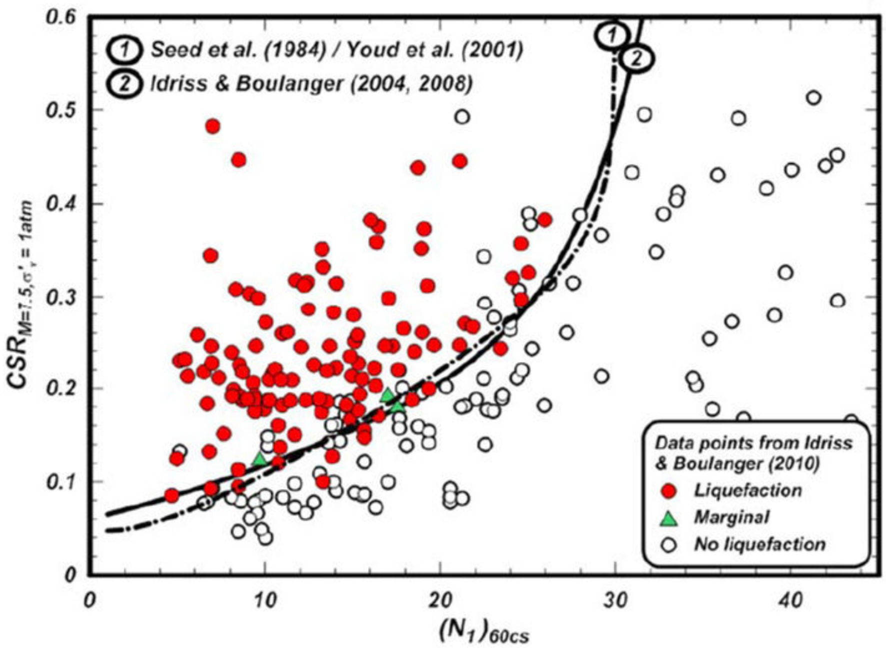


Figure 7: Correlations for cyclic resistance ratio (CRR) from SPT data (after Idriss & Boulanger (2010))

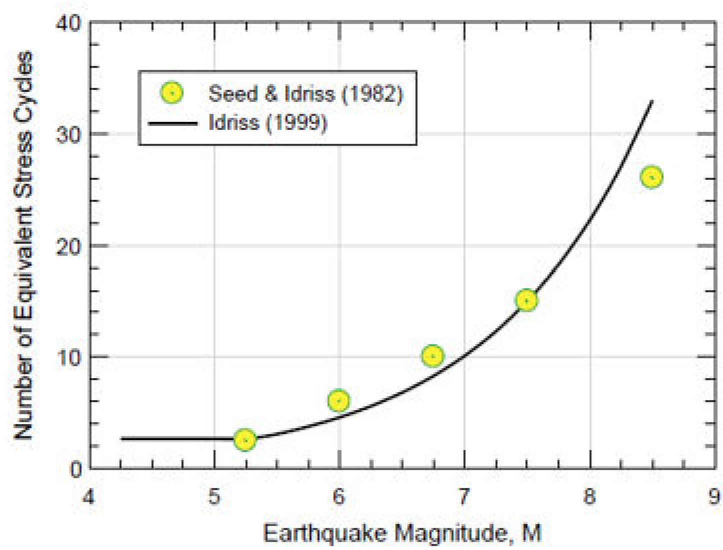


Figure 8: Mean number of equivalent uniform cycles at 65% of the peak stress versus earthquake magnitude (after Idriss & Boulanger (2008))

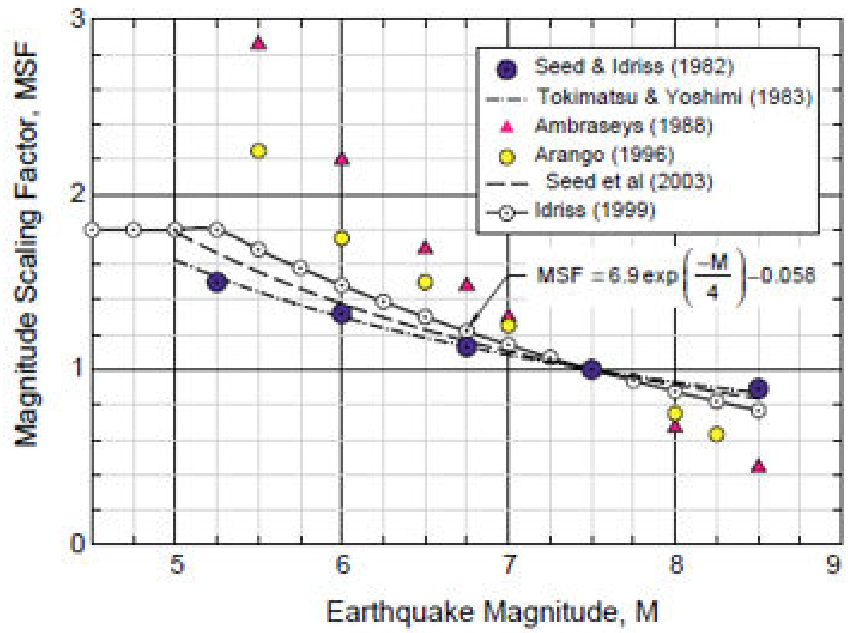


Figure 9: Magnitude scaling factors values proposed by various researchers (after Idriss & Boulanger (2008))

2.8.4 Atmospheric pressure p_A (pA) (kPa) (Default: 101.3 kPa)

The atmospheric pressure p_A should be specified by the user in the stress unit being used for the analysis. In order to be consistent with the original formulation the recommended value is 101.3 kPa.

2.8.5 Maximum and minimum void ratio e_{max} and e_{min} (emax and emin) (-) (Default:0.8;0.5)

The maximum and minimum void ratios e_{max} and e_{min} influence the computation of the relative state index ξ_R and the relationship between volume changes and the relative state index. The default values recommended by Boulanger & Ziotopoulou (2017) (on page 79) are 0.8 and 0.5, respectively.

2.8.6 Bounding surface parameter n^b (nb) (-) (Default:0.5)

The bounding surface parameter n^b controls the relative position of the bounding surface to the critical state surface dependent on the relative state index ξ_R (Eq. [10]). The default value is 0.5. This affects the plastic modulus, as well as the dilatancy and thus the peak effective friction angles. It should be noted that M^b for looser than critical states (i.e. $\xi_R > 0$) is computed using the value of $n^b / 4$.

2.8.7 Dilatancy surface parameter n^d (nd) (-) (Default:0.1)

The dilatancy surface parameter n^d controls the value of the stress ratio at which the contraction transitions to the dilation and vice versa. This transition is often referred to as phase transformation. The default value of n^d is 0.1. The value of 0.1 produces a phase transformation angle slightly smaller than φ_{cv} angle, which is consistent with the experimental data (Boulanger & Ziotopoulou, 2017 (on page 79)). Similarly to the bounding surface parameter n^b the value of M^d for loose of critical states (i.e. $\xi_R > 0$) is computed using the value of $4n^d$.

2.8.8 Critical state friction angle Φ_{cv} (phi$_{cv}$) (^{0}) (Default:33)

The critical state friction angle φ_{cv} defines the position of the critical state surface (i.e. the value of M stress ratio from Eq. [12]). The default value is 33 degrees.

2.8.9 Poisson's ratio ν (nu) (-) (Default: 0.3)

The Poisson's ratio ν . The default value is 0.3.

2.8.10 Critical state line parameters Q and R (Q, R) (-) (Default: 10, 1.5)

The parameters Q and R define the critical state line, see Eq. [9] and Figure 1 (on page 6). The default values for quartzitic sands per recommendations of Bolton (1986) (on page 79) are 10 and 1. Boulanger & Ziotopoulou(2017) (on page 79) use a slight increase in R to a value of 1.5 to lower the critical state line to better approximate typical results for direct simple shear loading.

2.8.11 Post shake switch (PostShake) (-) (Default:0)

The post shake switch is used to activate the reduction of elastic stiffness to simulate the post-shaking reconsolidation. According to Boulanger & Ziotopoulou(2017) (on page 79) the user should activate this feature only after the end of strong shaking. To properly simulate this in PLAXIS a copy of PM4Sand material in use should be created with the value of *PostShake* parameter equal to 1.0. Then the materials with *PostShake* equal to 0 and 1 should be interchanged as discussed in Section Post-shaking reconsolidation (on page 14).

2.8.12 Other model parameters

The values of the remaining model parameters of the original PM4Sand model are equal to the default values or are calculated from the index properties according to procedures described in Boulanger & Ziotopoulou (2017) (on page 79).

2.9 State parameters

The model uses around 30 state parameters as well as additional tracking variables. 3 state parameters are meant to be used by the user and therefore also have the clear identification, namely:

- σ_{v0} (sigv0): The initial vertical stress. It is automatically initialised at the beginning of the first phase in which the PM4Sand constitutive model is used and it is treated as a (fixed) state parameter. If needed, the user can reset the value of σ_{v0} as well as the remaining state variables by checking the 'Reset state variables' option in the phase settings.
- r_u (ru): The excess pore pressure ratio according to the vertical stress, calculated as shown in Eq. [56]:

$$r_u = 1 - \frac{\sigma_v}{\sigma_{v0}} \qquad Eq.\ [56]$$

- $r_{u,max}$ (ru,max): The maximum value of the excess pore pressure ratio r_u in the current phase.

2.10 Advice on the use of the model

- The model can only be used in 2D plane-strain analyses, because the out-of-plane stress σ_{zz} is ignored in the model formulation. The model still returns σ_{zz} using the linear elastic assumption for this component of stress following Eq. [7].
- The user is advised to use the manual time stepping with small time steps (substeps) otherwise the situation can arise with too big strain increments applied to the model which may result in time consuming stress integration and also a global divergence.

- Due to the plane-stress model formulation the only applicable test conditions in the PLAXIS Soil Test facility are the direct simple shear (DSS) and cyclic direct simple shear (CDSS) tests.
- In order to reduce the non-realistic concentrations of excess pore pressures and stresses, the user is advised to use the cavitation cut-off set at 100kPa during an undrained dynamic analysis, as well as a dynamic with consolidation analysis .This setting can be chosen in the *Phases* window under the *Deformation control parameters*.
- A better spread of pore-water pressure and less mesh dependency is often possible with the use of Dynamic analysis with Consolidation (i.e. Biot dynamics), which has been available since PLAXIS 2D 2018 version.
- The state variables can be explicitly initialised for a particular phase in the *Phases* window under the *Deformation control parameters* by enabling the option *Reset state variables*. Note that, this reinitialises the state variables of all constitutive models.
- Due to the known less accurate simulation capabilities of the model in static loading conditions particularly when the model is calibrated for dynamic loading, the recommended procedure of using the model in the calculations is to use other relevant material in place of the PM4Sand material in order to get the static (gravity, staged construction) phases of the calculations. At the dynamic phase, the material should be changed to PM4Sand. In this way the model internal variables will be initialised according to stress state of the previous static phase prior to the dynamic loading.

3 Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

3.1 The aim of the excercise:

The aim of this exercise is to simulate all the points on two published CSR-N curves generated by the original PM4Sand model (Boulanger & Ziotopoulou, 2015 (on page 79)) by the use of cyclic direct simple shear simulations in PLAXIS SoilTest facility. The tests are stress-controlled and in undrained conditions.

The chosen material is Ottawa sand at $D_R = 65~\%$. Ziotopoulou (2017) (on page 79) calibrated two material sets that were used in the LEAP project analyses (i.e. Case A and Case B material sets). The parameters used for cases A and B are given in Table 1 (on page 24) and Table 2 (on page 24), while the only difference between the two material sets is the contraction rate parameter h_{p0} as shown in Table 3 (on page 25).

Table 1: The calibrated parameters

Parameter	Value	Unit
D_{R0}	0.65	[-]
G_0	240	[-]
e_{max}	0.81	[-]
e_{min}	0.4915	[-]

Table 2: The parameters with default values

Parameter	Value	Unit
p_A	101.3	kPa
n^b	0.5	[-]
n^d	0.1	[-]

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

Parameter	Value	Unit
φ_{cv}	33	[^{0}]
ν	0.3	[-]
Q	10	[-]
R	1.5	[-]
PostShake	0	[-]

Table 3: The values of the contraction rate parameter h_{p0}

Parameter	Case A	Case B
h_{p0}	0.05	0.2

In Figure 10 (on page 25) the target CSR-N plots are shown from undrained cyclic stress-controlled DSS simulations using the original PM4Sand model (as reported by Ziotopoulou (2017) (on page 79)). Additionally, plots from simulations performed by the PM4Sand model implemented in PLAXIS by using the SoilTest facility are shown. These points (i.e. blue dots) represent the result of this exercise.

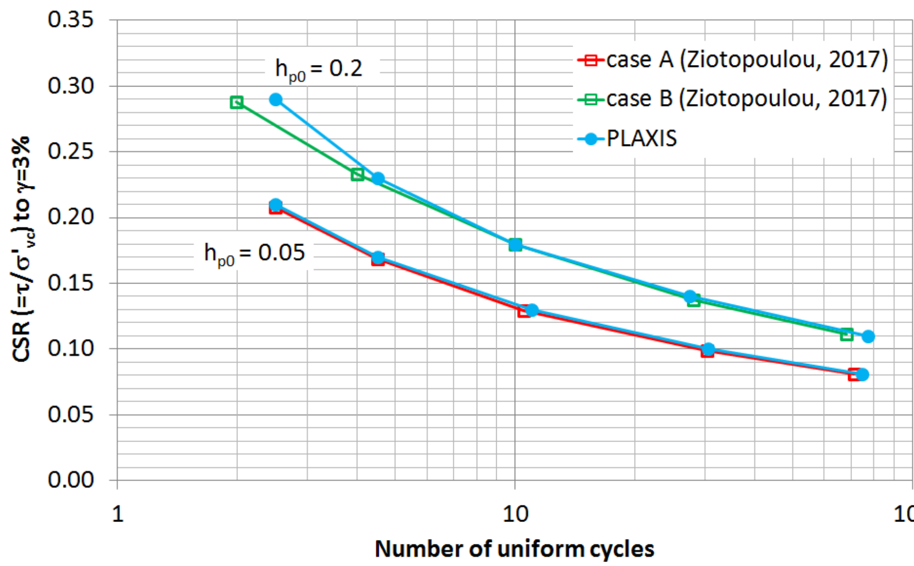


Figure 10: CSR-N relationships for case A and case B material sets. (modified from Ziotopoulou (2017))

The numbers used to draw the PLAXIS points on the plots are given in Table 4 (on page 26) and Table 5 (on page 26), for each material set respectively.

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

Table 4: CSR-N values at single amplitude shear strain γ=3% for Case A material

Point	CSR[-]	Shear Stress amplitude $\Delta\ \sigma_{xy}$ [kPa]	Number of uniform cycles N at $\gamma = 3\ \%$ [-]
1	0.21	21	2.5
2	0.17	17	4.5
3	0.13	13	11
4	0.1	10	30.5
5	0.081	8.1	74.5

Table 5: CSR-N values at single amplitude shear strain γ=3% for Case B material

Point	CSR[-]	Shear Stress amplitude $\Delta\ \sigma_{xy}$ [kPa]	Number of uniform cycles N at $\gamma = 3\ \%$ [-]
1	0.29	29	2.5
2	0.23	23	4.5
3	0.18	18	10
4	0.14	14	27.5
5	0.11	11	77

Step by step guide through the excercise are as follows:

1. Start PLAXIS Input.
2. Start a new project.
3. In the Project properties window, click Next.
4. In the *Model* tab sheet of the *Project properties* window, make sure that *Units* are set to [m], [kN] and [day], then click *OK*.
5. In the PLAXIS 2D Input window, click the *Materials* button in the toolbar.

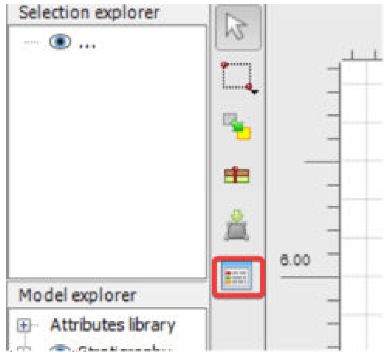


Figure 11: Input window showing Materials option

6. We will create one PM4Sand material set. In the *Material* sets window, click *New*....
7. In the *General* tab sheet of the Soil window, enter an *Identification* for the data set like "Ottawa sand" and then select *User-defined* as the Soil model (see the picture below). Other settings on this tab sheet are not relevant for this exercise.

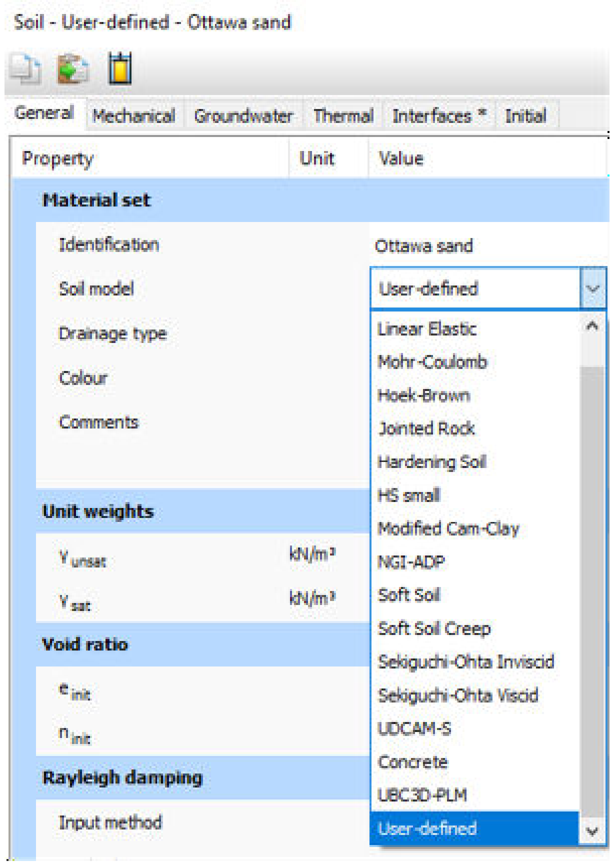


Figure 12: Material set with Ottawa sand selection

The *General* tab sheet should look like this:

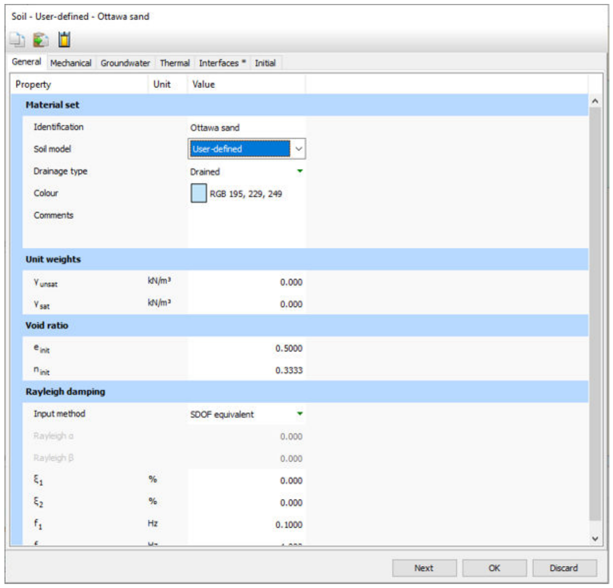


Figure 13: General Tab sheet

8. Click *Next* to get to the *Mechanical* tab sheet.
9. In the *Mechanical* tab sheet choose the *DLL file* pm4sand64.dll and *Model in DLL* PM4Sand. All the material parameters of the model will be shown with the values of 0.0.
10. Enter the following values of the parameters:

Parameter	Value	Unit
DR0	0.65	[-]
G0	240	[-]
hp0	0.05	[-]
pA	101.3	[kPa]
emax	0.81	[-]
emin	0.4915	[-]
nb	0.5	[-]
nd	0.1	[-]
phi_{cv}	33	[^{0}]
nu	0.3	[-]
Q	10	[-]
R	1.5	[-]

Parameter	Value	Unit
PostShake	0	[-]

The *Mechanical* tab sheet should look like in the following figure:

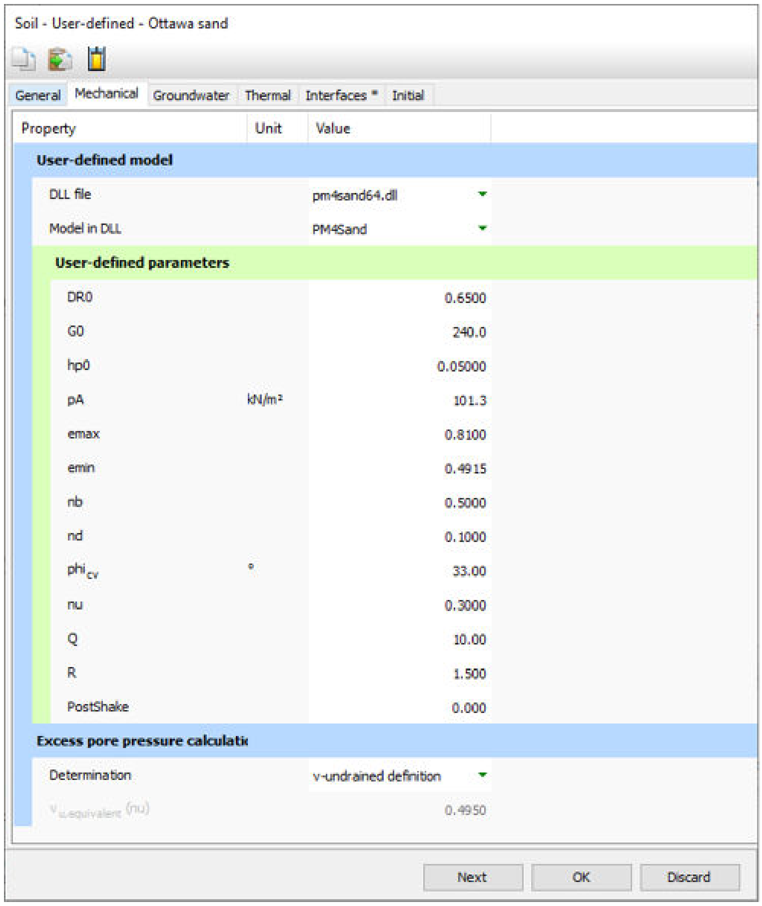


Figure 14: Mechanical Tab sheet

11. Go to the *Interfaces* tab sheet and enter imaginary values of 1 in the two fields (i.e. E_{oed}^{ref} and $c_{ref\ ,\ int\ er}$). The interfaces will not be used in this exercise, but numbers other than zeros are needed for PLAXIS to accept the data as a consistent data set.

The *Interfaces* tab sheet should look like the following:

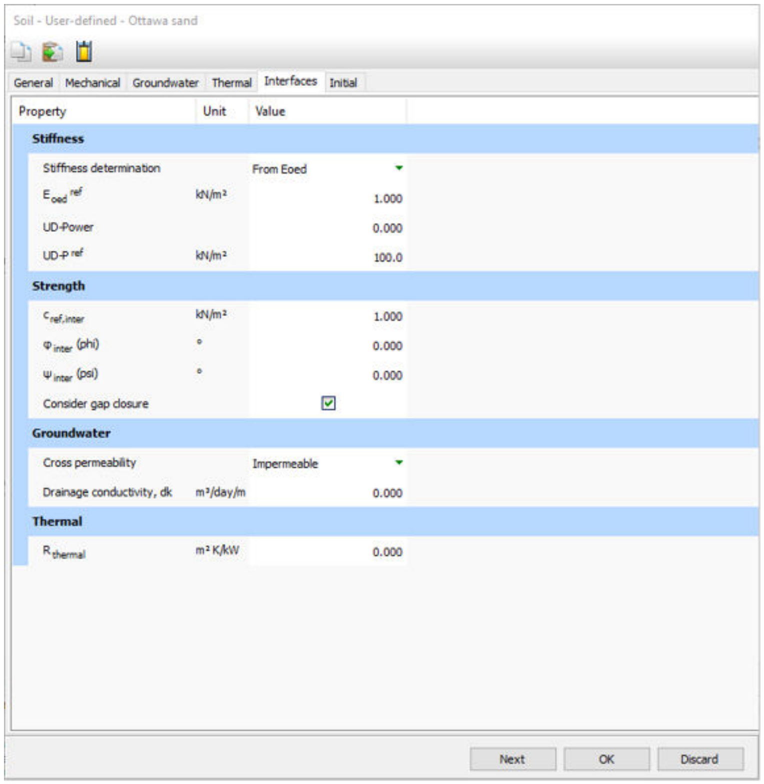


Figure 15: Interfaces Tab sheet

12. Click *OK* to finish the creation of the material set Ottawa sand. The *Material sets* window will look like this:

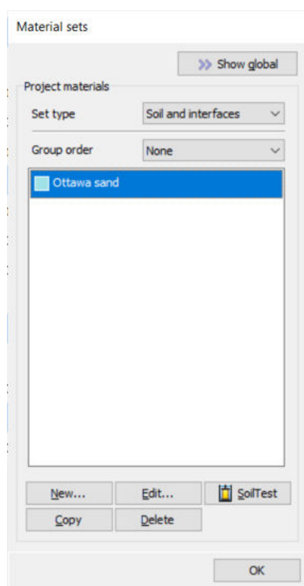


Figure 16: Material sets window

13. Click on the Ottawa sand material set and afterwards click on the SoilTest button as shown in the following figure:

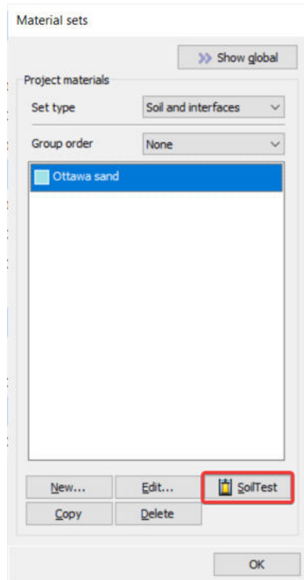


Figure 17: Ottawa sand Material set

The PLAXIS SoilTest facility will be opened.

14. In the PLAXIS SoilTest window choose the *CDSS* tab sheet.

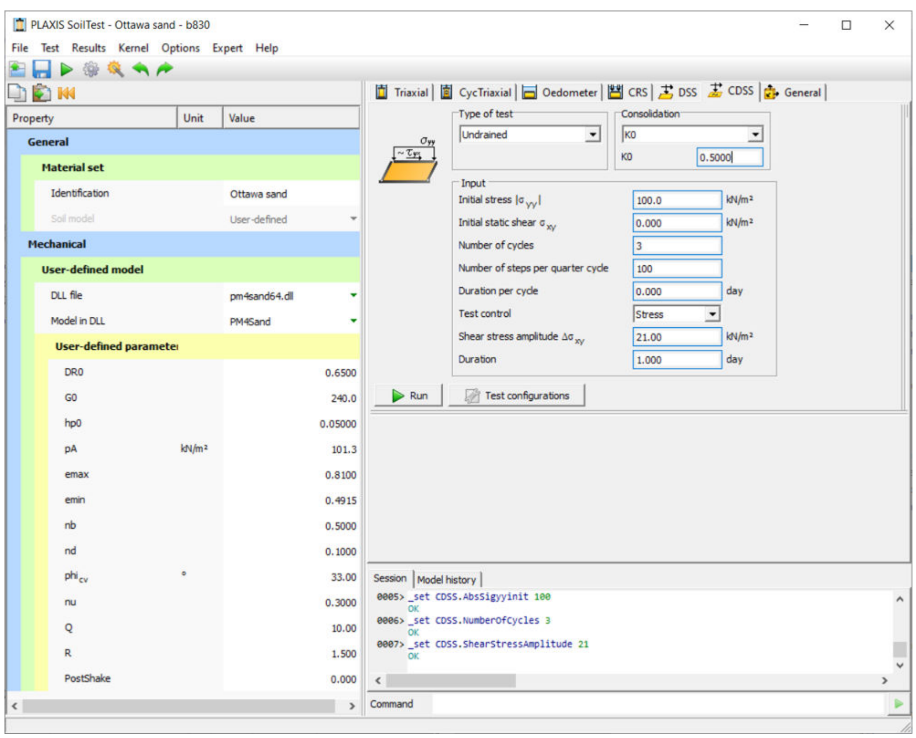


Figure 18: SoilTest Window

15. Enter the following test conditions in the fields:

Parameter	Value	Unit
Type of test	Undrained	[-]
Consolidation / K_0	0.5	[-]
Initial stress $\left\| \sigma'_{yy} \right\|$	100	kN/m^2
Initial static shear σ_{xy}	0	kN/m^2
Number of cycles	3	[-]
Number of steps per quarter cycle	100	[-]
Duration per cycle	Not relevant (0 day)	[-]
Test control	Stress	[-]
Shear stress amplitude $\Delta\ \sigma_{xy}$	21	kN/m^2
Duration	Not relevant (1 day)	[-]

Make sure that the K_0 value is set to 0.5, otherwise the liquefaction resistance will be higher than the target values from Figure 10 (on page 25).

16. Press the *Run* button. The simulation will run and plot the results when finished, as in the following figure:

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

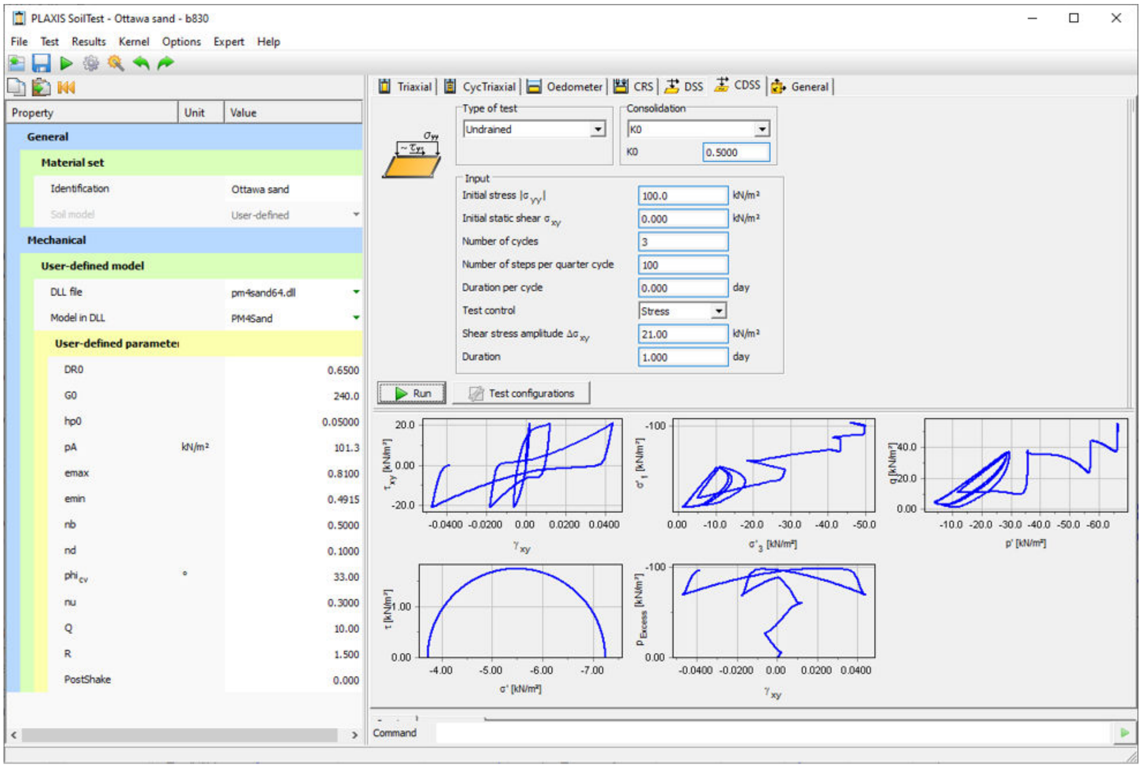


Figure 19: Result of SoilTest

17. Remove the non-relevant plots and keep only the $\tau_{xy} - \gamma_{xy}$ and $p_{Excess} - \gamma_{xy}$ plots. Do this by right clicking on them and selecting Hide chart.

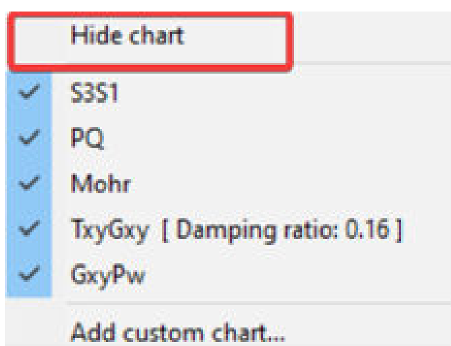


Figure 20: Hide chart

The SoilTest window will look like the following:

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

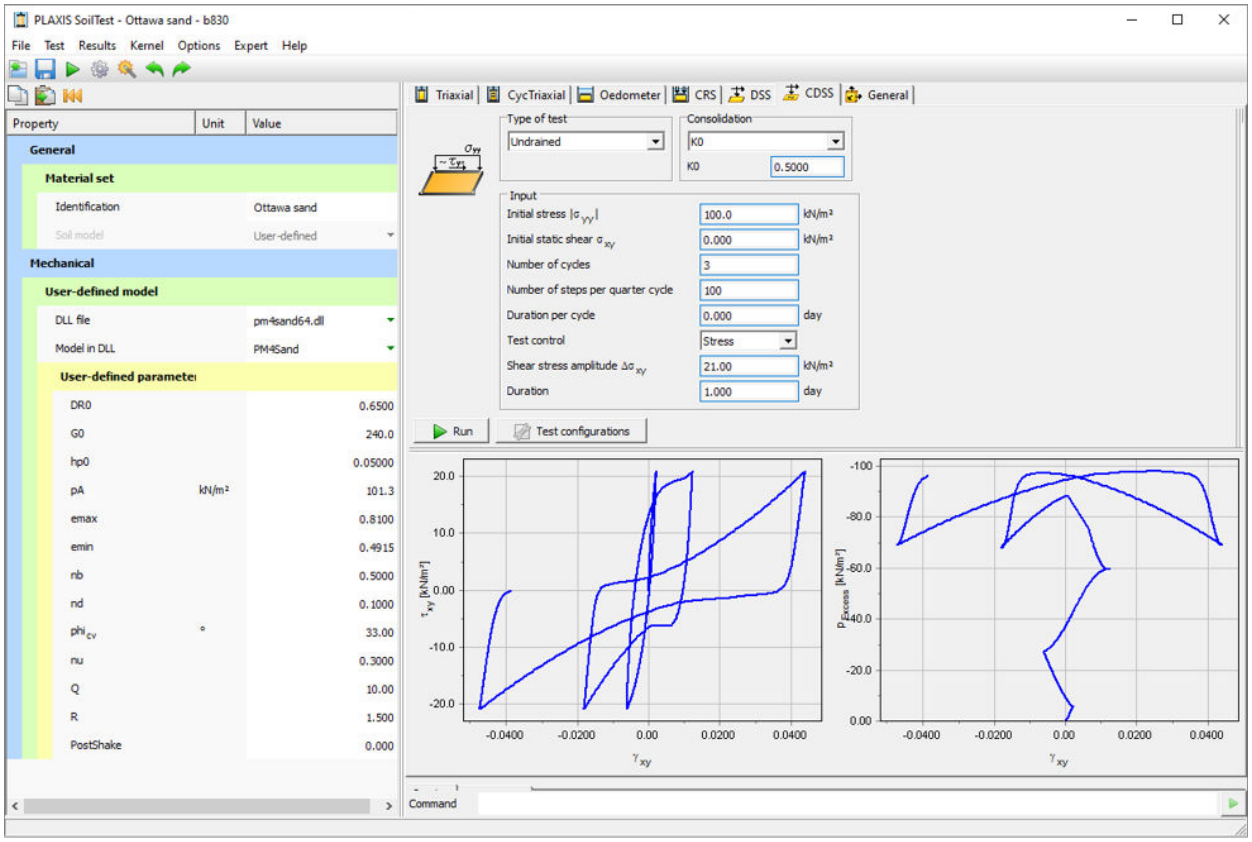


Figure 21: SoilTest Window

18. Add the $\tau_{xy} - \sigma'_{yy}$ plot by right clicking on the plot area and selecting Add custom chart... as:

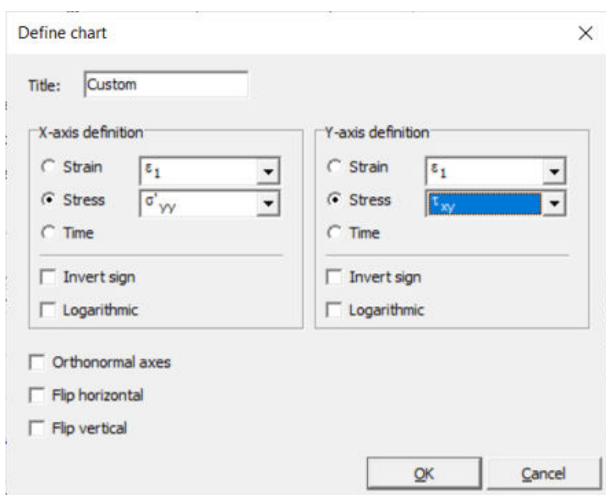


Figure 22: Adding a chart

Now the three characteristic plots are present in the window:

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

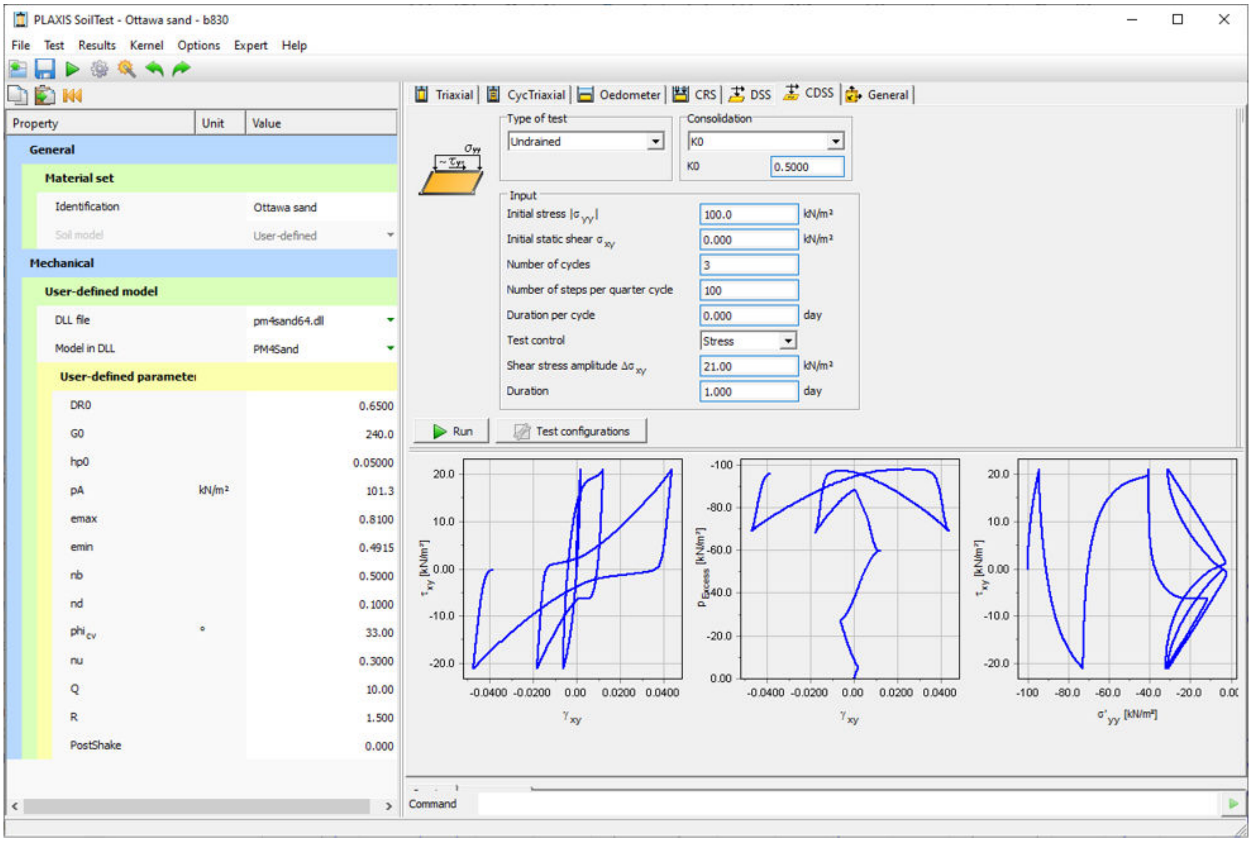


Figure 23: SoilTest window showing characteristic plots

From the $\tau_{xy} - \gamma_{xy}$ plot below it can be seen that the liquefaction triggering condition $|\gamma_{xy}| > 3\%$ is reached at approximately 2.5 cycles.

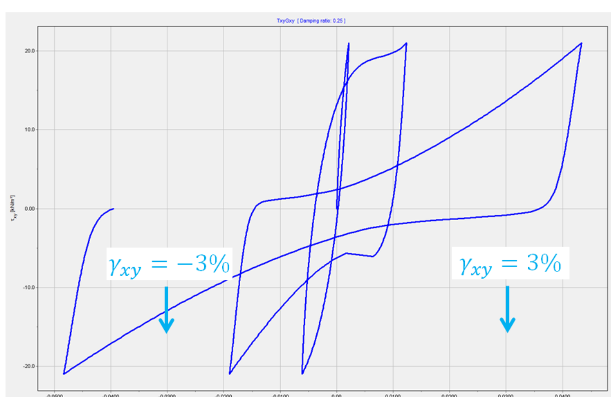


Figure 24: Plot showing Liquefaction at 2.5 cycles

Therefore, the first point in Table 4 (on page 26) can be confirmed to have N=2.5 cycles to liquefaction:

Exercise 1 : Simulation of CSR-N Response with the PM4Sand Model

The aim of the excercise:

Point	CSR	Shear stress amplitude $\Delta\ \sigma_{xy}$	Number of uniform cycles N at γ=3%
1	0.21	21	2.5

The point 2 from Table 4 (on page 26) is calculated by using $\Delta\ \sigma_{xy} = 17\ kN\ /\ m^2$ and Number of cycles at 4 and 5. If 4 cycles are chosen the following result is shown:

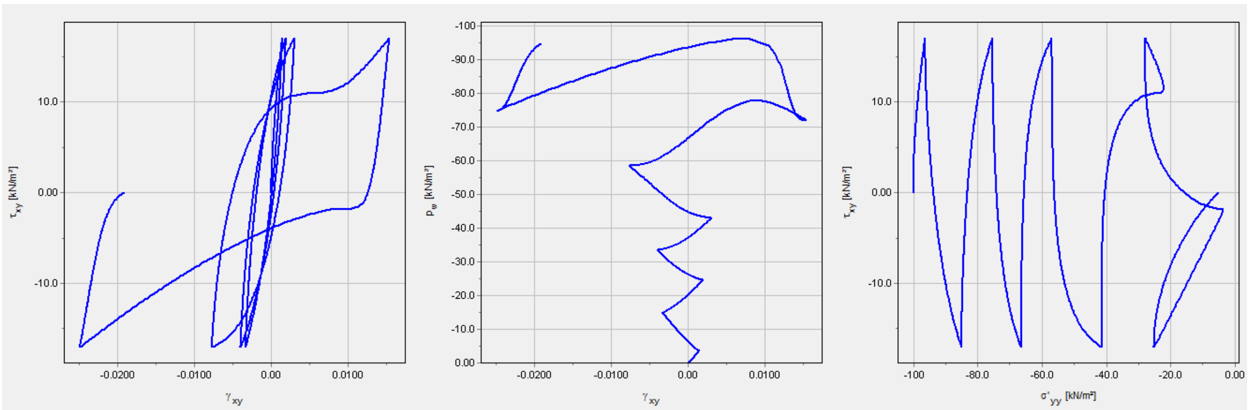


Figure 25: Results for 4 cycles

in which case liquefaction has not been reached yet, since $|\ \gamma_{xy}\ | < 3\ \%$. If 5 cycles are run, the following result is obtained:

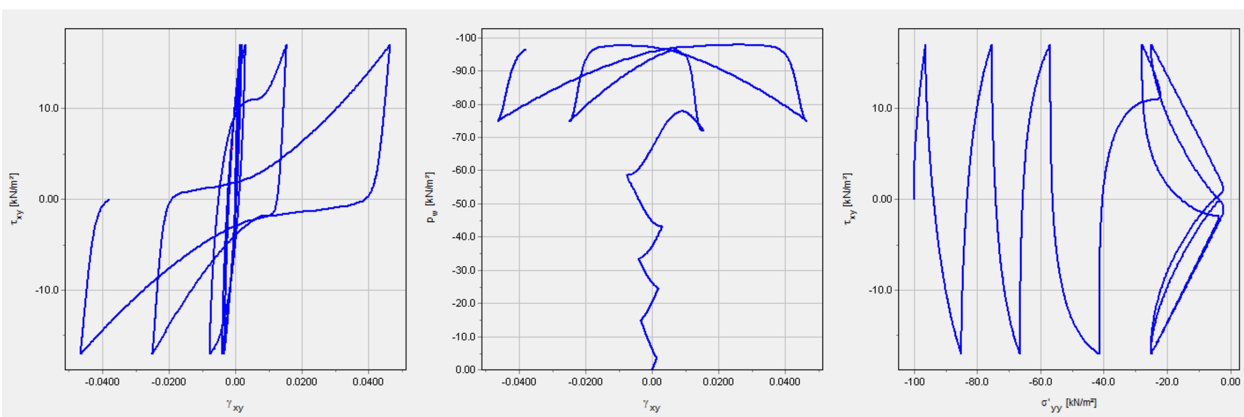


Figure 26: Results for 5 cycles

Therefore the result for point 2 is approximately 4.5 uniform cycles.

Point	CSR[-]	Shear Stress amplitude $\Delta\ \sigma_{xy}\ [kPa]$	Number of uniform cycles N at $\gamma = 3\ \%$
2	0.17	17	4.5

Point 3: The point 3 from Table 4 (on page 26) can be calculated by using $\Delta\ \sigma_{xy}\ = 13\ kN\ /\ m^2$ and Number of cycles 10 and 11. If 10 cycles are chosen, the following result is obtained:

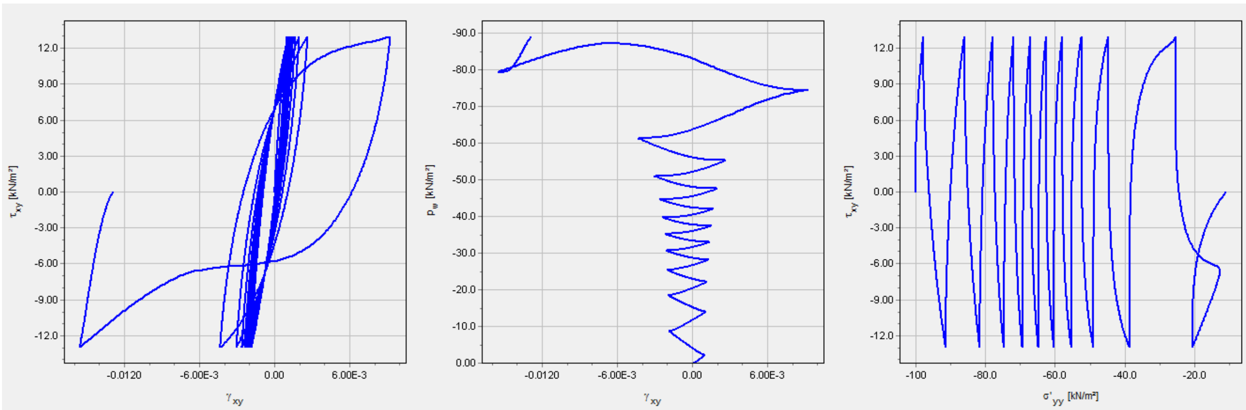


Figure 27: Result of point 3 for 10 cycles

While for 11 cycles the result is:

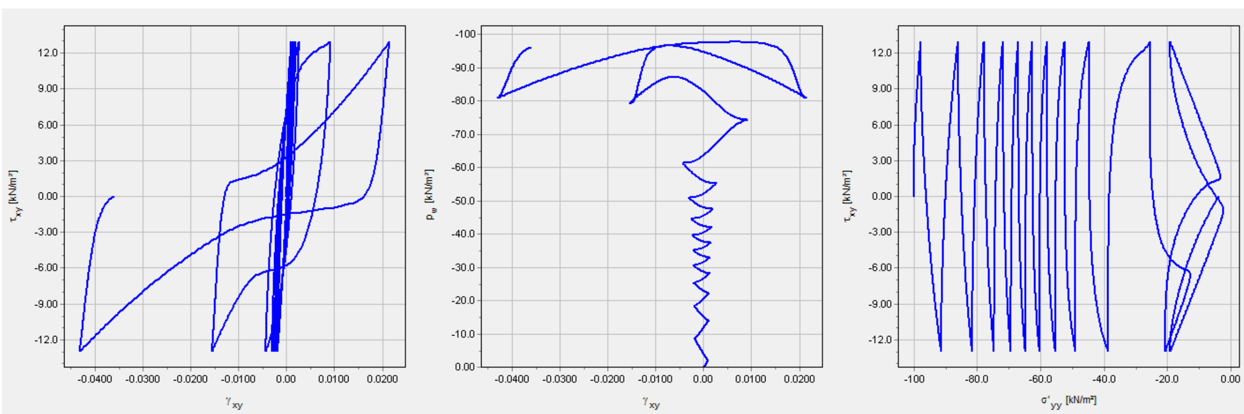


Figure 28: Result of point 3 for 11 cycles

Therefore the result for the point 3 is N=11:

Point	CSR[-]	Shear Stress amplitude $\Delta\ \sigma_{xy}\ [kPa]$	Number of uniform cycles N at $\gamma\ = 3\ \%$
3	0.13	13	11

Point 4:The point 4 from Table 4 (on page 26) can be calculated by using $\Delta\ \sigma_{xy}\ = 10\ kN\ /\ m^2$ and Number of cycles at 30 and 31. If 30 cycles are run, the following result is obtained:

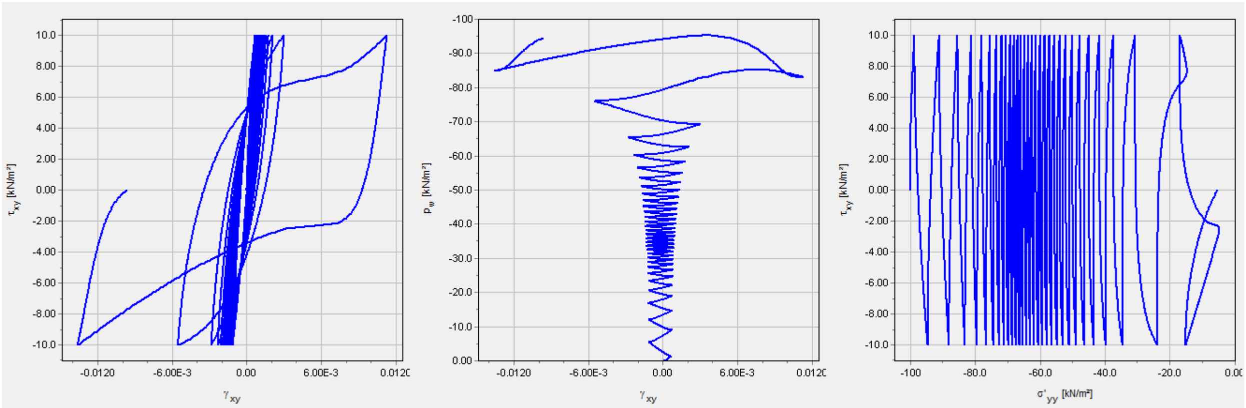


Figure 29: Result of point 4 for 30 cycles

while if 31 cycles are run, the result is:

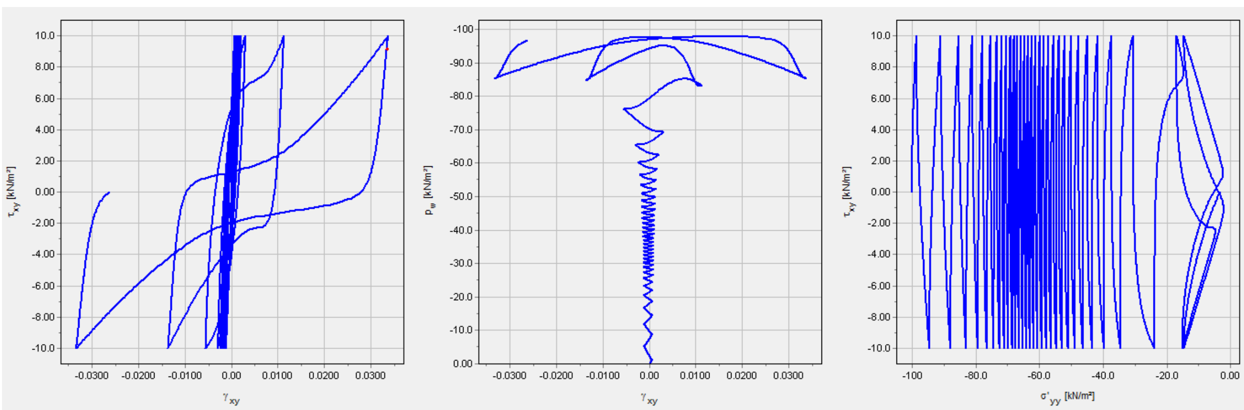


Figure 30: Result of point 4 for 31 cycles

Therefore the final result for the point 4 is N=30.5:

Point	**CSR[-]**	**Shear Stress amplitude** $\Delta\ \sigma_{xy}\ [kPa]$	**Number of uniform cycles N at** $\gamma\ =\ 3\ \%$
4	0.10	10	30.5

Point 5: The point 5 from Table 4 (on page 26) can be calculated by using $\Delta\ \sigma_{xy}\ = 8.1kN\ /\ m^{2}$ and Number of cycles from 74 to 75. If 74 cycles are chosen, the following result is obtained:

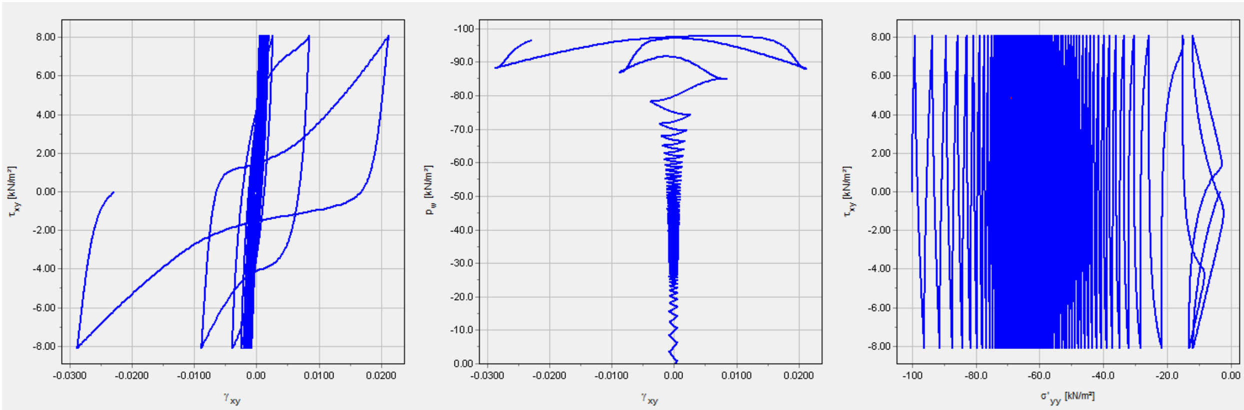


Figure 31: Result of point 5 for 74 cycles

And if 75 cycles are chosen, the result is:

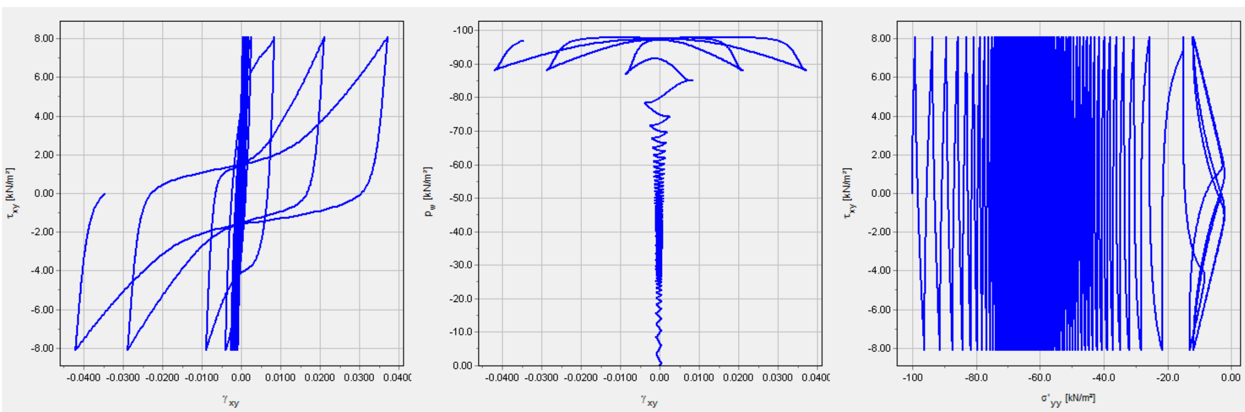


Figure 32: Result of point 5 for 75 cycles

Therefore the final result for the point 5 is approximately N=74.5:

Point	**CSR[-]**	**Shear Stress amplitude** $\Delta\ \sigma_{xy}\ [kPa]$	**Number of uniform cycles N at** $\gamma = 3\ \%$
5	0.081	8.1	74.5

Now the simulations will be performed with the higher contraction rate parameter h_{p0}. The effect of higher h_{p0} is in increasing the cyclic strength of the material. Therefore, the CSR-N curve in Figure 10 (on page 25) for the material with $h_{p0} = 0.2$ lies above the CRS-N for the material with $h_{p0} = 0.05$.

19. Insert the value 0.2 next to the h_{p0} parameter field (see figure below).

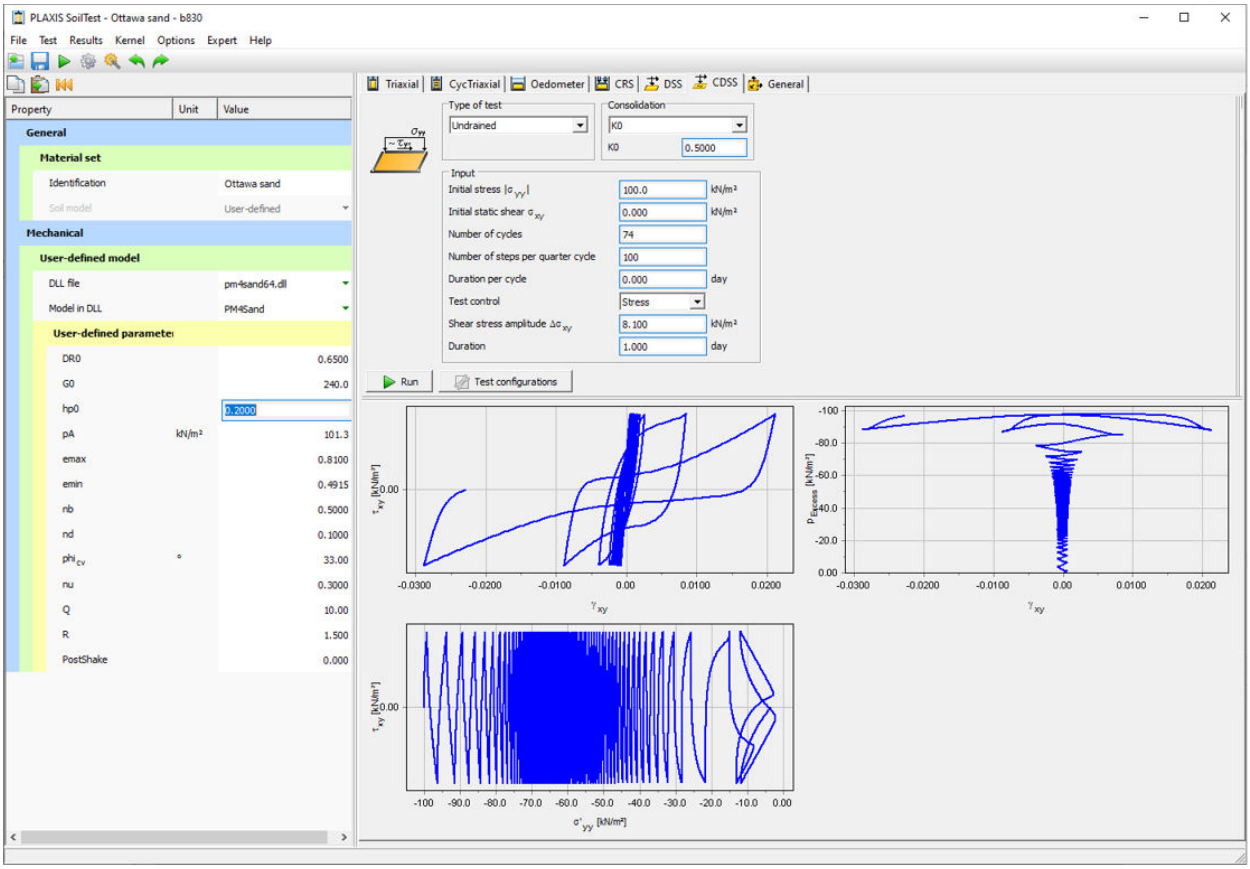


Figure 33: SoilTest Window with $h_{p0} = 0.2$

In the following pages, all the five points from the CSR-N curve for Case B will be simulated.

Point 1:

The point 1 from Table 5 (on page 26) can be calculated by using $\Delta \sigma_{xy} = 29\ kN / m^2$ and Number of cycles at 2 and 3.

If 2 cycles are simulated, the following result is obtained:

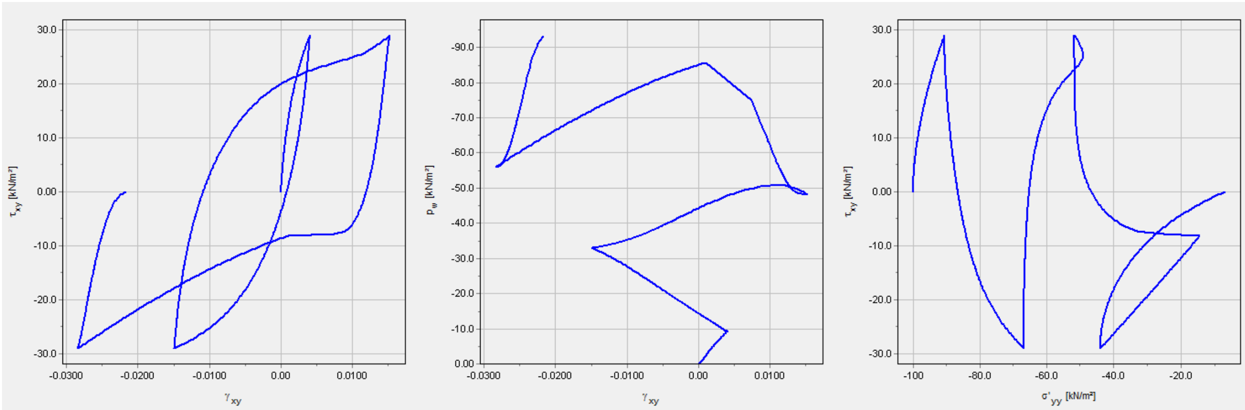


Figure 34: Result of point 1 for 2 cycles

while in the case of 3 cycles, the following result is obtained:

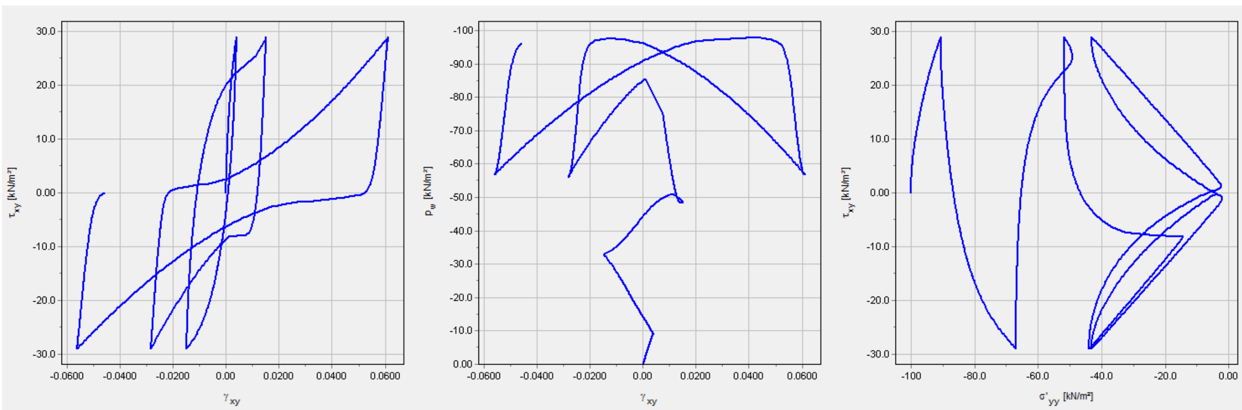


Figure 35: Result of point 1 for 3 cycles

The final result for the point 1 is approximately N=2.5:

Point	**CSR[-]**	**Shear Stress amplitude** $\Delta\ \sigma_{xy}\ [kPa]$	**Number of uniform cycles N at** $\gamma = 3\ \%$
1	0.29	29	2.5

The point 2 from Table 5 (on page 26) can be calculated by using $\Delta\ \sigma_{xy} = 23\ kN\ /\ m^2$ and Number of cycles at 4 and 5.The result for 4 cycles is:

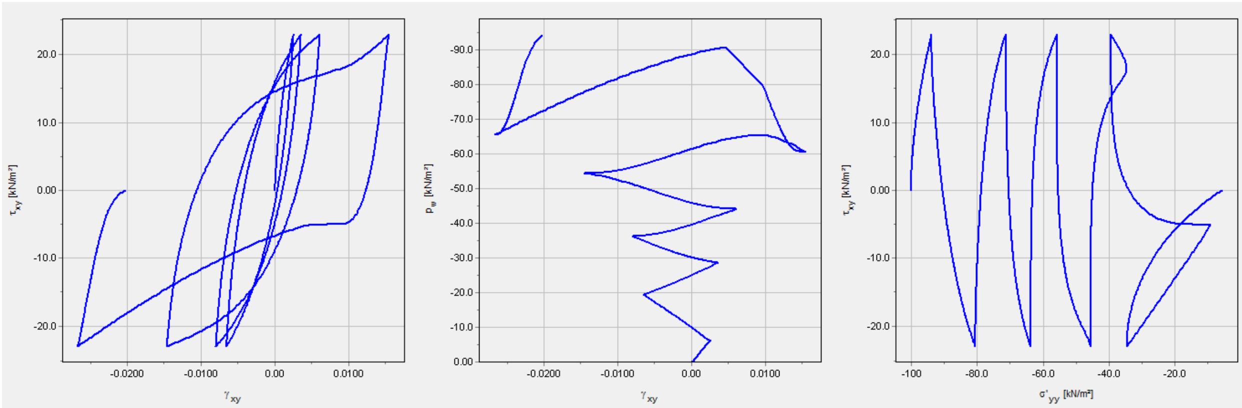


Figure 36: Result of point 2 for 4 cycles

and the result for 5 cycles is:

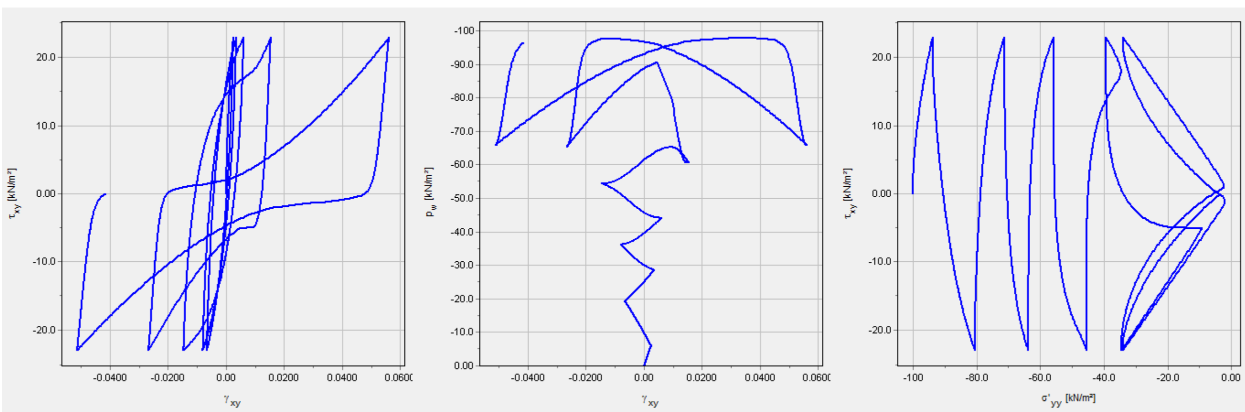


Figure 37: Result of point 2 for 5 cycles

Therefore, the final result for the point 2 is approximately N=4.5:

Point	**CSR[-]**	**Shear Stress amplitude** $\Delta\ \sigma_{xy}\ [kPa]$	**Number of uniform cycles N at** $\gamma = 3\ \%$
2	0.23	23	4.5

The point 3 from Table 5 (on page 26) can be calculated by using $\Delta\ \sigma_{xy} = 18\ kN\ /\ m^2$ and Number of cycles at 9 and 10. The result for 9 cycles is:

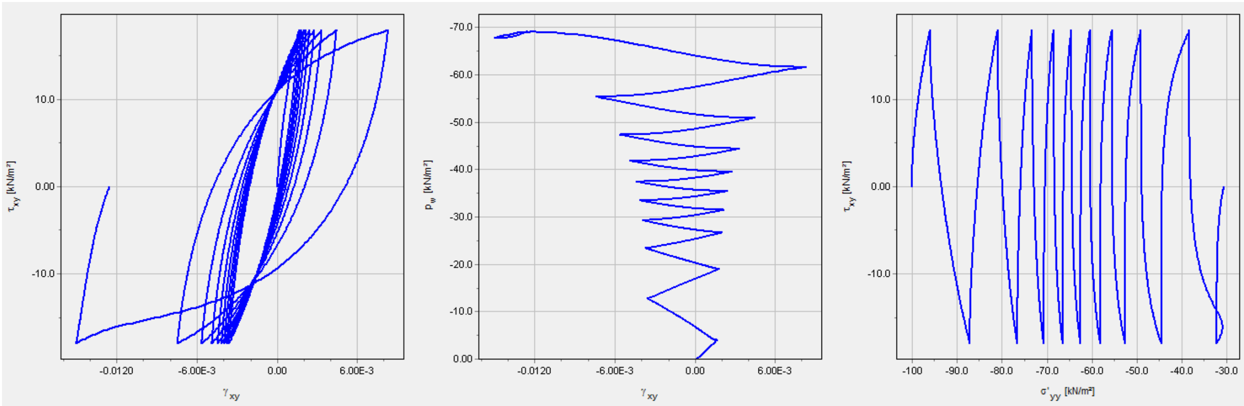


Figure 38: Result of point 3 for 9 cycles

And the result for 10 cycles is:

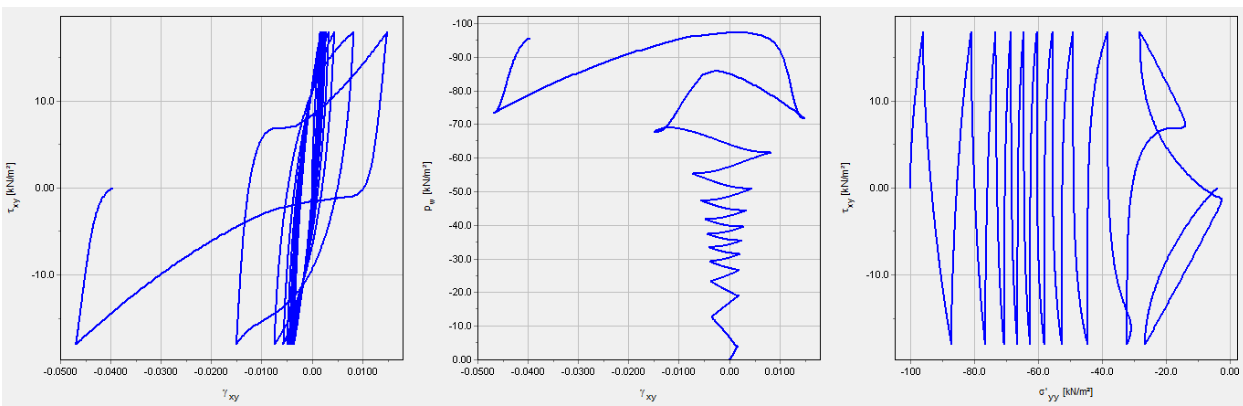


Figure 39: Result of point 3 for 10 cycles

The final result for the point 3 is N=10:

Point	**CSR[-]**	**Shear Stress amplitude $\Delta \sigma_{xy}$ $[kPa]$**	**Number of uniform cycles N at $\gamma = 3\ \%$**
3	0.18	18	10

The point 4 from Table 5 (on page 26) can be calculated by using $\Delta \sigma_{xy} = 14\ kN / m^2$ and Number of cycles at 27 and 28. The result for 27 cycles is:

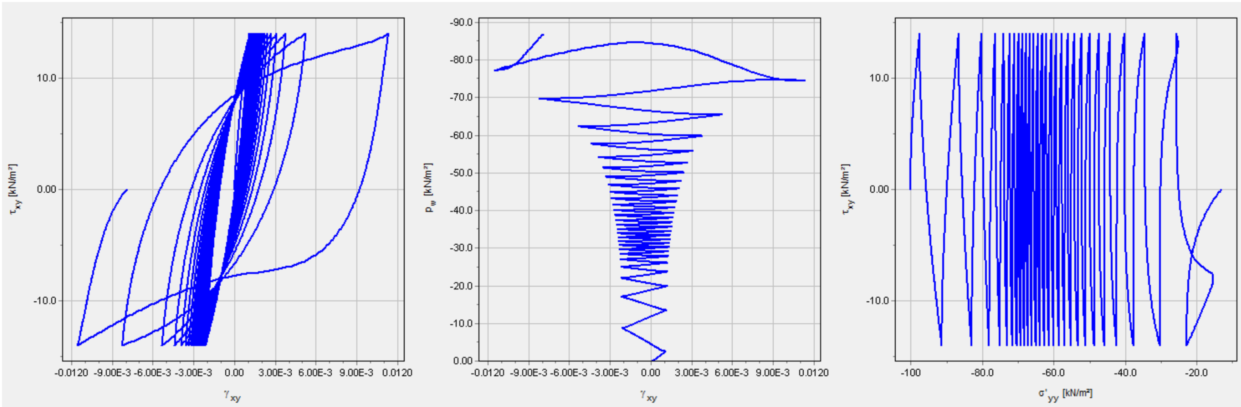


Figure 40: Result of point 4 for 27 cycles

And the result for 28 cycles is:

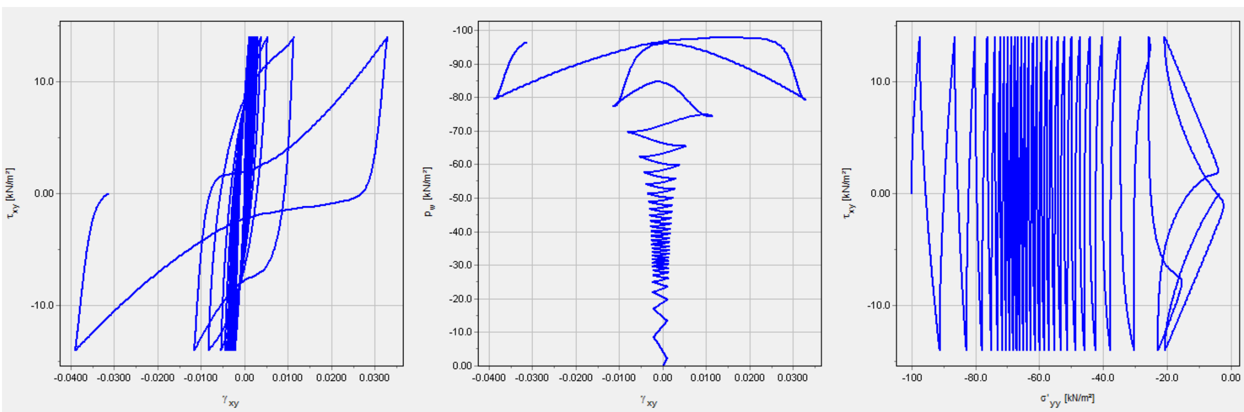


Figure 41: Result of point 4 for 28 cycles

Therefore, the final result for the point 4 is N=27.5:

Point	**CSR[-]**	**Shear Stress amplitude $\Delta \sigma_{xy} [kPa]$**	**Number of uniform cycles N at $\gamma = 3$ %**
4	0.14	14	27.5

The final point from Table 5 (on page 26) can be calculated by using $\Delta \sigma_{xy} = 11 \ kN / m^2$ and Number of cycles at 76 and 77. The result for 76 cycles is:

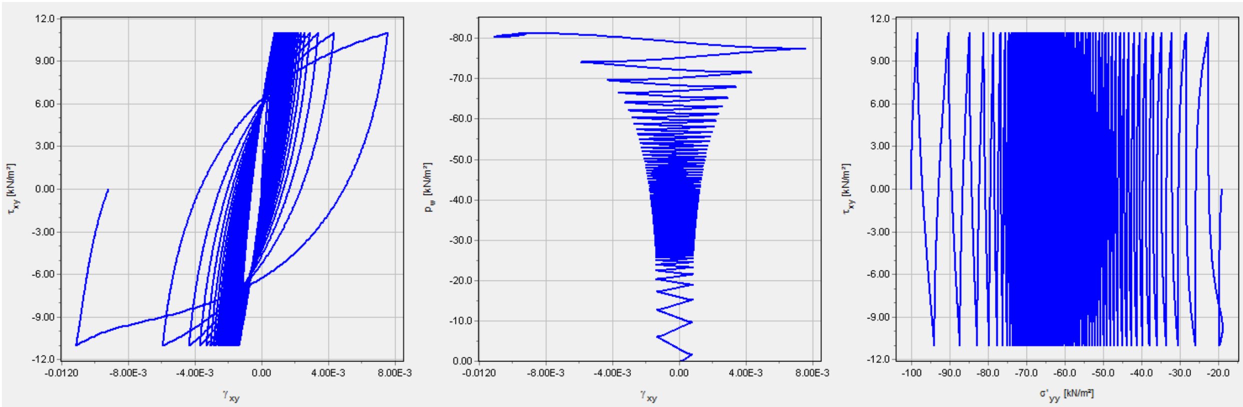


Figure 42: Result of point 5 for 76 cycles

and for 77 cycles is:

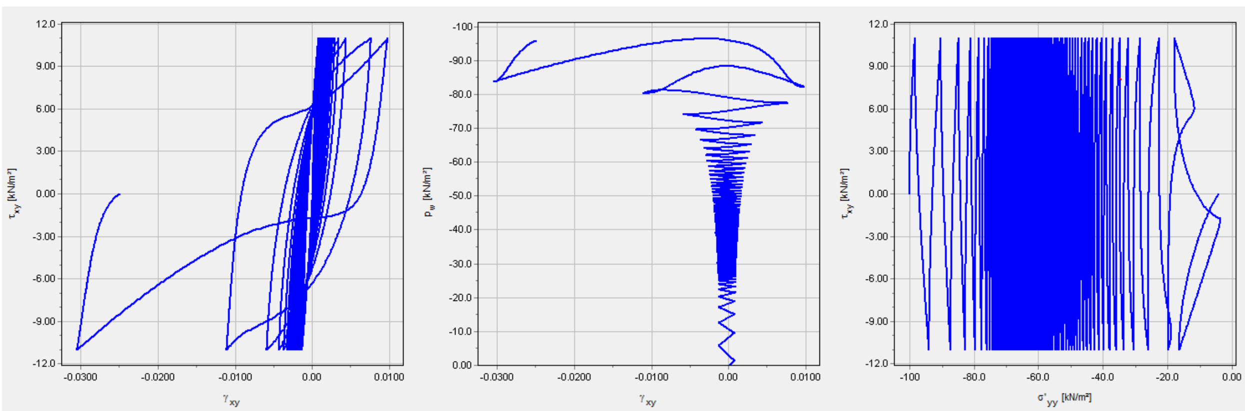


Figure 43: Result of point 5 for 77 cycles

Therefore, the final result for the point 5 is N=77:

Point	**CSR[-]**	**Shear Stress amplitude** $\Delta\ \sigma_{xy}\ [kPa]$	**Number of uniform cycles N at** $\gamma = 3\ \%$
5	0.11	11	77

4 Exercise 2: 1D Wave propagation analysis with the PM4Sand Model

4.1 The aim of exercise

The exercise aims at performing a dynamic numerical analysis using PLAXIS to predict the onset of liquefaction in a sandy layer modelled with the PM4Sand model.

The soil stratigraphy (Figure 44 (on page 46)) consists of an overconsolidated clay layer of medium compressibility that extends from the ground surface to 5m depth, followed by 10m of the sand layer with $D_R = 55\ \%$ and 25m of clay, until the bedrock is reached. The water table is assumed to be coincident with the ground surface level.

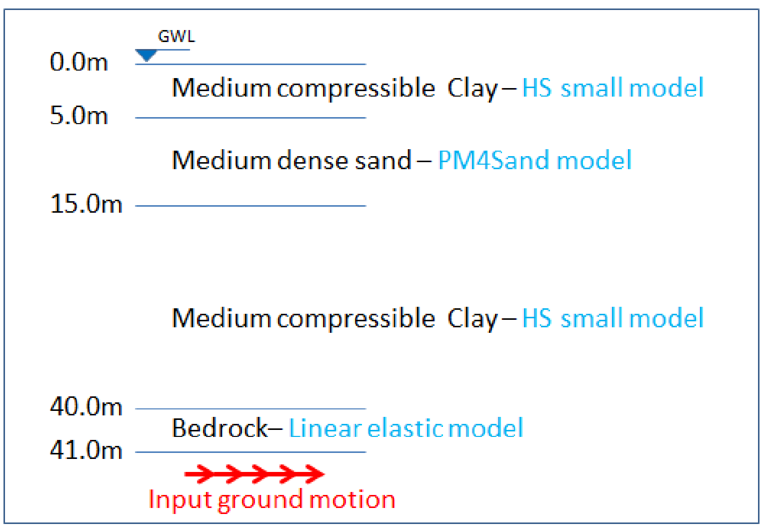


Figure 44: The soil stratigraphy used for the exercise

The clay material is modelled using the HS small model, while the sand material is modelled using the PM4Sand model. The bedrock layer of 1m thickness is modelled with the linear elastic material. The values of the material parameters for HS small model are given in Table 6 (on page 47) and material parameters for the bedrock in Table 7 (on page 48).

Table 6: HS small parameters for clay

Parameter	Symbol	Value	Unit
General			
Drainage type	-	Undrained (A)	-
Unsaturated unit weight	γ_{unsat}	19	kN/m^3
Saturated unit weight	γ_{sat}	21	kN/m^3
Rayleigh damping coefficient	α	0.096	-
Rayleigh damping coefficient	β	0.00079	-
Secant stiffness in standard drained TX test	E_{50}^{ref}	9000	kN/m^2
Tangent stiffness for primary oedometer loading	E_{oed}^{ref}	9000	kN/m^2
Unloading-reloading stiffness	E_{ur}^{ref}	27000	kN/m^2
Poisson's ratio	v_{ur}	0.2	-
Power for stress-level dependency of stiffness	m	1	-
Reference stress	p_{ref}	100	kN/m^2
Shear modulus at very small strains	G_0^{ref}	60000	kN/m^2
Shear strain at which G_s = $0.722G_0$	$\gamma_{0.7}$	0.0007	-
Cohesion	c'_{ref}	30	kN/m^2
Friction angle	φ'	26	º
Dilatancy angle	ψ	0	º
Cohesion increment	c_{inc}'	0	$kN/m^2/m$
Reference coordinate	y_{ref}	0	m

Parameter	Symbol	Value	Unit
General			
Tension cut-off	-	True	-
Tensile strength	-	0	kN/m^2
Normally consolidated earth pressure at rest	K_0^{nc}	0.5616	-
Failure ratio	R_f	0.9	-

Initial			
K_0 determination	-	Automatic	
Over-consolidation ratio	OCR	2	-

Table 7: Linear elastic parameters for bedrock

Parameter	Symbol	Value	Unit
Drainage type	-	Drained	-
Unsaturated unit weight	γ_{unsat}	22	kN/m^3
Saturated unit weight	γ_{sat}	22	kN/m^3
Young's modulus	E'_{ref}	$8x10^6$	kN/m^3
Poisson's ratio	ν (nu)	0.2	-

The values of the parameters for the PM4Sand model are given in Table 8 (on page 48). Only the primary parameters are given in Table 8 (on page 48), while the secondary have the default values given in Table 9 (on page 49).

Table 8: PM4Sand primary parameters for sand

Parameter	Symbol	Value	Unit
Drainage type	-	Undrained(A)	-
Unsaturated unit weight	γ_{unsat}	14	kN/m^3
Saturated unit weight	γ_{sat}	18	kN/m^3

Parameter	Symbol	Value	Unit
Rayleigh damping coefficient	α	0.096	kN/m^3
Rayleigh damping coefficient	β	0.00079	-
Initial relative density (D_{R0})	DR0	0.55	-
Shear modulus coefficient (G_0)	G0	677	-
Contraction rate (h_{p0})	hp0	0.40	-

Table 9: PM4Sand secondary (default) parameters

Parameter	Symbol	Value	Unit
Atmospheric pressure (pA)	pA	101.3	kPa
Maximum void ratio (e_{max})	emax	0.8	-
Minimum void ratio (e_{min})	emin	0.5	-
Bounding surface position according to $\xi_R(n^b)$	nb	0.5	-
Dilatancy surface position according to $\xi_R(n^d)$	nd	0.1	-
Critical state friction angle (φ_{cv})	phi_{cv}	33	-
Poisson's ratio	nu	0.3	-
Critical state line parameter	Q	10	-
Critical state line parameter	R	1.5	-
Post-shaking reconsolidation	PostShake	0	-
Earth pressure coefficient	K_0	0.5	-

Step by step guide through the exercise:

1. Start PLAXIS Input.
2. Start a new project.
3. In the Project properties window, click Next to open the Model tab sheet.
4. Define the dimensions of the calculation domain as:

x_{min}	0.0m
x_{max}	2.0m
y_{min}	0.0m
y_{max}	41.0m

5. Make sure that Units are set to [m], [kN] and [day]. Then click *OK*. The Model tab sheet should look like this:

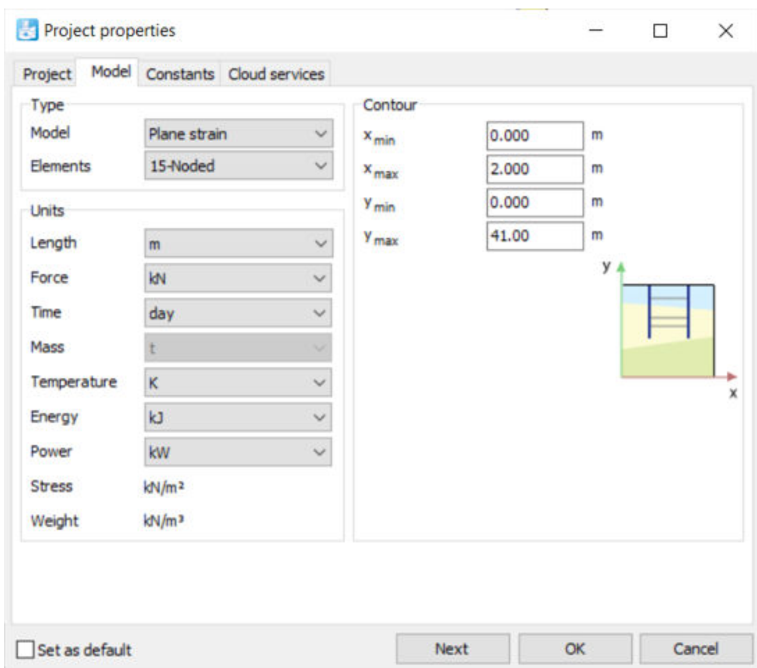


Figure 45: Model tab sheet

4.2 Creation of the materials

1. Open the *Material sets* window by clicking on *Show materials...* button.

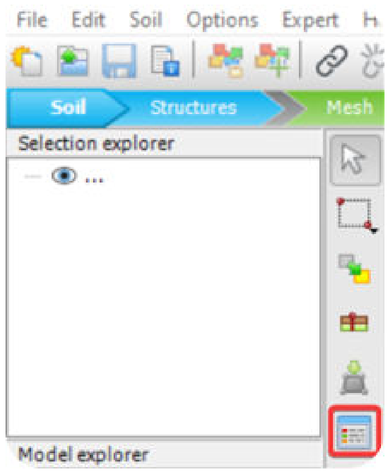


Figure 46: Material sets

2. Create the new material for the clay layers. Type *Identification* as *Clay*. Choose *Soil model HS small* and choose *Undrained (A)* drainage type. Enter the unsaturated and saturated unit weights $(\gamma_{unsat} = 19kN / m^3 \text{and} \;\; \gamma_{sat} = 21kN / m^3)$ and Rayleigh damping ratios (α=0.096, β=0.00079) after selecting *Direct* as *Input method*. The *General* tab sheet for *Clay* material should look like the following:

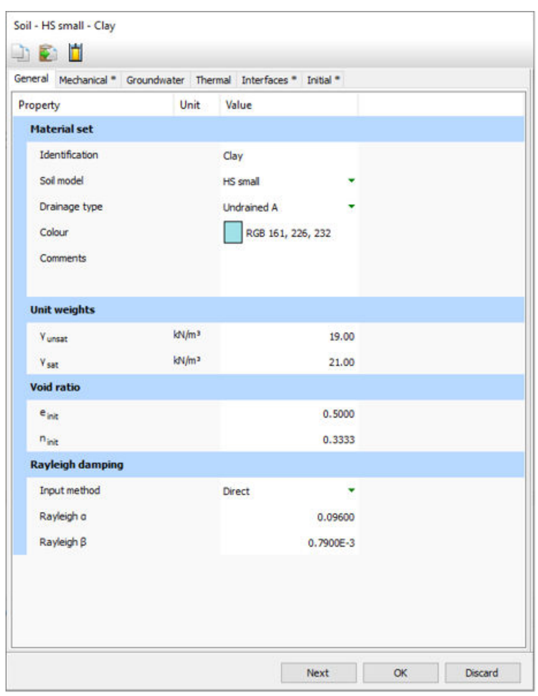


Figure 47: General tab sheet for clay

3. Go to the *Mechanical* tab sheet of HS small model and fill in the values. The filled tab sheet should look like the following:

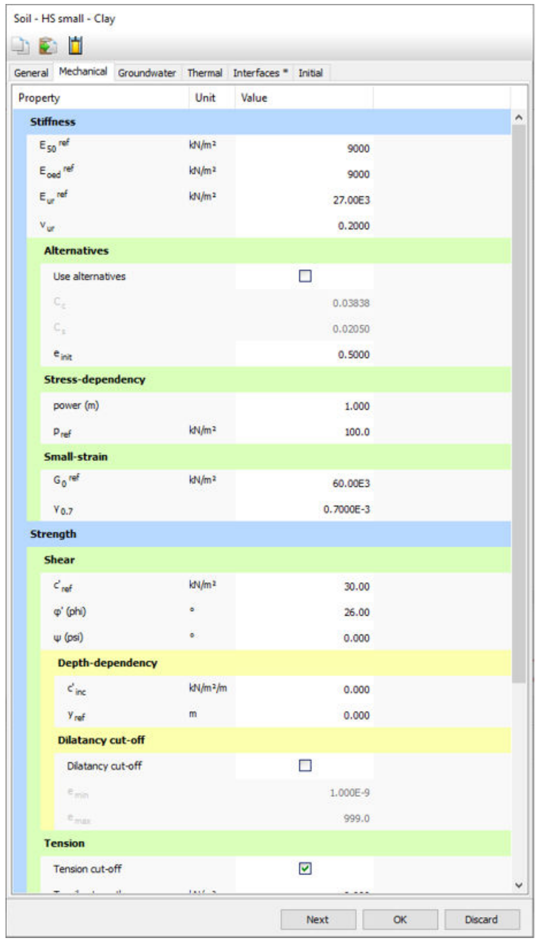


Figure 48: Mechanical tab sheet for clay

4. Go to the *Initial* tab sheet and insert the value of over-consolidation ratio *OCR* to be equal to 2 as in the following figure:

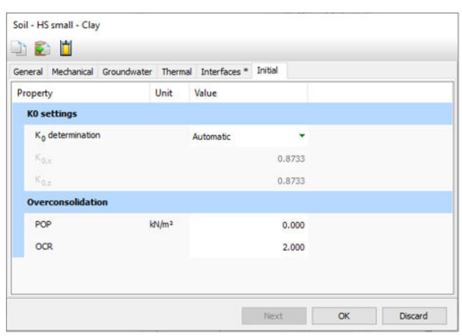


Figure 49: Initial tab sheet for clay

5. The creation of the *Clay* material is finished. Click *OK*.

6. Add a new material and name it *Bedrock*. Choose the *Linear elastic* soil model, drainage type as *Drained* and insert the unit weights $\left(\gamma_{unsat} = 22kN / m^3 and \ \ \gamma_{sat} = 22kN / m^3\right)$. The General tab should look like the following:

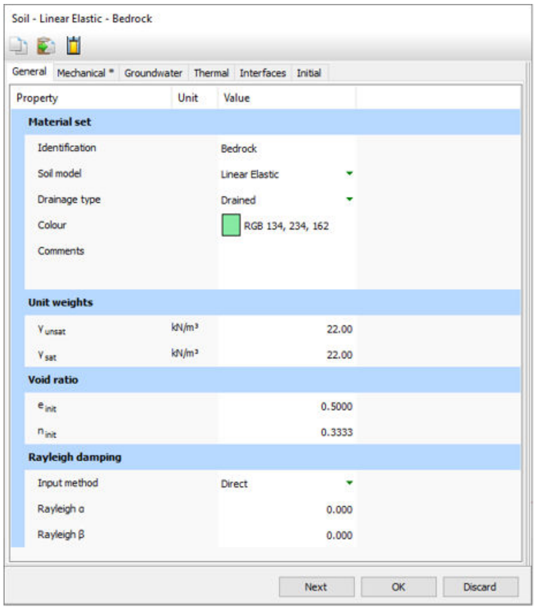


Figure 50: General tab sheet for bedrock

7. Go to the *Mechanical* tab sheet and E' and ν' values. The tab sheet should look like the following:

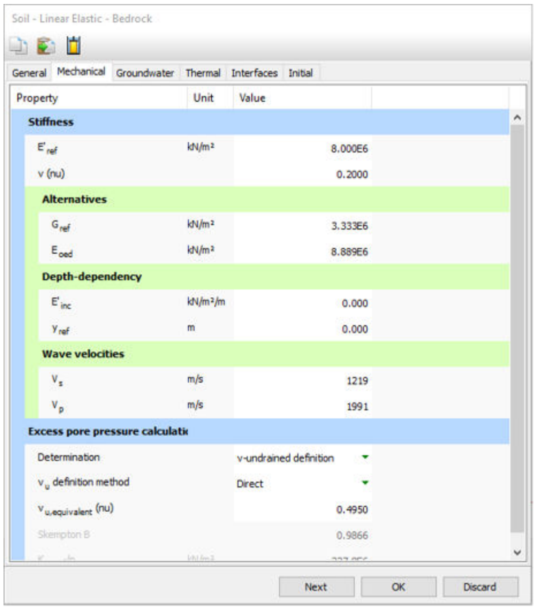


Figure 51: Mechanical tab sheet for bedrock

8. Press *OK* to finish the creation of the *Bedrock* material.

9. Create a new material and name it *Sand*. Choose the *User-defined* material model, choose *Undrained (A)* behaviour and fill in the unit weights $\left(\gamma_{unsat} = 14kN / m^3 and \ \ \gamma_{sat} = 18kN / m^3\right)$ and Rayleigh damping ratios (α=0.096,β= 0.00079) after selecting *Direct* as *Input Method*. The *General* tab sheet should look like the following:

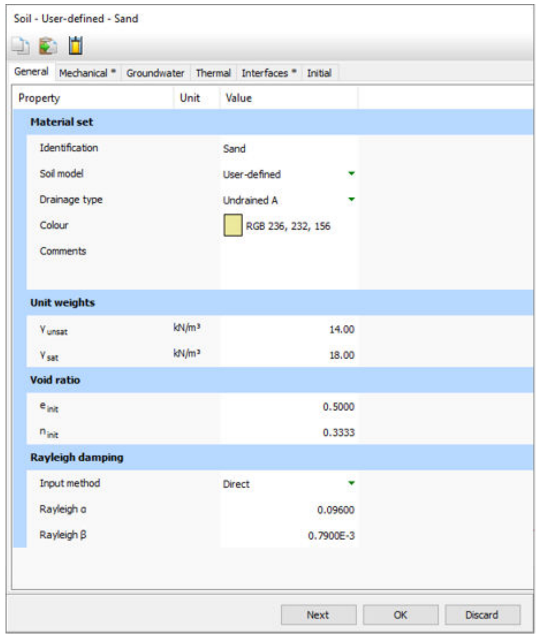


Figure 52: General tab sheet for sand

10. Go to the *Mechanical* tab sheet. Choose DLL file to be pm4sand64.dll and Model in DLL to PM4Sand. All the parameters of the PM4Sand model will appear. Fill them in. The *Mechanical* tab sheet should look like the following:

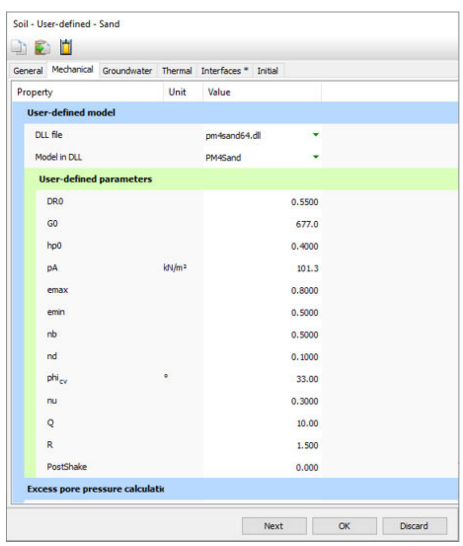


Figure 53: Mechanical tab sheet for sand

11. Go to *Interfaces* tab sheet and insert value 1 into $E_{oed}^{\ \ ref}$ and $c_{ref,inter}$ fields to make the model accepted as valid by PLAXIS, even though the interfaces will be not used for this model. The *Interfaces* tab should look like:

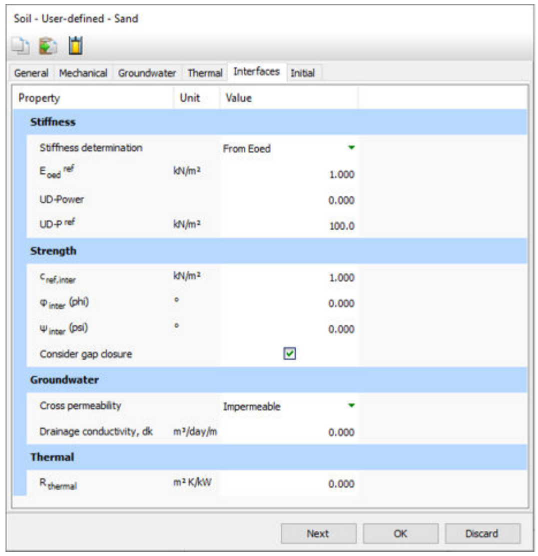


Figure 54: Interfaces tab sheet for sand

12. Go to the *Initial* tab and choose *Manual* K_0 determination and K_0 value to 0.5. The *Initial* tab should look like the following:

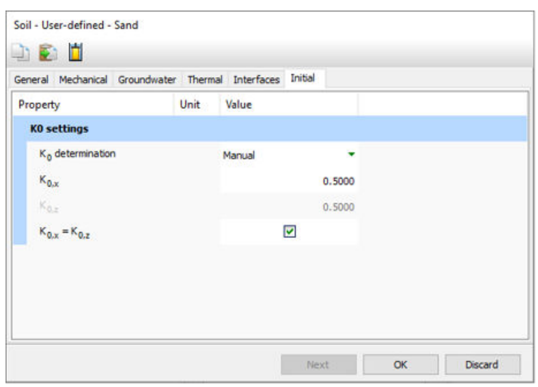


Figure 55: Initial tab sheet for sand

13. Click *OK* to finish the creation of the *Sand* material with PM4Sand model.

4.3 Creation of the layers

1. Create the borehole by clicking at the borehole icon in PLAXIS Input toolbar.

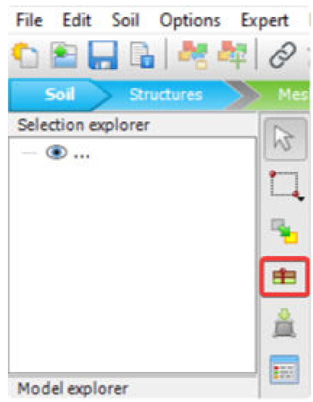
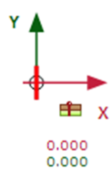


Figure 56: Create borehole

And click on the origin of the model:



2. The *Modify soil layers* window will appear. Define the stratigraphy as sketched in Figure 44 (on page 46). Click the *Add* button to add a layer. Assign the *Clay* material for the first layer and the *Top* and *Bottom* coordinates to 41.0m and 36.0m. The *Modify soil layers* window should look like this:

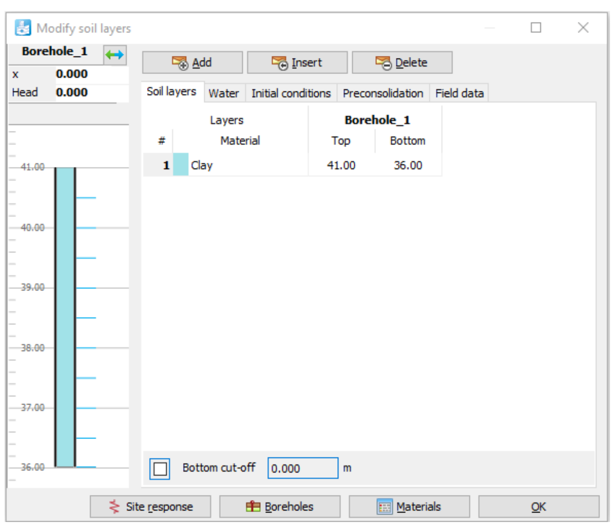


Figure 57: Modify soil layers

3. Add the next layer by clicking on the *Add* button. Assign the *Sand* material to it and the *Bottom* coordinate to 26.0m.
4. Add another layer. Assign the *Clay* material to it and the *Bottom* coordinate to 1.0m.
5. Add the last layer. Assign the *Bedrock* material to it and the *Bottom* coordinate to 0.0m.
6. Finally, assign the groundwater head to 41.0m in the Head field. This means that the groundwater level is assumed to be at the surface of the model. The complete information in the Modify soil layers window should look like the following:

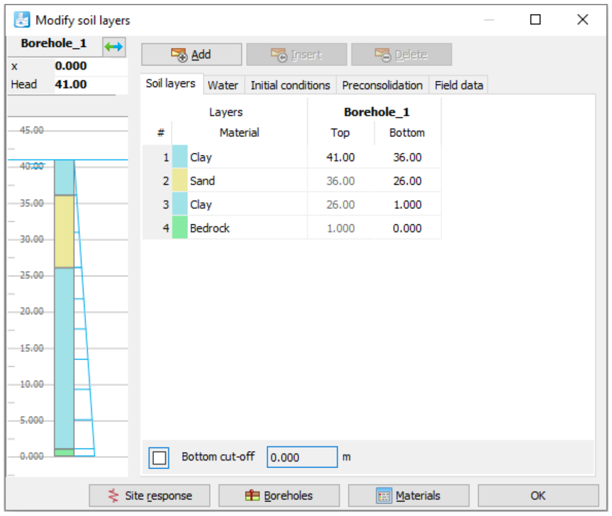


Figure 58: Modify soil layers with Material set

7. Click OK to finish the definition of layers.

4.4 Definition of the Earthquake Ground Motion

1. Go to the *Structures* mode in PLAXIS Input window by clicking on *Structures* button in the main toolbar as shown in the following figure:

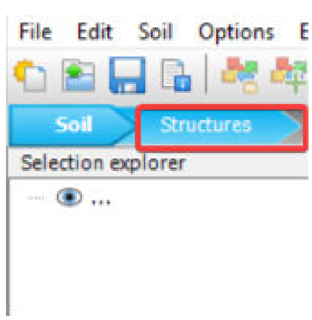


Figure 59: Structures mode

2. Zoom into the bottom of the model by using the mouse wheel (dragging and turning) and click on *Create line displacement* as shown below:

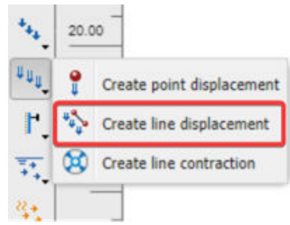


Figure 60: Create line displacement

3. Left click at the point (0.000, 0.000) and (2.000, 0.000), (and then right click to finish the insertion) to create line displacement at the bottom of the model as shown below:

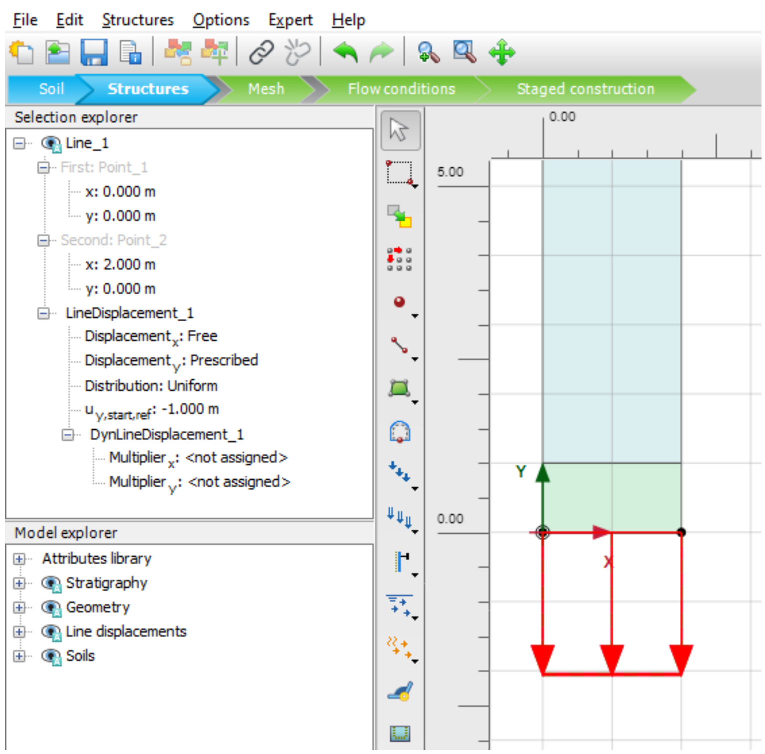


Figure 61: create line displacement

4. Click on the *Select* button to deactivate the line displacement creation, as shown below:

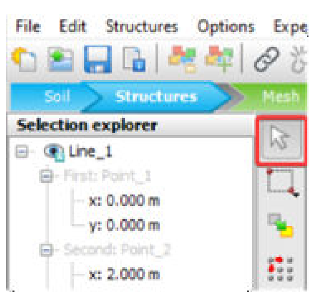


Figure 62: Select button

5. Click on the Line displacement at the bottom of the model and modify the definition of it in the left panel as follows:
    - $Displacement_x$: Prescribed
    - $Displacement_y$: Fixed
    - Distribution: Uniform
    - $U_{x,start,ref}$: 0.5000 m

    Considering that the boundary condition at the base of the model will be defined using a compliant base, the input signal has to be taken as half of the outcropping motion. Therefore the factor 0.5m is used as $U_{x,start,ref}$. The left panel should look like the following:

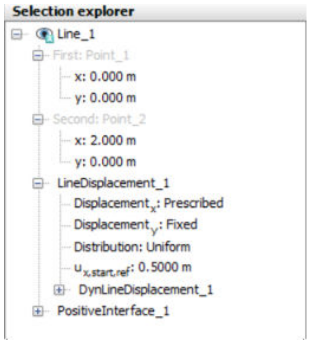


Figure 63: Selection explorer showing line displacement

6. Now the earthquake loading history will be assigned. Click next to the $Multiplier_x$ and at the + button, as shown below:

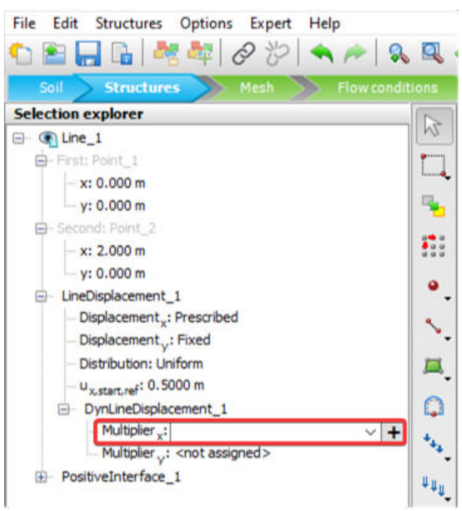


Figure 64: Earthquake loading history option with $Multiplier_x$

7. The Multipliers window will open.

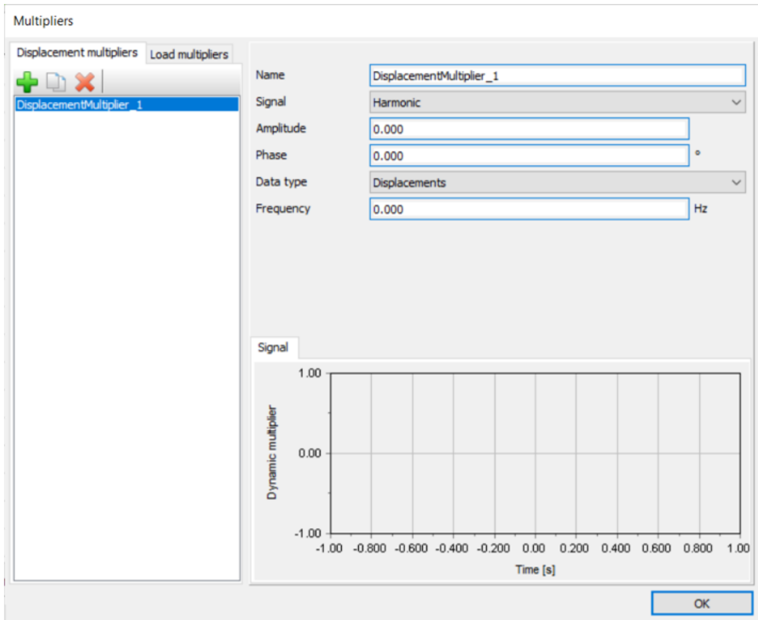


Figure 65: Multipliers window

8. Enter the name *Earthquake* and choose *Signal* as *Table*, *Data type* as *Accelerations* and *Drift correction* to *On*. The *Multipliers* window should look as shown below.

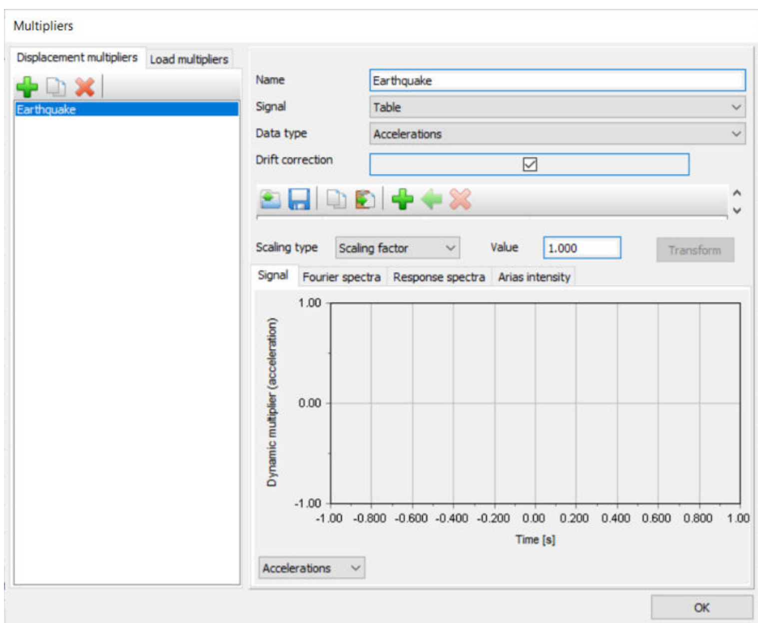


Figure 66: Multipliers window setting

9. Click on the button to import the input motion as follows.

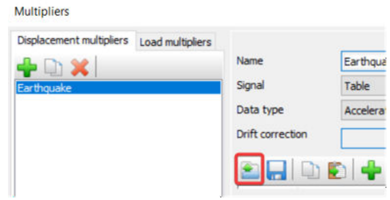
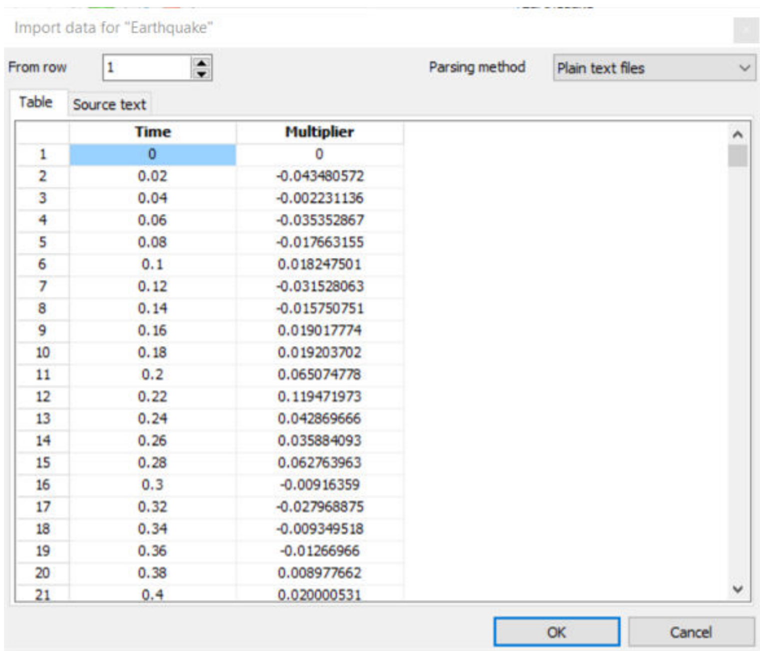


Figure 67: Button to import the input motion

10. The *Open* window will appear. Choose the file amax3_amax03g.txt with accelerogram data. (The file can be downloaded from [Bentley Communities]). The *Import data* for Earthquake window will appear with the imported 2 columns of data (i.e. *Time and Multiplier*). The *Import data* window is shown below.



	Time	Multiplier
1	0	0
2	0.02	-0.043480572
3	0.04	-0.002231136
4	0.06	-0.035352867
5	0.08	-0.017663155
6	0.1	0.018247501
7	0.12	-0.031528063
8	0.14	-0.015750751
9	0.16	0.019017774
10	0.18	0.019203702
11	0.2	0.065074778
12	0.22	0.119471973
13	0.24	0.042869666
14	0.26	0.035884093
15	0.28	0.062763963
16	0.3	-0.00916359
17	0.32	-0.027968875
18	0.34	-0.009349518
19	0.36	-0.01266966
20	0.38	0.008977662
21	0.4	0.020000531

Figure 68: Import data window

11. Press *OK* to close the *Import data* window. The imported data will be shown again in the *Multipliers* window together with the *Signal* plot. It can be seen that the maximum absolute value of the dynamic multiplier is 2.94 at 10.92s corresponding to approximately the peak horizontal acceleration of $3 m/s^2$ (i.e. 0.3g). A moment magnitude M_w equal to 6.9 characterises the provided accelerogram. The Multipliers window is shown below, together with the marked acceleration at 10.92s.

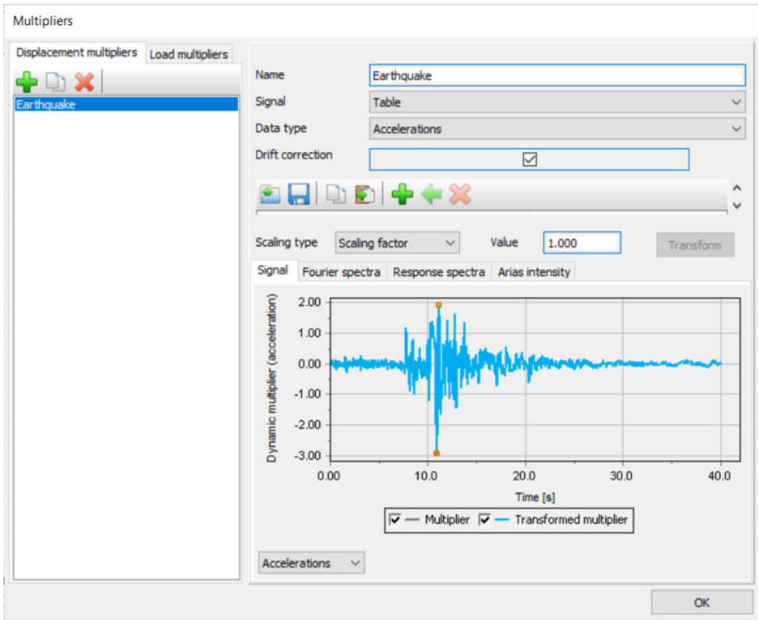


Figure 69: Multiplier window with imported data

12. Press *OK* to finish the definition of earthquake loading.

13. To model a compliant base, it is required to specify an interface at the bottom of the model. Therefore click at the *Create interface* button in the toolbar as shown in the following figure.

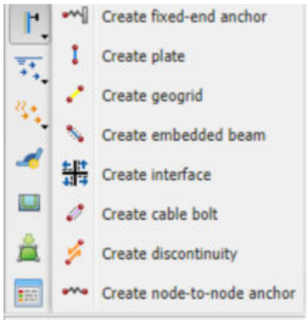


Figure 70: Create interface button

14. Create the interface, starting at the point (2.000, 0.000) towards the point (0.000, 0.000). In this way, the interface will be created with the properties of the bedrock side.

15. After finishing, the model at the bottom should look like in the following figure.

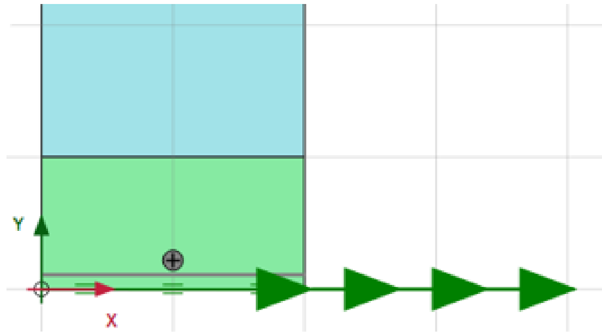


Figure 71: Model at the bottom showing interface

4.5 Creation of the finite element mesh

1. Click on the *Mesh* button of the main PLAXIS Input toolbar as shown in the figure:

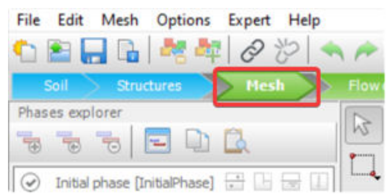


Figure 72: Mesh button

2. Click on the Generate mesh button as shown in the figure:

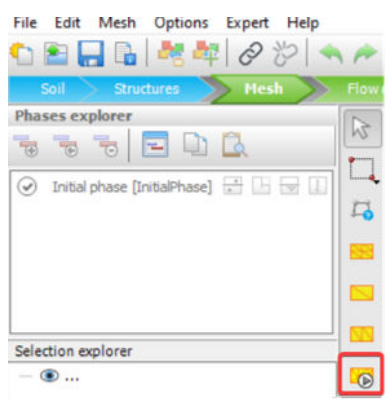


Figure 73: Generate mesh button

3. The *Mesh options* window will appear. Select the *Element distribution* option to *Very coarse* as shown below:

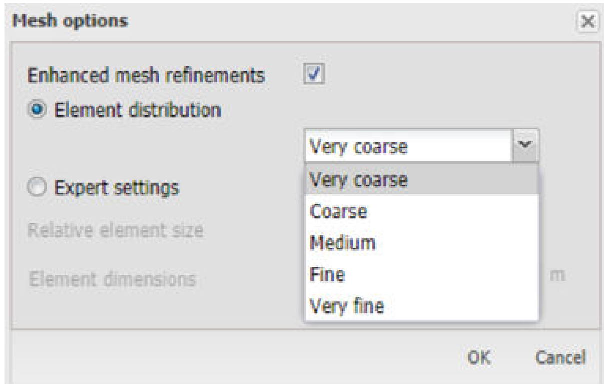


Figure 74: Element distribution option to Very coarse

4. Then click *OK* to generate the mesh. 37 finite elements are generated which is indicated in the bottom panel as follows:

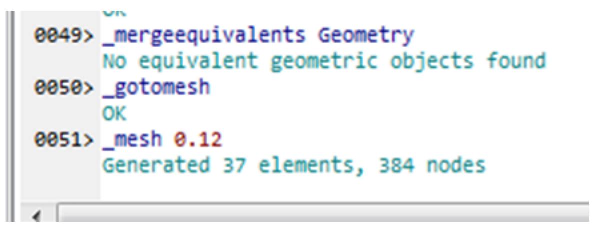


Figure 75: 37 finite elements generated mesh

5. The created finite element mesh can be viewed by clicking on the *View mesh* button as shown below:

Figure 76: View mesh button

6. The PLAXIS Output will open with the displayed generated mesh as shown in the following figure:

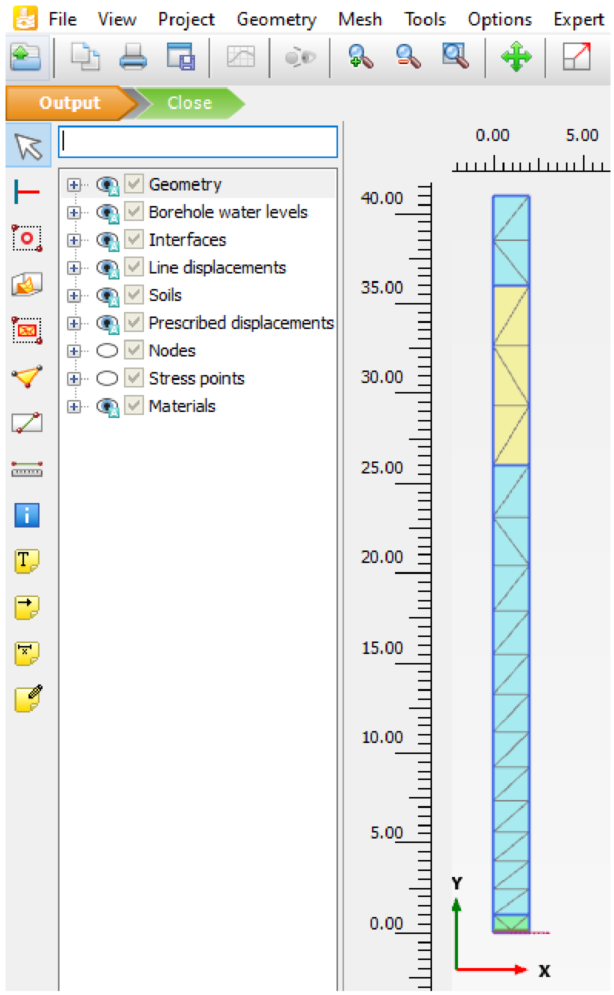


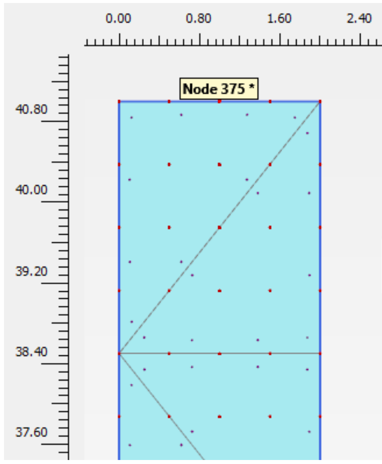
Figure 77: PLAXIS Output window

7. Click at the *Close* button in the main toolbar to get back to PLAXIS Input.
8. Lastly, we will define the Gauss points and nodes for plotting the curves. Click at the *Select points* for *curves* button as shown below:

Figure 78: Select points for curves button

9. The PLAXIS Output will open again with the connectivity plot and visible Gauss points and nodes. Zoom in with the mouse wheel and drag the model with the mouse wheel pressed. Click on the node (1.00, 41.00) at

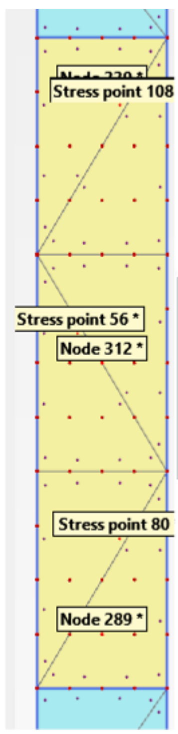
the top of the model (nodes are marked as red points). It will get marked with a node id (node 375) and is added to the database of points as shown below:



Name	X	Y	Selected	Data from
Node 375	1.00	41.00	Pre-calc	Soil_1_1

Figure 79: Selected node

10. Select three nodes in the sand layer at positions (1.00, 35.17), (1.00, 31.00) and (1.00, 26.83). Select also three Gauss points in the sand layer (Gauss points are marked as purple points), positioned at (1.27, 34.97), (0.62, 31.46) and (1.27, 28.30). The selected points in the sand layer are shown as follows, together with the table of all selected points:



Name	X	Y	Selected	Data from
Node 375	1.00	41.00	Pre-calc	Soil_1_1
Node 339	1.00	35.17	Pre-calc	Soil_2_1
Node 312	1.00	31.00	Pre-calc	Soil_2_1
Node 289	1.00	26.83	Pre-calc	Soil_2_1
Stress point 80	1.27	28.30	Pre-calc	Soil_2_1
Stress point 56	0.62	31.46	Pre-calc	Soil_2_1
Stress point 108	1.27	34.97	Pre-calc	Soil_2_1

Figure 80: Selected points

11. Click on the *Update* button to get back to the PLAXIS Input.

4.6 Flow conditions

1. Click on the Flow conditions button on the main PLAXIS Input toolbar.
2. The groundwater level has already been defined in the borehole. The initial steady state pore-water pressures can be seen by clicking on *Preview phase* button as shown below:

Figure 81: Preview phase button

3. The PLAXIS Output opens with the Steady-state pore water pressures in the form of shadings diagram as shown in the following figure:

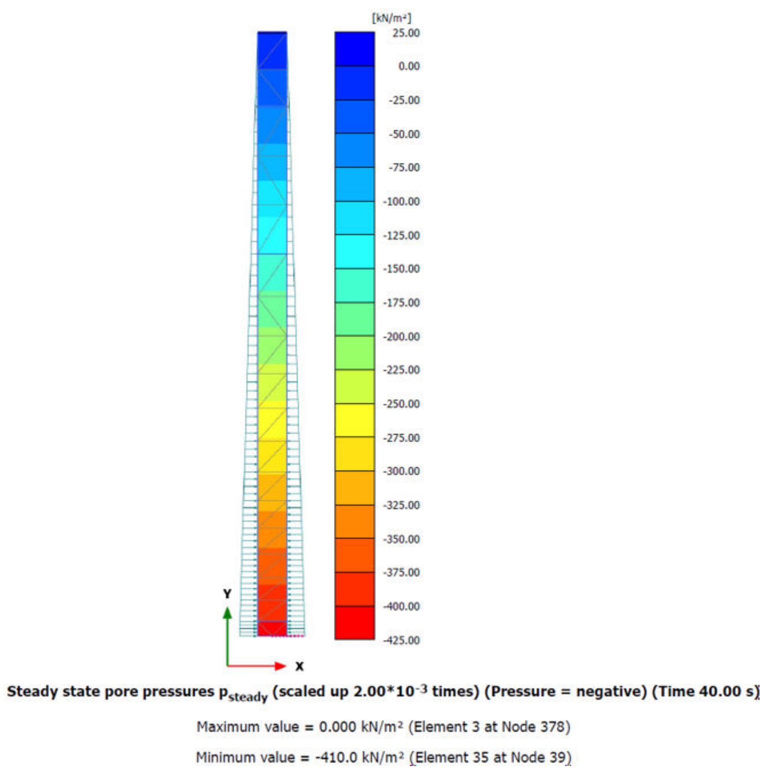


Figure 82: Steady-state pore water pressures

4. Press *Close* button at the main toolbar to get back to the PLAXIS Input program.

4.7 Definition of calculation stages, calculation settings and boundary conditions

1. Click on Staged construction button in the main PLAXIS Input toolbar.
2. Double click on the Initial phase at the bottom of the left panel as shown below:

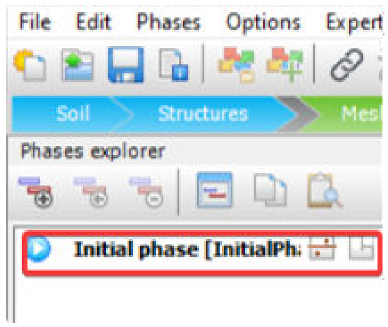


Figure 83: Initial phase button

3. The Phases window will open. You can change the ID of the phase to K0 conditions. Make sure that the Calculation type is set to K0 procedure as shown in the following figure:

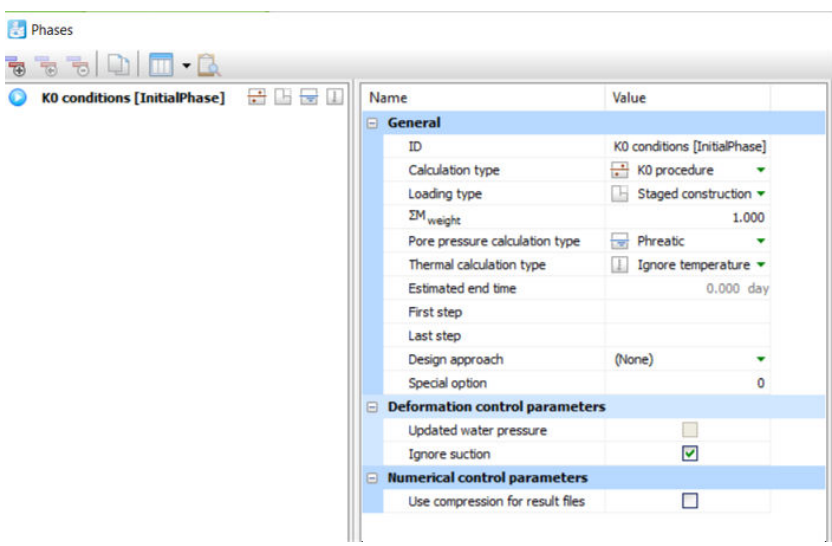


Figure 84: Calculation type set to K0 procedure

4. Add another phase by clicking on the + button on the toolbar of the Phases window.
5. Another phase has been added to the panel. Click on it as shown below:

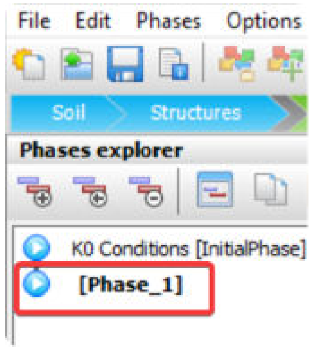


Figure 85: Adding phase

6. Make the following changes to the phase settings:
    - Type Earthquake as its ID.
    - Change the Calculation type to Dynamic.
    - Assign the Dynamic time interval to 40.0s.
    - Set the Max number of steps stored to 1000.
    - Deselect the Use default iter parameters.
    - Set the Time step determination to Manual.
    - Set the Max steps to 1000.
    - Set the Number of sub-steps to 10.

    Leave the other settings at the default values. The Phases window for the Earthquake phase should look like in the following figure:

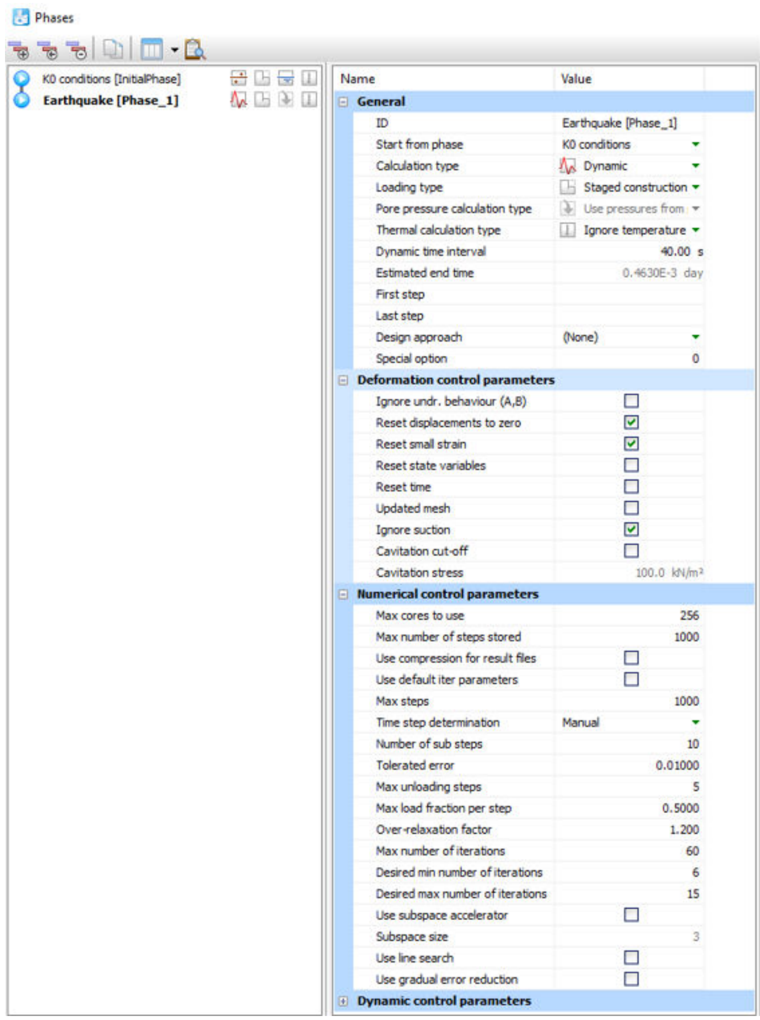


Figure 86: Phases window

7. When the settings are prepared, click OK to close the Phases window.
8. First make sure that the Earthquake phase is active by clicking on it in the panel, as shown below:

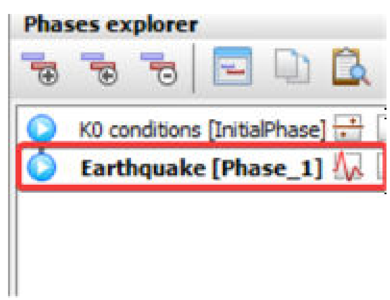


Figure 87: Earthquake phase is active

9. The deformation boundary conditions can be defined by expanding Model conditions -> Deformations in the Model explorer panel as shown below:

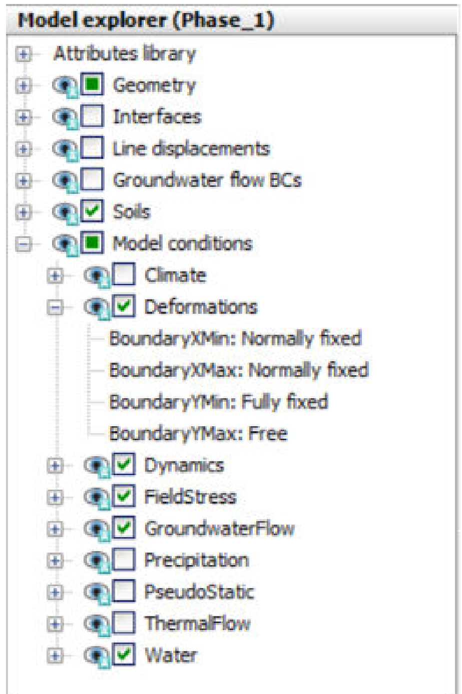


Figure 88: Model explorer

10. Set the deformation boundary conditions to the following values:

- *BoundaryXMin* to *Free*.
- *BoundaryXMax* to *Free*.
- *BoundaryYMin* and *BoundaryYMax* will be left to the default choices, namely *Fully fixed* and *Free*.

The correctly set deformation boundary conditions are shown in the following figure:

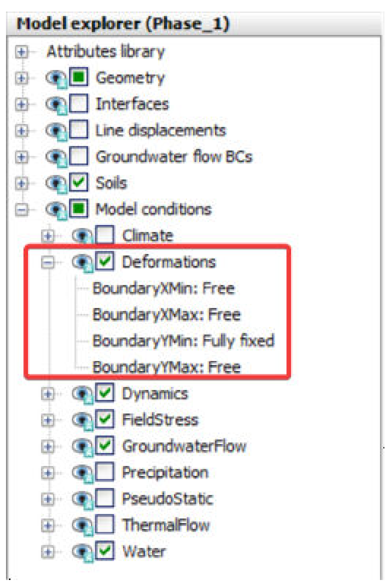


Figure 89: Deformation boundary conditions

11. To perform a 1D wave propagation analysis, the *Tied degrees of freedom* will be used on the left and right vertical boundaries of the problem domain and *Compliant base conditions* at the bottom. This can be done by clicking and expanding the *Model conditions -> Dynamics* option in the *Model explorer* panel. The following options must be set in the *Dynamics* sub items:

- *BoundaryXMin* to the *Tied degrees of freedom*.
- *BoundaryXMax* to the *Tied degrees of freedom*.
- *BoundaryYMin* to the *Compliant base*.

Other settings will be left to their default values. The *Model explorer* panel should look as follows:

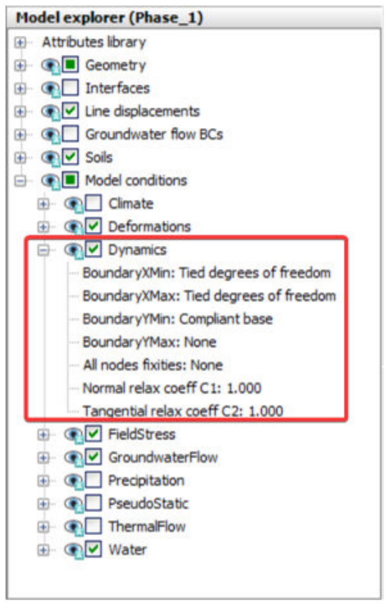


Figure 90: Model explorer showing degrees of freedom

4.8 Activation of the earthquake loading

1. Make sure that the Earthquake phase is active by clicking at it in the *Phases explorer* panel as shown below:

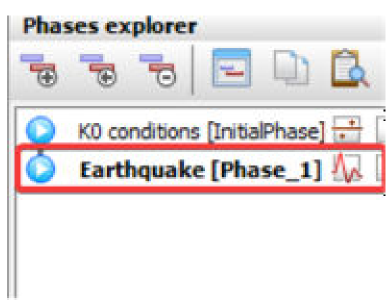


Figure 91: Earthquake phase

2. Go to the *Selection explorer* panel and activate the LineDisplacement_1_1. The *Selection explorer* panel should look as shown below and the arrows indicating the prescribed displacements will be coloured red:

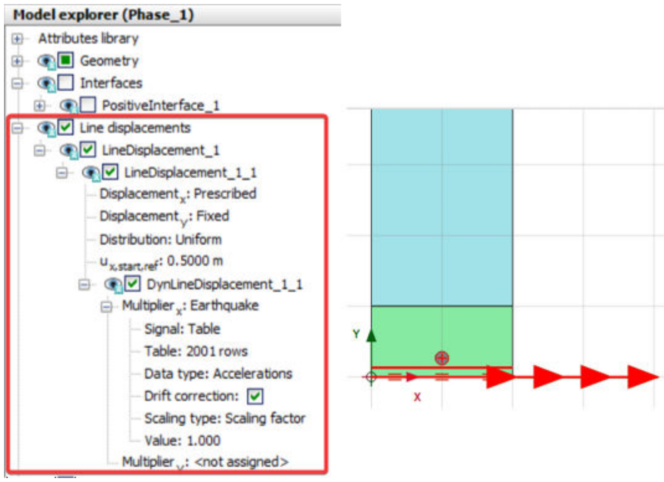


Figure 92: Selection explorer showing Line displacement

4.9 Running the calculation

1. Now the model is prepared to run the calculation. In order to start the calculation, press the *Calculate* button in the PLAXIS Input toolbar as shown below:

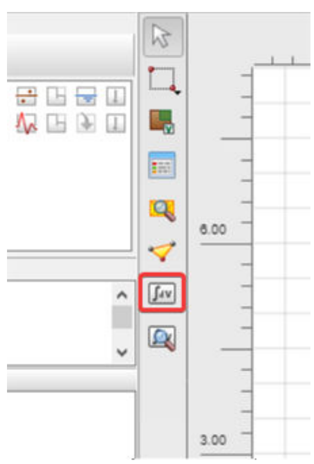


Figure 93: Calculate button in the PLAXIS Input toolbar

2. The *Active tasks* window will appear and both phases will be calculated as shown below:

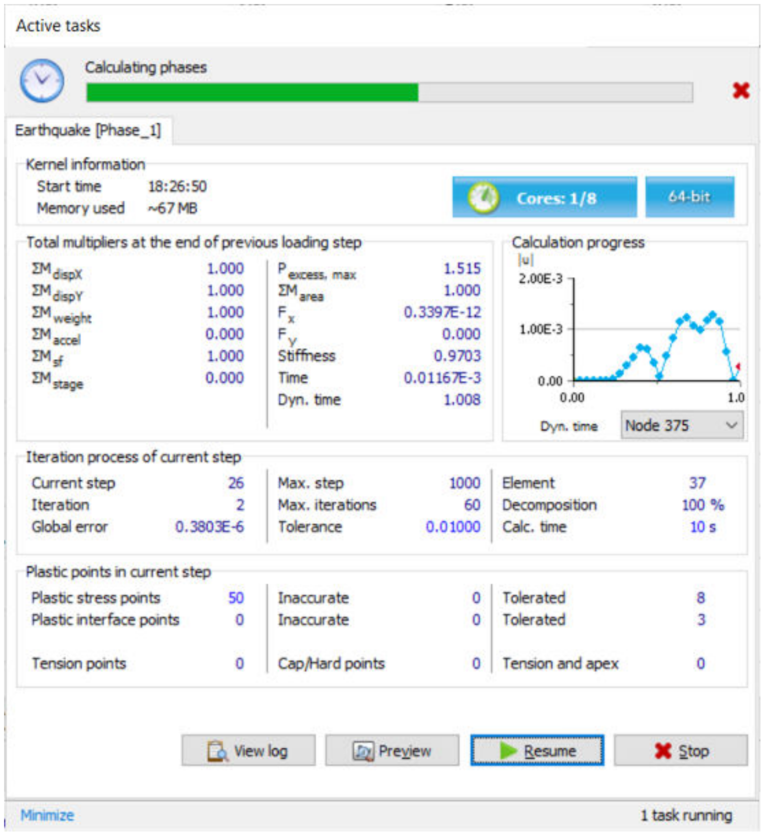


Figure 94: Active tasks window

3. If the calculation is successfully finished, the tick marks in front of phase labels in the *Phases explorer* will be in green colour as shown in the following figure:

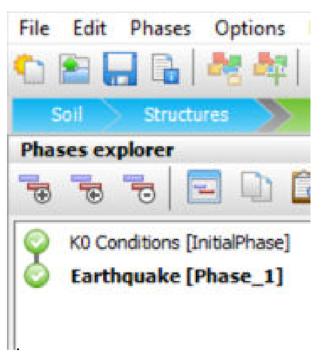


Figure 95: Calculation is successfully finished

4.10 Viewing the results of the calculation

1. To view the calculation results click on the *View calculation results* button on the PLAXIS Input toolbar as shown below, and the PLAXIS Output will open:

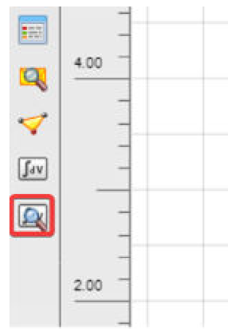


Figure 96: View calculation results button

2. The deformed mesh at the end of the Earthquake phase is shown. By clicking at the main menu *Stresses* -> *Pore pressures* -> p_{excess}, the excess pore pressure at the end of the analysis are shown, as shown below:

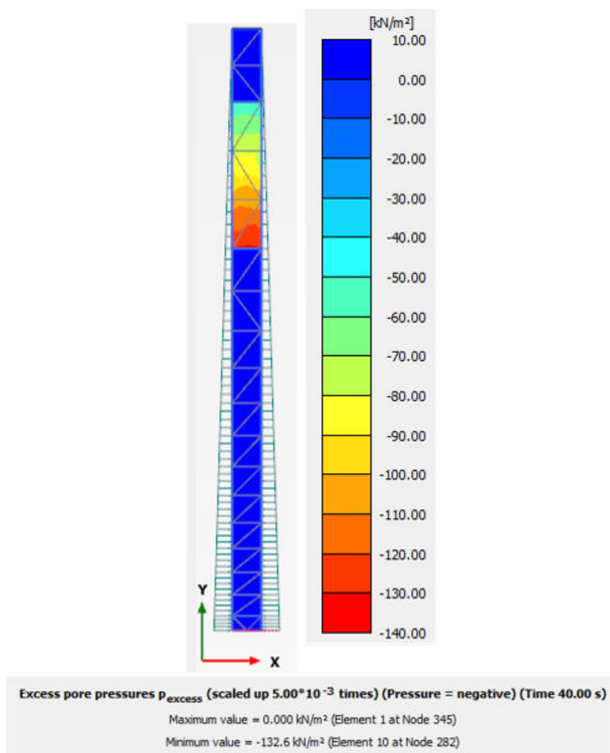


Figure 97: Deformed mesh showing excess pore pressures

3. By selecting *Stresses* -> *State parameters* -> *User-defined parameters* -> *[PM4Sand] ru*, max the maximum pore pressure ratio of the analysis in the *Sand* material is shown as below:

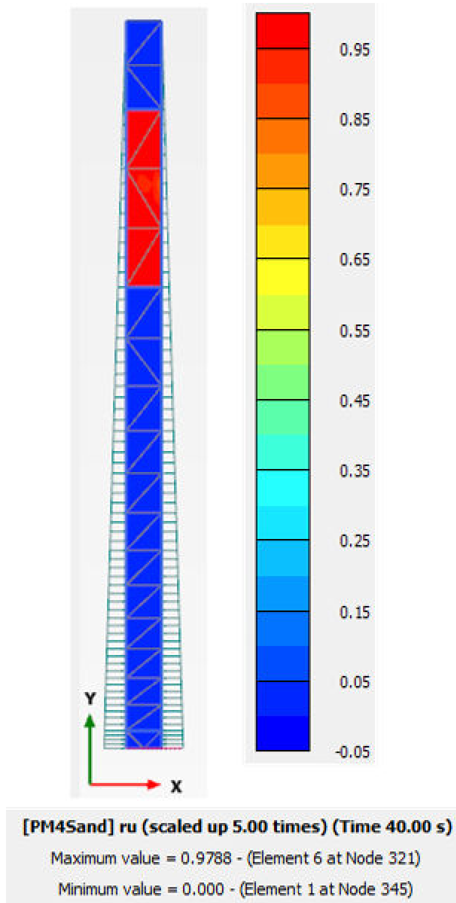


Figure 98: User-defined parameters

It can be observed that the whole Sand layer is completely liquefied.

4. At the end of this exercise, the power spectrum in acceleration can be calculated and visualised. In PLAXIS Output click at the *Curves manager* button (in the main toolbar) as shown below:

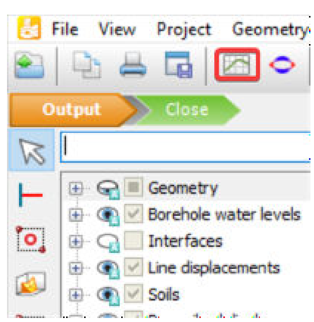


Figure 99: Curves manager button

5. The *Curves manager* window opens. Click on the *New* button.
6. The *Curve generation* window will open. Choose the tab sheet PSA and select a node at location (1.00, 41.00). It is the node at the ground surface of the model. Leave the settings of the *Curve generation* window as follows:

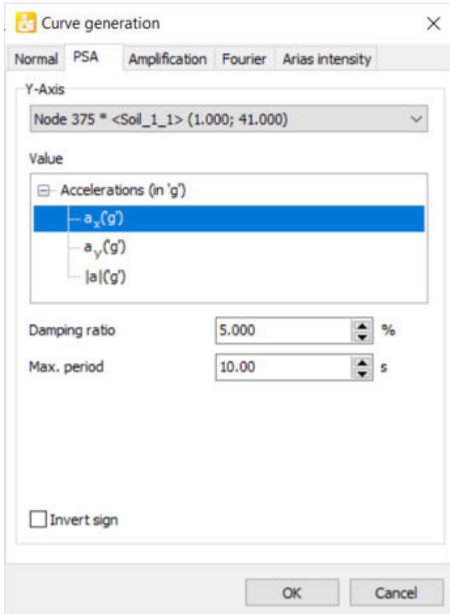


Figure 100: Curve generation window

7. Press *OK* and select the Earthquake phase in the *Select phases* window and press *OK*.
8. The PSA will be calculated and displayed as shown below:

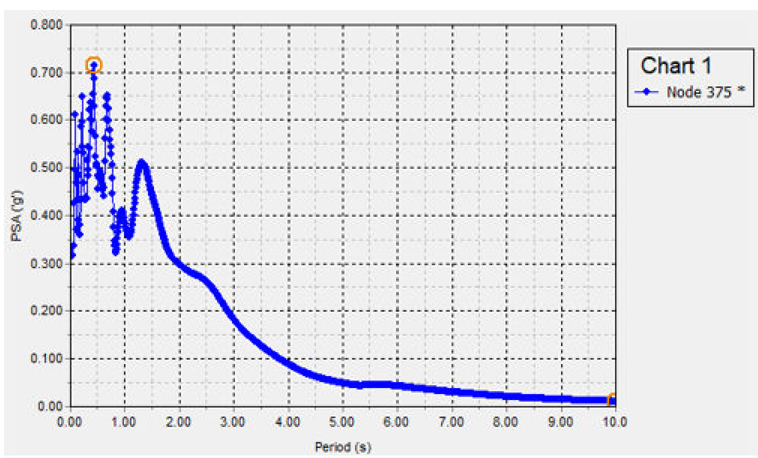


Figure 101: Calculated PSA

9. The development of the excess pore-pressure can be plotted by going to the *Curves manager* again, clicking the *New* button. In the *Curve generation* window the dynamic time can be chosen for the X-axis and the excess pore-pressure for the Y-axis as is shown below for the stress point at position (0.621, 31.455):

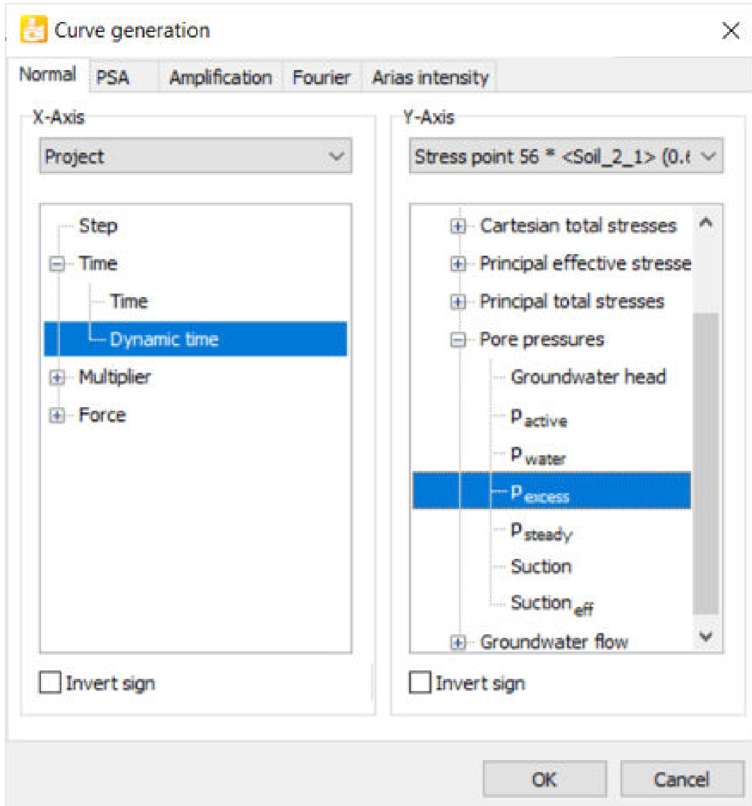


Figure 102: Curve generation window

10. The resulting excess pore-pressure development plot is shown below:

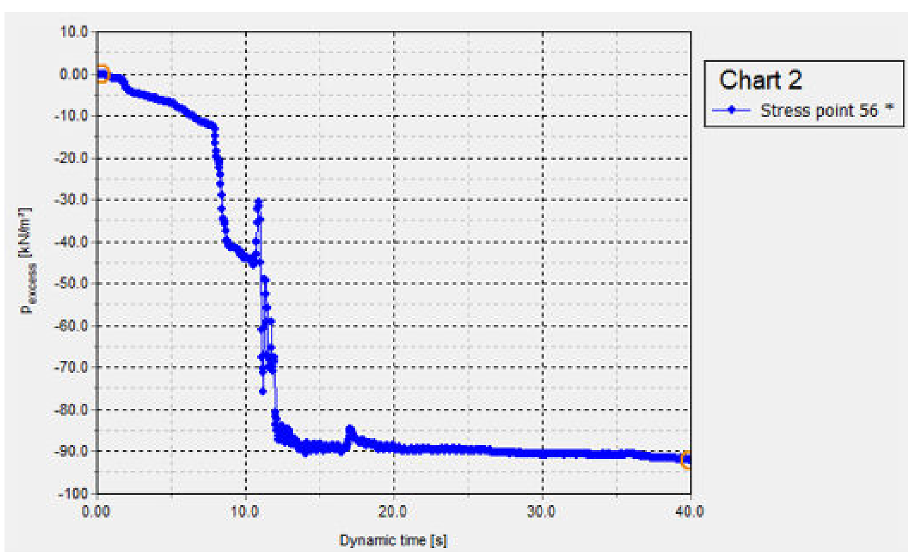


Figure 103: Plot showing excess pore-pressure development

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