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# Practical Methodology for Transitioning to a 3D Stability Analysis for Geotechnical Consultants



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# Practical Methodology for Transitioning to a 3D Stability Analysis for Geotechnical Consultants

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**ABSTRACT:** The analysis of slopes by 3D techniques such as the Limit Equilibrium Method (LEM) has received increased attention in recent years. The increased use has resulted in additional questions surfacing relating to the use of such analysis in consulting practice. Consulting engineers are interested in understanding the potential differences between 2D and 3D analysis as it relates to the modeling of either backward or forward analyses. It is generally understood that the calculated factor of safety (FOS) between a 2D and 3D analysis is different, and that the 3D FOS will be higher than the 2D FOS. It is also noted that the 2D analysis generally considers the analysis of a cylindrical failure surface with no shear resistance on the ends. This white paper defines the key differences between a 2D and a 3D analysis. A method of introducing the influence of ground surface topology in a 3D analysis as a second step and stratigraphy variation of a third influence is also be introduced. Lastly, there is a brief discussion related to the role of a 3D analysis within the existing design framework.

**KEYWORDS:** *3D slope stability, stability analysis, shear strength reduction, 2D stability analysis.*

## 1.0 INTRODUCTION

The traditional application of the limit equilibrium method (LEM) has been considered within the context of a 2D plane strain analysis. Geotechnical engineers have become accustomed to routinely using 2D slope stability as it is easy to perform. 2D analyses suffer from a number of fundamental limitations, foremost of which is: i) the slip shape is usually assumed to be cylindrical; ii) the ground surface geometry is usually assumed to be unchanged in the 3rd dimension; iii) the geo-strata are usually assumed to be unchanging in the 3rd dimension. Such assumptions have generally proven 2D analysis to be conservative with respect to a more accurate 3D factor of safety in the amounts between 10-50% (Domingos, 2016). If unsaturated aspects of slope stability are also considered, then the difference can be even higher (Zhang, 2015). Significant opportunity exists to optimize existing designs through the application of 3D slope stability analysis. It should be noted that the difference between a 2D and a calculated 3D factor of safety is different in each situation. It is generally impossible to assume a specific 2D/3D difference for a particular scenario without actually performing both 2D and 3d analyses.

The theory for the 3D LEM has been in existence for decades and is therefore not new. Early theoretical development efforts are available in research literature (Howland, 1977; Hungr, 1987, Hungr et al., 1989, and Fredlund, 1977). The primary limitation associated with all presented theory is that the slip direction is assumed to occur along the x-coordinate axis. Such a limitation introduces a significant problem for the practical application of a 3D slope stability analysis. This white paper presents an extension to the traditional 3D LEM which allows for the analysis of a sliding mass in any direction. Several benchmark examples are presented such that the implementation of the methodology is clearly illustrated. The technique may be applied to Bishop, Morgenstern-Price, GLE, Spencer and other limit equilibrium methods of analysis.

The application of a 3D slope stability limit equilibrium method of analysis requires consideration of the effects of: i) the **slip surface shape** being elliptical, block-based, or composite; ii) the effects of the

**ground surface geometry; iii) the effects of geo-strata in all dimensions.** This article examines the potential influence of all three effects. A recommended approach/methodology for the application of a 3D slope stability analysis within geotechnical engineering practise is also presented.

The application of software tools to analyze the stability in 3D has traditionally been highly limited. Recent developments allow for the easy application of 3D stability analysis to typical example problems (Fredlund, 2015; Reyes, 2014). The SVSLOPE software developed by SoilVision Systems Ltd. is utilized for the analysis presented in this white paper.

### 1.1 Continuity between 2D and 3D LEM

It is important to understand the difference between the slip shape analyzed in a 2D analysis as opposed to a 3D analysis. In a 2D analysis the slip shape is extended infinitely in the third dimension and shear forces on the end surfaces are not considered. An example 2D analysis is presented in Figure 1. The equivalent 3D analysis is shown in Figure 2 where there are no shear forces applied to the vertical end surfaces.

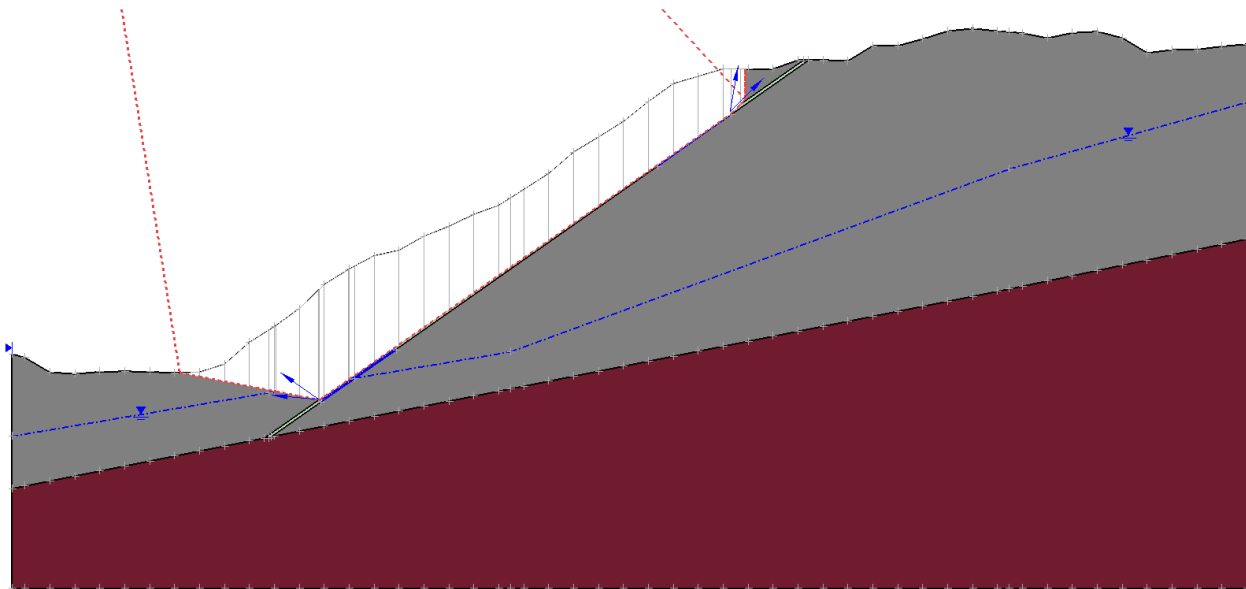


Figure 1 Example 2D slope stability analysis

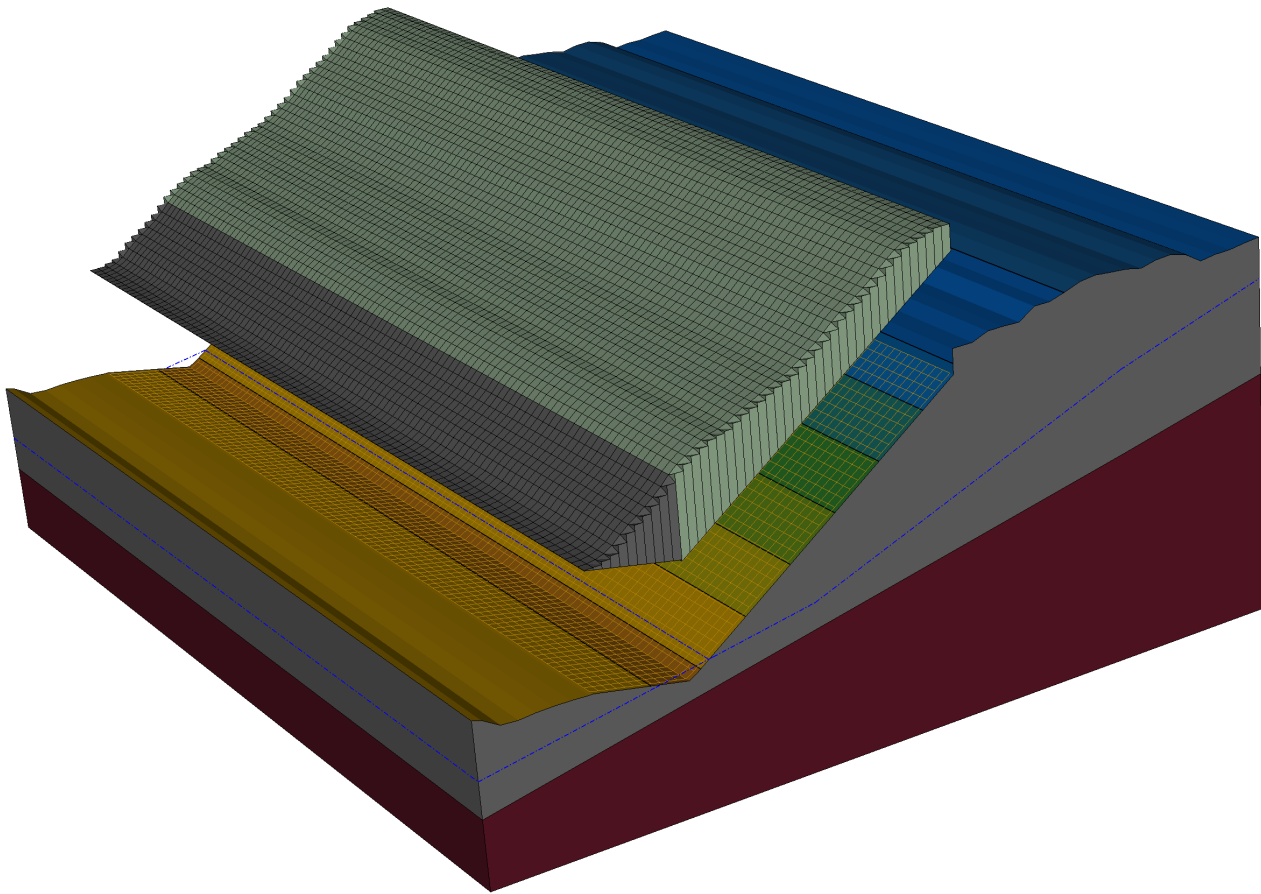


Figure 2 Equivalent 3D slope stability analysis with the slip surface exploded in the third dimension

It should be noted that a simple verification of a 3D analysis can be done by creating a 3D model of a 2D extruded slip surface while applying zero shear strength to the end-walls. This is a relatively straight forward method of verifying the 3D equivalent scenario of any 2D analysis. It also highlights the fundamental limitation of a 2D analysis which considers the slip to be of infinite length in the third dimension without considering the influence of shear forces on the end surfaces.

## 2.0 SLIP SURFACE SHAPE EFFECT

If an engineer desires to quantify the difference that slip surface shape makes on the analysis, then it is relatively easy to extrude a 2D profile model in the 3D space to determine the factor of safety of a slip surface assuming an ellipsoidal slip surface shape. It is recognized that not all 3D slip surfaces are of a perfect ellipsoidal shape but a majority of slips have an ellipsoidal shape. An ellipsoidal shape was considered to be a reasonable starting assumption for this white paper.

The benefit of this type of analysis is that it allows the engineer to determine the effect on the factor of safety (FOS) of moving from a 2D plane-strain analysis to a 3D analysis. When the slope is extruded to 3D then an aspect ratio for the ellipsoid must be found through a trial-and-error process. A number of example models were selected as benchmarks for this white paper and a few are presented in Figure 3 and Figure 4. These models were selected out of list of classic slope stability benchmarks compiled by SoilVision Systems Ltd.

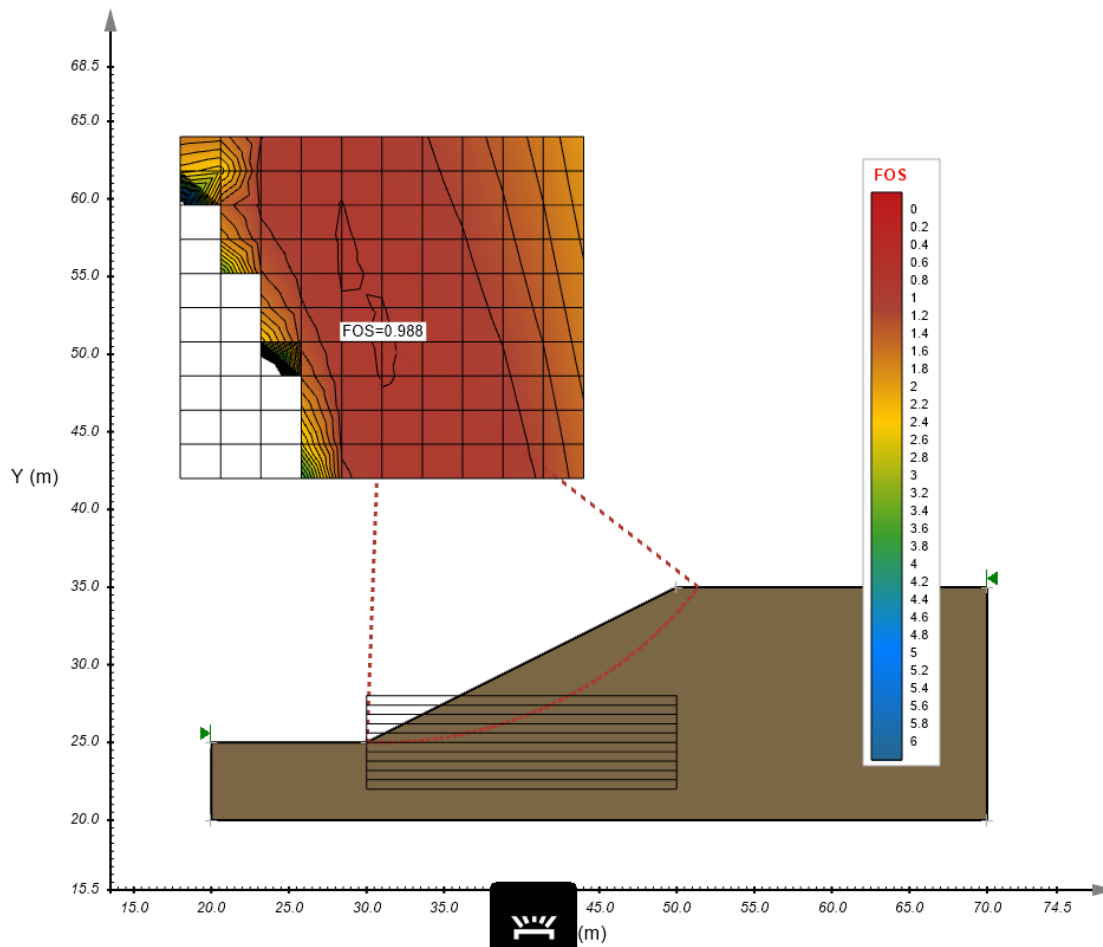
A common question related to 3D analysis is, "Will 3D analysis of an extruded profile tend to an ever wider and wider 3D elliptical slip surface as it tries to approach the 2D result (with its lower FOS)?".

One aspect to keep in mind in answering this question is the assumption regarding the slip surface shape. If the shape is cylindrical with small bevelled or rounded ends, then as the cylinder becomes longer and longer the effects of the shear on the ends theoretically become a smaller and smaller percentage in terms of their contribution. However, the “extended cylinder” described in the above argument is not a common slip surface failure shape in the field. If an elliptical slip surface failure shape is assumed then the results illustrate that the failure does not tend to be infinitely wide as previously assumed.

Figure 5 shows the results of extruding the VS\_24 model to an equivalent 3D numerical model. It can be seen from this extruded model that a wide range of aspect ratios for the ellipsoid have been tried (and the trial slip surfaces have been plotted) and the ellipsoid with the minimum FOS corresponds to an aspect ratio of 16.8 with a minimum FOS of 1.72 which is a 19.6% increase over the 2D FOS of 1.438. The results of extruding three such 2D models to 3D are shown in Table 1. It can also be seen from these results that the difference for simple and low-angle slopes is between the 2D and 3D FOS ranges of 4.3% to 20%.

**Table 1 Results of 2D profiles extruded to 3D**

Model	2-D		3-D			Diff
	FOS	Max Depth (m)	FOS	Aspect	Max Depth (m)	
VS_1	0.988		1.031	4.8	3.013	4.35%
VS_3	1.375	4.76	1.45	8.6	4.56	5.45%
VS_24	1.438	6.889	1.72	16.8	6.814	19.61%



**Figure 3 VS\_1 2D profile model**

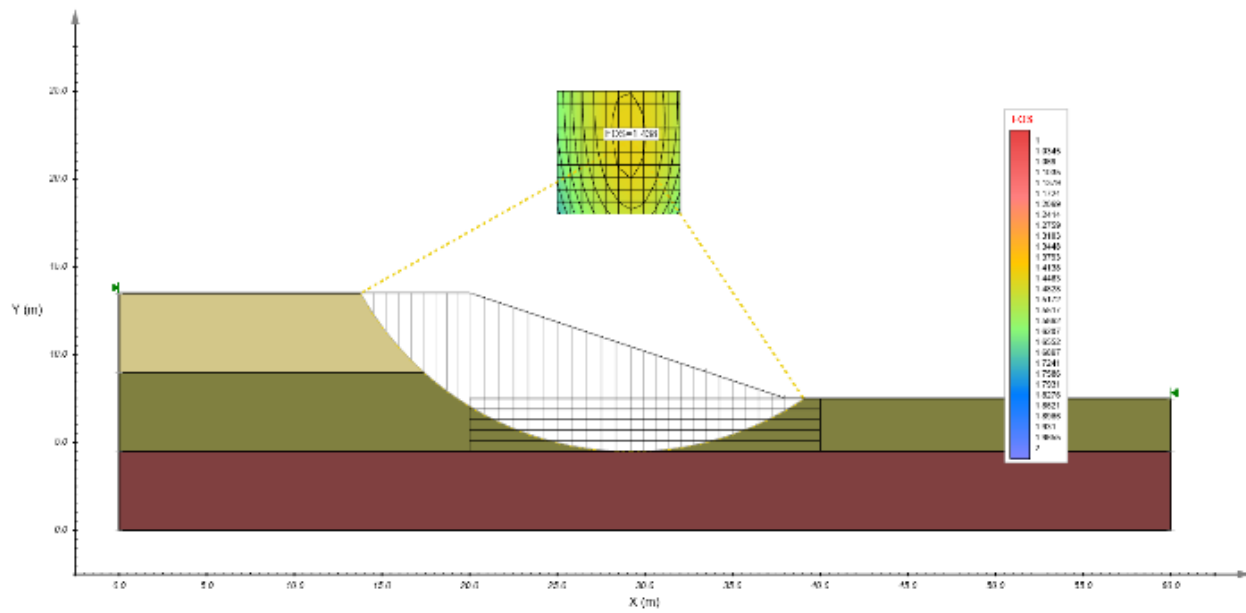


Figure 4 VS\_24 2D profile model

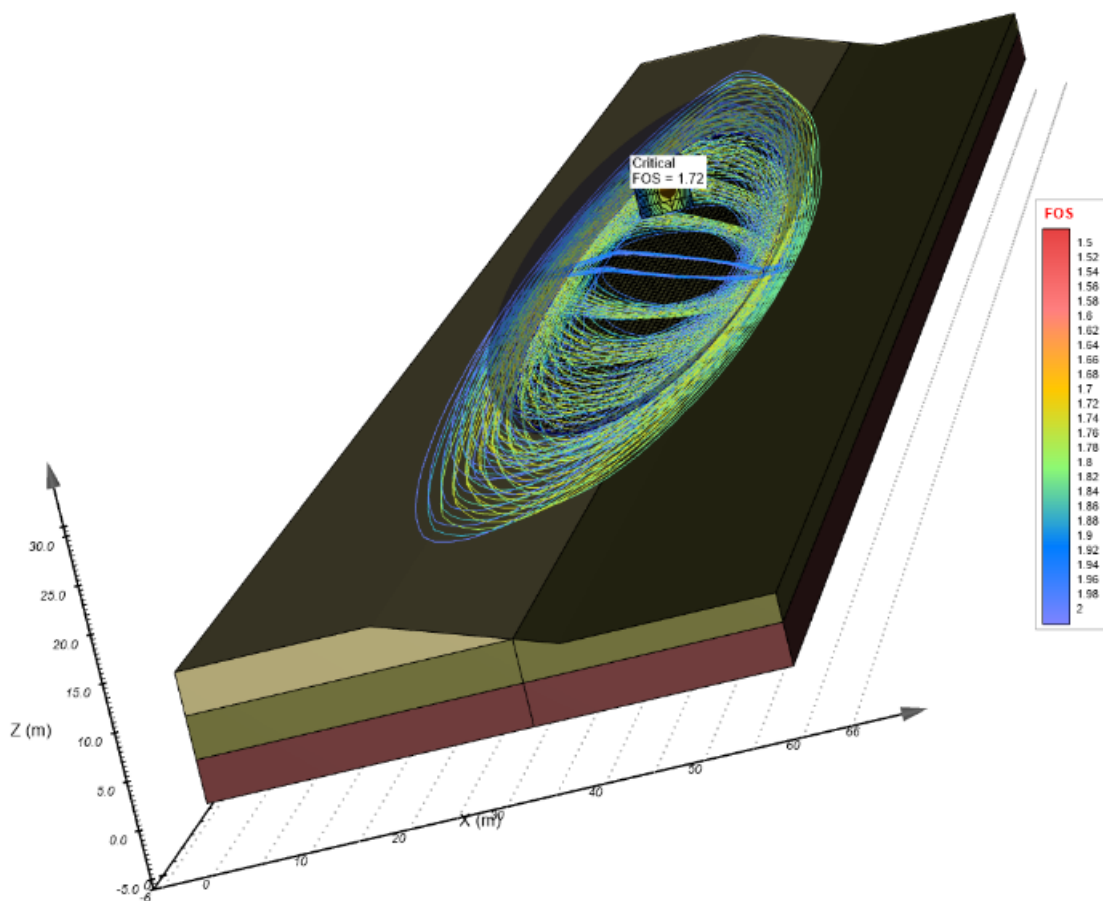


Figure 5 VS\_24 Extruded 3D model with critical slip surface

### 3.0 GROUND SURFACE TOPOGRAPHY VARIATIONS

The second primary reason why 3D analysis differs from 2D analysis is that there may be variation of the surface geometry or topology. Such variation can produce differences in the computed FOS and, with the emergence of LiDAR technology, clouds of surface topology data are now commonly available. Such variations can largely be grouped into surface shapes which are convex or concave. While it is recognized that typical topography may be highly variable, the general trends can be easily quantified with simple concave and convex shapes. Such an analysis has been performed (Zhang, 2015; Domingos, 2014) and the geometry for the comparison can be seen in Figure 6.

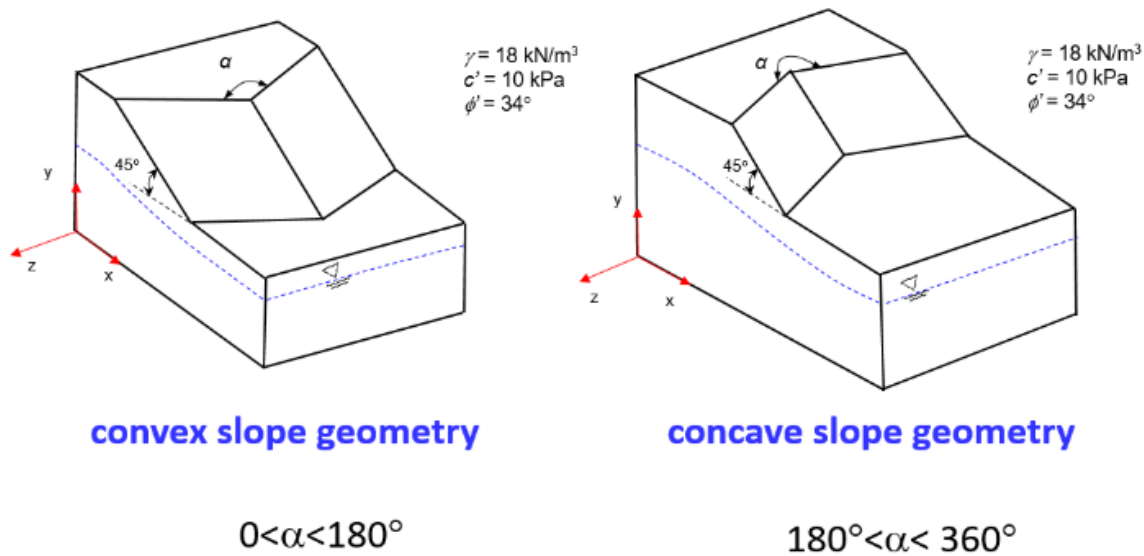


Figure 6 Convex and concave geometries used in 3D simulations (Zhang, 2015)

From this analysis of the convex and concave geometries it can be seen that: i) the consideration of such geometries results in an increase in the computed factor of safety (FOS) over a 2D scenario; ii) the consideration of unsaturated shear strengths further increases the computed FOS in 3D analysis. Thus it can also be surmised that the consideration of 3D geometries in a 3D model can often result in computed FOS values which are higher than 2D analysis values and have the potential to save on construction costs in the design of slopes.

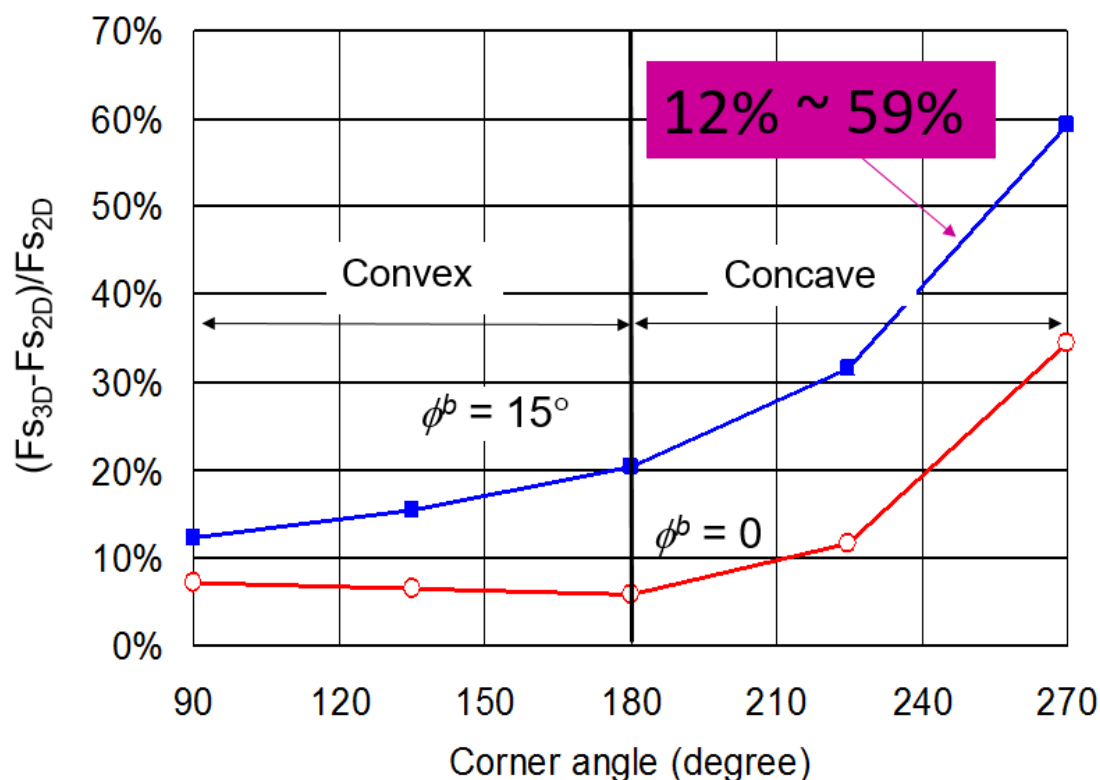


Figure 7 Differences between 2D and 3D computed FOS for convex and concave saturated/unsaturated slopes (Zhang, 2014)

The results of the influence of surface topology suggest that the consideration of convex and concave shapes may not be trivial. The consideration of 3D effects of topology are significant and useful in geotechnical engineering practice.

#### 4.0 EFFECTS OF GEO-STRATA IN ALL DIMENSIONS

A common question asked is, "What is the additional cost of moving to a 3D analysis?" First, it is relatively inexpensive to account for the effects of slip surface shape and 3D topology. Drilling additional boreholes to quantify geo-strata variation in 3D requires additional cost and the engineer must weigh the additional cost against the benefit achieved. The inclusion of additional boreholes in the analysis is relatively simple.

#### 5.0 PRACTICAL APPLICATIONS GUIDELINES

The current state of practice in geotechnical engineering has a long history of application and design of earth structures. It must be recognized that the existing limits of design of 1.3 and 1.5 have been developed in the context of an application of a 2D limit equilibrium numerical model. Common questions related to the application of a 3D numerical analysis to slope design are, "Can I use a 3D analysis and design for my same limits of 1.3 and 1.5?" or "Should we move to only use a 3D analysis?" There is recognition that further work is required to completely understand a 3D analysis but a number of principles can be developed which present a path forward in terms of a practical application of a 3D numerical model in geotechnical engineering practice.

Firstly, it is recommended that geotechnical engineers must begin by running both 2D and 3D numerical models to determine the potential differences between such analysis. There is not a single percentage that represents a consistent difference between a 2D and a 3D analysis and therefore the



geotechnical engineer must begin to become familiar with the results of the analysis in each engineering situation.

It is important to understand the logic followed by existing methods of analysis performed over the past few decades. The process may be illustrated in Figure 8. From this figure we may note that: i) the definition of failure at  $FOS=1.0$  does not change between 2D and 3D analysis, ii) A 3D FOS will typically be higher than a 2D FOS, and iii) when a 3D analysis is performed, the engineer is generally moving closer to field conditions and therefore moving closer to the real-world FOS.

It should also be understood that the true FOS is really comprised of a KNOWN and an UNKNOWN FOS. When a 2D analysis yields a value of 1.3, the known  $FOS_{2D}$  is 1.3 or 30%. The unknown  $FOS_{2D-3D}$  is the difference between the 3D and the 2D FOS. The geotechnical engineer traditionally has not known what the second  $FOS_{2D-3D}$  is but has a certain degree of comfort knowing the safety margin exists. However, the second unknown  $FOS_{2D-3D}$  can involve a difference between 10-60%. On both ends of this range further consideration is required. If the  $FOS_{3D}$  is 10% over the  $FOS_{2D}$  then it must be realized that there is little “cushion” over the known  $FOS_{2D}$ . If the  $FOS_{2D-3D}$  is closer to 60% or higher then there is significant over-design that may be costly.

When a 3D slope stability analysis is performed, the engineer is merely determining what the unknown  $FOS_{2D-3D}$  is that has existed all along but has remained unknown. This additional data gives the geotechnical engineer another crucial piece of information required for slope stability design.

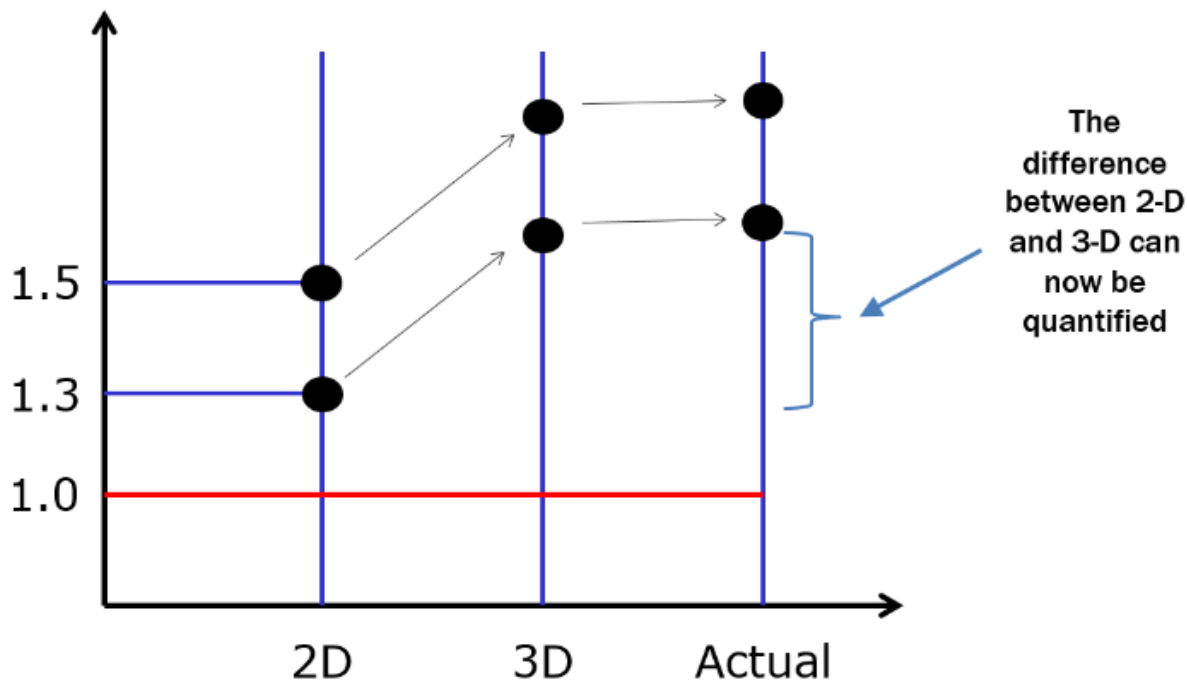


Figure 8 Illustration of 2D/3D analysis

Another common question is, “Can the engineer use a 3D analysis along with the existing factors of safety of 1.3 and 1.5?” It should be noted that the existing limits were developed based on the assumption for an additional and unknown conservatism. If a 3D analysis yields a  $FOS=1.3$ , the geotechnical engineer must realize that there is only a 30% over-design with no additional conservatism built in. It is suggested that geotechnical engineers should run both 2D and 3D analysis to become familiar with the true FOS limits which create sufficient comfort for designs.

Approaching a 3D analysis in steps may be an appropriate manner in which to educate the client about the 3D influences of the FOS calculation. As illustrated in this white paper the engineer can: i) first run a 2D analysis; ii) extrude the model to perform a simple 3D analysis; iii) then run a full 3D analysis considering the 3D geometry. This yields a progression that will educate both the geotechnical engineer and their client on the additional FOS contributed from representing the slip surface more accurately as well as from representing the topology and the geo-strata more accurately in the third dimension.

## 5.0 SUMMARY

As 3D analysis techniques become more common and easily applied the geotechnical engineer must understand the differences between a 2D and a 3D analysis. The primary differences in the 3D slope stability analysis lie in the fact that: i) the slip shape is more realistic; ii) the topology may be varying in the third dimension.

This white paper has illustrated how it is simple to extrude a numerical model and easily determine the influence of the improved slip shape. It can also be seen that it is quite straightforward to incur  $FOS_{3D}$ , which are 4% to 20% higher than the 2D plane strain equivalent models. Such an analysis can identify the specific influence of the slip surface shape on the factor of safety (FOS) calculations.

The influence of surface geometry on the calculated FOS can also be illustrated through the representation of typical concave or convex geometries. Such an analysis have been performed by Zhang (2014) and illustrated differences between 2D and 3D FOS values of between 8% to 60%. A higher difference was incurred when the unsaturated soil portion of the soil profile was considered.

This white paper presents a step-wise methodology for the application of 3D slope stability analysis in the practice of geotechnical engineering. This white paper also presents some foundational principles for the understanding of how a 3D analysis fits in the existing design limits of 1.3 and 1.5.

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