FREE VIBRATION ANALYSIS OF A DAM

This document describes an example that has been used to verify that the natural frequency of a vibrating dam, fixed at the base, is calculated correctly.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: A dynamic analysis is carried out to identify the natural frequencies of a free vibrating dam. The dam is 45.0 m high and has a 157.5 m wide base. The upstream (left) slope is 2:1 (horizontal:vertical) while the downstream (right) slope is 1.5:1. The base of the dam is fixed in all directions and the rest of the dam is allowed to vibrate only in the horizontal direction by use of prescribed line displacements (fixed in vertical direction). A static point load is acting horizontally rightward at the crest. The model geometry is presented in Figure 1.

In PLAXIS 3D the model is extended by 1 m in the y-direction (plane strain conditions). Prescribed surface displacements are used in order to force the dam to vibrate only in horizontal direction. The default viscous dynamic boundaries in y-direction are set to *None*.

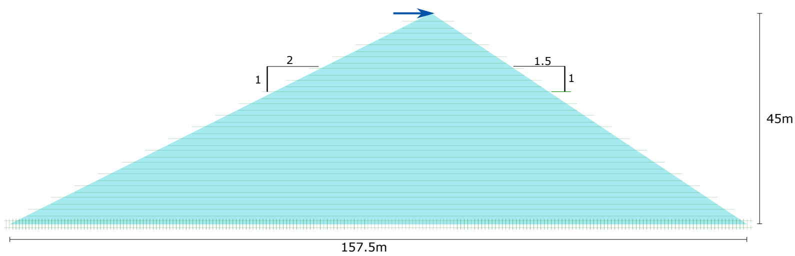

Figure 1 Model geometry (PLAXIS 2D)

Materials: The dam is constructed of compacted clayey soil with shear wave velocity (V_s) equal to 365.8 m/s (1200 ft/s). The values of Rayleigh α and β are selected to be zero to avoid any material damping. The adopted material parameters are:

Soil: Linear elastic Drained γ = 18 kN/m^3 ν' = 0.2 G = 245.5 $\times 10^3$ kN/m^2

Meshing: The prescribed line/surface displacements used to force the dam to vibrate horizontally result in a well-formed structured mesh. In both PLAXIS 2D and PLAXIS 3D models, the *Fine* option is selected for the *Element distribution*. The crest of the dam is refined with a *Coarseness factor* of 0.2. The resulting mesh is shown in Figure 2 and Figure 3.

Calculations: The self-weight of the dam is not considered for generation of the initial stresses in the initial phase ($\sum M_{weight}$ is selected equal to 0). A plastic analysis (*Staged construction*) is performed in Phase 1, with a static load equal to 1000 kN/m acting at the crest (Figure 1).

In Phase 2, the free vibration analysis is performed. The time interval is selected to be 10 s and the displacements field obtained in Phase 1 is taken into account. The static load is deactivated in order to allow for free vibrations. The *time step determination* is set to

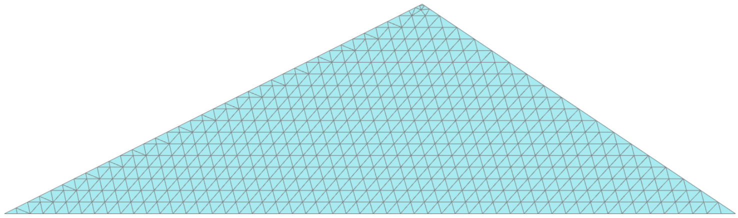

Figure 2 Generated mesh (PLAXIS 2D)

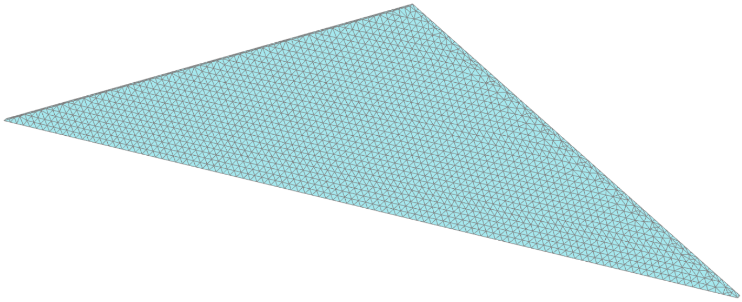

Figure 3 Generated mesh (PLAXIS 3D)

semi-automatic and the *Max steps* are set equal to 250. The *Mass matrix* value is selected equal to 1, considering a consistent mass matrix. The *Newmark alpha* and *Newmark beta* coefficients for the numerical time integration are defined as 0.25 and 0.5 respectively (average acceleration method).

Output: The horizontal displacement u_x for a point located at the crest of the dam is plot against time. The results for both PLAXIS 2D and PLAXIS 3D are presented in Figure 4 and they are in good agreement.

The corresponding *Power spectrum* is plot against frequency in Figure 5. Two distinct peaks are observed in which the power spectral density is concentrated. The corresponding frequencies are 2.9 Hz and 6.4 Hz respectively. PLAXIS 2D and PLAXIS 3D results are in perfect agreement.

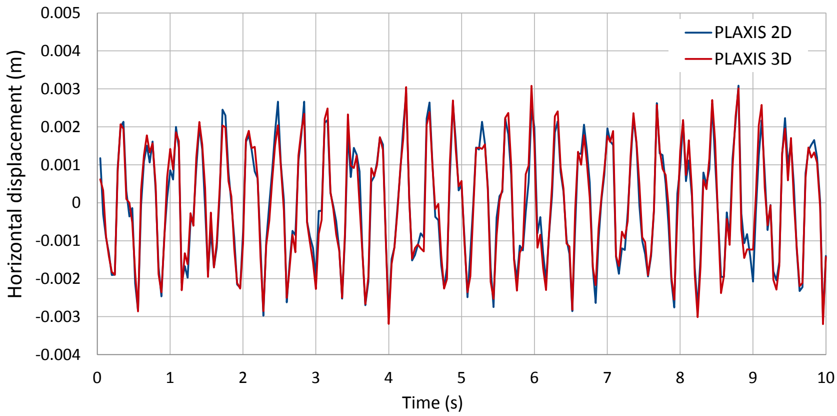

Figure 4 Time history of displacement on top of the dam obtained in PLAXIS

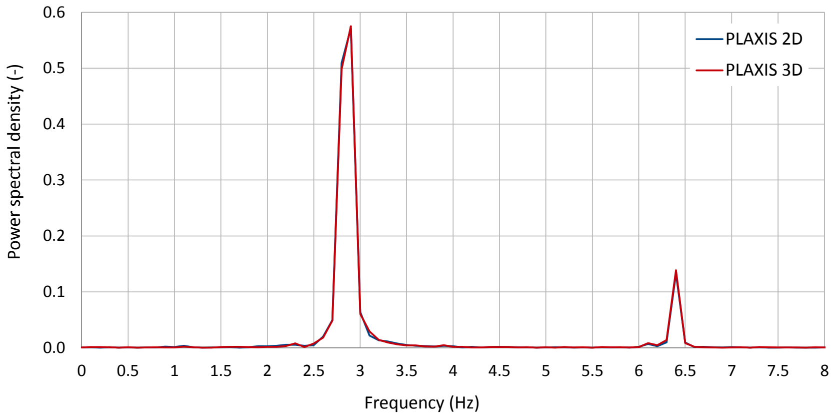

Figure 5 Power spectral density versus frequency on top of the dam obtained in PLAXIS

Verification: The natural frequencies of a free vibrating dam are calculated as (Lambe & Whitman, 1979):

$$\omega_n = \frac{V_s}{H}\frac{\beta_n}{8}(4+m)(2-m) \tag{1}$$

where, ω_n is the n^{th} natural frequency of the dam in rad/s, H is the height of the dam, β_n is the n^{th} root of a period relation for the first five modes of vibration (Kramer, 1996), m is a stiffness parameter which relates the shear modulus of soil at depth z to the average shear modulus at the base of the dam: $G(z) = G_b(z/H)^m$.

The results obtained with PLAXIS are verified against the closed form analytical solution. The first two natural frequencies (n = 1, n = 2) are reported to be 3.1 Hz and 7.0 Hz (Kramer, 1996). The values obtained from the analysis in PLAXIS (Figure 5) are in close agreement with the analytical solution.

REFERENCES

[1] Kramer, S.L. (1996). Geotechnical earthquake engineering. Prentice Hall, New Jersey.

[2] Lambe, T.W., Whitman, R.V. (1979). Soil Mechanics. John Wiley and Sons.