SHEAR DEFORMATION IN STRUCTURAL ELEMENTS

PLAXIS takes into account the shear deformation in structural elements subjected to bending. For the verification of the resulting shear deformation different support conditions are considered. The beams are clamped at the bottom and horizontally supported at the top. A bending moment is applied at the top of the beams.

Used version:

- PLAXIS 2D - Version 2018.0

Geometry: In PLAXIS the structures cannot be used individually. A soil cluster is used to create the geometry. Note that the properties of the material assigned to the soil do not affect the results as the clusters will be deactivated in the calculation phase (Phase 1).

In PLAXIS 2D a plane strain model is used, in which six beams with length equal to 4 m are simulated. A point fixity in x-direction and a bending moment of 100 kNm/m (in counterclockwise direction) are applied at the top of each beam. The bottom of the beams touches the bottom model boundary and it is automatically clamped. Figure 1 illustrates the model geometry.

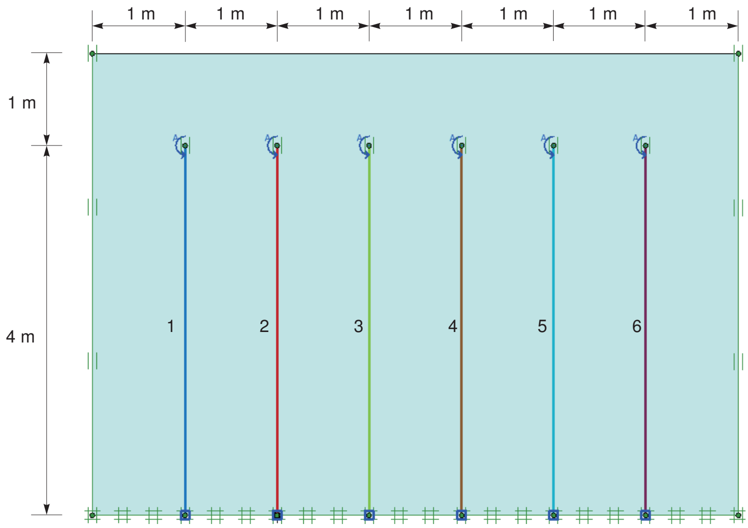

Figure 1 Model geometry in PLAXIS 2D

Materials: The beams are denoted as plates in PLAXIS 2D. In total six different plate materials are used, assigned to each one of the beams from 1 to 6. The selected material properties which are in common for every beam are:

Plate: Elastic (isotropic) γ = 0 kN/m/m EI = 1.2 $\times$ 10^3 kNm^2/m

The varying plate properties are the axial stiffness *EA*, the thickness *d* and the Poisson's ratio ν. Table 1 presents the selected values for each one of those parameters per plate.

Meshing: The *Coarse* option is selected for the *Element distribution*. The elements representing the beam are refined with a *Coarseness factor* equal to 0.25.

Calculations: In the Initial phase, zero initial stresses are generated by using the *K0*

Table 1 Varying material properties of the plates

Material property	Plate 1	Plate 2	Plate 3	Plate 4	Plate 5	Plate 6	Unit
EA	$3.6 \cdot 10^5$		$9.0 \cdot 10^4$		$3.6 \cdot 10^3$		kN/m
d	0.2		0.4		2.0		m
ν	0.0	0.2	0.0	0.2	0.0	0.2	–

procedure (γ = 0). A new calculation phase is introduced (Phase 1) and the *Calculation type* is set to *Plastic analysis*. The soil cluster is deactivated, while the plates, the point fixities and the bending moments are activated.

Output: The resulting bending moments at each beam are plotted in Figure 2. The values at both edges of each beam are specified as well.

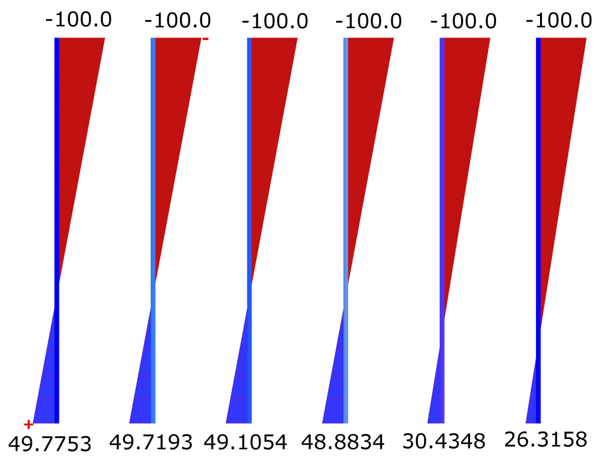

Figure 2 Distribution of bending moments (kNm/m)

The influence of the various selected d/L ratios and Poisson's ratios on the resulting bending moment at the bottom of the beams and on the horizontal reaction force at its top is presented in Table 2.

Verification: Analyzing the simple bending of a beam (plate), the problem can be separated in two loading cases, transverse loading (load case A) and bending (load case B), as shown in Figure 3.

The deformation in x-direction at the top is calculated according to the small deformation theory as:

$$u = u_A + u_B = \frac{FL^3}{3EI} + \frac{ML^2}{2EI} = 0 \tag{1}$$

If shear deformation is taken into account, the deformation in Case A becomes:

$$u_A = \frac{FL^3}{3EI} + \frac{FL}{kGA} \tag{2}$$

where k is a shape factor equal to 5/6 for rectangular beams. The combined deformation

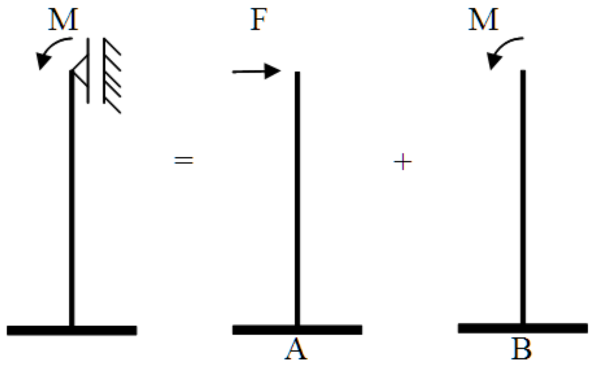

Figure 3 Bending analysis of a beam

becomes:

$$u = u_A + u_B = \frac{FL^3}{3EI} + \frac{FL}{kGA} + \frac{ML^2}{2EI} = 0 \tag{3}$$

Substituting $I/A = d^2/[12(1-\nu^2)]$ and $E/G = 2(1+\nu)$, and assuming a rectangular cross section ($k = 5/6$), Eq. (3) is modified as:

$$\frac{FL}{M} = -\frac{3}{2}\frac{1}{1+\frac{3}{5}\frac{1}{1-\nu}\left(\frac{d}{L}\right)^2} \tag{4}$$

The bending moment at the bottom of the beam is:

$$M_{clamp} = M + FL \tag{5}$$

Hint: For beam elements $I/A = d^2/12$

An overview of the analytical results based on the small deformation theory and the results obtained in PLAXIS 2D small deformation analysis is presented in Table 2. PLAXIS and analytical results are in perfect agreement.

Note that disregarding shear deformation the resulting bending moment M_{clamp} would be 50 kNm/m and the reaction force F_{top} would be 37.5 kN/m.

Table 2 Results overview (small deformation theory)

d (m)	ν (-)	PLAXIS 2D		Analytical	
		F_{top} (kN/m)	M_{clamp} (kNm/m)	F_{top} (kN/m)	M_{clamp} (kNm/m)
0.2	0.0	37.4438	49.7753	37.4438	49.7753
	0.2	37.4298	49.7193	37.4298	49.7193
0.4	0.0	37.2763	49.1054	37.2763	49.1054
	0.2	37.2208	48.8834	37.2208	48.8834
2.0	0.0	32.6087	30.4348	32.6087	30.4348
	0.2	31.5789	26.3158	31.5789	26.3158