

Biaxial Analysis of General Shaped Base Plates

R. GONZALO ORELLANA ¹

Summary: A linear model is used for the contact stresses calculation between a steel base plate and a concrete foundation. It is also considered that the anchor rods tensions vary linearly. Furthermore, strain compatibility is proposed. The non-linear system has as variables: the neutral axis angle, the maximum anchor rods tension, the bearing length and optionally the maximum stress in the concrete. The Newton-Raphson's method is used to solve the system.

Key words: Base plate; Biaxial; Compressive; Newton-Raphson

INTRODUCTION

Base plate design is an essential part of any structural project. All available bibliography related to the analysis and subsequent design of base plates considers uniaxial behavior. However, since many structural models are tridimensional, the reactions obtained from the structure have moments about both axes, as shown in Figure 1. In this article uniaxial methods are extended to handle biaxial problems.

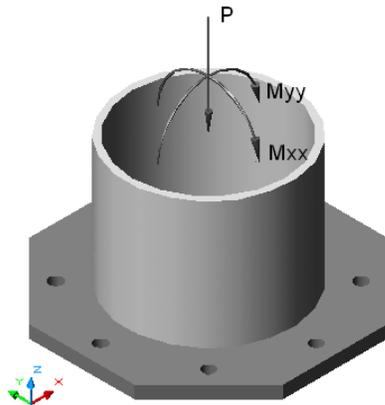


Figure 1. Biaxially loaded base plate

To solve biaxial problems a non-linear equations system is needed, because concrete can only handle compression forces and anchor rods can only handle tension.

In general, the system variables are: the maximum compressive concrete stress, maximum anchor rods tension, neutral axis angle and bearing length. A common practice is to fix maximum compressive concrete stress by the adopted design code. In this case; we have a three equations and three variables system. In addition, a new equation could be included: strain compatibility; that is to say, the base

plate is considered as fully rigid and the strains in compression and tension zones have a lineal relationship.

METHOD DEVELOPMENT

To develop the methodology we will take as an example an octagonal base plate with an inscribed circle radius of 10 in (8.28 in per side approx.). An opening is made, also octagonal, with an inscribed radius of 4 in (3.31 in per side approx.). 8 anchor rods of 5/8" are included symmetrically in a circular placement with an 8.5 in circle radius.

An important step is the borders definition. The external borders are defined counterclockwise and the openings clockwise as shown in Figure 2. Then, each line segment is taken in the same way for all defined borders.

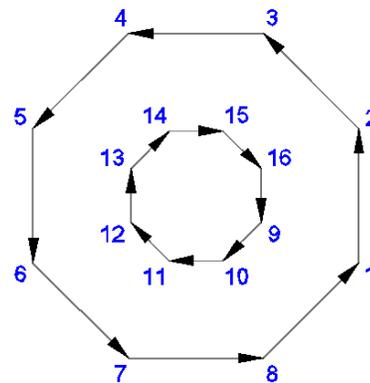


Figure 2. Borders definition

Linear models are proposed for concrete compressive stresses and anchor rods tension forces, see Figure 3.

The next step for numerical solution is to transform the base plate border and anchorage geometry to a local coordinate system, see Figure 3.

¹ Civil engineer,
personal e-mail: gonchio@hotmail.com,
work e-mail: gonzalo.orellana@bentley.com

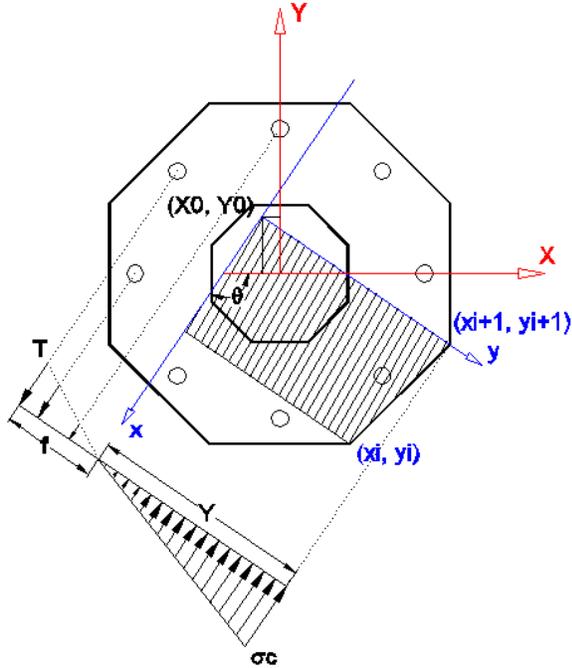


Figure 3. Linear model and local coordinate system

where,

- T = maximum tension force on anchor rods.
- σ_c = maximum compressive stress.
- θ = neutral axis (NA) angle.
- Y = bearing length.
- f = maximum anchor rod distance from NA.

The three variables of the biaxial problem are: bearing length, maximum anchor rod tension and the neutral axis angle. A simplification is to consider that the maximum concrete stress reaches the maximum concrete bearing strength by the adopted design code. The Equation (1), that has a limit for the term $\sqrt{A_2/A_1} \leq 2$, shows the maximum design bearing strength adopted by the American Concrete Institute (ACI).

$$\phi f_{p(max)} = \phi 0.85 f'_c \sqrt{\frac{A_2}{A_1}} \quad (1)$$

where,

- $f_{p(max)}$ = maximum design bearing strength, Section 10.14, ACI 318-08.
- ϕ = compression resistance factor 0.65 by Section 9.3, ACI 318-08.
- f'_c = specified compressive strength of concrete.
- A_1 = base plate area.
- A_2 = maximum area of the portion of the supporting surface that is geometrically

similar to and concentric with the loaded area.

To solve the non-linear system, the equations must also be three: the vertical forces equilibrium, Equation (2), the moment equilibrium about X axis, Equation (3) and the moment equilibrium about Y axis, Equation (4). It is mandatory to calculate the equations in the same coordinate system.

In addition, optionally, a fourth variable could be included with its respective equation.

$$P - \sum_{i=1}^n P_i - \sum_{i=1}^m T_i = 0 \quad (2)$$

$$M_{xx} - \sum_{i=1}^n M_{ixx} - \sum_{i=1}^m T_i Y_i = 0 \quad (3)$$

$$M_{yy} - \sum_{i=1}^n M_{iyy} - \sum_{i=1}^m T_i X_i = 0 \quad (4)$$

where,

- P = applied axial force.
- M_{xx} = applied moment about X global axis.
- M_{yy} = applied moment about Y global axis.
- P_i = axial force for segment i .
- M_{ixx} = moment about X global axis for segment i .
- M_{iyy} = moment about Y global axis for segment i .
- n = total segment number.
- T_i = force tension in anchor rod i .
- (X_i, Y_i) = global coordinates of anchor rod i .
- m = total anchor rods number.

Instead of considering the maximum concrete compressive stress constant by the design code, we could take it as a fourth variable and add the strain compatibility, Equation (5), to the equation system to reach equilibrium.

$$\sigma_c - \frac{T Y}{a n f} = 0 \quad (5)$$

where,

- T = maximum tension force on anchor rods.
- a = maximum tension anchor rod area.
- σ_c = maximum compressive stress.
- Y = bearing length.
- f = maximum anchor rod distance from NA.
- n = elasticity ratio.

$$n = \frac{E_s}{E_c} \quad (6)$$

- E_s = modulus of elasticity of steel.
- E_c = modulus of elasticity of concrete.

To calculate the tension, in the anchor rod i , we use a linear interpolation employing its local coordinate y , as shown in Equation (7).

$$T_i = \frac{T}{f} y_i \leq 0 \quad (7)$$

where,

- T_i = tension in anchor rod i .
- T = maximum tension force on anchor rods.
- f = maximum anchor rod distance from NA.
- y_i = local coordinate y of anchor rod i .

The compressive concrete stress for node i of a border segment is calculated by linear interpolation as shown in Equation (8).

$$\sigma_i = \frac{\sigma_c}{Y} y_i \geq 0 \quad (8)$$

where,

- σ_i = compressive stress at node i .
- σ_c = maximum compressive stress.
- Y = bearing length.
- y_i = local coordinate y of node i .

The equivalent force, moment about local x axis and moment about local y axis for a line segment between node i and node $i+1$ can be determined by the Equations (9), (10) and (11) respectively.

$$P_i = \frac{1}{2}(x_{i+1} - x_i)y_i\sigma_i + \frac{1}{2}(x_{i+1} - x_i)(y_{i+1} - y_i)\sigma_i + \frac{1}{6}(x_{i+1} - x_i)(y_{i+1} - y_i)(\sigma_{i+1} - \sigma_i) \quad (9)$$

$$M_{ixx} = \frac{1}{2}(x_{i+1} - x_i)y_i\sigma_i\frac{2}{3}y_i + \frac{1}{2}(x_{i+1} - x_i)(y_{i+1} - y_i)\sigma_i\frac{1}{3}(y_{i+1} + 2y_i) + \frac{1}{6}(x_{i+1} - x_i)(y_{i+1} - y_i)(\sigma_{i+1} - \sigma_i)\frac{1}{2}(y_{i+1} + y_i) \quad (10)$$

$$M_{iyy} = \frac{1}{2}(x_{i+1} - x_i)y_i\sigma_i\frac{1}{2}(x_{i+1} + x_i) + \frac{1}{2}(x_{i+1} - x_i)(y_{i+1} - y_i)\sigma_i\frac{1}{3}(2x_{i+1} + x_i) + \frac{1}{6}(x_{i+1} - x_i)(y_{i+1} - y_i)(\sigma_{i+1} - \sigma_i)\frac{1}{4}(3x_{i+1} + x_i) \quad (11)$$

where,

- P_i = axial force for segment i .
- M_{ixx} = moment about local x axis for segment i .
- M_{iyy} = moment about local y axis for segment i .
- σ_i = compressive stress for node i .

- σ_{i+1} = compressive stress for node $i+1$.
- (x_i, y_i) = local coordinates for node i .
- (x_{i+1}, y_{i+1}) = local coordinates for node $i+1$.

NEWTON-RAPHSON METHOD

This article explains briefly a method for a non-linear equations system based on Newton-Raphson, applied to a biaxial base plate problem case.

The problem is to obtain variable values, such that the functions proposed are balanced.

$$f_i(\sigma_c, \theta, Y, T) = 0 \quad (12)$$

We denote by X the entire variables vector, then each function could be expanded by Taylor series.

$$f_i(X + \delta X) = f_i(X) + \sum_{j=1}^4 \frac{\partial f_i}{\partial x_j} \delta x_j + O(\delta X^2) \quad (13)$$

Terms δX^2 and higher are neglected. Then, we obtain a set of four lineal equations to determine the correction vector δX , which moves the functions closer to zero simultaneously on each iteration. Since the system converges, the term in the Equation (13) $f_i(X + \delta X) = 0$, then we get,

$$\sum_{j=1}^4 \frac{\partial f_i}{\partial x_j} \delta x_j = -f_i(X) \quad (14)$$

The Equation (14) could be written in matrix notation as,

$$\begin{bmatrix} \frac{\partial P}{\partial \sigma_c} & \frac{\partial M_{xx}}{\partial \sigma_c} & \frac{\partial M_{yy}}{\partial \sigma_c} & \frac{\partial \sigma}{\partial \sigma_c} \\ \frac{\partial P}{\partial \theta} & \frac{\partial M_{xx}}{\partial \theta} & \frac{\partial M_{yy}}{\partial \theta} & \frac{\partial \sigma}{\partial \theta} \\ \frac{\partial P}{\partial Y} & \frac{\partial M_{xx}}{\partial Y} & \frac{\partial M_{yy}}{\partial Y} & \frac{\partial \sigma}{\partial Y} \\ \frac{\partial P}{\partial T} & \frac{\partial M_{xx}}{\partial T} & \frac{\partial M_{yy}}{\partial T} & \frac{\partial \sigma}{\partial T} \end{bmatrix} \begin{bmatrix} \delta \sigma_c \\ \delta \theta \\ \delta Y \\ \delta T \end{bmatrix} = - \begin{bmatrix} P \\ M_{xx} \\ M_{yy} \\ \sigma \end{bmatrix} \quad (15)$$

These matrices must be calculated by a lineal equations solver like the Gauss Method. Thus, we get the values δx to modify the variables in the next iteration,

$$x_{n+1} = x_n + \delta x \quad (16)$$

The iterative process stops when δx values are less than the tolerance and the variables' values do not change.

To determine the partial derivatives a numerical procedure is used as shown in Equation (17),

$$\frac{\partial f_i}{\partial x_j} = \frac{f_i(X + \Delta x_j) - f_i(X)}{\Delta x_j} \quad (17)$$

donde,

$$\Delta x_j \rightarrow 0 = \text{differential of variable } j.$$

PRACTICAL APPLICATION

To continue with the numerical example, we applied an external axial compression load of 90 kip, a moment about global X axis of 300 kip-in and another moment about global Y axis of 440 kip-in to the base plate. The steel of the base plate has a modulus of elasticity equal to 29000 ksi and the modulus of elasticity of the supporting concrete is 3122.02 ksi.

$$n = \frac{E_S}{E_C} = \frac{29000 \text{ ksi}}{3122.02 \text{ ksi}} = 9.289$$

Considering the strain compatibility for current example we will use four equations with four variables.

An important step to solve a non-linear equation system using the Newton-Raphson method, is to use an adequate set of initial values for the variables.

We suggest the following initial values for the problem.

$$\sigma_{c0} = 3.06 \text{ ksi}$$

We used an initial neutral axis angle as a function of applied loads.

$$\theta_0 = \tan^{-1} \left(\frac{M_{YY}}{M_{XX}} \right) - 180^\circ =$$

$$\tan^{-1} \left(\frac{440 \text{ ksi}}{300 \text{ ksi}} \right) - 180^\circ = -124.287^\circ$$

As initial bearing length we take the distance between the origin and the maximum compressed fiber of the base plate border.

$$Y_0 = 10.596 \text{ in}$$

The maximum tension in the anchor rod depends on another set of initial values and could be solved from Equation (5).

$$\begin{aligned} T_0 &= \frac{n \sigma_{c0} a f_0}{Y_0} = \\ &= \frac{9.289 \times 3.06 \text{ ksi} \times 0.306 \text{ in}^2 \times 8.352 \text{ in}}{10.596 \text{ in}} = \\ &= 6.856 \text{ kip} \end{aligned}$$

The next step is to determine the equivalent forces and moments in the local coordinate system for the concrete compression and anchor rods tension.

$$\begin{aligned} \sum_{i=1}^n P_i + \sum_{i=1}^n T_i &= \\ &= -17.893 \text{ kip} + 195.434 \text{ kip} = \\ &= 177.541 \text{ kip} \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^n M_{ixx} + \sum_{i=1}^n T_i y_i &= \\ &= 1232.358 \text{ kip} \cdot \text{in} + 118.91 \text{ kip} \cdot \text{in} = \\ &= 1351.269 \text{ kip} \cdot \text{in} \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^n M_{iyy} + \sum_{i=1}^n T_i x_i &= \\ &= 0.001 \text{ kip} \cdot \text{in} + 0.001 \text{ kip} \cdot \text{in} = \\ &= 0.002 \text{ kip} \cdot \text{in} \end{aligned}$$

Next, forces and moments must be transformed to the global coordinate system to obtain the equilibrium differences.

$$\begin{aligned} P - \sum_{i=1}^n P_i - \sum_{i=1}^m T_i &= \\ &= 90 \text{ kip} - 177.541 \text{ kip} = -87.541 \text{ kip} \end{aligned}$$

$$\begin{aligned} M_{xx} - \sum_{i=1}^n M_{ixx} - \sum_{i=1}^m T_i Y_i &= \\ &= 300 \text{ kip} \cdot \text{in} - 761.19 \text{ kip} \cdot \text{in} = \\ &= -461.19 \text{ kip} \cdot \text{in} \end{aligned}$$

$$\begin{aligned} M_{yy} - \sum_{i=1}^n M_{iyy} - \sum_{i=1}^m T_i X_i &= \\ &= 440 \text{ kip} \cdot \text{in} - 1116.409 \text{ kip} \cdot \text{in} = \\ &= -676.409 \text{ kip} \cdot \text{in} \end{aligned}$$

$$\begin{aligned} \sigma_c - \frac{T Y}{a n f} &= \\ &= 3.06 \text{ ksi} - \frac{6.856 \text{ kip} \times 10.596 \text{ in}}{0.306 \text{ in}^2 \times 9.289 \times 8.352 \text{ in}} = \\ &= 0 \text{ ksi} \end{aligned}$$

This procedure must be repeated until the equations converge to zero using a numerical method for a non-linear equations system. In the current example the Newton-Raphson method was employed.

After six iterations the system converges and the results below are calculated:

$$\sigma_c = 1.124 \text{ ksi}$$

$$\theta = -124.119^\circ$$

$$Y = 13.319''$$

$$T = 1.354 \text{ kip}$$

In Figure 4, the base plate to concrete contact stresses are shown. The steel anchor rods tension forces are displayed in Figure 5.

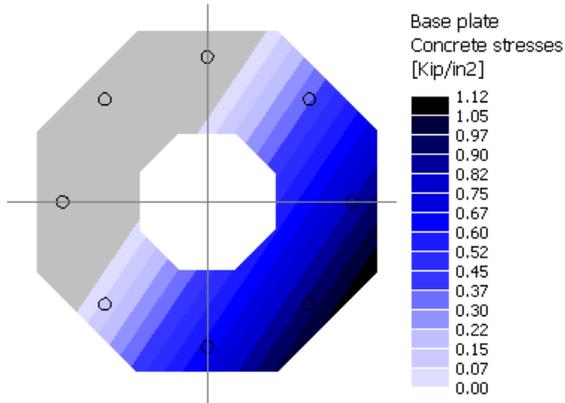


Figure 4. Concrete stresses

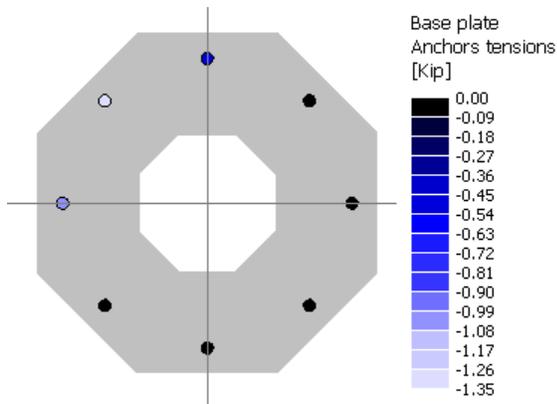


Figure 5. Anchor rods tension forces

REFERENCES

1. Blodgett, Omer W., *Design of Welded Structures*, 1982.
2. Dake, Richard M. and Elkin, Sharon J., *Beam-Column Base Plate Design LRFD Method*, 1999.
3. Rodríguez, J. A. and Aristizabal, J. Dario, *Contact Pressure Caused by a Rigid Footing of Any Shape Subjected to Eccentric Axial Load*, 2004
4. *American Institute of Steel Construction (AISC) Manual*, 13th Edition, 2005
5. *Base Plate and Anchor Rod Design*, Steel Design Guide 1, 2nd Edition, 2006

CONCLUSIONS

Since the developed procedure is an extension of uniaxial methods, widely accepted for the analysis of base plates, it also gives accurate results for moments about one axis.

Another advantage, of the presented biaxial procedure calculation, is to be an alternative to the finite element method (FEM) for the analysis of special projects with arbitrary base plate shapes with openings. This method could be easily abstracted into a computational algorithm.