THEIS AND THIEM SOLUTION

This document verifies that groundwater flow principles are correctly implemented in PLAXIS. Assuming a horizontal confined aquifer, the radial flow towards a well is studied under transient and steady-state flow conditions. PLAXIS results are compared with the analytical solutions presented by Theis (1935) and Thiem (1906).

Used version:

- PLAXIS 2D - Version 2018.0
- PLAXIS 3D - Version 2018.0

Geometry: The model geometry in PLAXIS 2D is presented in Figure 1. An axisymmetric model with 15-noded elements is used. To simulate a well with radius R_{well} equal to 0.1 m, the model's left boundary is placed 0.1 m away from the axis of symmetry. To simulate an approximately infinite aquifer, the model length in x-direction equals 1000 m, while the height of the soil column is 5 m. To model the aquifer as confined, the top and bottom groundwater flow boundaries are set to *Closed* (impervious). To generate the initial phreatic level, a *head* equal to 15 m is prescribed at the right and left groundwater flow boundaries. To simulate radial flow towards a well, the left groundwater flow boundary is set to *Outflow* with *Uniform* and *Constant* discharge equal to 0.03183 m/s. Under the assumption that model's radius (1000 m) is adequate to approximate an infinite aquifer, the *head* at the right groundwater flow boundary remains equal to 15 m.

In PLAXIS 3D, taking advantage of the model's symmetry, only one-quarter of the geometry is modelled. The model is extended 1000 m in both x and y directions, while its depth in vertical z-direction equals 5 m. The top and bottom groundwater flow boundaries are set to *Closed* (impervious). A *head* equal to 15 m is prescribed at the outer (rear and right) groundwater flow boundaries to generate the initial phreatic level. The inner (left and front) groundwater flow boundaries are set to be *Closed*. A cylinder with radius equal to 0.1 m is used to simulate the well, covering the whole depth of the soil column. Its outer surface is set to *Outflow* with *Uniform* and *Constant* discharge equal to 0.03183 m/s. Figure 2 illustrates the model geometry in PLAXIS 3D.

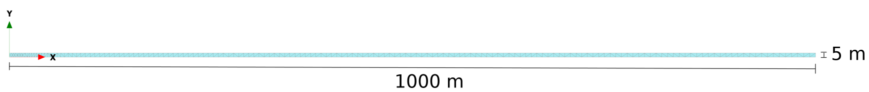

Figure 1 Model geometry and generated mesh (PLAXIS 2D)

Materials: The soil is modelled as *Drained*, *Linear elastic*. The adopted material parameters are:

Soil:	Linear elastic	Drained	γ = 0 kN/m^3	e_{init} = 0.5
	E' = 30000 kN/m^2	ν' = 0.3	k = 0.01 m/s	

Meshing: In PLAXIS 2D, the *Very Fine* option is selected for the *Element distribution*, while in PLAXIS 3D, the *Medium* option is used. Two refinement zones are used in both PLAXIS 2D and PLAXIS 3D. The first one is at the vicinity of the well, covering an area/volume up to 10 m away of its center. A *Coarseness factor* of 0.1 is used for the mesh refinement zone 1. The second zone covers the remaining area/volume up to 100

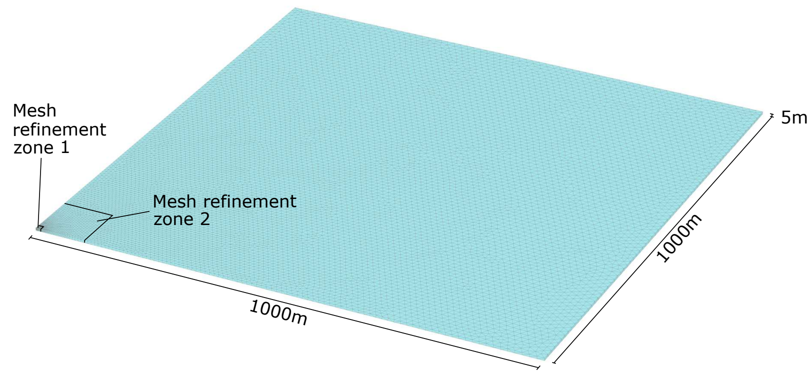

Figure 2 Model geometry and generated mesh (PLAXIS 3D)

m away from the well's center. A *Coarseness factor* of 0.5 is used for the mesh refinement zone 2. The generated mesh is illustrated in Figures 1 and 2 for PLAXIS 2D and PLAXIS 3D respectively.

Calculations: The calculations are performed using the *Flow only* calculation type in the Initial Phase. In Phase 1, the *Transient groundwater flow* is used as the pore pressure calculation type and the *Time interval* is set to 1500 s. The groundwater flow boundary conditions are adjusted as explained above to allow for radial flow towards the well. In PLAXIS 3D, the soil cluster within the cylinder that represents the well is deactivated. The *Tolerated error* is set equal to 0.001, the *First time step* is set to 0.1 s, the *Min time step* is set to 0.01 s and the *Max time step* is set to 100 s.

An additional phase (Phase 2) is used to check the results of the steady-state groundwater flow analysis. For that purpose the *Steady state groundwater flow* is used as the pore pressure calculation type.

Output: For both PLAXIS 2D and PLAXIS 3D, the groundwater head at the end of Phase 1 is identical to the groundwater head obtained in Phase 2. As a result it is concluded that the time interval used in Phase 1 (1500 s) is sufficient to achieve steady-state conditions. Figure 3 depicts the corresponding results for PLAXIS 2D (a) and PLAXIS 3D (b).

Verification: Theis (1935) presented an analytical expression for the transient drawdown of two-dimensional radial flow towards a discharge well in an infinite, homogeneous aquifer. The analytical solution gives the transient hydraulic head as a function of distance from the well's center (r) and time (t):

$$h(r,t) = h_0 - \frac{Q}{4\pi T} W(u) \qquad (1)$$

in which the dimensionless time parameter u is given as:

$$u = \frac{r^2 S}{4Tt} \qquad (2)$$

In Eq. (1), h_0 is the initial hydraulic head, Q the discharge rate of the well, T the aquifer transmissivity and $W(u)$ the well function (exponential integral). The latter is

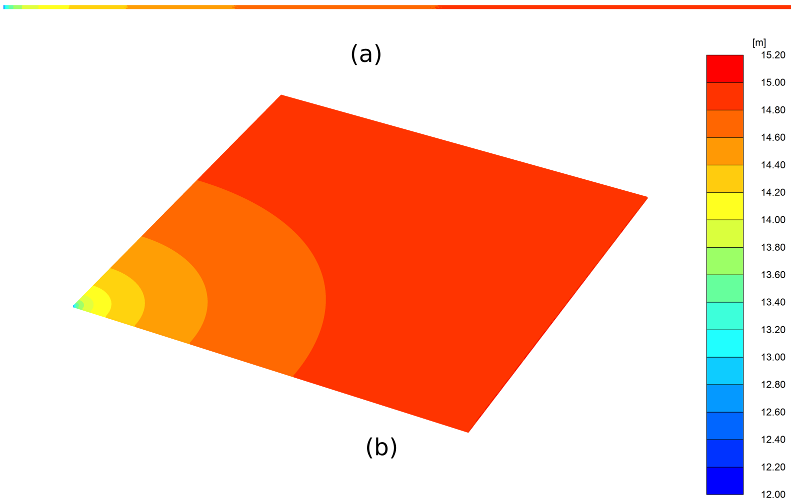

Figure 3 Groundwater head at steady-state conditions in PLAXIS 2D (a) and PLAXIS 3D (b)

approximated as:

$$W(u) = -0.577216 - \ln(u) - \sum_{\alpha=1}^{\infty} \frac{(-u)^{\alpha}}{\alpha\,\alpha!} \tag{3}$$

Hint: The well discharge rate Q is linked to the outflow discharge q used in PLAXIS as: $q = \frac{Q}{2\pi R_w d}$. Thus, for q = 0.03183 m/s, R_w = 0.1 m and d = 5 m, the well discharge rate is Q = 0.1 m^3/s.

The aquifer's transmissivity T is given as:

$$T = kd \tag{4}$$

in which, k is the permeability and d the thickness of the aquifer.

In Eq. (2), S stands for the aquifer's storativity. For a confined and homogeneous aquifer it is given as:

$$S = S_s d \tag{5}$$

in which S_s is the volumetric specific storage:

$$S_s = \gamma_w \frac{n}{K_w} \tag{6}$$

In Eq. (2), γ_w is the unit weight of the water (10 kN/m^3), n is the porosity and K_w the water bulk modulus.

For long discharge times, Theis' solution is unsuitable to produce steady-state results. It reduces to the analytical formulation proposed by Thiem (1906) for steady-state radial flow conditions:

$$h(r) = h_0 - \frac{Q}{2\pi T} \ln\left(\frac{R}{r}\right) \tag{7}$$

in which R is the radius of influence, i.e. the distance at which the head equals h_0 (assumed 1000 m as stated above).

Figure 4 presents PLAXIS results of the transient flow calculations for a point located at distance r from the well center, with r equal to 10 m. PLAXIS 2D and PLAXIS 3D are in perfect agreement. The analytical approach of Theis is plot for the same point. For short discharge times (up to approximately 250 s), PLAXIS results are in good agreement with Theis' solution. As time increases, the hydraulic head computed in PLAXIS reaches a steady-state value equal to 13.534 m, while Theis' solution indicates an on-going drawdown, resulting in a deviation of 1.59 % for t = 1500 s.

Similar results are observed in Figure 5, in which the transient flow calculations are plotted for a point located at r equal to 100 m. PLAXIS results indicate a steady-state value of 14.267 m for the hydraulic head and, for t = 1500 s, Theis' solution deviates 1.51 %.

The steady-state PLAXIS results along the whole model (1000 m) are compared to the result of the Thiem's analytical formulation in Figure 6. It is concluded that PLAXIS results are in good agreement with Thiem's solution.

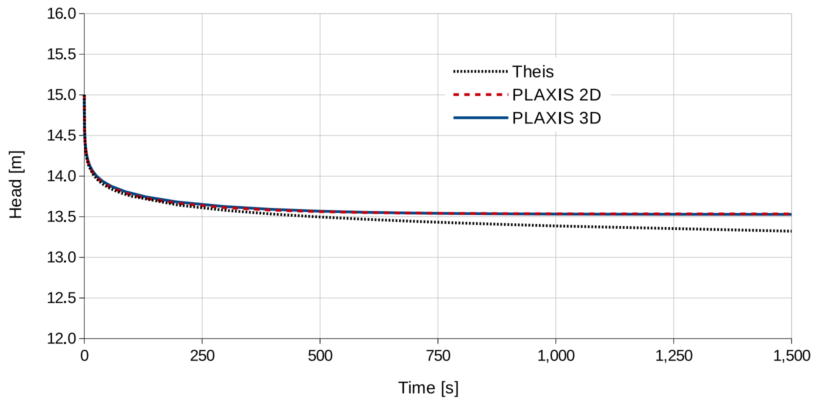

Figure 4 Head variation over time (r = 10 m)

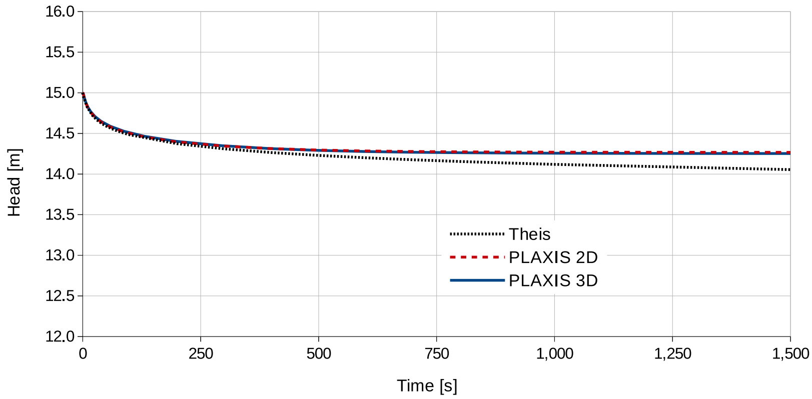

Figure 5 Head variation over time (r = 100 m)

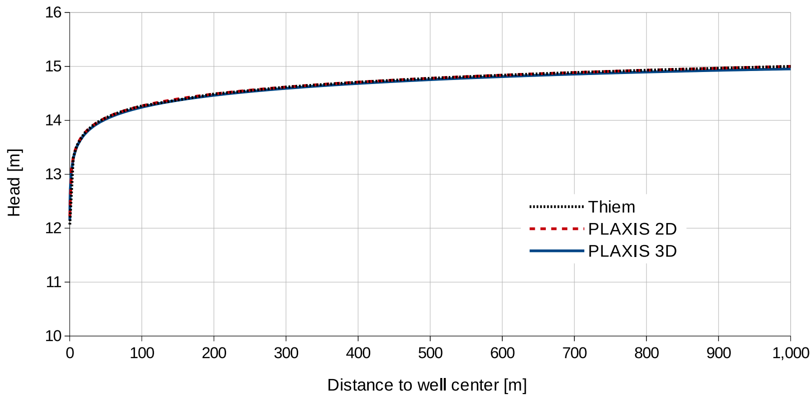

Figure 6 Head variation over distance from the well center at steady-state conditions

REFERENCES

[1] Theis, C.V. (1935). The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. Eos Transactions, American Geophysical Union, 16(2), 519–524.

[2] Thiem, G. (1906). Hydrologische methoden. Gebhardt, Leipzig.