NATURAL FREQUENCY OF A SIMPLY SUPPORTED BEAM

In this document the natural frequency of a simply supported beam is studied. The natural frequency of a system is the frequency that characterizes its response under free vibrations condition.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: In PLAXIS, structures cannot be used individually. A soil cluster is needed to generate the finite element mesh. Note that the properties of the material assigned to the soil will not affect the results as the soil cluster will be deactivated for the calculations.

In order to compute the natural frequency with PLAXIS, a distributed load equal to 1 kN/m is statically applied to the structure. Subsequently, the load is removed and the structure is allowed to vibrate freely.

In PLAXIS 2D, a 10 m long beam (placed within the soil cluster) is used to model the structure. *Point displacements* are used to hinge the beam at both edges. The left edge is fully fixed; the right edge is fixed vertically and it is free in horizontal direction. A *Line load* equal to 1 kN/m/m is applied to the beam. Figure 1 illustrates the model geometry.

To simulate the analogous problem in PLAXIS 3D, the structure is modeled as a plate 10 m long and 1 m wide (placed within the soil cluster). *Line displacements* are used at both edges in x-direction (x_{min} and x_{max}). The left edge is fully fixed; the right edge is fixed in both z (vertical) and y (horizontal, out-of-plane) directions, while it is free in x (horizontal, in-plane) direction. A *Surface load* equal to 1 kN/m^2 is applied to the beam.

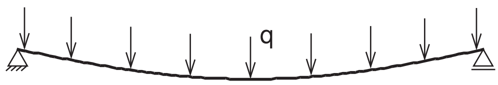

Figure 1 Simply supported beam

Materials: As mentioned above, arbitrary material properties are assigned to the soil cluster. An *Elastic* plate material is used with the following material parameters:

Plate: Elastic (isotropic) d=0.1 m EA=$1.2 \cdot 10^6$ kN EI=1000 kNm^2

w=0.981 kN/m/m $\nu = 0.0$

Meshing: In both PLAXIS 2D and PLAXIS 3D the *Very fine* option is selected for the *Element distribution*. The mesh is locally refined at the structural elements with a *Coarseness factor* equal to 0.25.

Calculations: The soil is deactivated in the Initial phase of the calculations, while the plate and the *Point/Line displacements* are activated. The *Calculation type* of the Initial phase is set to *Gravity loading*. In Phase 1 a *Plastic analysis* is performed, in which the point/line displacements together with the distributed load are activated. In Phase 2 the *Calculation type* is set to *Dynamic* and the distributed load is deactivated. The *Time interval* is selected equal to 2.5 s. The *Time step determination* is set to *Semi-automatic* and the *Maximum steps* of the analysis are selected to be 1000. The default values of the remaining parameters are valid.

Output: In Figure 2 the vertical displacement of a point located at the center of the beam is plot against time. The graph starts with an initial displacement equal to 0.1302 m caused by the removed load.

The first natural period T_1 of the beam can be determined by calculating the time interval needed for a full vibration cycle: T_1 = 0.637 s.

The corresponding natural frequency is: $f_1 = \frac{1}{T_1}$ = 1.57 Hz.

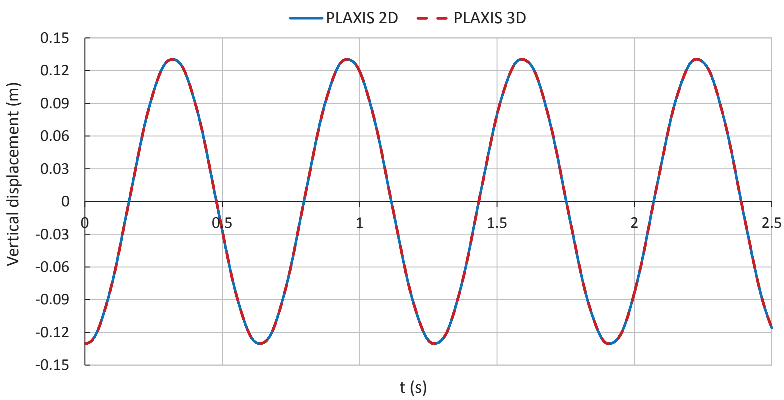

Figure 2 Vertical displacement versus time

Verification: The maximum vertical displacement at the center of a simply supported beam of length L with flexural rigidity EI, loaded by a distributed load q, is given by Eq. (1):

$$\delta_{max} = \frac{5qL^4}{384EI} \tag{1}$$

Based on Eq. (1), δ_{max} equals 0.1302 m, which is in perfect agreement with PLAXIS results (Figure 2).

A simply supported beam has an infinite number of fundamental modes of vibration. The natural frequency of the first mode of vibration (excluding transversal shear and rotational effects) is given by Eq. (2).

$$f_1 = \frac{1}{2\pi}\left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{\rho}} \tag{2}$$

where

$$\rho = \frac{m}{V} = \frac{w}{gV} \tag{3}$$

in which m is the mass per unit of length, w is the weight per unit of length, V is the volume per unit of length and g is the gravity acceleration (9.81 m/s^2).

According to Eq. (2), the first natural frequency and period are respectively: f_1 = 1.57 Hz, T_1 = 0.637 s.

The analytical results are in perfect agreement with the ones obtained in PLAXIS.