This document describes an example that has been used to verify that stresses are calculated correctly during the expansion of a cylindrical cavity in an elastic perfectly-plastic cohesive soil (Figure 1). Both small and large displacement calculations are performed.

Geometry: A cylindrical cavity of initial radius \( a_0 \) is expanded to radius \( a \) under an internal pressure \( p \) which is triggered by a Prescribed displacement. The radius of the elastic-plastic boundary is denoted as \( r_p \). An axisymmetric model is considered. The model geometry and the selected boundary conditions are presented in Figure 2. The value of \( a_0 \) is 1.0 m. The width of the soil layer is selected to be 64\( a_0 \) and its thickness equals 10\( a_0 \). Since the theoretical solution is based on an infinite continuum, a correcting material cluster is added to the soil perimeter, extended by 64\( a_0 \).

Materials: The soil is assumed to be nearly incompressible with an angle of friction \( \phi' \) equal to zero. The ratio \( G/c' \) equals 100 and the Poisson's ratio is 0.495. The Tension cut-off option is deactivated. Regarding the correcting layer, a Poisson's ratio of 0.25 and a Young's modulus of 5\( E'/12 \) are assigned, where \( E' \) is the soil Young's modulus. The selected material properties of the correcting layer are presented by Burd & Houlsby (1990). The adopted material parameters are:
Soil: Mohr-Coulomb \( G = 100 \text{ kN/m}^2 \) \( \nu' = 0.495 \) \( c' = 1.0 \text{ kN/m}^2 \)

Correcting layer: Linear elastic \( E' = 124.6 \text{ kN/m}^2 \) \( \nu' = 0.25 \)

**Meshing:** The *Medium* option is selected for the *Element distribution*. The left boundary of the model, where the prescribed displacement is applied, is refined with a *Coarseness factor* of 0.1. Equal refinement is selected for the boundary line between the soil and the correcting layer. The resulting mesh is shown in Figure 3.

![Generated mesh](image)

**Calculations:** In the Initial phase zero initial stresses are generated by using the *K0 procedure* as *Calculation type* \( (\gamma = 0) \). The small displacement calculation is performed in the first phase with *Plastic analysis*. The *Reset displacements to zero* option is selected. The prescribed displacement is activated and is set equal to 4 m. The *Tolerated error* is selected equal to 0.001. The large displacement calculation is similarly defined in Phase 2, starting from the Initial phase as well. The *Updated mesh* option is selected.

**Output:** In order to obtain the cavity pressure \( p \) (radial stress) from PLAXIS, the computed force per unit radian acting on the cavity surface should be divided by the thickness of the soil layer times the cavity radius. Note that in the small displacement calculation the cavity radius is constant and equal to \( a_0 \). In the large displacement calculation, the cavity radius increases from \( a_0 \) to \( a \) as the calculation evolves. The obtained cavity pressure is normalized over the soil cohesion \( c' \). The variation of \( p/c' \) over the normalized radial displacement \( (a - a_0)/a_0 \) is presented in Figure 4. PLAXIS results for both small and large displacements are plotted.

![Variation of normalised cavity pressure over normalised radial displacement](image)
**Verification:** The present problem has been studied by various researchers and a theoretical solution exists for both large and small displacements (Sagaseta, 1984). The analytical solution is obtained as:

Small displacement solution:

\[
\begin{align*}
 r_p^2 &= 2 \left( \frac{G}{c} \right) a_0 (a - a_0) \\
 p &= \frac{2G(a - a_0)}{a_0} \\
 p &= c' - 2c' \ln \left( \frac{a_0}{r_p} \right)
\end{align*}
\]

elasto-plastic boundary radius

\[
\begin{align*}
 p &= \frac{2G(a - a_0)}{a_0} \\
 p &= c' - 2c' \ln \left( \frac{a_0}{r_p} \right)
\end{align*}
\]

for \( r_p < a_0 \)

for \( r_p > a_0 \)

Large displacement solution:

\[
\begin{align*}
 r_p^2 &= \frac{1}{\eta^2} (a^2 - a_0^2) \\
 p &= GF(\eta) \\
 p &= GF(\eta) + 2c' \ln \left( \frac{\eta}{\eta_r} \right)
\end{align*}
\]

elasto-plastic boundary radius

\[
\begin{align*}
 p &= GF(\eta) \\
 p &= GF(\eta) + 2c' \ln \left( \frac{\eta}{\eta_r} \right)
\end{align*}
\]

for \( r_p < a \)

for \( r_p > a \)

where:

\[
\begin{align*}
 \eta^2 &= \frac{a^2 - a_0^2}{a^2} \\
 \eta_r^2 &= 1 - \exp \left( \frac{-c'}{G} \right) \\
 F(\eta) &= \eta^2 + \frac{\eta^4}{4} + \frac{\eta^6}{9} + ...
\end{align*}
\]

The analytical results are presented in Figure 4. It is concluded that they are in good agreement with PLAXIS results.

With respect to the elasto-plastic boundary radius \( r_p \) when the cavity is expanded to radius \( a = 5 \) m, analytical and numerical results are compared in Table 1 and they are found to be in good agreement as well.

**Table 1** Comparison between analytical and PLAXIS results regarding \( r_p \) when \( a = 5 \) m

<table>
<thead>
<tr>
<th>Approach</th>
<th>Analytical</th>
<th>PLAXIS 2D</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small displacements</td>
<td>28.28 m</td>
<td>28.01 m</td>
<td>1.0</td>
</tr>
<tr>
<td>Large displacements</td>
<td>49.11 m</td>
<td>48.85 m</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The plastic region (plastic points distribution) for the small and large displacement approach is illustrated in Figure 5 and Figure 6 correspondingly.
REFERENCES
