ONE-DIMENSIONAL EVAPORATION

This document describes an example that has been used to verify the unsaturated flow capabilities of PLAXIS. The problem involves steady-state and transient solutions of the suction head for the case of uniform constant evaporation in unsaturated media.

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: In PLAXIS 2D, a soil cluster is used to create the geometry with a width of 0.1 m and a height of 3.0 m. Geometry lines to be used in local refinement and controlling of the mesh are introduced dividing the geometry model into three equal parts. In PLAXIS 3D, the same geometry is modelled ($0.1 \times 0.1 \times 3.0$ m^3). Geometry surfaces are used to divide the model in 3 consecutive parts.

The groundwater table is located at the bottom which is used as the reference level. The ground surface (top boundary) is exposed to an outflow rate of q = 0.002 m/day. For transient calculations, the initial negative pore water pressure head distribution is considered to be hydrostatic. The soil has a water characteristic curve given by Eq. (1) and a permeability function given by Eq. (2).

$$S = S_{res} + (S_{sat} - S_{res}) \cdot e^{-\alpha \cdot \psi} \tag{1}$$

$$k(\psi) = k_{sat} \cdot e^{-\alpha \cdot \psi} \tag{2}$$

where α is a fitting parameter. The negative pore water pressure head distribution is calculated using the steady-state option. The same problem is reanalysed using the transient calculation option with different time intervals leading to a final time of 5 days. The side groundwater flow boundaries are set to *Closed*. The bottom boundary of the model (y = 0) is set to *Head*, with reference height h_{ref} equal to 0.0 m. Figure 1 illustrates the model geometries in PLAXIS 2D and PLAXIS 3D.

Materials: The selected material parameters for the soil are:

Soil: Linear elastic Undrained (A) γ = 0 kN/m^3 E' = 48810 kN/m^2 ν' = 0.2

The groundwater flow properties are given in Table 1. A *User-defined* groundwater data set is used. Eqs. (1) and (2) are discretized and a *Spline* function is used to obtain the soil-water retention curves, i.e. relative permeability and degree of saturation versus pressure head (Figure 2).

Table 1 Soil properties

Description	Symbol	Unit	Value
Permeability of saturated soil	k_{sat}	m/day	1.0
Saturation of saturated soil	S_{sat}	-	1.0
Residual saturation	S_{res}	-	0.23
Porosity	n	-	0.4
Fitting parameter	α	m^{-1}	2.0
Equivalent bulk modulus of water	K_w / n	kN/m^2	$2.0 \cdot 10^6$

Meshing: In both PLAXIS 2D and PLAXIS 3D, the *Coarseness factor* for the top section

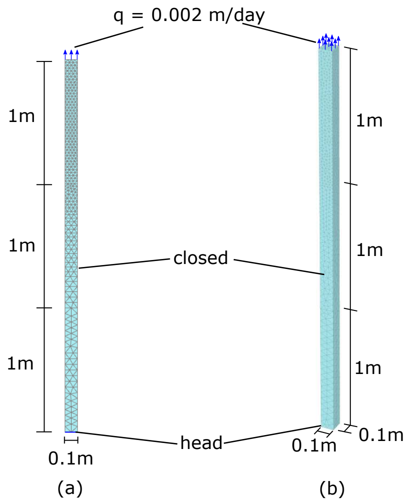

Figure 1 Problem geometry, generated mesh and groundwater flow boundary conditions in PLAXIS 2D (a) and PLAXIS 3D (b)

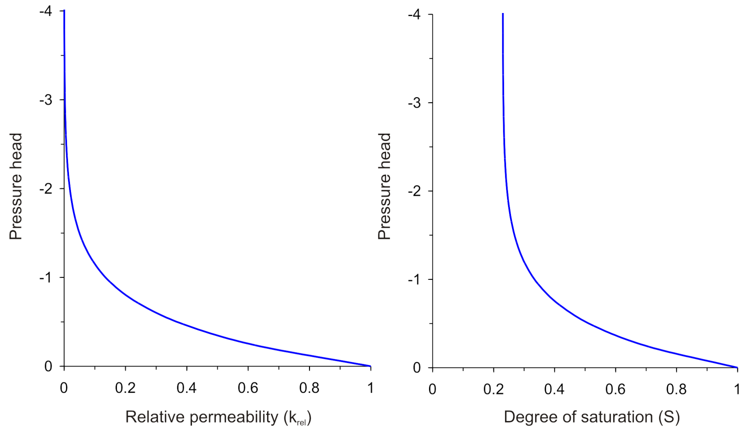

Figure 2 Relative permeability and degree of saturation versus pressure head

is set to 0.25 and for the middle and bottom sections is set to 0.5. The *Element distribution* used in PLAXIS 2D is *Very fine*, while in PLAXIS 3D is *Fine*. Figure 1 depicts the generated finite element mesh in PLAXIS 2D and PLAXIS 3D.

Calculations: The *Calculation type* is set to *Flow only*. In Phase 1, the *Pore pressure calculation type* is set to *Steady state groundwater flow*. Phase 2 starts from the Initial phase as well, but the *Pore pressure calculation type* is *Transient groundwater flow*. The *Transient groundwater flow* calculation is performed up to a total time interval of 5 days, with intermediate time intervals of 0.1, 0.25, 0.5 and 1.0 days (in total 5 calculation Phases).

Output: Figure 3 shows the calculated negative pore water pressure head distribution at steady-state condition in PLAXIS 2D and PLAXIS 3D. To obtain PLAXIS results, a vertical cross section is used at the middle of the model.

Verification: The problem of one-dimensional unsaturated water flow can be described by the following differential equation for the negative pore water pressure head ϕ_p:

$$\frac{\partial}{\partial y}\left[k(\phi_p)\cdot\left(\frac{\partial\phi_p}{\partial y}+1\right)\right]=c(\phi_p)\cdot\frac{\partial\phi_p}{\partial t} \tag{3}$$

The analytical solution of Eq. (3) at steady-state, i.e negative pore water pressure head as a function of vertical position (Gardner, 1958), for the particular permeability function in Eq. (2), is given by Eq. (4).

$$\phi_p=-\frac{1}{\alpha}\cdot\ln\left[\left(1-\frac{q}{k_{sat}}\right)e^{-\alpha y}+\frac{q}{k_{sat}}\right] \tag{4}$$

This solution is presented by the dashed line in Figure 3. It can be seen that the numerical solution perfectly matches the analytical solution.

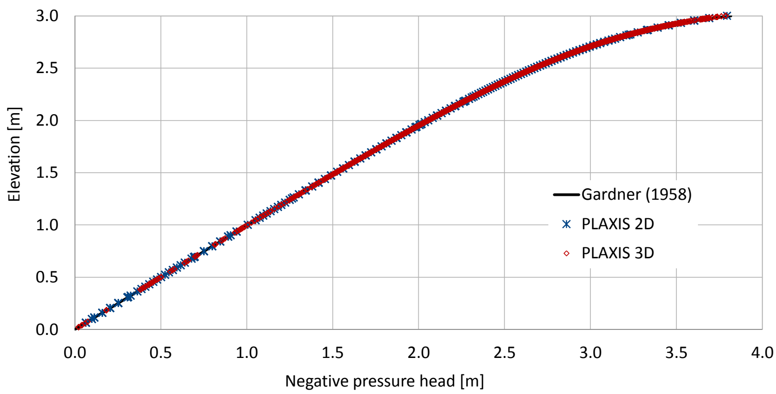

Figure 3 Negative pore water pressure head at steady-state condition

The analytical solution of Eq. (3) in the case of transient flow, i.e negative pore water pressure head ϕ_p as a function of time and vertical position (Srivastava & Yeh, 1991), for the particular water characteristic and permeability functions given by Eqs. (1) and (2), is given by Eq. (5).

$$\phi_p=\frac{\ln R}{\alpha} \tag{5}$$

where:

$$\begin{aligned}
R &= \frac{q}{k_{sat}}-\left(\frac{q}{k_{sat}}-1\right)e^{-y^*}-\frac{4q}{k_{sat}}e^{(L^*-y^*)/2}\cdot e^{-t^*/4}\cdot\sum_{i=1}^{\infty}\frac{\sin(\lambda_i y^*)\sin(\lambda_i L^*)e^{-\lambda_i^2 t^*}}{1+(L^*/2)+2\lambda_i^2 L^*}\\
t^* &= \frac{\alpha k_{sat} t}{n(S_{sat}-S_{res})}\\
y^* &= \alpha y\\
L^* &= \alpha L
\end{aligned} \tag{6}$$

where n is porosity, λ_i is the i^{th} root of the characteristic equation and t is the time.

$$\tan(\lambda\cdot L^*)+2\cdot\lambda=0 \tag{7}$$

The analytical and numerical solutions under transient conditions are presented in Figure 4 for PLAXIS 2D and Figure 5 for PLAXIS 3D. This figure shows a good agreement between the numerical and analytical solution. It is also interesting to notice that Gardner's solution for steady state is obtained after 5 days of continuous evaporation.

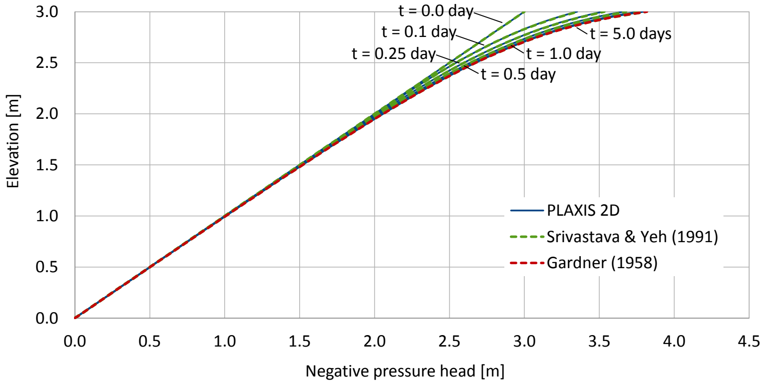

Figure 4 Negative pressure head for different time intervals t (PLAXIS 2D)

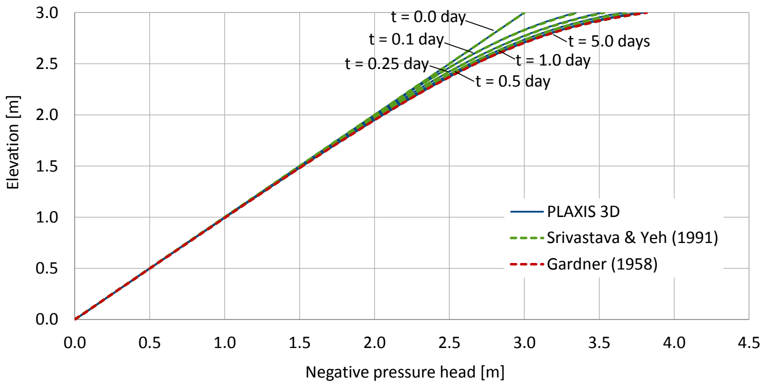

Figure 5 Negative pressure head for different time intervals t (PLAXIS 3D)

REFERENCES

[1] Gardner, W.R. (1958). Some steady-state solutions of the unsaturated moisture flow equation with applications to evaporation from a water table. Soil Science, 85, 228–232.

[2] Srivastava, R., Yeh, J. (1991). Analytical solutions for one-dimensional, transient infiltration toward the water table in homogeneous and layered soils. Water Resources Research, 27, 753–762.