BENDING OF AXISYMMETRIC PLATES

In an axisymmetric analysis, plates may be used as circular. This document describes an example that has been used to verify the bending of axisymmetric plates in PLAXIS 2D. This verification example involves a uniformly distributed load *q* on a circular plate. In one case the plate can rotate freely at the boundary and in the other case the plate is clamped, as indicated in Figure 1.

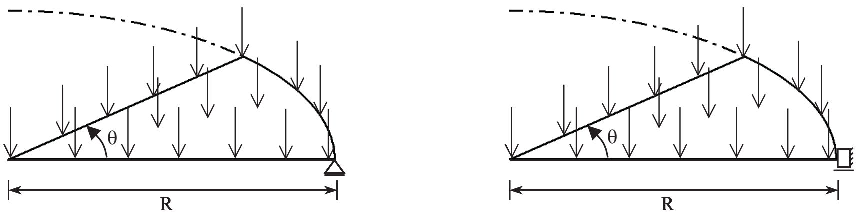

Figure 1 Loading scheme for testing axisymmetric plates

Used version:

- PLAXIS 2D - Version 2018.0

Geometry: In PLAXIS the structures cannot be used individually. A soil cluster is used to create the geometry (Figure 2). Note that the properties of the material assigned to the soil do not affect the results as the clusters will be deactivated in the calculation phases. The assignment of a soil material to the clusters is required before generating the mesh.

An axisymmetric model is used in PLAXIS 2D. Due to symmetry, half the geometry is modelled. The length of the plate is 1 m. A uniformly distributed load *q* acts vertically downward on the plate. Its magnitude equals 1 kN/m/m. The right corner of the plate is vertically fixed in the first case and fully fixity (vertical displacement and rotation) in the second case. The left corner of the plate is horizontally fixed in both the cases.

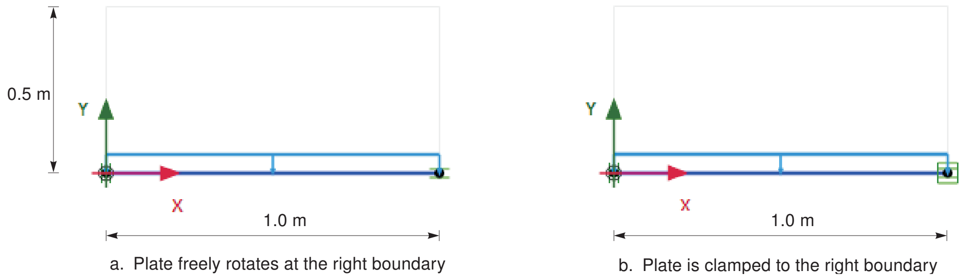

Figure 2 Model geometry and boundary conditions

Materials: The material properties adopted for the plate are:

EA = 1200 kN/m EI = 1 kN m^2/m ν = 0.0

Meshing: The *Medium* option is selected for the *Element distribution* of the *Global coarseness*. The mesh is locally refined by a *Coarseness factor* of 0.25 at the geometry line in which the plate material is assigned.

Calculations: In the Initial phase zero initial stresses are generated by using the K0 procedure ($\gamma = 0$). A new calculation phase is introduced (Phase 1) and the *Calculation type* is set to *Plastic analysis*. The *Reset displacements to zero* option is selected and the *Tolerated error* is set to 0.001. In this phase the soil clusters are deactivated and the plates are activated. Default fixities are deactivated.

Output: The results of both calculations are presented in Figures 3 and 4.

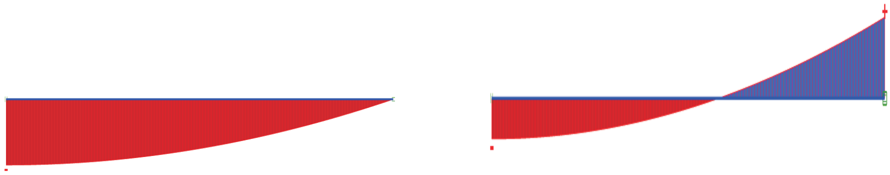

Figure 3 Computed distribution of moments (PLAXIS 2D)

Figure 4 Computed displacements (PLAXIS 2D)

Verification: For the situation of a circular plate with a uniformly distributed load one can elaborate and solve a differential equation. The analytical solutions for this equation depends on the boundary conditions.

For the plate with free rotation at the right boundary (hinged), the solution reads:

Settlement:

$$w = \frac{qR^4}{64D}\left(\frac{5+\nu}{1+\nu} - \frac{6+2\nu}{1+\nu}\frac{r^2}{R^2} + \frac{r^4}{R^4}\right) \quad \text{with} \quad D = \frac{EI}{1-\nu^2} \tag{1}$$

Moments:

$$m_{rr} = \frac{qR^2}{16}\left((3+\nu) - (3+\nu)\frac{r^2}{R^2}\right) \tag{2}$$

For the plate which is clamped t the right boundary, the solution reads:

Settlement:

$$w = \frac{qR^4}{64D}\left(1 - \frac{r^2}{R^2}\right)^2 \quad \text{with} \quad D = \frac{EI}{1-\nu^2} \tag{3}$$

Moments:

$$m_{rr} = \frac{qR^2}{16}\left((1+\nu) - (3+\nu)\frac{r^2}{R^2}\right) \tag{4}$$

Comparison of the analytical and PLAXIS 2D results is presented in Table 1 and 2 for the hinged and the clamped plate respectively. The settlement difference is mainly due to shear deformation, which is included in the numerical solution but not in the analytical solution. PLAXIS results are in good agreement with the analytical solution.

Table 1 Comparison between analytical and PLAXIS 2D results for the hinged plate

Location	Analytical solution	PLAXIS 2D	Error (%)
$r = 0$	w = 0.078125 m	w = 0.078625 m	0.6
	m_{rr} = 0.1875 kNm/m	m_{rr} = 0.187501 kNm/m	0.0
$r = R/2$	w = 0.055664 m	w = 0.056039 m	0.7
	m_{rr} = 0.140625 kNm/m	m_{rr} = 0.140625 kNm/m	0.0

Table 2 Comparison between analytical and PLAXIS 2D results for the clamped plate

Location	Analytical solution	PLAXIS 2D	Error (%)
$r = 0$	w = 0.015625 m	w = 0.016125 m	0.3
	m_{rr} = 0.0625 kNm/m	m_{rr} = 0.062501 kNm/m	0.0
$r = R/2$	w = 0.008789 m	w = 0.009164 m	4.3
	m_{rr} = 0.015625 kNm/m	m_{rr} = 0.015625 kNm/m	0.0
$r = R$	m_{rr} = -0.125 kNm/m	m_{rr} = -0.125 kNm/m	0.0