Sources of Nonlinear Deformations in Light Frame Wood Shear Walls

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Abstract

The wood shear wall deflection formula as specified in the IBC Eq. (23-2) is critical in understanding the diaphragm rigidity, inter-story drift and serviceability issues resulting from excessive drift. The formula has four deflection components which are derived independently and superimposed to get the overall deflection under a given loading. Under static loading a shear wall system acts as a combined unit which is composed of framing, sheathing, nailing, and hold-down connections. As observed in experiments the response of the system under such a loading is non-linear and coupled and hence cannot be simply broken into simpler components as specified in the equation. This paper presents a parametric study of these components for various shear walls. A virtual work based methodology is utilized to uncouple these components under static loading. It is shown that each component contribution is nonlinear and not just the nails as specified in the formula. The nonlinearity arises due to the nature of the problem which is not represented in the simplified formula. It is also found that the simplified formula is conservative for deflection calculation of shear walls but may not be conservative for computing wood diaphragm flexibility.

Introduction

The International Building Code (IBC, 2006) classifies a wood structural diaphragm as rigid or flexible based on the prediction of deflections of diaphragm and shear walls. To make this prediction the code specifies two simple equations for diaphragms and shear walls. These equations govern the nature of lateral load distribution applied to a wood structural system and hence differences caused by the inaccurate prediction made by these equations may be critical in seismic design. In regions where wind design governs, the inaccurate prediction of inter-story drift from these equations may cause serviceability problems. This paper undertakes the study of this IBC specified deflection formula for shear walls which is shown in Eq. (1). As shown, the shear wall deflection formula has four components: (a) bending from chords which are acting as flanges, (b) shear from wall sheathing, (c) nail slip causing relative movement between the sheathing and the framing, and (d) slip in hold-down anchorage causing relative movement between the chords and the foundation. These components represent the contribution from various structural elements constituting the shear wall. This simplified formula shows the response of different structural elements as uncoupled whereas in reality due to significant redundancy and complex behavior they are coupled. In fact, the whole formula is derived by summing up independently derived component contributions which is only applicable to linear systems (Breyer et al., 1999). Various experiments on wood shear walls have shown that their lateral load-displacement behavior is nonlinear and the nonlinearity kicks in at very small load levels as the wall starts to deflect laterally (Foschi, 1982; Easley et al., 1982; McCutcheon, 1985; White and Dolan, 1985). This nonlinear behavior is due to the nail connection between sheathing panels and framing studs. The third component in Eq. (1) is supposed to capture this nonlinear effect. The missing component from this equation is the effect of interaction between structural elements constituting the shear wall. This simplified formula does not represent this interaction and hence it may be inaccurate. This is a subject of discussion in this paper which focuses on capturing the interaction of framing, nails and sheathing. The study neglects the slip in hold down anchorage and hence fourth term in Eq. (1) is set to zero. Table 1 shows the nail slip equations for nails used in the study. This table is adapted from Table A-2 of APA (2007) and is equivalent to UBC (1997) Table 23-2-K and IBC (2006) Table 2305.2.2(1).

\[
\Delta = \frac{8vH^3}{EAb} + \frac{vH}{Gt} + 0.75He_a + da \frac{H}{b}
\]

where,

- \(v\) is lateral load per unit length of wall (lb/ft),
- \(H\) is height of wall (ft),
- \(b\) is width of wall (ft),
- \(E\) is modulus of elasticity of boundary element (psi),
- \(Gt\) is panel rigidity through the thickness (lb/in),
- \(A\) is area of cross-section of boundary elements (in\(^2\)),
- \(d_a\) is vertical elongation of overturning anchorage (in),
Table 1: Nail Slip Equations

<table>
<thead>
<tr>
<th>Fastener</th>
<th>Minimum Penetration (in)</th>
<th>For Maximum Loads up to</th>
<th>Approximate Slip $e_n$, $e_{n/2}$ (in)</th>
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</thead>
<tbody>
<tr>
<td>6d common nail</td>
<td>1-1/4</td>
<td>180</td>
<td>(V_n/434)$^{0.764}$ (V_n/345)$^{0.764}$</td>
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<tr>
<td>(0.113” x 2”)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8d common nail</td>
<td>1-7/16</td>
<td>220</td>
<td>(V_n/857)$^{0.588}$ (V_n/616)$^{0.588}$</td>
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<tr>
<td>(0.131” x 2-1/2”)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>10d common nail</td>
<td>1-5/8</td>
<td>260</td>
<td>(V_n/977)$^{0.688}$ (V_n/669)$^{0.688}$</td>
</tr>
<tr>
<td>(0.148” x 3”)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Fabricated green/tested dry (seasoned); fabricated dry/tested dry. $V_n$: fastener load.
(b) Values are based on Structural-I panels fastened to Group-II lumber, specific gravity 0.50 or greater. Increase slip by 20% when panels are not Structural-I.
(c) ASD basis.

Objective and Scope

The objective of this work is to study the accuracy of first three terms in the commonly used Eq. (1) above. A parametric study of this deflection equation is performed by varying different parameters in shear wall analytical models. The shear walls are modeled using the finite element approach where each individual nail is modeled along with sheathing panels and framing members.

A detailed finite element model is used here as it helps to incorporate various components into the model and also to capture their response. This is also required for the computation of component contribution under lateral loading. The computation of component contribution is accomplished using a virtual work based approach also known as “Displacement Participation Factor” approach. This methodology has been used in the past to compute component contribution in a framed structural system (Velivasakis and DeScenza, 1983; Baker, 1990; Charney, 1990, 1991, 1993; Kim and Shin, 2006). The use of this approach was extended to finite elements by Charney et al., (2005) where the authors used it to calculate the effective shear area of steel wide flange sections. The same approach was used by Charney and Pathak (2008) and was extended to solid finite elements to study the sources of elastic deformations in steel beam-column subassemblages. However, so far this approach is only utilized for linear elastic systems. This paper extends the approach to nonlinear inelastic systems for the first time. A few examples verifying the accuracy of this approach are also presented.

A nonlinear static pushover analysis is performed on all the shear wall models and individual component contributions are computed. These computed contributions from nails, framing and panels are used in comparison with the corresponding component in the formula shown in Eq. (1). A significant difference between the nail response is expected as the nail properties used in the finite element model are extracted from the most recent results of the cyclic nail tests (Ekiert and Hong, 2006) which were conducted at the University at Buffalo as part of the NEESWood benchmark shake table test program (Filiatrault et al., 2007).

Shear Wall Finite Element Model

Various finite element models of wood shear walls have been proposed by researchers in the past and their experimental validation has also been performed (Dolan, 1989; Dolan and Foschi, 1991). The modeling methodology described in this section has been validated by the author (2008) using the experimental results from Dolan (1989) and analytical results from the ABAQUS model of Judd (2005). The shear wall finite element modeling details used in this study is depicted in Fig. (1). The horizontal and vertical studs are modeled using frame elements. The side studs and top plates are modeled using a single frame element having double thicknesses. The vertical frames have moment releases at their connections with the plate and sill frames. The horizontal blocking frames also have moment releases at their connections with the vertical studs. The material for the stud frames is assumed to be isotropic. The sheathing is modeled using shell elements and is connected to frame elements on the perimeter and inside region using two node link elements having an oriented spring pair (Judd and Fonseca 2005). The oriented spring pair represents the nailing between sheathing panel and framing. The direction in which each nail moves when the initial loading is applied is set as the initial orientation for the nail in their oriented spring pair connection. This initial orientation changes as the analysis proceeds but in this study it is kept constant throughout the analysis. The edge interaction between the adjacent sheathings is not considered in the modeling. The material for sheathing is defined as orthotropic and the spring properties for the oriented spring pair are defined using Modified Stewart Hysteresis model. This hysteresis model was developed by Fozl and Filiatrault (2001) as a part of the CASHEW wood frame project and has previously been used in modeling dowel type connections in wood structures. This is a ten parameter model and its force-deformation behavior under monotonic and cyclic loading is as shown in Fig. (2). The monotonic loading portion of this model was initially proposed by Foschi (1977) for modeling connections and requires only six parameters which may be obtained from the experimental data to characterize the curve. The region $J_0J_1$ and $J_3J_4$ are the only subject of interest here as all the models are subjected to only positive monotonic loading. Mathematically, these paths are modeled as exponential as shown in Eq. (2) which is typically observed in the experiments and explains the very early onset of nonlinear behavior in shear wall response.
Virtual work is defined as the “work done by either virtual forces acting through real displacement or by real forces acting through virtual displacement”. In this definition the force could also be a moment, and displacement could also be a rotation. The principle of virtual forces and displacements are applicable to both rigid and deformable systems. This application study requires only the knowledge of principle of virtual forces and hence here it discusses only that. To demonstrate the calculation of DISPAR using this principle a two-story framed structure as shown in Fig. (3) is considered. A real force $R$ is applied at the top location 1 which causes the structure to deform generating strain energy in the system. The goal here is to obtain the horizontal displacement at location 2.

To accomplish this it is assumed that there is a virtual force $\nu$ already present and the structure is in equilibrium when $R$ gets applied. In this case, the total external work is done by real force $R$ acting through displacement $\Delta_1$ and virtual force $\nu$ acting through displacement $\Delta_2$. As the virtual force is assumed to be already present when real force $R$ is applied the internal work is done by virtual stresses $\{\bar{\sigma}\}$ over the real strains $\{\varepsilon\}$ in addition to work done by real stresses $\{\sigma\}$ over the real strain $\{\varepsilon\}$. The total internal work done is over the entire volume of the structural system. The total external and internal work done is shown in Eqns. (3) and (4), and as both of them must be the same, these two terms can be equated to obtain Eq. (5). Also, as the magnitude of virtual force is arbitrary, it can be set to zero resulting in Eq. (6). So mathematically, it can be seen that there is a one to one correspondence between real and virtual terms in left and right hand side of Eq. (5) and hence the virtual terms can also be set equal to each other as shown in Eq. (7). This is the concept of virtual forces and has been found very useful in computing displacements in structural systems. The virtual force term can take any value but often a unit force is used and that is why this method is also sometimes referred to as unit force method.

\[
P = (P_0 + r_1K_0\delta)(1 - e^{-\frac{K_0}{P_0}\delta}), \text{if } \delta \leq \delta_{\text{ult}} \tag{2}
\]

\[
W_e = \frac{1}{2}R\Delta_1 + \nu\Delta_2 \tag{3}
\]

\[
W_i = \int_{\text{VOL}} \frac{1}{2} \{\sigma\}^T\{\varepsilon\}dV + \int_{\text{VOL}} \{\bar{\sigma}\}^T\{\varepsilon\}dV \tag{4}
\]

\[
\frac{1}{2}R\Delta_1 + \nu\Delta_2 = \int_{\text{VOL}} \frac{1}{2} \{\sigma\}^T\{\varepsilon\}dV + \int_{\text{VOL}} \{\bar{\sigma}\}^T\{\varepsilon\}dV \tag{5}
\]

\[
\frac{1}{2}R\Delta_1 = \int_{\text{VOL}} \frac{1}{2} \{\sigma\}^T\{\varepsilon\}dV \tag{6}
\]
The approach discussed so far works well for linear systems as no step loading or iteration is required, however when a system is nonlinear the loading cannot be applied in one step to obtain a displacement solution. Nonlinear system solution requires that a step loading be applied and some kind of convergence iteration (Newton-Raphson) be performed within a load step to maintain equilibrium. In such a situation, the displacement participation of an element at any given load level is the accumulated displacement participation from all previous load steps. The displacement participation at each load step must be calculated in the current state of the system for the method to still be valid. The implementation of this strategy for large complex structural systems is a challenge when real and virtual loads are not acting at the same location and direction. In cases where real and virtual loads are at the same location and direction one may easily use the magnitude of virtual force to be the same as the real force. Other cases may require additional exercises to not perturb the current state of the system when the virtual force is applied. The discussion and solution to the latter is beyond the scope of this paper as this application study is only interested in the displacement at the location and direction of real loading. To obtain the DISPAR equation for displacement at the point of real loading, Eq. (8) is written in an incremental form and the result is shown in Eq. (9). This equation in incremental format is for the plane problem. The derivation of Eq. (8) from Eq. (7) is left as an exercise for the reader.
frame structural system which has all linear elements and the same formulation is valid for structures with nonlinear elements.

\[
\Delta_i = \sum_{j=1}^{\text{NumLoadSteps}} \delta \Delta^j_i
\]

\[
= \sum_{j=1}^{\text{NumLoadSteps}} \sum_{n=1}^{\text{NumColumns}} \left( \sum_{l=1}^{\text{NumBeams}} \left[ \frac{1}{\delta R^j} \int_0^R \left( \delta P_{c, i}^j(x) \delta P_{e, i}^j(x) \right) \frac{dx}{EA_c} + \delta M_{c, i}^j(x) \delta M_{e, i}^j(x) \frac{dx}{EI_c} \right] + \delta V_{i}^j(x) \delta V_{j}^i(x) \frac{dx}{GA_c} \right)
\]

(9)

To demonstrate the virtual work application to nonlinear systems, a simple example of a cantilever steel column with a prismatic I section is now presented. The column is shown in Fig. (4) and is subjected to a gradually increasing horizontal load at its tip. Yielding starts at the point of maximum stress which occurs at a section with maximum moment, which in a case of cantilever lies at its support.

To simplify the problem, cross-section yielding at the column support is idealized with a discrete rotational spring as shown in Fig. (5). The Moment-Rotation property of the spring is shown in the adjoining plot. It is desired to determine the displacement \( \Delta_T \) at the column tip due to horizontal load \( V \).

In this given loading situation, there are only three components contributing to displacement \( \Delta_T \) i.e. flexure deformation in the column (\( \Delta_{CF} \)), shear deformation in the column (\( \Delta_{CS} \)), and flexure deformation in the support spring (\( \Delta_{SF} \)). Their sum is represented in Eq. (10), and Eq. (11) presents a detailed familiar version of it when principle of virtual forces is applied. Eq. (11) assumes that the rotational spring is always linear elastic but it can easily be extended to bilinear spring as shown in Eq. (12).

\[
\Delta_T = \Delta_{CF} + \Delta_{CS} + \Delta_{SF}
\]

(10)

\[
\Delta_T = \frac{1}{V} \left( \int_0^R \frac{VH}{EI} \frac{dx}{x} + \int_0^R \frac{VH}{GA} \frac{dx}{x} + \frac{VH}{K_r} \right)
\]

(11)

\[
\Delta_T = \frac{VH^3}{3EI} + \frac{VH}{GA} + \frac{VH}{K_r} \quad \text{when} \; VH \leq M_y
\]

(12)

\[
\Delta_T = \frac{VH^3}{3EI} + \frac{VH}{GA} + \frac{M_y H}{K_r} + \left( \frac{VH^2 - M_y^2}{\alpha K_r} \right) \quad \text{when} \; VH > M_y
\]

This solution is applied to the problem in discussion using the geometric and material properties from Table 2. Fig. (6) shows various component contributions under the incremental horizontal loading. It can be seen that during the initial loading a majority of the displacement contribution comes from column flexure but after reaching a certain load level, the spring flexure contribution takes over. This transition does not happen at the yielding of rotational spring but at a slightly higher load level. The transition point can easily be shown to be a function of strain hardening ratio \( \alpha \) in this case.

| Table 2: Column Material and Geometric Properties |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Column Properties | E (kips/in^2) | G (kips/in^2) | I (in^4) | A (in^2) | H (in) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| E  | 29000 | 11154 | 1550 | 10.2 | 120 |
| G  | 11154 | 1550 | 10.2 | 120 |
| I  | 1550 | 10.2 | 120 |
| A  | 10.2 | 120 |
| H  | 120 |
| Rotational Spring Property | K_r (kips-in/ rad) | 3600000 | 0.02 | 72000 | 7200 |
| α  | 0.02 | 72000 | 7200 |
| K_r (kips-in/ rad) | 3600000 | 0.02 | 72000 | 7200 |
This example is statically determinate so a closed form solution is readily obtained and the components are uncoupled. In the case of a redundant and indeterminate structural system like a wood shear wall there is no closed form solution but the displacement participation theory is applicable. The DISPAR equation for walls has contribution from frames, panels, nails and anchorage. The formulation presented here will not consider any contribution from anchorage; the remaining terms are shown in Eq. (13). The components are defined along the local axes of the finite elements used in representing the structural elements. The loading applied is in the plane of the wall and none of the elements are assumed to be stressed out-of-plane, and hence out-of-plane terms can be omitted from the formulation. In the analysis performed on shear walls, the forces are readily available for frame elements and nail springs, and stresses are available for panel shells, hence the displacement participation is represented in terms of forces and stresses respectively. These equations are derived for uniformly incremental real loading $\delta P$ and are shown in Eqns. (17), (18) and (19). Two nomenclatures are defined to represent incremental force and stress responses. $\Delta R^{i,j}$ component, direction nomenclature is used for incremental force response of the $i^{th}$ element at the $j^{th}$ load step of real load for a particular component in a given element local axis direction. Similarly $\Delta \sigma^{i,j}$ direction, $\Delta \tau^{i,j}$ direction, represent stress responses in the $i^{th}$ element at the $j^{th}$ load step in its local axis direction for the real loading. As the goal here is to obtain the displacement at the location of real load, the virtual response can be replaced with the real response in the DISPAR equations. The DISPAR obtained for an incremental load is referred here as incremental (δ) DISPAR and the total displacement at a load level is the cumulative sum of the all component DISPARs calculated at previous load increments.

$$\Delta \tau = \Delta_{\text{frame}} + \Delta_{\text{Panel}} + \Delta_{\text{Nail}}$$  \hspace{1cm} (13)

$$\Delta_{\text{Frame}} = \Delta_{\text{Axial,xx}} + \Delta_{\text{shear,yy}} + \Delta_{\text{shear,zz}} + \Delta_{\text{Flexure,yy}} + \Delta_{\text{Flexure,zz}} + \Delta_{\text{Torsion,xx}}$$  \hspace{1cm} (14)

$$\Delta_{\text{Panel}} = \Delta_{\text{Axial,xx}} + \Delta_{\text{Axial,yy}} + \Delta_{\text{Axial,zz}} + \Delta_{\text{Shear,xy}} + \Delta_{\text{Shear,zy}} + \Delta_{\text{Shear,xz}}$$  \hspace{1cm} (15)

$$\Delta_{\text{Nail}} = \Delta_{\text{Axial,xx}} + \Delta_{\text{Shear,yy}} + \Delta_{\text{Shear,zz}} + \Delta_{\text{Flexure,yy}} + \Delta_{\text{Flexure,zz}} + \Delta_{\text{Torsion,xx}}$$  \hspace{1cm} (16)

As the frame and shell elements are linear, the stiffness terms in Eqns. (17) and (18) do not change with load but the stiffness term in Eq. (19) for nails is load dependent and is shown with the superscript $j$. As previously mentioned, the nonlinear action begins at very early stages of loading. At each load step there is a redistribution of forces between framing, nails and the sheathing panel which may cause other components contribution to be nonlinear as well. This is the behavior which is the subject of this study and in the author’s knowledge can only be captured using the aforementioned virtual work technique.
Nonlinear Virtual Work Verification

This section presents an example verification study of the analysis program used in the parametric analysis. The WoodFrameSolver is a finite element analysis program which has previously been extensively verified for the linear virtual work approach (Pathak, 2008). The nonlinear extension is a recent implementation and is computationally very intensive for finite element models. The nonlinear virtual work computational complexity is roughly the number of load steps times the linear virtual work computational complexity for the same problem. This is just a note and not a barrier anymore given the kind of computing speed and memory space available in the computers today. To verify the nonlinear virtual work implementation, a comparison is made between the displacements obtained from the finite element and the DISPAR analyses. The DISPAR solution is computed by integrating the forces and stresses which is obtained from finite element analysis. It is to be noted that result from both approaches may completely match but still be inaccurate because the solution accuracy depends upon the realistic representation of the problem.

The results obtained for the earlier cantilever example are matched with the DISPAR results obtained using finite element analysis. The problem is modeled using a single frame element with a rotational spring at its support and load is applied incrementally. The results using the two approaches are presented in Fig. (7) where a perfect match can be seen at all the load levels between stiffness, DISPAR and hand calculation. One should note here that the DISPAR and hand calculation use the same formulae and hence are expected to match, however the match between stiffness (finite element) and DISPAR completes the verification. The beauty of the virtual work method lies in its ability to decouple various responses which are coupled in a stiffness solution.

Parametric Study

Shear wall’s stiffness is a function of various parameters, and using all may result in an overwhelming number of models. For this reason, this study is limited only to nail type, nail spacing, number of vertical studs and number of sheathing panels. The study is performed on 8 ft by 8 ft shear walls which are most commonly used in practice. The primary grouping of the models is done based on the nail type and within each primary group nail spacing, number of panels and number of vertical studs are varied. As the parameters used here generate multiple combinations, a system is defined to quickly identify a wall. A wall is always tagged using an unique identification in the form GSPF where G stands for group, S stands for nail spacing, P stands for number of panels and F stands for number of frames.

Table 3 presents all the 36 combinations of walls generated for analyses using the parameters. All the walls are made of two double side studs, interior studs, a double top plate, a bottom sill, mid level blocking and OSB sheathing. The sheathing to framing connection is made using galvanized common nails always spaced at 12” center to center in the field. The framing material is considered to be Hem-Fir and double side studs and top plates are modeled using a single cross-section of double thickness. The material and section properties used in the analysis are shown in Table 4. The loading is applied on shear wall models in small increments and final loading is restricted close to the ultimate loading capacity of individual shear walls. Fig. (8) shows an example shear wall from group 1, with 4” nail spacing, 2 sheathing panels and 9 vertical studs. Using the nomenclature above this wall has a unique identification of 1429.
Table 3: Wall Unique Identification (GSPS: G-Group, S-Nail Spacing, P-Number of Panels, and F-Number of Studs)

<table>
<thead>
<tr>
<th>#</th>
<th>10d</th>
<th>8d</th>
<th>6d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1619</td>
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</tr>
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<td>12</td>
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Table 4: Shear Wall Properties used in Finite Element Analysis

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>DIMENSION</th>
<th>PROPERTIES</th>
<th>FINITE ELEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOP PLATE, SIDE STUD</td>
<td>b x h = 3 in x 3.5 in</td>
<td>E = 1400 kip/in², μ = 0.33</td>
<td>2 node frame</td>
</tr>
<tr>
<td>BOTTOM PLATE, INTERIOR STUD</td>
<td>b x h = 1.5 in x 3.5 in</td>
<td>E = 1400 kip/in², μ = 0.33</td>
<td>2 node frame</td>
</tr>
<tr>
<td>OSB PANEL</td>
<td>thickness = 7/16 in</td>
<td>E = 714 kip/in², G = 218 kip/in², μ = 0.33</td>
<td>4 node shell</td>
</tr>
</tbody>
</table>

NAIL

<table>
<thead>
<tr>
<th>10d</th>
<th>K_b (kip/in)</th>
<th>P_0 (kip)</th>
<th>r_l</th>
<th>δ_ult (in)</th>
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<tbody>
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<td>0.33</td>
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Discussion of Results

All the finite element models of shear walls are analyzed and the DISPAR of frame, panels and nails is computed at all load levels. A similar computation is also performed using Eq. (1) for corresponding components at corresponding load levels for comparison purposes. The load displacement curve obtained using DISPAR and stiffness solutions is found to be closely matching for all the walls and is only presented here for a few walls from group 2 in Fig. (9).

Frame Component

The frame components obtained from models in different wall groups are compared in Fig. (10). The curve obtained from the frame component of Eq. (1) is linear as compared to the corresponding response obtained from finite element models which look linear up to a certain load level and then depicts a highly nonlinear response. The finite element frame response includes shear, bending and axial flexibility of the framing. The nonlinear response after a certain load level can be attributed to highly nonlinear distribution of forces between nail and framing. The frame component using the code formula is same for all the walls studied in this paper. This is because it is only dependent upon the lateral load, geometry of shear wall, modulus of elasticity of exterior studs and their area of cross-section. However, this is not true for the DISPAR results of the frame elements as can be seen in the figure. It is also observed that the frame response using Eq. (1) is underestimated which may be due to cantilever assumption (stiffer system) and the absence of shear and axial flexibility of framing in the derivation of the frame component. The walls in group 3 are modeled using 6d nails which start to yield earlier compared to 8d and 10d nails, hence an early nonlinear action is seen in frames in this group.
followed by wall group 2 and then 1. Theoretically, if more nails are added to the system the more stiff the system would get. This provides a higher margin for linear distribution of forces between panel, nail and framing. This behavior is also observed in all the groups between their walls using different nail spacing i.e. smaller the nail spacing, more the number of nails used and hence more linear response is obtained from the framing.

Panel Component

The panel components obtained from models in the different wall groups is compared in Fig. (11). The panel component from Eq. (1) is same for all the walls and is found to be close to corresponding DISPAR computations. The DISPAR computation from all the finite element models do not show any significant variation between their responses when corresponding walls between different groups are compared. The code specified and finite element responses are almost aligned during the early stages of loading where nonlinear action is small. This seems to change though but not very significantly at the higher load levels for individual walls. It can be seen that panel component behavior is similar to the frame component but nonlinear response is not at all prominent at higher loading levels like it is for frames. This may be due to more linear distribution of forces in the panels compared to framing when incremental loading is applied.

Nail Component

The nail components obtained from different models are compared in Fig. (12). Using Eq. (1) the nail contribution for a given nail type and given loading, is the same for all the models which have the same perimeter nail spacing. The nail contribution is defined using the power curves shown in Table 1 and were derived empirically. The contribution from finite element analysis is derived for nails which are represented using exponential curves shown in Fig. (2). In addition, the computation of DISPAR from nails incorporates the interaction between nails, panel and framing. As expected a complete mismatch is seen between the response between Eq. (1) and the DISPAR computation.

The nail component contribution from Eq. (1) is both underestimated and overestimated at the smaller and larger load levels respectively. The only deviation from this trend is seen in walls of group 1 which use 10d nails. In this case, the IBC formula always underestimates the nail component contribution. It is known that the code formula ceases to be valid after a certain load threshold in the nail but the nails in the finite element models are observed to carry more loads. This is due to the significant discrepancy which exists between the code formula and the more recently published nail-slip data.
**Figure 11:** Panel Contribution comparison between IBC and DISPAR response

**Figure 12:** Nail Contribution comparison between IBC and DISPAR response
**Total DISPAR and IBC computed Static Drift**

The total contributions obtained from different models are compared in Fig. (13). The DISPAR response simulates the nonlinear inelastic behavior of shear walls but the code response presented here is elastic. The code deflection response can be translated to an inelastic response by multiplying it by deflection amplification factor, $C_d$ obtained from the Table 12.2-1 in ASCE 7-05. Using the $C_d$ value of 4, it is noted that IBC formula is always conservative when comparing deflections computed at same load levels. It is also found that the nails in the shear wall finite element models analyzed here are able to carry more loads compared to what is specified for same nails in Table 1 which limits its use only up to a certain load. This is because there is a redistribution of nail forces in finite element models, causing nails to see lesser forces at the same load level than what is used in Eq. (1). It is also noted that each nail in the analytical model sees a different force compared to the uniform force assumed for nails in the formula. The load redistribution effect is missing in the simplified equation.

Eq. (1) is also used in calculating wood diaphragm flexibility which is computed as the ratio of maximum deflection in diaphragm to average drift in shear walls. This ratio is not deflection amplification factors dependent if $C_d$ is the same for both diaphragm and wall. The deflection results for wall group 1 using Eq. (1) is always conservative and hence may affect the diaphragm flexibility calculation. The conclusion is not very confident as further study needs to be performed on the diaphragms, the results of which may offset the error. The static drift using Eq. (1) for wall groups 2 and 3 are both underestimated and overestimated at lower and higher load levels respectively. Hence, diaphragm flexibility measured using deflections from an overdesigned wall from wall group 2 and 3 may also be erroneous.

**Summary, Conclusions and Future Work**

This paper presents an application study of nonlinear virtual work for the first time. Linear virtual work method has been identified in the past as a remarkable tool for computing displacements and contribution to it from various components in a structure. This paper has extended the approach in a nonlinear structural analysis framework using finite elements. Various light frame wood shear walls are modeled using finite elements and a comparison is made between displacement contributions obtained using virtual work and the more simplified formula obtained in IBC. It is observed that the component contribution from framing elements is nonlinear as compared to what is specified in the code based formula which is also always underestimated.
The underestimation is primarily due to the cantilever assumption of the framing and the missing shear and axial flexibility contribution when deriving the frame component. The panel contribution is found to be reasonably accurately represented by the IBC formula. The comparison of nail contribution between the models and code shows that nail contribution is underestimated and overestimated at lower and higher loading respectively. The reason for this is because the nail empirical formula is not on par with current nail models used in finite element models. The latter is calibrated from the latest experimental research. The IBC inelastic drift is found to be conservative for all the models when compared to the corresponding sum of nail, frame and panel DISPAR. The analysis also shows that diaphragm flexibility may be affected using the code formula but needs further study using diaphragms. Overall, more models need to be analyzed with anchorage also included in the analysis for a more complete study.

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